Unitary Coupled-Channel Model for Heavy Meson Decays into Three Mesons

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Hadron Spectroscopy

⇒ Key information for confinement physics
Hadron Spectroscopy

data analysis $\Rightarrow$ hadron properties

$(J^P_C, \text{mass, width, branching ratios, ..})$

Reliable analysis tool is essential!
Light Flavor Meson Spectroscopy  (relevant to JLab 12 GeV programs)

* data

* analysis tool
**Light Flavor Meson Spectroscopy**  (relevant to JLab 12 GeV programs)

E.g., E852 (BNL) \[ \pi^- p \rightarrow \pi^+ \pi^- \pi^- p \]  Chung et al., PRD 65, 072001 (2001)
Light Flavor Meson Spectroscopy (relevant to JLab 12 GeV programs)

e.g., E852 (BNL) \( \pi^- p \rightarrow \pi^+ \pi^- \pi^- p \)  
Chung et al., PRD 65, 072001 (2001)

- \( \pi \) subsystem forms a resonance
- 3rd \( \pi \) is a spectator
**Isobar model**

E852 (BNL), Chung et al., PRD 65, 072001 (2001)

* $L = 0, 1, 2$

* For $R = f_0(980)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$

  $\implies$ Breit-Wigner form  
  \[ A_R = \frac{F_{R \to \pi\pi}}{m^2_R - m^2_{\pi\pi} - i m_R \Gamma_R(m_{\pi\pi})} \]

* For $R = \sigma$

  $\implies$ K-matrix model  
  [e.g., Au, Morgan, Pennington, PRD 35, 1633 (1987)]

* $A_{M^* \to \pi\pi\pi} = \sum_R a_R e^{i\phi_R} A_R + \text{(background)}$
Questions

* coupled-channels?
* 3-body unitarity?
Coupled-channel effect

e.g., $\pi^0 N \rightarrow \pi \pi^0 N$  
[\pi N, \eta N, \pi \Delta, \rho N, \sigma N$ coupled-channels]

Kamano et al., PRC79 025206 (2009)
3-body unitarity requires ... Z-diagrams
Question to be addressed

How 3-body unitarity makes a difference in extracting hadron properties from data?

Method

1. Construct a unitary and an isobar models

2. Fit them to the same Dalitz plot

3. Extract and compare $M^*$ properties from them
   (pole position, coupling strength to decay channels)
Coupled-Channels Model

Kamano, Nakamura, Sato, Lee, PRD 84 114019 (2011)

\[ M^* \rightarrow \pi R \rightarrow \pi\pi\pi \]

Channels \( R : f_0(600), f_0(980), \rho(760), f_2(1270), .. \)

\( R \) : resonance in \( \pi\pi \) scattering amplitude (not Breit-Wigner form)

(I) Develop \( \pi\pi \) model

(II) Develop \( \pi R \) interaction

(III) Solve \( \pi R \) scattering equation
Simple $\pi\pi$ model

Coupled-channel scattering equation for $\pi\pi$ partial wave ($L, I$)

$$t_{i,j}^{LI}(p', p; W) = V_{i,j}^{LI} + \sum_k \int_0^\infty q^2 dq \ V_{i,k}^{LI}(p', q; W) \frac{1}{W - E_k(q) + i\epsilon} t_{k,j}^{LI}(q, p; W)$$

$$E_{\pi\pi}(q) = 2\sqrt{m_\pi^2 + q^2}$$

$$V_{i,j}^{LI}(p', p; W) = \sum_R f_{R,i}^{LI}(p') \frac{1}{W - m_R} f_{R,j}^{LI}(p)$$

$$f_{R,i}^{LI}(p) = \frac{g_{R,i}}{\sqrt{m_\pi}} \frac{1}{1 + (c_{R,i}p)^2} \left( \frac{p}{m_\pi} \right)^L$$
Phase and inelasticity of $\pi\pi$ amplitude

$L = I = 0 \ (2 \ R)$

$L = I = 1 \ (2 \ R)$

$L = 2, I = 0 \ (1 \ R)$

[Data: Gayer et al. (1974); Hyams et al. (1973); Batley et al. (2008)]
Pole positions in $\pi\pi$ amplitude

