Parity violating asymmetry, dipole polarizability, and the neutron skin thickness in $^{48}\text{Ca}$ and $^{208}\text{Pb}$

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Motivation:

The importance of determining isovector properties in nuclei

- **In the past** (and also in the present), *neutron properties* in stable medium and heavy nuclei have been mainly measured by using *strongly interacting probes*.

[↓]

**Limited knowledge of isovector properties**

- **At present,**
  - the use of *rare ion beams* has opened the possibility of measuring properties of *exotic nuclei*
  - *parity violating elastic electron scattering* (PVES), a *model independent technique*, has allowed to estimate the *neutron radius* of a stable heavy nucleus like $^{208}$Pb

[↓]

**Promising perspectives** for the near future
Motivation:

It is possible to connect observables with general isovector properties of the nuclear effective interaction?

Example:
Mean-Field predictions show a clear correlation between $\Delta r_{np}$ of a medium and heavy nucleus and the density slope of the symmetry energy ($L = 3\rho_0 \partial_\rho S(\rho)|_{\rho_0} = 3\rho_0 p_0$).

More generally within MF, it has been found a semi-empirical law: $a_{\text{sym}}(A) \approx S(\rho_A)$ with $\rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}) \Rightarrow$

direct and clear connection of any ground state isospin sensitive observable with the parameters of the EoS.

Following the same example: $\Delta r_{np}^{\text{total}}(A, I) = \Delta r_{np}^{\text{bulk}}(A, I) + \Delta r_{np}^{\text{surface}}(A, I)$

$$\Delta r_{np}^{\text{bulk}}(A, I) \approx \frac{2r_0}{3J} L \left(1 - \epsilon_A K_{\text{sym}} \frac{2L}{2L}\right) \epsilon_A A^{1/3} (I - I_C)$$

Motivation:

Observables, processes and observations known to be correlated with the isovector properties of the nuclear effective interaction

- **Binding energies**
- **Neutron distributions** (proton elastic scattering, antiprotonic atoms, parity violating asymmetry,...)
- **Heavy Ion Collisions** (EoS — transport models)
- **Neutron Star properties**: mass-radius relation, transition density crust-core, composition,... (observational data).
- Low-energy dipole response (?)
- Isovector GQR [see PRC 87, 034301 (2013)!]
- Isoscalar Giant Resonances along isotopic chains (?)
- ...
Parity violating elastic electron scattering in $^{48}\text{Ca}$ and $^{208}\text{Pb}$
From previous talks, we have seen that,

- **Electrons** interact by exchanging a $\gamma$ or a $Z_0$ boson.
- While **protons** couple basically to $\gamma$, **neutrons** do it to $Z_0$.
- **Ultra-relativistic electrons**, depending on their helicity, interact with the nucleons $V_{\pm} = V_{\text{Coulomb}} \pm V_{\text{Weak}}$.
- **Ultra-relativistic electrons** moving under the effect of $V_{\pm}$ where **Coulomb distortions** are important $\Rightarrow$ solution of the Dirac equation via the Distorted Wave Born Approximation (DWBA).
- **Input for the calculation:** $\rho_n$ and $\rho_p$ ... and nucleon form factors for the e-m and the weak neutral current...

**Refs:**
**PREx and CREx measure:** model-independently the *parity* violating asymmetry,

\[ A_{pv} = \frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega} \]

at 1.06 GeV and for a single angle (\(\sim 5\) deg.) in \(^{208}\text{Pb}\) and at 2.20 GeV and for a single angle (\(\sim 4\) deg.) in \(^{48}\text{Ca}\)

\(\rho_n\) of \(^{208}\text{Pb}\) and \(^{48}\text{Ca}\) are the quantities to be determined, a precise determination of \(\Delta r_{np}\) would constrain the density dependence of the symmetry energy around saturation.
Qualitatively,

$A_{pv}$ within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi \alpha \sqrt{2}} \left[ 4 \sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right]$$

... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$-\frac{q^2}{6} \left( \langle r_n^2 \rangle - \langle r_p^2 \rangle \right) = -\frac{q^2}{6} \left[ \Delta r_{np} (\langle r_n^2 \rangle^{1/2} + \langle r_p^2 \rangle^{1/2}) \right]$$

$$= -\frac{q^2}{6} \left( 2\langle r_p^2 \rangle^{1/2} \Delta r_{np} + \Delta r_{np}^2 \right)$$

variation of $A_{pv}$ at a fixed $q$ dominated by the variation of $\Delta r_{np}$. $F_p(q)$ well fixed by experiment.
**Pb: direct correlations**

DWBA; no radiative corrections or strange quark effects included


**MF correlations allows to determine $\Delta r_{np}$ and $L$ without direct assumptions on $\rho$, PREx-II and PV-RAPTOR expected accuracy $\rightarrow$ constrain on $L$**

Different experiments on proton elastic scattering and antiprotonic atoms agrees with the correlation
48Ca: direct correlations within MF including radiative corrections and strange quark effects

$A_{pv}$ decreases by around 0.005 ppm with an error of about 0.01 - 0.02 ppm when $G_E^s(Q^2)$ is included.

