

CHIRAL DYNAMICS AND THE CASCADE SPECTRUM

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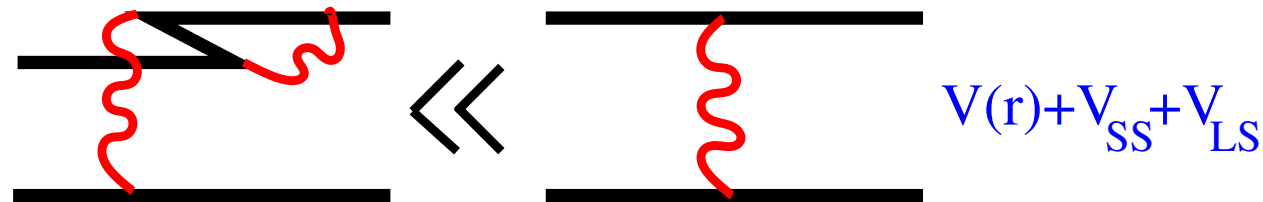
Contents of the Talk

- Chiral symmetry of QCD and its spontaneous breaking.
- Low and high-lying baryon spectra.
- The chiral constituent quark model.
- Chiral symmetry restoration in excited hadrons.
- Chiral and $U(1)_A$ restorations in the semiclassical regime.
- Summary.

Nonrelativistic versus relativistic.

If fermions are heavy, then creation of pairs from the vacuum is suppressed and the gauge field is reduced to the potential:

Physics of atoms; positronium; heavy quarkonium.



If fermions are light, $m_u, m_d \ll \Lambda_{QCD}, M_{hadrons}$, then creation of pairs from the vacuum is NOT suppressed and hadrons are many-body systems:

$$\bar{q}q + \bar{q}q\bar{q}q + \bar{q}q\bar{q}qg + \dots$$

Quantum fluctuations are crucially important. Chiral and $U(1)_A$ symmetries are broken. Description is much more difficult: proper effective degrees of freedom should be found (quasiparticles, etc).

Strange quarks: $m_s \sim \Lambda_{QCD}$. Which type of description should be applied?

Spontaneous breaking of chiral symmetry in QCD.

In QCD $m_u, m_d \ll \Lambda_{QCD}$. Hence to a good approximation.

$$U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

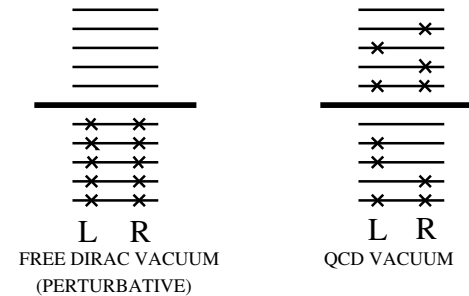
$U(1)_A$ is explicitly broken by the axial anomaly (quantum fluctuations).

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V.$$

Why? No chiral symmetry in the vacuum.

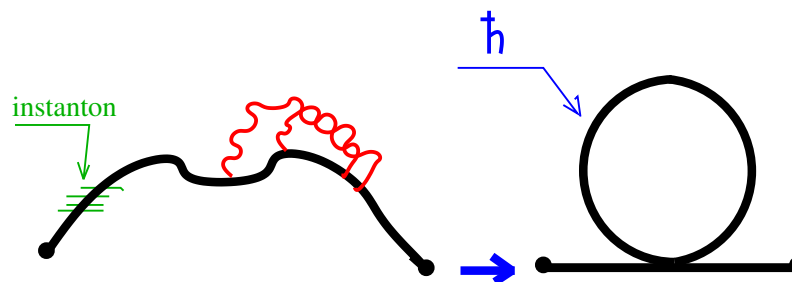
(i) No parity doublets low in the spectrum.

