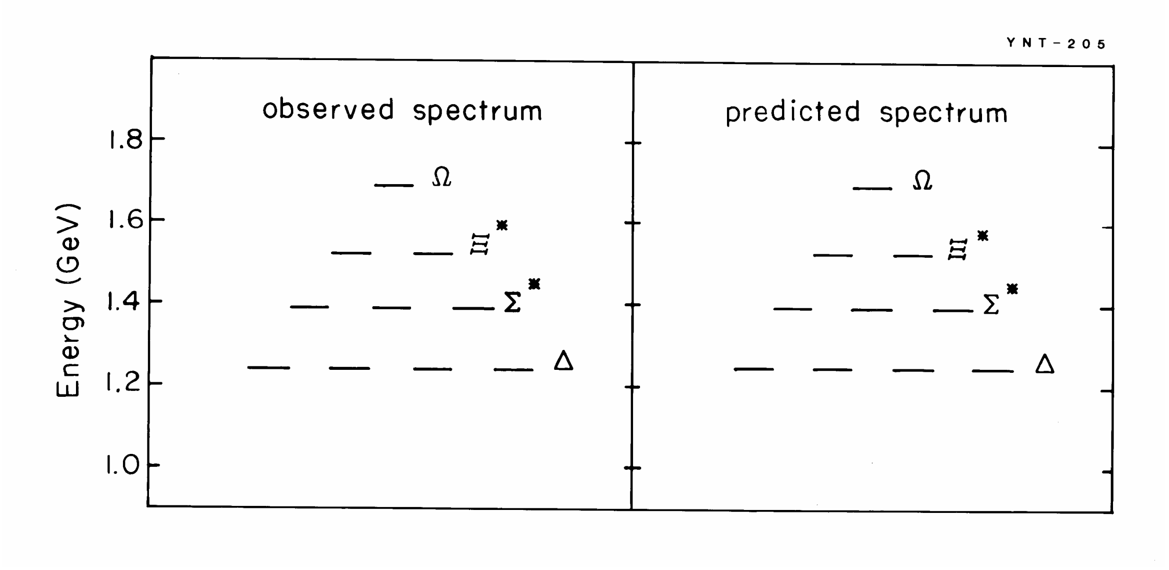
ALGEBRAIC MODELS OF HADRONS: STRANGE BARYONS

Francesco Iachello

Yale University

SYMMETRY APPROACH

The symmetry approach to hadron structure was introduced by Gell'Mann and Ne'eman in 1962: Flavor symmetry $SU_f(3)$.
This approach exploits the symmetries of the interactions to give explicit analytic formulas for all observable quantities in terms of quantum numbers characterizing the states of the system.
It is a very powerful method for complex (composite) systems.
It led, among other things, to the discovery of the Ω particle.



In the 1960's, it was applied to the internal flavor-spin degrees of freedom. The internal dynamical group was assumed (Gürsey and Radicati) to be

$$SU_{sf}(6) \supset SU_f(3) \otimes SU_s(2)$$

leading to the mass formula

$$M(Y;I,I_3;S) = M_0 + aY + b[I(I+1) - \frac{Y^2}{4}] + cS(S+1)$$

Introduction of color in the 1970's (QCD) extended the internal group to

$$G_i := SU_f(3) \otimes SU_s(2) \otimes SU_c(3)$$

In the 1970's, it was suggested that the symmetry approach could be extended to include space degrees of freedom and applied to the spectroscopy of nuclei (Arima and Iachello, 1975) and molecules (Iachello, 1981). The basic ingredient in this approach is the dynamical algebra G. Hence the name algebraic approach given to it.

Renewed interest in hadron spectroscopy in the 1990's, stimulated an algebraic treatment of space degrees of freedom in hadron structure and led to Algebraic Models of Hadrons.

Algebraic models of physical systems assume that a problem with n degrees of freedom has a dynamical group U(n+1). The number of degrees of freedom (excluding translations) for a system with N constituents is n=3N-3.

One thus obtains:

For $q\bar{q}$ mesons, M, G_r:=U(4)

For qqq baryons, B, G_r:=U(7)

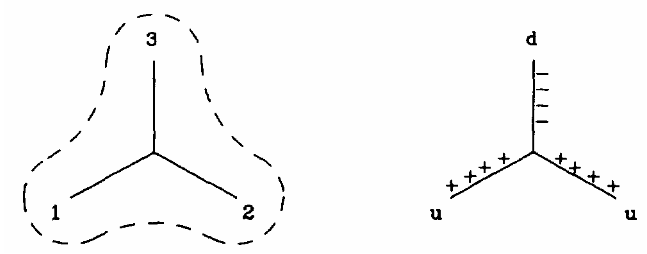
The full algebraic structure of hadrons is assumed to be

$$G = G_r \otimes G_i$$

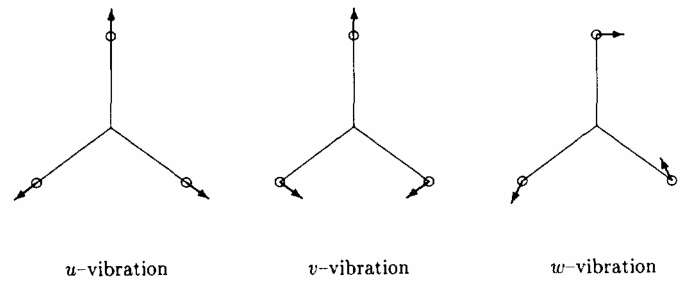
The advantage of the algebraic method is that all results can be obtained in explicit analytic form thus affording a straightforward comparison with experiment.
The disadvantage of the method is that its mathematical underpinning is rather complex.