Used $G_E^s(Q^2)$ from PRC 76, 025202 (2007) by Liu, McKeown, and Ramsey-Musolf

In the two tested models, spin-orbit effects shifts to lower values the $A_{pv}$ consistently by about 0.07 ppm. This predicts a reduction of $\Delta r_{np}$ of about 0.05 fm.

Charge density distributions including spin orbit effects provided by J. Piekarewicz (FSU).
**48Ca: Estimation of three-neutron forces effects in comparison with other corrections**

Shell Model calculations based on $\chi$EFT with NN to N3LO (fixed to scattering data) and 3N to N2LO (fixed to $B$ tritium and $R$ of alpha particle) **provided by J. Menendez (TU Darmstadt).**

Three-neutron forces used here shifts downwards the $A_{p\nu}$ by about **0.05 ppm** (very similar to spin-orbit effect)
Isovector static dipole polarizability
**Definition:** $\alpha_D$

- The linear response or dynamic polarizability of a nuclear system excited from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the action of an external oscillating dipolar field of the form $(Fe^{iwt} + F^\dagger e^{-iwt})$: 

\[
F_D = \frac{Z}{A} \sum_i^N r_n Y_{1M}(\hat{r}_n) - \frac{N}{A} \sum_i^Z r_p Y_{1M}(\hat{r}_p)
\]

- is proportional to the **static dipole polarizability**, $\alpha_D$, for small oscillations

\[
\alpha_D = \frac{8\pi}{9} e^2 m_{-1} = \frac{8\pi}{9} e^2 \sum_{\nu} \frac{|\langle \nu | F_D | 0 \rangle|^2}{E}
\]

where $m_{-1}$ is the inverse energy weighted moment of the strength function,

\[
S_D(E) = \sum_{\nu} |\langle \nu | F_D | 0 \rangle|^2 \delta(E - E_\nu)
\]
Mean-Field + RPA results for $^{208}\text{Pb}$

J. Piekarewicz, B. K. Agrawal, G. Colò, W. Nazarewicz, N. Paar, P.-G. Reinhard, X. Roca-Maza and D. Vretenar,

Mean-Field + RPA results for $^{48}\text{Ca}$

$$\Delta r_{np} \text{ (fm)}$$

$$\alpha_D \text{ (fm}^3\text{)}$$

Data on relativistic models provided by N. Paar and D. Vretenar
Conclusions:

- A precise and **model-independent** determination of $\Delta r_{np}$ in $^{48}$Ca and $^{208}$Pb via PVES experiments would **probe** at the same time the density dependence of the nuclear **symmetry energy** and the relevance of **three neutron-forces** in $^{48}$Ca. Eventually, it can also provide indirect indications on the impact of 3N in $^{208}$Pb.

- We demonstrate a close **linear correlation** between $A_{pv}$ and $\Delta r_{np}$ within the same framework in which the $\Delta r_{np}$ is correlated with $L$.

- Other **experiments** fairly **agree** with the **correlation** between $A_{pv}$ and $\Delta r_{np}$.
Conclusions:

- The estimated corrections to the
  \[ A_{pv} \approx A_{pv}^0 \times [1 - 0.005 \text{(strange)} - 0.03(s-o)] \]
  where \( A_{pv}^0 \) is the result from DWBA calculations with a given neutron and proton density distributions convoluted with experimental electromagnetic form factors and weak neutral current form factors including radiative corrections, **indicate a reduction of about the 3%**.

- In addition, the inclusion of **3N-forces** would change the neutron density producing a **reduction in** \( A_{pv}^0 \) of a few %.
Conclusions:

- Families of modern energy density functionals show an almost linear correlation between $\alpha_D$ and $\Delta r_{np}$ while the correlation gets worst when models based on different grounds are also taken into account.

- $A_{pv}$ and $\alpha_D$ are complementary observables that may set tight constraints on the density dependence of the symmetry energy.
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