(ii) $\langle 0 | \bar{q}q | 0 \rangle \approx (-240 \text{ MeV})^3$

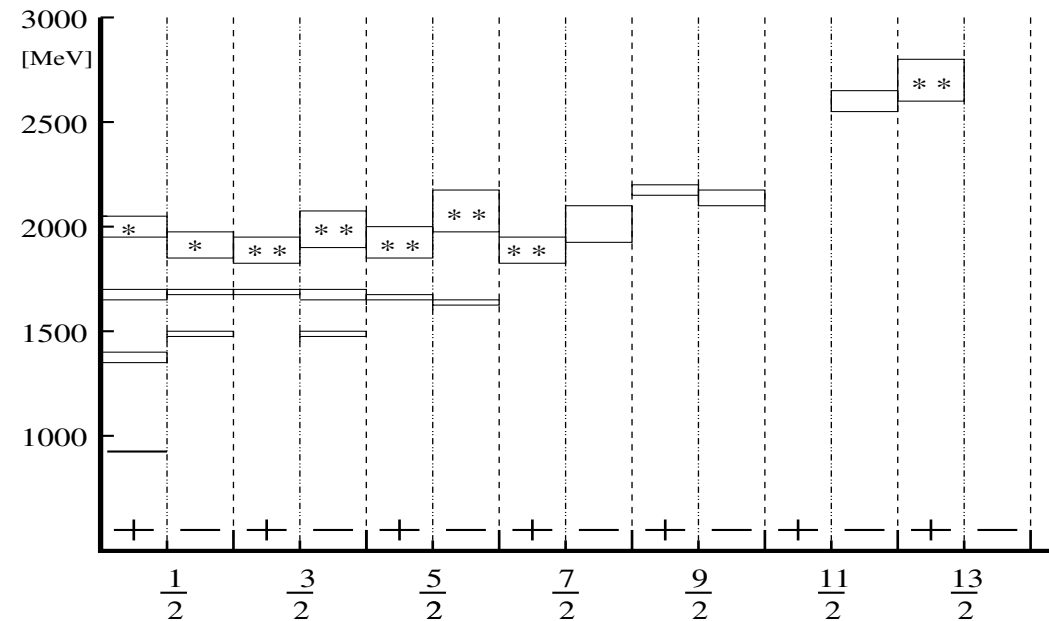


The most general origin are quantum fluctuations of quark fields.

$$\langle \bar{q}q \rangle = -Tr \lim_{x \rightarrow 0_+} \langle 0 | T \{ q(0) \bar{q}(x) \} | 0 \rangle \longleftrightarrow \hbar$$



Low and high-lying baryon spectra.



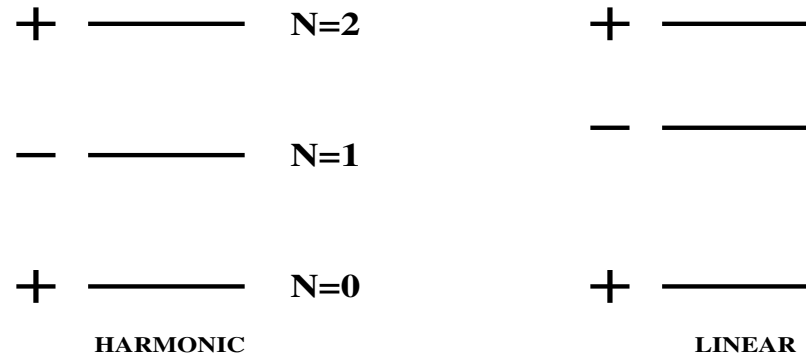
Low-lying spectrum: spontaneous breaking of chiral symmetry dominates physics.

High-lying spectrum: parity doubling indicates the onset of the new physics regime -
chiral symmetry restoration in excited hadrons.

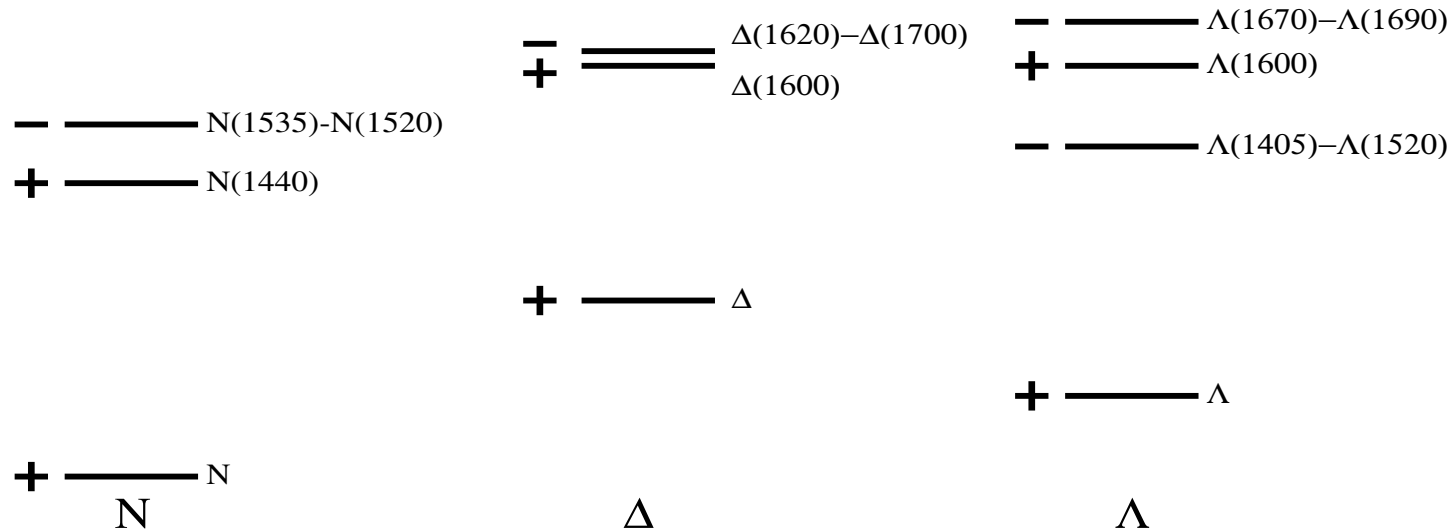
L.Ya.G., 2000 ; T.D. Cohen and L.Ya.G., 2002

In strange hadrons chiral symmetry is EXPLICITLY broken by the massive strange quark. The chiral symmetry restoration should NOT be so pronounced!

Where is a key problem for low-lying baryons?