ALGEBRAIC MODEL OF BARYONS
SPACE PART

It is assumed that baryons are composed of three constituent parts with a geometric (string-like) configuration



Excitations of this configuration are characterized by four quantum numbers:
Rotations: L
Vibrations: (n_u,n_v,n_w)



The vibrational part of the mass squared operator is taken to be linear in the quantum numbers

$$M^2_{vib} = \kappa_1 n_u + \kappa_2 n_v + \kappa_3 n_w$$

For systems with S_3-invariance, $\kappa_2 = \kappa_3$ and

$$M^2_{vib} = \kappa_1 v_1 + \kappa_2 v_2$$

where $v_1 = n_u; v_2 = n_v + n_w$

The rotational part of the mass squared operator is also taken to be linear (Regge behavior)

$$M^2_{rot} = \alpha L$$

The total space part of the mass squared operator is then

$$M^2_{space} = \kappa_1 v_1 + \kappa_2 v_2 + \alpha L$$

This part is characterized by 3 parameters κ_1, κ_2 and α.
(α is the slope of the Regge trajectory)

SPIN-FLAVOR PART

The spin-flavor part is characterized by the quantum numbers

$$\left| \begin{array}{ccccc} SU_{sf}(6) \supset & SU_f(3) \otimes & SU_s(2) \supset & SU_I(2) \otimes & U_Y(1) \otimes SU_s(2) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [f_1\, f_2\, f_3] & [g_1\, g_2] & S & I & Y \end{array} \right\rangle$$

An alternative notation for $SU_{sf}(6)$ is

$[f_1, f_2, f_3]$	*Symmetrytype*	dim
$[3,0,0]$	*Symmetric*	56
$[2,1,0]$	*Mixedsymmetry*	70
$[1,1,1]$	*Antisymmetric*	20

An alternative notation for $SU_f(3)$ is

$[g_1, g_2]$	*name*	dim
$[3,0]$	*decuplet*	10
$[2,1]$	*octet*	8
$[0,0]$	sin *glet*	1

The decomposition of $SU_{sf}(6)$ representations into $SU_f(3) \otimes SU_s(2)$ is

$$S = [56] \supset {}^2 8 \oplus {}^4 10$$
$$M = [70] \supset {}^2 8 \oplus {}^4 8 \oplus {}^2 10 \oplus {}^2 1$$
$$A = [20] \supset {}^2 8 \oplus {}^4 1$$

The spin-flavor mass squared operator is taken to be a linear function of the Casimir operators with eigenvalues

$$M_{sf}^2 = a\left[2 f_1(f_1+5) + 2 f_2(f_2+3) + 2 f_3(f_3+1) - \frac{1}{3}\left(f_1+f_2+f_3\right)^2 - 45\right]$$
$$+b\left[\frac{3}{2}\left(g_1(g_1+2) + g_2^2 - \frac{1}{3}(g_1+g_2)^2 - 9\right)\right] + c[S(S+1) - \frac{3}{4}]$$
$$+d[Y-1] + e[Y^2-1] + f[I(I+1) - \frac{3}{4}]$$

It is characterized by 6 parameters a,b,c,d,e,f.
The total mass squared operator is:

$$M^2 = M_0^2 + M_{space}^2 + M_{sf}^2$$

characterized by 9 parameters (M_0^2=0.882 GeV^2 is fixed to the mass of the nucleon)

Fit to 48 resonances with r.m.s. δ=33 MeV

TABLE II

Values of the Parameters in the Mass Formula of Eq. (4.1) in GeV^2

Parameter	Present	Ref. [3]
M_0^2	0.882	0.882
κ_1	1.204	1.192
κ_2	1.460	1.535
α	1.068	1.064
a	−0.041	−0.042
b	0.017	0.030
c	0.130	0.124
d	−0.449	
e	0.016	
f	0.042	
δ(MeV)	33	39