<table>
<thead>
<tr>
<th></th>
<th>Re[(M_R)] (MeV)</th>
<th>−Im[(M_R)] (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ours</td>
<td>PDG</td>
</tr>
<tr>
<td>(f_0) (600)</td>
<td>430</td>
<td>400 − 1200</td>
</tr>
<tr>
<td>(f_0) (980)</td>
<td>1000</td>
<td>980 ± 10</td>
</tr>
<tr>
<td>(f_0) (1370)</td>
<td>1350</td>
<td>1200 − 1500</td>
</tr>
<tr>
<td>(\rho) (760)</td>
<td>770</td>
<td>775.5 ± 0.3</td>
</tr>
<tr>
<td>(\rho) (1700)</td>
<td>1610</td>
<td>1550 − 1780</td>
</tr>
<tr>
<td>(f_2) (1270)</td>
<td>1250</td>
<td>1275 ± 1.2</td>
</tr>
</tbody>
</table>
Quasi two-particle ($\pi R$) interaction

3 $\pi$ Z-graph

$M^*$ graph
Quasi two-particle ($\pi R$) scattering equation

$$\pi \quad t \quad R' \quad = \quad \quad \quad \quad + $$

$M^*$ decay amplitude (unitary model)

$M^*$ decay amplitude (isobar model)
Case Study : $\gamma p \rightarrow M^* n \rightarrow \pi^+ \pi^+ \pi^- n$ (CLAS 6, GlueX)

How 3-body unitary makes a difference in extracting $M^*$ properties from data?

Nakamura, Kamano, Sato, Lee, in preparation
$M^*$ propagator  (unitary model)

\[ G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) \]
$M^*$ propagator (unitary model)

\[
\begin{align*}
    &\pi -\longrightarrow \quad R' \\
    \text{R} &\quad -\longrightarrow \quad \pi'
\end{align*}
\]

\[
\begin{align*}
    \rho &\quad +\quad t \\
    \text{R} &\quad +\quad t
\end{align*}
\]

\[G^{-1}(W) = W - M_{M^*}^0 - \Sigma_{M^*}(W)\]

Pole position: $M_R \Rightarrow G^{-1}(M_R) = 0$

Solved with analytic continuation to the unphysical Riemann sheet

$M^*$ propagator  (unitary model)

$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$

$M^*$ propagator  (isobar model)

$G^{-1}(W) = W - i \frac{\Gamma(W)}{2}$

but any phenomenological parametrization should be fine ...
\( M^* \) propagator (unitary model)

\[
\begin{align*}
\pi \rightarrow R' & \quad R' \rightarrow \pi' \\
\text{This work} : & \quad G^{-1}(W) = W - M_{M^*}^0 - \Sigma_{M^*}(W) - \Delta M_{M^*}^0
\end{align*}
\]

\( M^* \) propagator (isobar model)

\[
\pi \rightarrow R' \rightarrow \pi' \\
\text{Pole position} : M_R(\text{isobar}) \sim M_R(\text{unitary}) + \Delta M_{M^*}^0
\]
Production amplitude

\[ \gamma \xrightarrow{\rho} M^* \]
\[ p \xrightarrow{\pi^+} n \]

Simple assumptions:

* \( t \)-channel \( \pi \)-exchange

* Vector-dominance of \( \gamma\pi M^* \) coupling

Not realistic but good enough

interested in effect of 3-body unitarity implemented in \( M^* \) propagation and decay
Kinematics

[cf. CLAS 6, PRL 102, 102002 (2009)]

* $E_\gamma = 5$ GeV

* $t = -0.4$ GeV$^2$

* $0.8$ GeV $\leq W \leq 2$ GeV ; $W : 3 \pi$ invariant mass

* $3 \pi$ orientation
  (Euler angles : $\alpha, \beta, \gamma$)

\[ \begin{align*}
  \alpha, \beta & \text{ fixed ; } 0 \leq \gamma \leq 2\pi \\
  \Rightarrow \text{Photon excites a polarized } M^* \text{ that decays to a certain } \gamma \text{ more often}
\end{align*} \]
Procedure