A sequence of positive and negative parity levels with confining interaction.



Low-lying spectra of nucleon, Δ -resonance and Λ -hyperon.

The chiral constituent quark model.

Consequences of $SB\chi S$, $\langle \bar{q}q \rangle = -(240 MeV)^3$:

- (i) Valence quarks acquire dynamical (constituent) mass through their coupling to the quark condensate;
- (ii) Practically massless Goldstone bosons (pion,...) appear as a collective quark-antiquark mode.

Physics insight from **Nambu and Jona-Lasinio** .

The axial current conservation requires a coupling of the constituent quark with the pion field.

The low-lying baryons can be approximated at low momenta as systems of three confined quasiparticles (constituent quarks) with the 'residual' interaction mediated by the Goldstone boson field - **L.Ya.G. and D.O.Riska, 1996**

$$- \sum_{i < j} V(r_{ij}) \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Output: correct low-lying baryon spectra.

The chiral constituent quark model.

$$-\sum_{i<j} V(r_{ij}) \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j \longrightarrow -\sum_{i<j} C \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Positive parity states:

Octet (N, Λ, Σ, Ξ); Octet* ($N(1440), \Lambda(1600), \dots$): **-14C**

Decuplet ($\Delta, \Sigma^*, \Xi^*, \Omega$); Decuplet* ($\Delta(1600), \dots$): **-4C**

Negative parity states:

$N(1535) - N(1520)$: **-2C**

$\Lambda(1670) - \Lambda(1690)$: **-2C**

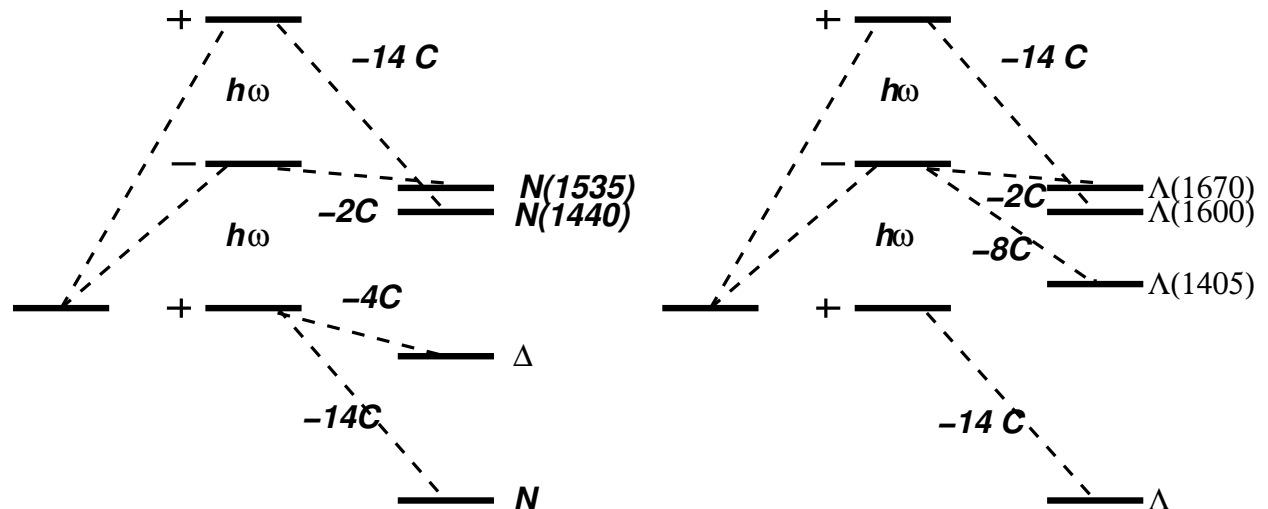
$\Lambda(1405) - \Lambda(1520)$: **-8C**

$\Delta - N$ splitting:

$$C = 29.3 \text{ MeV}$$

$N(1440) - N$ splitting:

$$\hbar\omega = 250 \text{ MeV}$$



Spectra have been obtained from the numerical solution of Faddeev equations

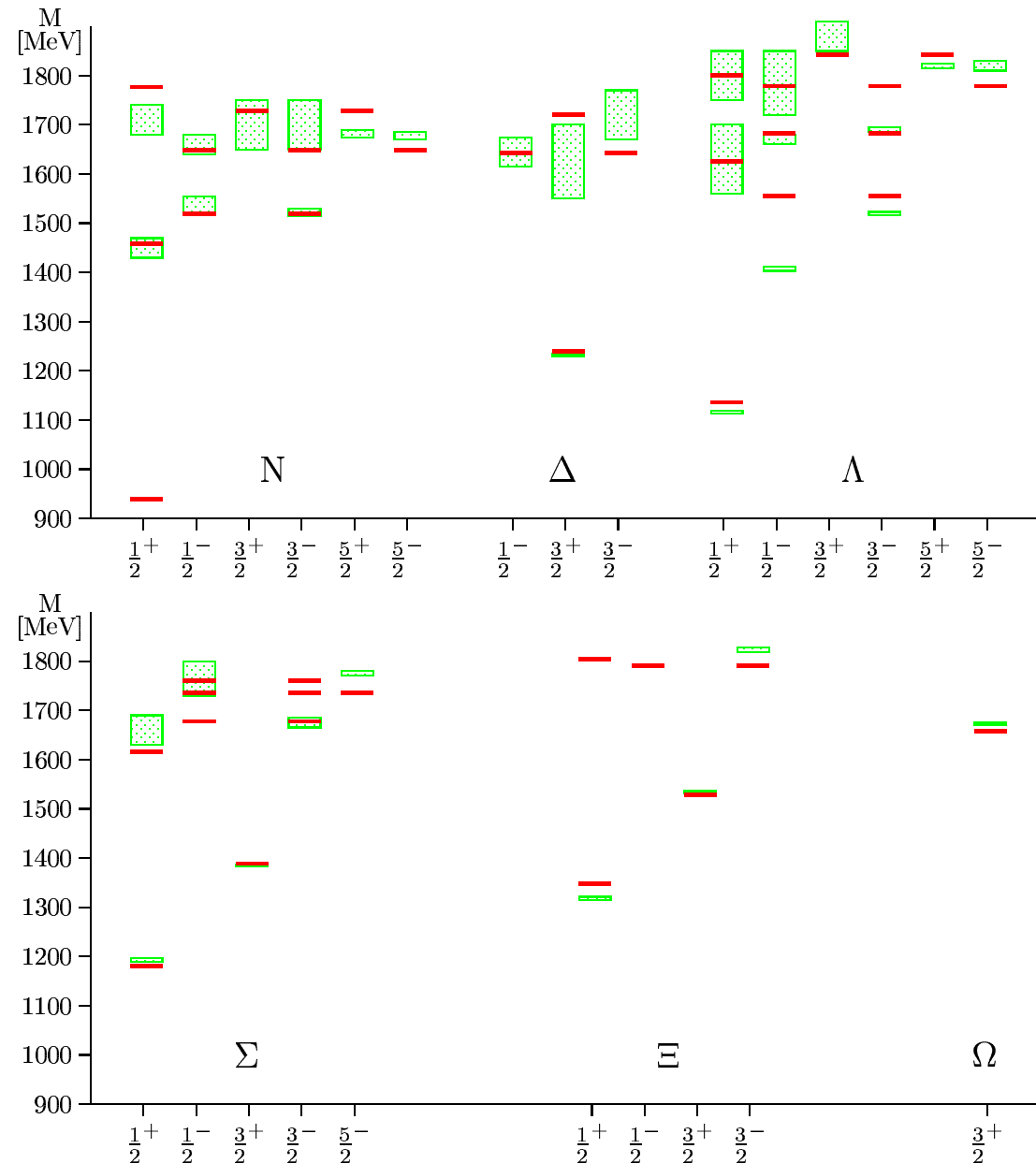
L.Ya.G., Z.Papp, W.Plessas, PLB, 1996

and stochastic variational approach

L.Ya.G., W.Plessas, K.Varga, R.F.Wagenbrunn, PRD, 1998

CAN THIS BE EXTRAPOLATED

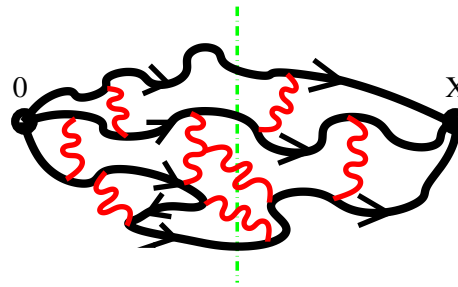
TO THE Ξ and Ω SPECTRA ???



Chiral symmetry restoration by definition.

Hadrons can be seen as intermediate states in the two-point correlation function:

$$\Pi = i \int d^4x e^{iqx} \langle 0 | T \{ J_\alpha(x) J_\alpha^\dagger(0) \} | 0 \rangle.$$

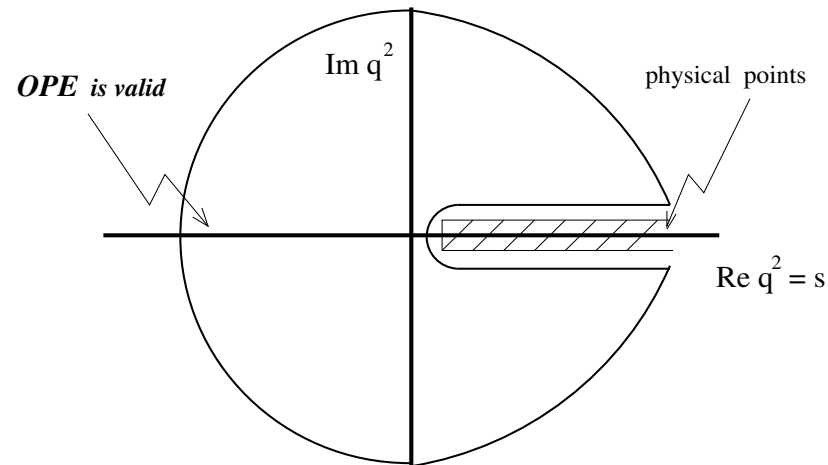


Consider two interpolators $J_1(x)$ and $J_2(x)$ such that $J_1(x) = U J_2(x) U^\dagger$ where $U \in SU(2)_L \times SU(2)_R$. If $U|0\rangle = |0\rangle$, then spectra of hadrons with the quantum numbers **1** and **2** must be identical (**Wigner-Weyl mode**).