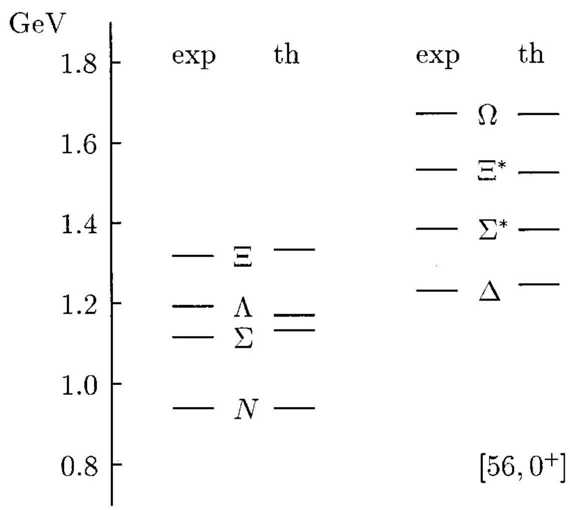


TABLE III

Mass Spectrum of Nonstrange Baryon Resonances in the Oblate Top Model

Baryon $L_{2I,2J}$	Status	Mass	State	(v_1, v_2)	M_{calc}
$N(939)\ P_{11}$	****	939	$^2 8_{1/2}[56, 0^+]$	(0, 0)	939
$N(1440)\ P_{11}$	****	1430-1470	$^2 8_{1/2}[56, 0^+]$	(1, 0)	1444
$N(1520)\ D_{13}$	****	1515-1530	$^2 8_{3/2}[70, 1^-]$	(0, 0)	1563
$N(1535)\ S_{11}$	****	1520-1555	$^2 8_{1/2}[70, 1^-]$	(0, 0)	1563
$N(1650)\ S_{11}$	****	1640-1680	$^4 8_{1/2}[70, 1^-]$	(0, 0)	1683
$N(1675)\ D_{15}$	****	1670-1685	$^4 8_{5/2}[70, 1^-]$	(0, 0)	1683
$N(1680)\ F_{15}$	****	1675-1690	$^2 8_{5/2}[56, 2^+]$	(0, 0)	1737
$N(1700)\ D_{13}$	***	1650-1750	$^4 8_{3/2}[70, 1^-]$	(0, 0)	1683
$N(1710)\ P_{11}$	***	1680-1740	$^2 8_{1/2}[70, 0^+]$	(0, 1)	1683
$N(1720)\ P_{13}$	****	1650-1750	$^2 8_{3/2}[56, 2^+]$	(0, 0)	1737
$N(2190)\ G_{17}$	****	2100-2200	$^2 8_{7/2}[70, 3^-]$	(0, 0)	2140
$N(2220)\ H_{19}$	****	2180-2310	$^2 8_{9/2}[56, 4^+]$	(0, 0)	2271
$N(2250)\ G_{19}$	****	2170-2310	$^4 8_{9/2}[70, 3^-]$	(0, 0)	2229
$N(2600)\ I_{1,11}$	***	2550-2750	$^2 8_{11/2}[70, 5^-]$	(0, 0)	2591
$\Delta(1232)\ P_{33}$	****	1230-1234	$^4 10_{3/2}[56, 0^+]$	(0, 0)	1246
$\Delta(1600)\ P_{33}$	***	1550-1700	$^4 10_{3/2}[56, 0^+]$	(1, 0)	1660
$\Delta(1620)\ S_{31}$	****	1615-1675	$^2 10_{1/2}[70, 1^-]$	(0, 0)	1649
$\Delta(1700)\ D_{33}$	****	1670-1770	$^2 10_{3/2}[70, 1^-]$	(0, 0)	1649
$\Delta(1905)\ F_{35}$	****	1870-1920	$^4 10_{5/2}[56, 2^+]$	(0, 0)	1921
$\Delta(1910)\ P_{31}$	****	1870-1920	$^4 10_{1/2}[56, 2^+]$	(0, 0)	1921
$\Delta(1920)\ P_{33}$	***	1900-1970	$^4 10_{3/2}[56, 2^+]$	(0, 0)	1921
$\Delta(1930)\ D_{35}$	***	1920-1970	$^2 10_{5/2}[70, 2^-]$	(0, 0)	1946
$\Delta(1950)\ F_{37}$	****	1940-1960	$^4 10_{7/2}[56, 2^+]$	(0, 0)	1921
$\Delta(2420)\ H_{3,11}$	****	2300-2500	$^4 10_{11/2}[56, 4^+]$	(0, 0)	2414

Note. The masses are given in MeV. The experimental values are taken from [13].

TABLE IV

Mass Spectrum of Strange Baryon Resonances

Baryon $L_{I,2J}$	Status	Mass	State	(v_1, v_2)	M_{calc}
$\Sigma(1193)\ P_{11}$	****	1193	$^{2}8_{1/2}[56, 0^{+}]$	(0, 0)	1170
$\Sigma(1660)\ P_{11}$	***	1630-1690	$^{2}8_{1/2}[56, 0^{+}]$	(1, 0)	1604
$\Sigma(1670)\ D_{13}$	****	1665-1685	$^{2}8_{3/2}[70, 1^{-}]$	(0, 0)	1711
$\Sigma(1750)\ S_{11}$	***	1730-1800	$^{2}8_{1/2}[70, 1^{-}]$	(0, 0)	1711
$\Sigma(1775)\ D_{15}$	****	1770-1780	$^{4}8_{5/2}[70, 1^{-}]$	(0, 0)	1822
$\Sigma(1915)\ F_{15}$	****	1900-1935	$^{2}8_{5/2}[56, 2^{+}]$	(0, 0)	1872
$\Sigma(1940)\ D_{13}$	***	1900-1950	$^{2}8_{3/2}[56, 1^{-}]$	(0, 1)	1974
$\Sigma^*(1385)\ P_{13}$	****	1383-1385	$^{4}10_{3/2}[56, 0^{+}]$	(0, 0)	1382
$\Sigma^*(2030)\ F_{17}$	****	2025-2040	$^{4}10_{7/2}[56, 2^{+}]$	(0, 0)	2012
$\Lambda(1116)\ P_{01}$	****	1116	$^{2}8_{1/2}[56, 0^{+}]$	(0, 0)	1133
$\Lambda(1600)\ P_{01}$	***	1560-1700	$^{2}8_{1/2}[56, 0^{+}]$	(1, 0)	1577
$\Lambda(1670)\ S_{01}$	****	1660-1680	$^{2}8_{1/2}[70, 1^{-}]$	(0, 0)	1686
$\Lambda(1690)\ D_{03}$	****	1685-1690	$^{2}8_{3/2}[70, 1^{-}]$	(0, 0)	1686
$\Lambda(1800)\ S_{01}$	***	1720-1850	$^{4}8_{1/2}[70, 1^{-}]$	(0, 0)	1799
$\Lambda(1810)\ P_{01}$	***	1750-1850	$^{2}8_{1/2}[70, 0^{+}]$	(0, 1)	1799
$\Lambda(1820)\ F_{05}$	****	1815-1825	$^{2}8_{5/2}[56, 2^{+}]$	(0, 0)	1849
$\Lambda(1830)\ D_{05}$	****	1810-1830	$^{4}8_{5/2}[70, 1^{-}]$	(0, 0)	1799
$\Lambda(1890)\ P_{03}$	****	1850-1910	$^{2}8_{3/2}[56, 2^{+}]$	(0, 0)	1849
$\Lambda(2110)\ F_{05}$	****	2090-2140	$^{4}8_{5/2}[70, 2^{+}]$	(0, 0)	2074
$\Lambda(2350)\ H_{09}$	***	2340-2370	$^{2}8_{9/2}[56, 4^{+}]$	(0, 0)	2357
$\Lambda^*(1405)\ S_{01}$	****	1402-1410	$^{2}1_{1/2}[70, 1^{-}]$	(0, 0)	1641
$\Lambda^*(1520)\ D_{03}$	****	1518-1520	$^{2}1_{3/2}[70, 1^{-}]$	(0, 0)	1641
$\Lambda^*(2100)\ G_{07}$	****	2090-2110	$^{2}1_{7/2}[70, 3^{-}]$	(0, 0)	2197
$\Xi(1318)\ P_{11}$	****	1314-1316	$^{2}8_{1/2}[56, 0^{+}]$	(0, 0)	1334
$\Xi(1820)\ D_{13}$	***	1818-1828	$^{2}8_{3/2}[70, 1^{-}]$	(0, 0)	1828
$\Xi^*(1530)\ P_{13}$	****	1531-1532	$^{4}10_{3/2}[56, 0^{+}]$	(0, 0)	1524
$\Omega(1672)\ P_{03}$	****	1672-1673	$^{4}10_{3/2}[56, 0^{+}]$	(0, 0)	1670