1. Determine parameters of unitary model with reasonable input

2. Generate mock data with the unitary model

3. Fit the data with isobar model

4. Compare $M^*$ properties from the two models
Partial wave, $M^*$'s in unitary model

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$M^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{++}$</td>
<td>$a_1(1230)$, $a_1(1700)$</td>
</tr>
<tr>
<td>$2^{++}$</td>
<td>$a_2(1320)$, $a_2(1700)$</td>
</tr>
<tr>
<td>$2^{--}$</td>
<td>$\pi_2(1670)$, $\pi_2(1800)$</td>
</tr>
<tr>
<td>$1^{--}$</td>
<td>$\pi_1(1600)$</td>
</tr>
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</table>

[cf. CLAS 6, PRL 102, 102002 (2009)]
Determination of $M^*$ parameters for unitary model

* $M^*$ bare mass

* $M^* \to \pi R$ bare coupling and cutoff

$M^*$ propagator

$$G^{-1}(W) = W - M_{M^*}^0 - \Sigma_{M^*}(W)$$

$M^* \to \pi R$ vertex function

$$F(p) \propto \frac{C}{\sqrt{E_R E_\pi}} \left( \frac{\Lambda^2}{p^2 + \Lambda^2} \right)^{2+(L/2)} p^L$$
Determination of $M^*$ parameters for unitary model

- $M^*$ bare mass
- $M^* \rightarrow \pi R$ bare coupling and cutoff

$^3P_0$ model

Flux-tube model for $\pi_1(1600)$

- Partial width $\Rightarrow$ coupling
- Cutoff is set to 1 GeV

Barnes et al., PRD 55, 4157 (1997)

Isgur et al., PRL 54, 869 (1985)
<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>decay modes</th>
<th>$\Gamma_{q\bar{q}}$</th>
<th>$\Gamma_{\text{hybrid}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{++}$</td>
<td>$a_1(1230) \rightarrow \pi\rho(770)$</td>
<td>540.</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$a_1(1700) \rightarrow \pi f_0(1300)$</td>
<td>2.</td>
<td>6.</td>
</tr>
<tr>
<td></td>
<td>$\pi\rho(770)$</td>
<td>57.</td>
<td>30.</td>
</tr>
<tr>
<td></td>
<td>$\pi\rho(1465)$</td>
<td>41.</td>
<td>0.</td>
</tr>
<tr>
<td></td>
<td>$\pi f_2(1275)$</td>
<td>39.</td>
<td>70.</td>
</tr>
<tr>
<td>$2^{++}$</td>
<td>$a_2(1318) \rightarrow \pi\rho(770)$</td>
<td>55.</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$a_2(1700) \rightarrow \pi\rho(770)$</td>
<td>104.</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\pi f_2(1275)$</td>
<td>20.</td>
<td>-</td>
</tr>
<tr>
<td>$2^{-+}$</td>
<td>$\pi_2(1670) \rightarrow \pi\rho(770)$</td>
<td>118.</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\pi f_2(1275)$</td>
<td>75.</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\pi_2(1800) \rightarrow \pi f_0(1300)$</td>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td></td>
<td>$\pi\rho(770)$</td>
<td>162.</td>
<td>8.</td>
</tr>
<tr>
<td></td>
<td>$\pi f_2(1275)$</td>
<td>86.</td>
<td>50.</td>
</tr>
<tr>
<td>$1^{-+}$</td>
<td>$\pi_1(1600) \rightarrow \pi\rho(770)$</td>
<td>-</td>
<td>8.</td>
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<td>$\Gamma_{\text{hybrid}}$</td>
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Difference between $\Gamma_{q\bar{q}}$ and $\Gamma_{\text{hybrid}}$

$\Rightarrow$ Coupling strength to decay channel is a key information to understand the nature of hadron structure

We use $\Gamma_{q\bar{q}}$ except for $\pi_1(1600)$
$W$-dependence of integrated Dalitz plots from unitary model
Dalitz plot from unitary model

\[ W = 1 \text{ GeV} \text{ near } a_1(1230) \text{ peak} \]
Dalitz plot from unitary model

\[ W = 1.24 \text{ GeV} \] near \( a_2(1320) \) peak
Dalitz plot from unitary model

\[ W = 1.76 \text{ GeV} \text{ near } \pi_2(1800) \text{ peak} \]
Fit with isobar model

Error:

- Data for the same $W$ have the same error
- At each $W$, the error is assigned by 5% of the highest peak

Fit:

1. Fit with real couplings of $M^* \rightarrow \pi R$
2. Allow couplings complex
3. Include $W$-dependent flat non-interfering background
   (2, 3 are common in isobar-model analysis)