Spontaneous breaking of chiral symmetry in the vacuum implies that the spectra of **1** and **2** are different (**Nambu - Goldstone mode**). However, it may happen that the noninvariance of the vacuum becomes irrelevant (unimportant) high in the spectrum. Then the chiral symmetry will be restored in the high-lying hadrons.

Effective chiral symmetry restoration or chiral symmetry restoration of the second kind .

$$\text{Causality} \rightarrow \text{analyticity} \rightarrow \Pi(q^2) = \frac{1}{\pi} \int ds \frac{\rho(s)}{s - q^2 - i\epsilon}$$



At unphysical points the **OPE** guarantees that the effects of the spontaneous breaking of chiral symmetry (quark condensates of different dimensions) are suppressed by $1/q^n, n > 0$. The same must be true in the physical region at large s :

$$\rho_1(s \rightarrow \infty) \rightarrow \rho_2(s \rightarrow \infty), \quad \text{where} \quad J_1(x) = U J_2(x) U^\dagger$$

If the spectrum is quasidiscrete, then **approximate chiral multiplets**.

Shifman, 2005: The slowest possible rate of the symmetry restoration in large N_c in mesons $\sim 1/n^{3/2}$.

A simple pedagogical example.

2-dim harm. osc.: $H = a_x^\dagger a_x + a_y^\dagger a_y + 1$.

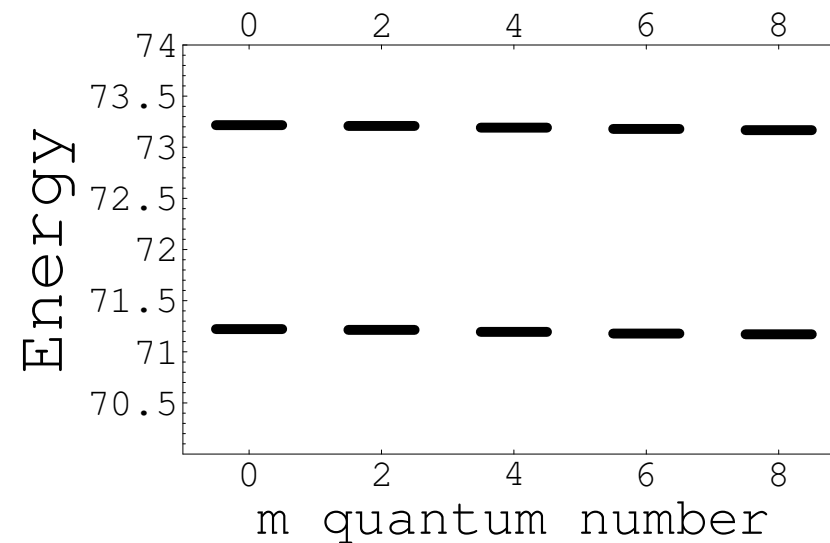
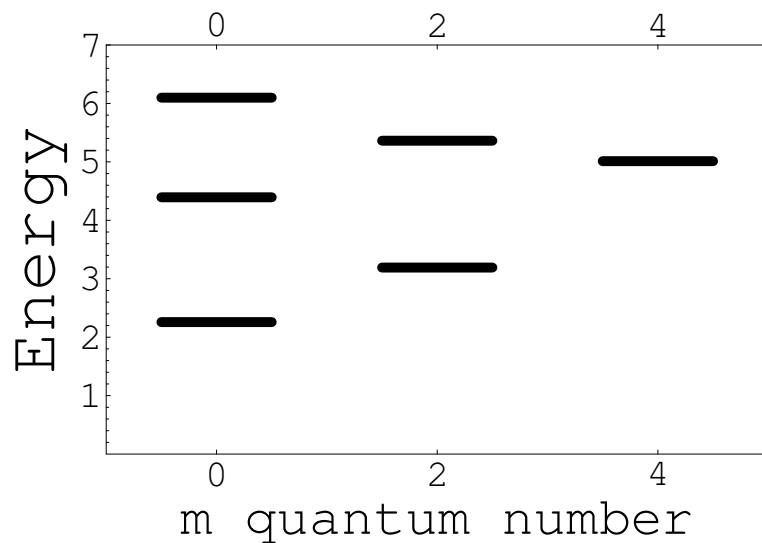
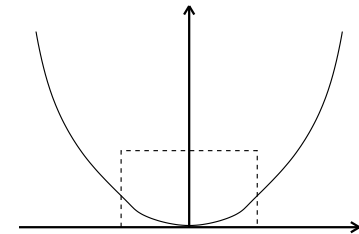
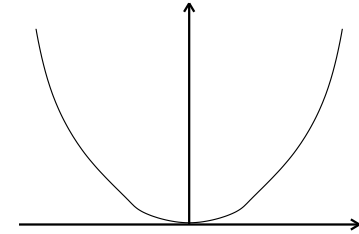
Symmetry: $SU(2) \times U(1)$

$E_{N,m} = (N + 1); m = N, N - 2, \dots, -N$.