Note. The masses are given in MeV. The experimental values are taken from [13]. The Ξ resonances are denoted by $L_{2I,2J}$.

The extent to which the experimental values disagree from the calculated values is an indication of physics beyond the standard qqq $SU_f(3)$ model.

Discrepancies:

The main discrepancy is for the singlet states $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(2100)$

Explanation:

Additional interactions in singlet states? U(1) problem?

Quasi-bound molecular states?

Missing states:

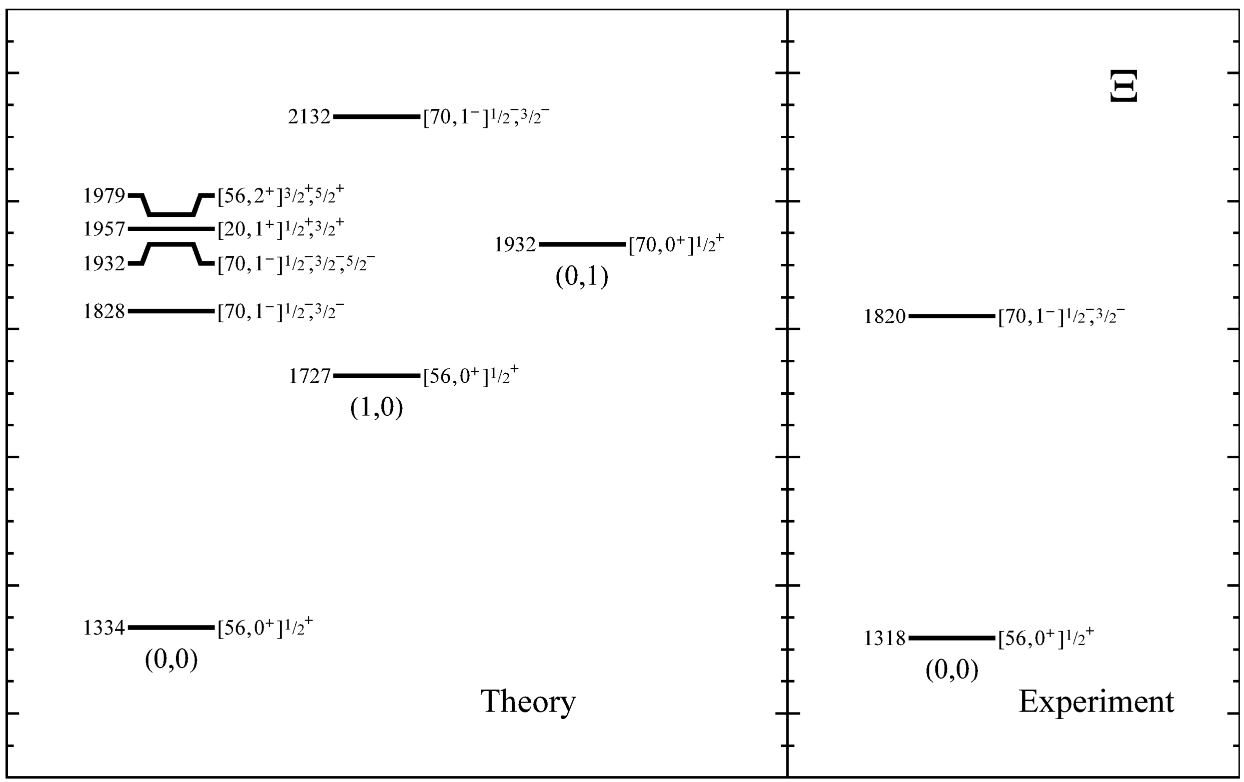
The antisymmetric states $^{2}8[20,1^{+}]$ are entirely missing from the experimental spectrum

Explanation:

Quark-Diquark structure of baryons? qQ

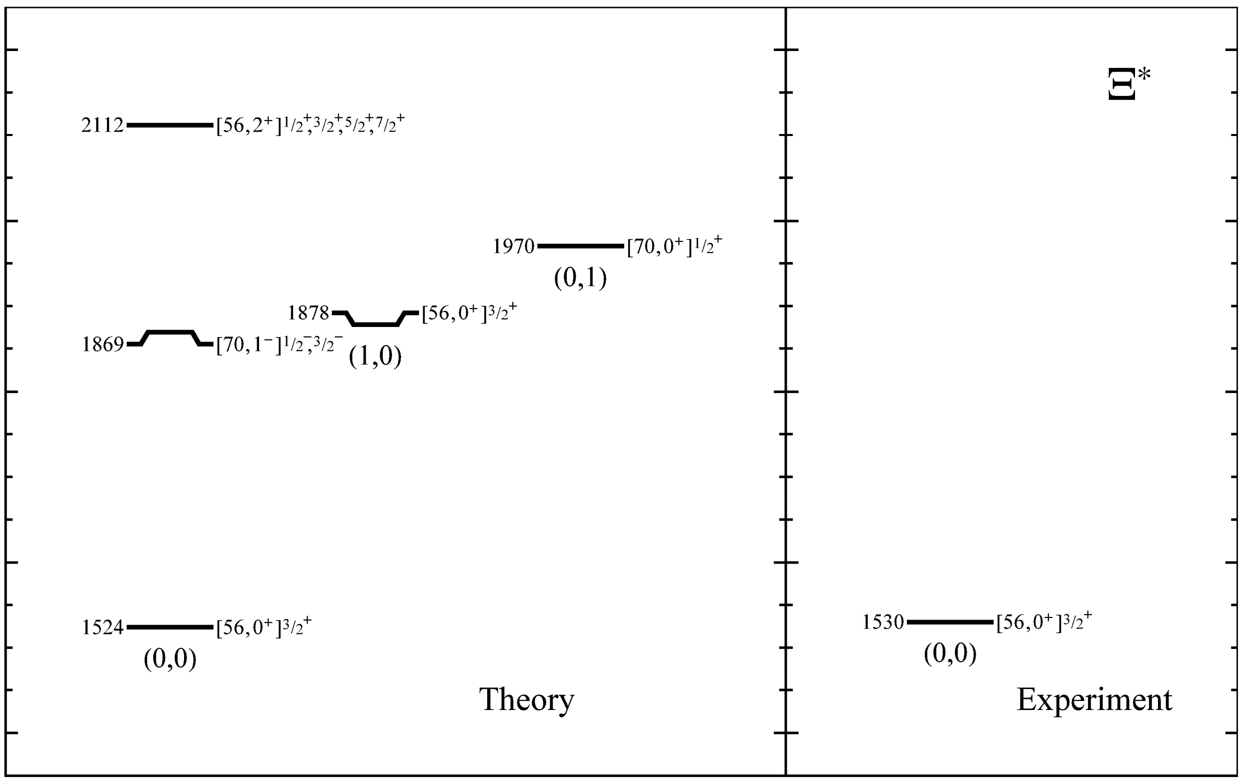
Cascade Physics: A deeper look into the structure of hadrons

Comparison between experimental and predicted spectrum of octet cascades Ξ



New experiments?

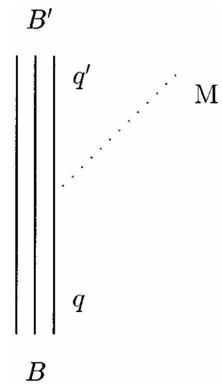
Comparison between the experimental and predicted spectrum of decuplet cascades Ξ^*



New experiments?

STRONG DECAY WIDTHS

$B \to B'+M$



These can be calculated assuming a form for the operator inducing the transition

$$H_{strong} = \frac{1}{(2\pi)^{3/2}(2k_0)^{3/2}} \sum_{j=1}^{3} X_j^M \left[2g(s_j \cdot k)e^{-ik\cdot r_j} + hs_j \cdot \left(p_j e^{-ik\cdot r_j} + e^{-ik\cdot r_j} p_j \right) \right]$$

with 2 parameters g and h

New ingredient in the calculation: Clebsch-Gordan coefficients of $SU_f(3)$
Selection rules: tests of flavor symmetry

Results for octect and decuplet baryons emitting an octet and singlet pseudoscalar meson (with mixing angle θ_P=-23º)

g=1.164 fm; h=-0.094 fm

Comparison between experimental and predicted strong decay widths of nucleon resonances

TABLE XIII

Strong Decay Widths of Three- and Four-star Nucleon Resonances in MeV

Baryon	$N\pi$	$N\eta$	ΣK	ΛK	$\Delta\pi$	$\Sigma^* K$
$N(1440)\ P_{11}$	108	—	—	—	0	—
	227 ± 67				87 ± 30	
$N(1520)\ D_{13}$	115	1	—	—	12	—
	67 ± 9				24 ± 7	
$N(1535)\ S_{11}$	85	0	—	—	23	—
	79 ± 38	74 ± 39			$< 1 \pm 1$	
$N(1650)\ S_{11}$	35	8	—	0	24	—
	121 ± 34	11 ± 6		12 ± 7	7 ± 5	
$N(1675)\ D_{15}$	31	17	—	0	123	—
	72 ± 12			$< 1 \pm 1$	88 ± 14	
$N(1680)\ F_{15}$	41	0	—	0	5	—
	84 ± 9				13 ± 7	
$N(1700)\ D_{13}$	5	4	0	0	225	—
	10 ± 7			$< 2 \pm 2$		
$N(1710)\ P_{11}$	85	8	0	1	34	—
	23 ± 17			23 ± 21	41 ± 33	
$N(1720)\ P_{13}$	31	0	0	0	10	—
	23 ± 11			12 ± 11		
$N(2190)\ G_{17}$	34	11	1	7	25	1
	68 ± 27					
$N(2220)\ H_{19}$	15	1	0	2	5	0
	65 ± 28					
$N(2250)\ G_{19}$	7	9	9	0	40	2
	38 ± 21					
$N(2600)\ I_{1,11}$	9	3	0	3	7	1
	49 ± 20					

Note. The experimental values are taken from [13]. Decay channels labeled by — are below threshold.