$\frac{\chi^2}{(#\ of\ data)} < 0.5$ is achieved
Fit with isobar model

W=1 GeV

W=1.24 GeV

unitary model without Z

W=1.76 GeV
Fit with isobar model

\[ M^2_{\pi^+\pi^0} + M^2_{\pi^+\pi^0} = \text{unitary model without Z isobar-fit} \]

\[ W=1\text{GeV} \]

\[ W=1.24\text{GeV} \]

\[ W=1.76\text{GeV} \]
Question I didn’t explicitly ask

Can isobar model extract partial wave amplitudes of the unitary model?
\[ \frac{d^4 \sigma}{dt \, dW \, d\alpha \, d\beta} \left( \frac{\mu b}{\text{GeV}^3 \text{sr}} \right) \]

\begin{align*}
1^{++} a_1 & \quad \text{(unitary isobar)} \\
2^{+} \pi_2 & \quad \text{(unitary isobar)} \\
2^{++} a_2 & \\
1^{-} \pi_1 &
\end{align*}
**Q**: Can isobar model extract partial wave amplitudes of the unitary model?

**A**: To a good extent, yes.

**Comments**

- Not so in kinematics where a partial wave amplitude plays a minor role
- Analyzing polarized observables may have helped
**$M^*$ pole positions**

$\Delta M^0_{M^*}$ (MeV) : pole position shift in the isobar model

$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) - \Delta M^0_{M^*}$$

<table>
<thead>
<tr>
<th></th>
<th>$a_1(1260)$</th>
<th>$a_1(1700)$</th>
<th>$a_2(1320)$</th>
<th>$a_2(1700)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-22.37 - 23.21i$</td>
<td>$8.48 - 4.46i$</td>
<td>$0.15 - 0.04i$</td>
<td>$-1.03 - 0.30i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\pi_2(1670)$</th>
<th>$\pi_2(1800)$</th>
<th>$\pi_1(1600)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.51 + 0.38i$</td>
<td>$-3.07 - 3.42i$</td>
<td>$0.57 + 0.33i$</td>
</tr>
</tbody>
</table>

Here, 3-body unitarity effect is moderate
Couplings of $a_1(1260)$ to decay channels

<table>
<thead>
<tr>
<th></th>
<th>Unitary</th>
<th>Isobar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1(1260)$</td>
<td>→ $\pi f_0(1300)$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>→ $\pi f_0(2400)$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>→ $\pi \rho(770)$</td>
<td>24.6</td>
</tr>
<tr>
<td></td>
<td>→ $\pi \rho(1700)$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>→ $\pi f_2(1270)$</td>
<td>-</td>
</tr>
</tbody>
</table>

Rather large change in $M^*$ couplings to decay channels

$\Leftarrow$ Large Z-graph effect in $a_1(1260)$ region
## Couplings of $a_2(1320)$ to decay channels

<table>
<thead>
<tr>
<th>Couplings</th>
<th>Unitary</th>
<th>Isobar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2(1320) \rightarrow \pi \rho(770)$</td>
<td>1.0</td>
<td>0.9 $- 0.1,i$</td>
</tr>
<tr>
<td>$\quad \rightarrow \pi f_2(1270)$</td>
<td>-</td>
<td>1.4 $- 0.1,i$</td>
</tr>
</tbody>
</table>

Still rather large change in $M^*$ couplings to decay channels

even though Z-graph effect on Dalitz plot in $a_2(1320)$ region seems moderate
Conclusion

Q: How 3-body unitary makes a difference in extracting $M^*$ properties from data?

Method

1. Construct a unitary and an isobar models
2. Fit them to the same Dalitz plot
3. Extract and compare $M^*$ properties from them
Conclusion

Q: How 3-body unitary makes a difference in extracting $M^*$ properties from data?

A: It (and thus Z-diagrams) makes a significant difference in extracting dynamical aspect of $M^*$ properties, i.e., coupling strength to decay channel

Key information to understand the hadron structure
Conclusion

Q : How 3-body unitary makes a difference in extracting $M^*$ properties from data ?

A : It (and thus Z-diagrams) makes a significant difference in extracting dynamical aspect of $M^*$ properties, i.e., coupling strength to decay channel

Q : What about pole position ?

A : Moderate. Sometimes non-negligible.