Add a symmetry breaking interaction:

$V_{SB} = A\Theta(r - R)$.

No $SU(2)$ symmetry.



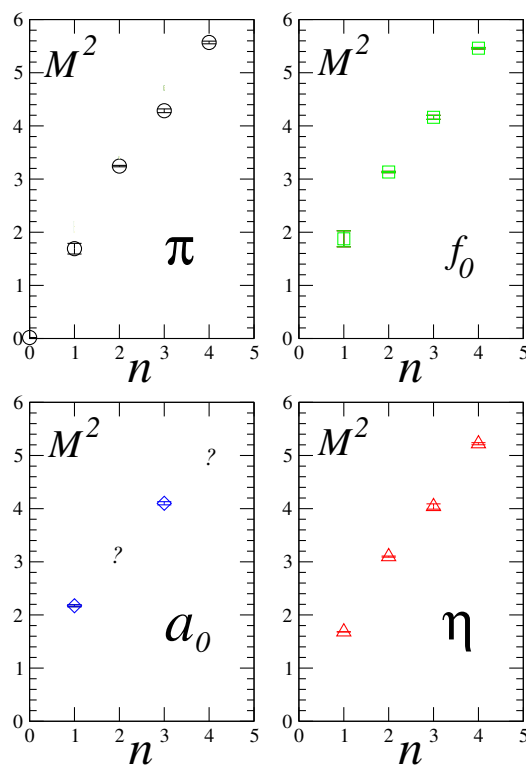
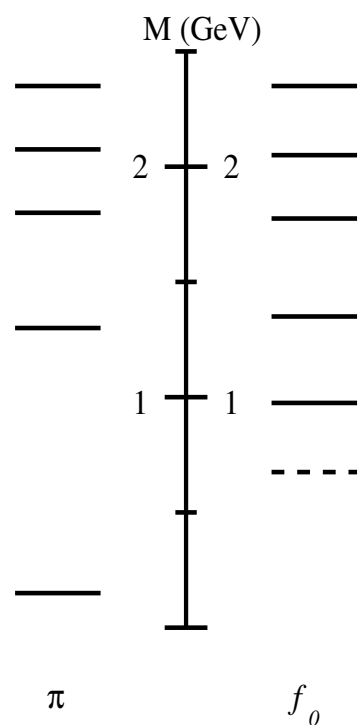
Chiral multiplets of excited mesons.

Chiral partners ($(1/2, 1/2)$ representation of $SU(2)_L \times SU(2)_R$):

$\pi(I, J^{PC} = 1, 0^{-+})$ and $f_0(I, J^{PC} = 0, 0^{++})$,
 $a_0(I, J^{PC} = 1, 0^{++})$ and $\eta(I, J^{PC} = 0, 0^{-+})$.

$$j_\pi(x) = \bar{q}(x)\vec{\tau}i\gamma_5q(x) \longleftrightarrow j_{f_0}(x) = \bar{q}(x)q(x),$$

$$j_{a_0}(x) = \bar{q}(x)\vec{\tau}q(x) \longleftrightarrow j_\eta(x) = \bar{q}(x)i\gamma_5q(x).$$



Chiral classification of excited baryons.

$$SU(2)_L \times SU(2)_R$$

Irreducible representation: (I_L, I_R)

Parity operation: $L \longleftrightarrow R$. Hence $(I_L, I_R) \longleftrightarrow (I_R, I_L)$.

Irreducible representation of the parity-chiral group: $(I_L, I_R) \oplus (I_R, I_L)$.

It contains states with isospins $I = |I_L - I_R|, \dots, I_L + I_R$ of both parities.

For $I \leq 3/2$ there are 3 possibilities:

(i) $(1/2, 0) \oplus (0, 1/2)$: doublets in N ($I^P = 1/2^+, 1/2^-$).

(ii) $(3/2, 0) \oplus (0, 3/2)$: doublets in Δ ($I^P = 3/2^+, 3/2^-$).

(iii) $(1/2, 1) \oplus (1, 1/2)$: quartets in N and Δ ($I^P = 1/2^+, 1/2^-, 3/2^+, 3/2^-$).

If the strange quark is included, then $SU(3)_L \times SU(3)_R$ should be considered.

Chiral classification of excited baryons.

$J = \frac{1}{2} :$	$N^+(2100)(*)$	$N^-(2090)(*)$	$\Delta^+(1910)$	$\Delta^-(1900)$
$J = \frac{3}{2} :$	$N^+(1900)$	$N^-(2080)$	$\Delta^+(1920)$	$\Delta^-(1940)(*)$
$J = \frac{5}{2} :$	$N^+(2000)$	$N^-(2200)$	$\Delta^+(1905)$	$\Delta^-(1930)$
$J = \frac{7}{2} :$	$N^+(1990)$	$N^-(2190)$	$\Delta^+(1950)$	$\Delta^-(2200)(*)$
$J = \frac{9}{2} :$	$N^+(2220)$	$N^-(2250)$	$\Delta^+(2300)$	$\Delta^-(2400)$
$J = \frac{11}{2} :$?	$N^-(2600)$	$\Delta^+(2420)$?
$J = \frac{13}{2} :$	$N^+(2700)$?	?	$\Delta^-(2750)$
$J = \frac{15}{2} :$?	?	$\Delta^+(2950)$?