Comparison between experimental and calculated strong decay widths of Λ resonances

TABLE XVI

Strong Decay Widths of Λ Resonances in MeV

Baryon	$N\bar{K}$	$\Sigma\pi$	$\Lambda\eta$	ΞK	$\Sigma^*\pi$	Ξ^*K
$\Lambda(1600)\ P_{01}$	25	21	—	—	0	—
	34 ± 25	53 ± 51				
$\Lambda(1670)\ S_{01}$	44	9	0	—	14	—
	8 ± 3	15 ± 9	9 ± 5			
$\Lambda(1690)\ D_{03}$	100	16	0	—	14	—
	15 ± 4	18 ± 7				
$\Lambda(1800)\ S_{01}$	0	80	5	—	19	—
	98 ± 40	seen			seen	
$\Lambda(1810)\ P_{01}$	62	9	0	—	15	—
	53 ± 42	38 ± 34			seen	
$\Lambda(1820)\ F_{05}$	23	13	0	0	3	—
	48 ± 7	9 ± 3			6 ± 2	
$\Lambda(1830)\ D_{05}$	0	77	16	0	101	—
	6 ± 3	47 ± 22			$> 13 \pm 4$	
$\Lambda(1890)\ P_{03}$	19	12	0	0	10	—
	36 ± 22	8 ± 6			seen	
$\Lambda(2110)\ F_{05}$	0	10	4	2	120	1
	30 ± 21	50 ± 33			seen	
$\Lambda^*(1405)\ S_{01}$	—	0	—	—		
		50 ± 2				
$\Lambda^*(1520)\ D_{03}$	10	28	—	—		
	7 ± 1	7 ± 1				
$\Lambda^*(2100)\ G_{07}$	18	22	4	2		
	53 ± 24	$\sim 9 \pm 4$	$< 3 \pm 3$	$< 3 \pm 3$		

Note. The notation and source are the same as for Table XIII.

Discrepancies:

N(1535), Σ(1750) have a large measured width into Nη (calculated small)

Explanation:

These states are quasi-molecular Nη states

Λ(1405) has a large observed width into Σπ (calculated zero)

Explanation:

This state is a quasi-molecular state

Strong decay widths of Ξ resonances observed so far are in reasonable agreement with calculations

TABLE XVII

Strong Decay Widths of Ξ Resonances in MeV

Baryon	$\Sigma\bar{K}$	$\Lambda\bar{K}$	$\Xi\pi$	$\Xi\eta$	$\Sigma^*\bar{K}$	$\Xi^*\pi$	$\Xi^*\eta$	ΩK
$\Xi(1820)\ D_{13}$	30 7 ± 4	18 7 ± 4	6 2 ± 2	—	—	3 7 ± 4	—	—
$\Xi^*(1530)\ P_{13}$	—	—	22 10 ± 1	—	—	—	—	—

Note. The notation and source are the same as for Table XIII.

New experiments?

Cascade physics:

TABLE XXI

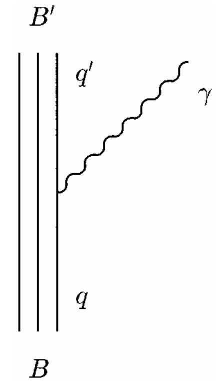
Strong Decay Widths for Missing Ξ Resonances in MeV

Ξ	(v_1, v_2)	Mass	$\Sigma\bar{K}$	$\Lambda\bar{K}$	$\Xi\pi$	$\Xi\eta$	$\Sigma^*\bar{K}$	$\Xi^*\pi$	$\Xi^*\eta$	ΩK
$^{2}8_{1/2}[70, 1^-]$	(0, 0)	1828	11	10	4	—	—	6	—	—
$^{4}8_{1/2}[70, 1^-]$	(0, 0)	1932	14	24	119	0	1	6	—	—
$^{4}8_{3/2}[70, 1^-]$	(0, 0)	1932	3	4	17	0	2	34	—	—
$^{4}8_{5/2}[70, 1^-]$	(0, 0)	1932	15	23	100	0	2	24	—	—
$^{2}8_{J}[20, 1^+]$	(0, 0)	1957	0	0	0	0	0	0	—	—
$^{2}8_{3/2}[56, 2^+]$	(0, 0)	1979	8	1	1	0	0	2	—	—
$^{2}8_{5/2}[56, 2^+]$	(0, 0)	1979	20	1	1	0	0	1	—	—
$^{2}8_{3/2}[70, 2^+]$	(0, 0)	2100	25	9	3	1	5	10	0	—
$^{2}8_{5/2}[70, 2^+]$	(0, 0)	2100	47	16	4	2	3	7	0	—
$^{2}8_{J}[70, 2^-]$	(0, 0)	2100	0	0	0	0	0	0	0	—
$^{4}8_{1/2}[70, 2^+]$	(0, 0)	2191	11	14	60	1	2	3	0	0
$^{4}8_{3/2}[70, 2^+]$	(0, 0)	2191	5	7	30	1	11	18	0	0
$^{4}8_{5/2}[70, 2^+]$	(0, 0)	2191	3	3	13	0	19	30	0	0
$^{4}8_{7/2}[70, 2^+]$	(0, 0)	2191	12	15	59	2	13	19	0	0
$^{4}8_{J}[70, 2^-]$	(0, 0)	2191	0	0	0	0	0	0	0	0
$^{2}8_{1/2}[56, 0^+]$	(1, 0)	1727	0	0	2	—	—	0	—	—
$^{2}8_{1/2}[70, 1^-]$	(1, 0)	2132	3	2	1	0	1	0	0	—
$^{2}8_{3/2}[70, 1^-]$	(1, 0)	2132	5	3	1	0	1	0	0	—
$^{4}8_{1/2}[70, 1^-]$	(1, 0)	2222	3	6	22	0	0	0	0	0
$^{4}8_{3/2}[70, 1^-]$	(1, 0)	2222	0	1	3	0	0	1	1	1
$^{4}8_{5/2}[70, 1^-]$	(1, 0)	2222	3	5	17	0	0	1	0	1
$^{2}8_{J}[20, 1^+]$	(1, 0)	2244	0	0	0	0	0	0	0	0
$^{2}8_{1/2}[70, 0^+]$	(0, 1)	1932	24	11	3	0	0	4	—	—
$^{4}8_{3/2}[70, 0^+]$	(0, 1)	2030	7	10	44	0	4	15	—	—
$^{2}8_{1/2}[56, 1^-]$	(0, 1)	2076	34	2	2	1	2	5	—	—
$^{2}8_{3/2}[56, 1^-]$	(0, 1)	2076	52	3	3	1	1	4	—	—
$^{2}8_{1/2}[70, 1^-]$	(0, 1)	2191	5	1	0	1	6	5	1	0
$^{2}8_{3/2}[70, 1^-]$	(0, 1)	2191	6	1	0	2	5	5	0	0
$^{2}8_{J}[70, 1^+]$	(0, 1)	2191	0	0	0	0	0	0	0	0
$^{4}8_{1/2}[70, 1^-]$	(0, 1)	2278	1	0	2	0	1	1	0	1
$^{4}8_{3/2}[70, 1^-]$	(0, 1)	2278	0	0	0	0	8	8	2	6
$^{4}8_{5/2}[70, 1^-]$	(0, 1)	2278	0	0	1	0	5	4	1	5
$^{4}8_{J}[70, 1^+]$	(0, 1)	2278	0	0	0	0	0	0	0	0
$^{2}8_{J}[20, 1^-]$	(0, 1)	2300	0	0	0	0	0	0	0	0

Note. The notation is the same as for Table XVIII.

ELECTROMAGNETIC DECAY WIDTHS

$B \rightarrow B' + \gamma$



Calculated assuming a transition operator

$$H_{em} = 2\sqrt{\frac{\pi}{k_0}} \sum_{j=1}^{3} \mu_j e_j \left[k s_{j,-} e^{-ik\cdot r_j} + \frac{1}{2g_j}\left(p_{j,-} e^{-ik\cdot r_j} + e^{-ik\cdot r_j} p_{j,-} \right) \right]$$

with $SU_f(3)$ symmetry for all octet and decuplet baryons (no free parameters)

Discrepancies:
$\Lambda(1405)$ has an observed width of 10 ± 4 keV into $\Sigma^0\gamma$ calculated to be 156 keV
Explanation:
Quasi molecular state?

Cascade Physics:

TABLE XXX

Radiative Decay Widths of Baryons in keV. Systematic and Statistical Errors Are Added Quadratically

	$\Gamma(B \to B' + \gamma)$				
$B \to B' + \gamma$	Ref. [41]	Ref. [42]	Present	Exp.	
$\Sigma^0 \to \Lambda + \gamma$			8.6	8.6 ± 1.0	[37]
$\Delta^+ \to p + \gamma$	430 ± 150	350	343.7	672 ± 56	[13]
$\Delta^0 \to n + \gamma$	430 ± 150	350	341.5		
$\Sigma^{*,+} \to \Sigma^+ + \gamma$	100 ± 26	105	140.7		
$\Sigma^{*,0} \to \Sigma^0 + \gamma$	17 ± 4	17.4	33.9		
$\Sigma^{*,-} \to \Sigma^- + \gamma$	3.3 ± 1.2	3.6	0.0		
$\Sigma^{*,0} \to \Lambda + \gamma$		265	221.3		
$\Xi^{*,0} \to \Xi^0 + \gamma$	129 ± 29	172	188.2		
$\Xi^{*,-} \to \Xi^- + \gamma$	3.8 ± 1.2	6.2	0.0		
$\Lambda^*(1405) \to \Lambda + \gamma$			116.9	27 ± 8	[13]
$\Lambda^*(1405) \to \Sigma^{*,0} + \gamma$			0.0		
$\Lambda^*(1405) \to \Sigma^0 + \gamma$			155.7	10 ± 4	[13]
				23 ± 7	[13]
$\Lambda^*(1520) \to \Lambda + \gamma$			85.1	134 ± 23	[34, 38]
				33 ± 11	[39]
$\Lambda^*(1520) \to \Sigma^{*,0} + \gamma$			0.0		
$\Lambda^*(1520) \to \Sigma^0 + \gamma$			180.4	47 ± 17	[39]

New experiments?