$(1/2, 1) \oplus (1, 1/2)$?? The parity doublets in the nucleon spectrum persist at ~ 1.7 GeV (no doublets yet in the delta spectrum). Then independent $(1/2, 0) \oplus (0, 1/2)$ and $(3/2, 0) \oplus (0, 3/2)$ doublets. If in addition $(1/2, 1) \oplus (1, 1/2)$, then there are still missing doublets.

In the Ξ and Ω spectra the parity doublets should NOT be so pronounced because of large explicit chiral symmetry breaking by the strange quark!

What is the most fundamental reason of symmetry restoration? (L.Ya.G.,2004)

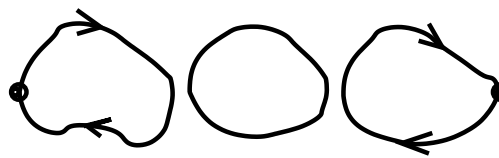
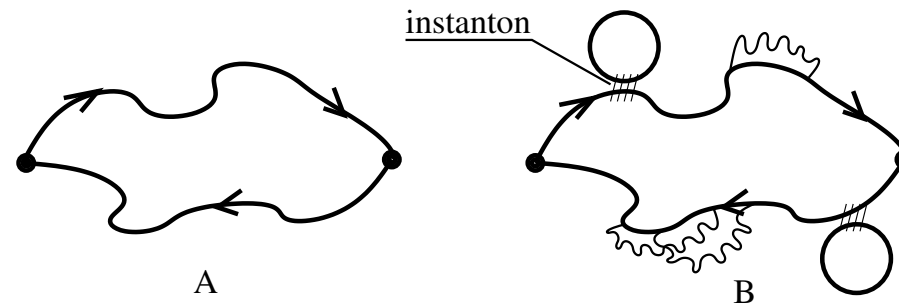
$$\Pi(x, y) = \frac{1}{Z} \int DA_\mu D\Psi D\bar{\Psi} e^{\frac{iS(\bar{\Psi}, \Psi, A)}{\hbar}} J(x) J(y)^\dagger$$

If $\hbar \rightarrow 0$, then only the CLASSICAL TRAJECTORY survives:

$$\delta S(\bar{\Psi}_{cl}, \Psi_{cl}, A_{cl}) = 0 \quad \Leftarrow \text{CLASSICAL EQUATION OF MOTION}$$

At the classical level both chiral and $U(1)_A$ symmetries are manifest.

Their breaking comes from the quantum fluctuations and starts from the one loop order .



If $S \gg \hbar$, then the **SEMICLASSICAL EXPANSION** is valid:

- the classical contribution $\rightarrow (\frac{\hbar}{S})^0$
- the one-loop contribution $\rightarrow (\frac{\hbar}{S})^1$
- the two-loop contribution $\rightarrow (\frac{\hbar}{S})^2$

Chiral and $U(1)_A$ breakings start from the one-loop order. If $S \gg \hbar$, the quantum fluctuations become suppressed and chiral and $U(1)_A$ symmetries get restored.

In hadrons with large n (radial quantum number) or large J , $S \gg \hbar$.

Do we reach a semiclassical regime? At $N_c = \infty$ it is an exact statement (for mesons).

- The mesons are narrow states and the two-point function is saturated by the bound states only.
- The spectrum is infinite.

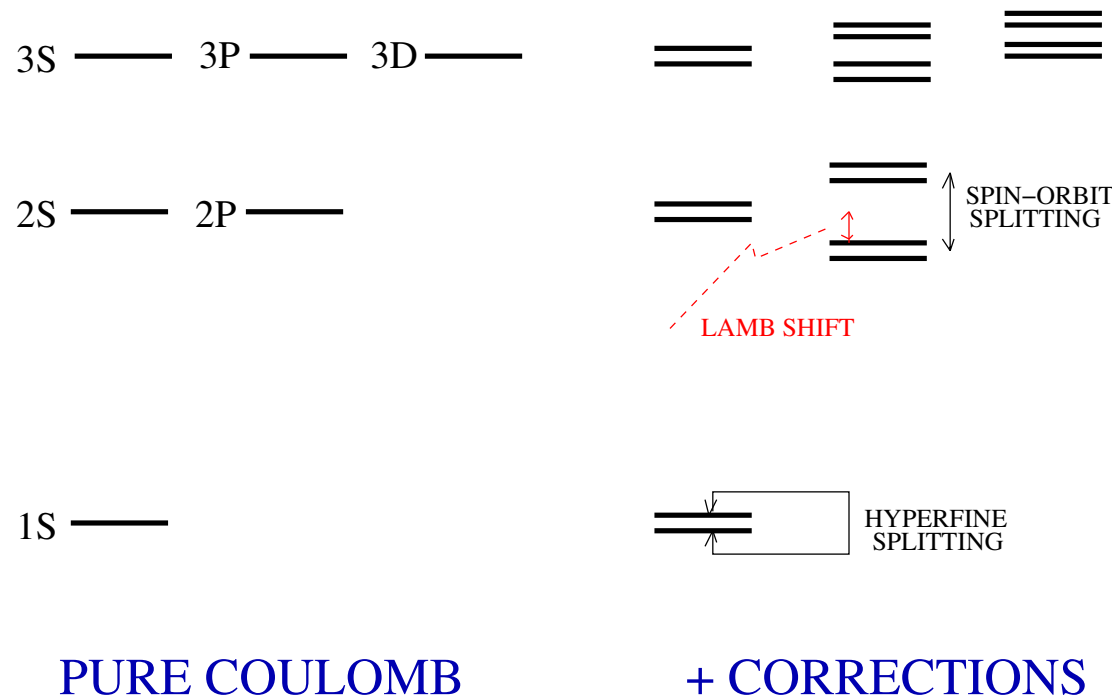
Then we can excite a meson with an arbitrary large action S .