Comments on electromagnetic decays:

(a) Diagonal breaking of $SU_f(3)$ symmetry was also studied (unpublished). Cascade physics:

[Widths in keV]

$B \to B' + \gamma$	$SU_f(3)$	$Broken SU_f(3)$
$\Xi^{*,0} \to \Xi^0 + \gamma$	188.2	146.6
$\Xi^{*,-} \to \Xi^- + \gamma$	0.0	3.1

(b) Magnetic moments were also studied. Cascade physics:

[Moments in μ_N]

	$SU_f(3)$	$Broken SU_f(3)$	Exp
Ξ^0	-1.86	-1.43	-1.25
Ξ^-	-0.93	-0.49	-0.65

A major problem occurs for cascades: Violation of the $SU_f(3)$ rule

$$(\mu_{\Xi^-} - \mu_{\Xi^0}) = \frac{1}{5}(\mu_p - \mu_n) = \frac{1}{4}(\mu_{\Sigma^+} - \mu_{\Sigma^-}) \qquad \text{Th}$$

$$0.599 \neq 0.941 \cong 0.905 \qquad \text{Exp}$$

Comment on electromagnetic mass splittings:

(a) These can be studied by adding to the mass squared operator (unpublished)

$$M_{em}^2 = d' I_3 + d'' I_3 (Y-1)$$

Values in MeV; d' and d'' fixed to *

$m_n - m_p$	1.29 ± 0.00	$1.29*$
$m_{\Sigma^-} - m_{\Sigma^0}$	4.80 ± 0.03	4.04
$m_{\Sigma^-} - m_{\Sigma^+}$	8.08 ± 0.03	$8.08*$
$m_{\Xi^-} - m_{\Xi^0}$	6.48 ± 0.24	6.79

CONCLUSIONS

Questions in hadron physics that can be elucidated with cascades:

Origin of low-lying excitation $[56,0^+](1,0)$

$N(1440)$-$\Sigma(1660)$-$\Lambda(1600)$-$\Xi(?)$

$\Xi(?)$ predicted at 1727 MeV $I(J^P)=1/2(1/2^+)$

Existence of more complex configurations:

(a) Quasi-molecular B-η (Nefkens)?

$N(1535)$-$\Sigma(1750)$-$\Lambda(1690)$-$\Xi(?)$

$\Xi(?)$ predicted at 1885 MeV $I(J^P)=1/2(1/2^-)$

Tests of flavor symmetry:

(a) $\Xi^{*,-}$ ✹ $\Xi^{-}+\gamma$

Predicted to be zero

Speculative configurations:

(a) Pentaquarks

S=-2 pentaquark I=3/2 Ξ^{--}?

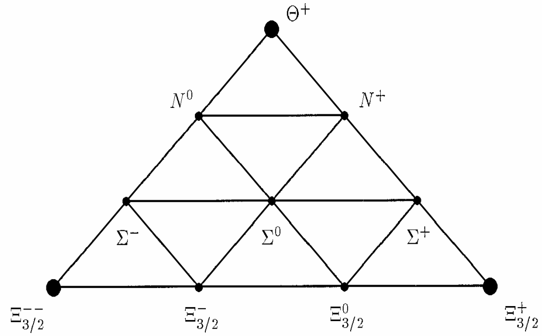
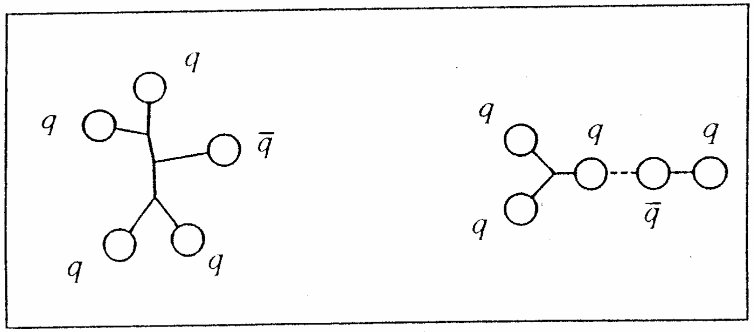


Figure 1: $SU(3)$ antidecuplet. The isospin-hypercharge multiplets are $(I, Y) = (0, 2)$, $(\frac{1}{2}, 1)$, $(1, 0)$ and $(\frac{3}{2}, -1)$. Exotic states are located at the corners and are indicated with •.

The existence of configurations more complex than qqq and qqbar is one of the crucial problems in hadron physics. In particular, five quark configurations, may play an important role in baryon spectroscopy (cascade physics?)



SUMMARY

A comprehensive algebraic calculation of q^3 baryons, B, is now available.
R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. **236**, 69 (1994)
R. Bijker, F. Iachello and A. Leviatan, Phys. Rev. D **55**, 2862 (1997)
R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. **284**, 89 (2000)*

[A comprehensive algebraic calculation of $q\bar{q}$ mesons, M, is also available.]
F. Iachello, N.C. Mukhopadhyai, and L. Zhang, Phys. Rev. D **44**, 898 (1991)
F. Iachello and D. Kusnezov, Phys. Rev. D **45**, 4156 (1992)
C. Gobbi, F. Iachello, and D. Kusnezov, Phys. Rev. D **50**, 2048 (1994)

A comprehensive calculation of B-M quasi-molecular states is needed.
[A partial algebraic calculation of octet-η meson quasi-molecular states is available.]
F. Iachello, in N^* Physics, Proc. of the Fourth CEBAF/INT Workshop, T.S.H. Lee and W.Roberts, eds., World Scientific, p. 78 (1997)
[A comprehensive algebraic calculation of M-M and B-B quasi-molecular states is also needed.]

Speculative configurations:

(a) Tetraquarks and Pentaquarks
[A partial algebraic calculation of pentaquarks is available]
R. Bijker *et al.*, arXiv:hep-ph/0409022v1 (2004)

(b) Gluonic states (glueballs) and composite states of quarks and gluons
[No algebraic calculation is available at the present time]