For any large S there always exist such N_c that the isolated mesons with such an action do exist and can be described semiclassically.

Restoration of the Coulomb symmetry in Hydrogen.

If $n \gg 1$ or $J \gg 1 \implies$ the semiclassical regime approaches \implies quantum fluctuations get suppressed \implies **classical symmetry** gets restored.

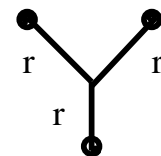
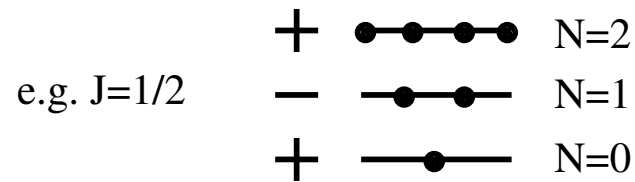
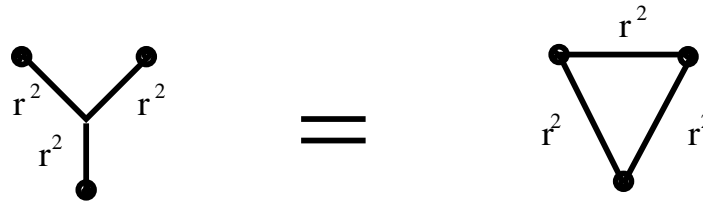
HYDROGEN SPECTRUM



$$\text{Lamb shift} \sim 1/n^3 .$$

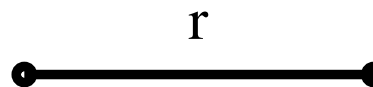
Can simple potential models explain doubling?

BARYONS



'Missing states'

MESONS



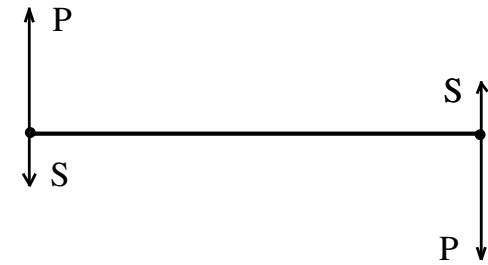
$$P = -(-1)^L;$$

e.g. $\pi \rightarrow^1 S_0$ $f_0 \rightarrow^3 P_0$

Chiral symmetry restoration and the string picture.

What is a model for excited hadrons? Assume :

- (i) the field in the string is of pure color-electric origin
- (ii) the valence quarks have a definite chirality



Then:

- (i) The hadrons that belong to the same intrinsic quantum state of the string with quarks falling into the same parity-chiral multiplet must be degenerate.
- (ii) The total parity of the hadron is a product of parity of the string in the given quantum state and the parity of the specific parity-chiral configuration of the quarks at the ends of the string.

Other implications:

- (i) The spin-orbit interaction of quarks with the fixed chirality is absent (spin-orbit operator and chirality operator do not commute)
- (ii) The tensor interaction is absent ($\vec{\sigma}(i) \cdot \vec{r}(i) = 0$; $\vec{\sigma}(i) \cdot \vec{r}(j) = 0$)

1. There are clear indications from phenomenology and theory that physics of the low-lying and the high-lying hadrons is rather different. The low-lying hadrons are strongly affected by the spontaneous breaking of chiral and $U(1)_A$ symmetries, while in the high-lying states these chiral symmetry breakings become irrelevant (chiral symmetry restoration).
2. While for the low-lying baryons the idea of quasiparticles (constituent quarks) interacting strongly with the "pion" field is fruitful, in the high-lying hadrons a string (flux-tube) picture with valence quarks with definite chirality at the ends of the string is probably correct. In the highly excited strange hadrons the EXPLICIT chiral symmetry breaking should be taken into account and the chiral multiplets will NOT be so pronounced.
3. A fundamental origin of chiral symmetry restoration is that effects of quantum fluctuations of the quark fields must vanish at large n and J and the semiclassical description becomes adequate.
4. Physics of the high-lying states is a pure physics of confinement. A vigorous and systematic experimental and theoretical study of the high-lying hadrons is required.