Probing the Nuclear Symmetry Energy and Neutron Skin from Collective Excitations

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What are relationships between the properties of collective modes of excitation, symmetry energy and neutron skin thickness? Which dynamic observables constrain the neutron distribution and neutron skin thickness in finite nuclei?

Applications of dipole excitations and antianalog giant dipole resonance, based on recent experimental data and relativistic nuclear energy density functional, in constraining the neutron-skin thickness in nuclei, nuclear matter symmetry energy at saturation density and slope of the symmetry energy.

 Assessing statistical correlations by means of covariance analysis involving response properties of nuclei, nuclear matter properties, and ground state properties. Theoretical uncertainties in modeling various observables.

The theory framework is the relativistic nuclear energy density functional, which provides a unified microscopic description of the structure of finite nuclei. In the limit of small amplitude vibrations, this framework enables a fully self-consistent relativistic quasiparticle RPA (RQRPA) to analyze giant resonances, low-energy multipole response in weakly-bound nuclei, dynamics of exotic modes of excitation, weak interaction rates, neutrino-nucleus reactions, etc.

In order to explore the evolution of the excitation spectra as a function of the density dependence of the symmetry energy, a set of interactions is used, that span a broad range of values for the symmetry energy at saturation density ($J$) and the slope parameter ($L$).

Nuclear matter energy per part.:

$$E(\rho, \alpha) = E(\rho, 0) + S_2(\rho) \alpha^2 + \ldots$$

$$\alpha = (N - Z)/A$$

Symmetry energy term:

$$S_2(\rho) = J \cdot L\epsilon + \ldots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{dr} \bigg|_{\rho_0}$$
Constraining the symmetry energy from dipole polarizability

- Theoretical constraints on the symmetry energy at saturation density ($J$) and slope of the symmetry energy ($L$) from dipole polarizability ($\alpha_D$) using relativistic nuclear energy density functionals

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$

- Exp. data from polarized proton inelastic scattering, $\alpha_D = 18.9(13) \text{ fm}^3/e^2$
  A. Tamii et al., PRL. 107, 062502 (2011)

\[ \begin{align*}
J &= (32.6 \pm 1.4) \text{ MeV} \\
L &= (50.9 \pm 12.6) \text{ MeV}
\end{align*} \]
The pygmy dipole strength for $^{68}\text{Ni}$ and comparison to experimental data

- $\gamma$ decay from Coulomb excitation of $^{68}\text{Ni}$ at 600 MeV/nucleon (INFN,GSI,...)

<table>
<thead>
<tr>
<th>$S_{\text{EW}}(E1)$</th>
<th>% EWSR (TRK)</th>
<th>$B(E1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[e^2\text{fm}^2\text{MeV}]$</td>
<td>(TRK)</td>
<td>$[e^2\text{fm}^2]$</td>
</tr>
<tr>
<td>RNEDF</td>
<td>14.4</td>
<td>5.9 %</td>
</tr>
<tr>
<td>Exp.</td>
<td>$12.2 \pm 3.7$</td>
<td>$(5\pm1.5)%$</td>
</tr>
</tbody>
</table>

9.05 MeV: coherent neutron transitions:
- $2p3/2 \rightarrow 3s1/2$
- $2p1/2 \rightarrow 2d3/2$
- $1f5/2 \rightarrow 2d3/2$
- $2p3/2, 2p1/2, 1f5/2$: 12 excess neutrons above N=Z core

EXCESS NEUTRONS OSCILLATION MODE

O. Wieland et al., PRL 102, 092502 (2009)
Various constraints from dipole excitations on the slope of the symmetry energy (L) and symmetry energy at saturation density (J)

- Energy weighted pygmy dipole strength in $^{68}$Ni, $^{132}$Sn, $^{208}$Pb
- Dipole polarizability in $^{208}$Pb
- PDR/GDR B(E1) strength in $^{132}$Sn, $^{130}$Sn (Klimkiewicz et al. 2007)

Based on exp. data on PDR & $\alpha_D$

$^{68}$Ni: O. Wieland et al, PRL. 102, 092502 (2009)
$^{132,130}$Sn: A. Klimkiewicz et al., PRC 76, 051603 (R) (2007)
$^{208}$Pb: I. Poltoratska et al., PRC 85, 041304 (R) (2012)
$^{208}$Pb ($\alpha_D$): A. Tamii et al., PRL 107, 062502 (2011)
Constraining the symmetry energy at saturation density and slope of the symmetry energy from various approaches:

Also see M. B. Tsang et al., PRC 86, 015803 (2012)
Pierson product-moment correlation

\[ c_{AB} = \frac{|\Delta A \Delta B|}{\sqrt{\Delta A^2 \Delta B^2}} \]

Talks W. Nazarewicz
J. Piekarewicz

**Relativistic functionals:**
- Relativistic point coupling model (PC-min1)
- Relativistic model with density dependent meson-nucleon couplings (DDME-min1)

- The model parameters (9) of both interactions are constrained by the same set of (17) nuclei and their properties: binding energies, charge radii, diffraction radii, surface thickness (N.P.)

- (for relativistic EDFs – FSUGold also see talk by J. Piekarewicz)

**Non-relativistic functionals (Skyrme):**
- SLy5 (X. Roca-Maza, G. Colò)

- SV-min
  - (Skyrme type parameterization embraces nuclear bulk properties for selected semimagic nuclei)
  - (P.-G. Reinhard, W. Nazarewicz)
Covariance analysis allows calculation of theoretical uncertainty of any physical quantity of interest. E.g., proton and neutron distribution radii ($r_p, r_n$), neutron skin thickness ($r_{np}$).

<table>
<thead>
<tr>
<th></th>
<th>$r_p$ (fm)</th>
<th>$r_n$ (fm)</th>
<th>$r_{np}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV-min</td>
<td>5.444 ± 0.005</td>
<td>5.614 ± 0.039</td>
<td>0.170 ± 0.036</td>
</tr>
<tr>
<td>FSUGold</td>
<td>5.469 ± 0.035</td>
<td>5.676 ± 0.041</td>
<td>0.207 ± 0.037</td>
</tr>
<tr>
<td>DDME-min1</td>
<td>5.523 ± 0.003</td>
<td>5.658 ± 0.041</td>
<td>0.194 ± 0.040</td>
</tr>
</tbody>
</table>

(P.G.Reinhard)  
(J. Piekarewicz)  
(N.P.)

<table>
<thead>
<tr>
<th></th>
<th>$r_{np}$ (fm) (EXP.)</th>
</tr>
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<tbody>
<tr>
<td>PREX</td>
<td>0.33 ± 0.17</td>
</tr>
<tr>
<td>(p,p)</td>
<td>0.156 ± 0.025</td>
</tr>
<tr>
<td>(α,α’)</td>
<td>0.12 ± 0.07</td>
</tr>
<tr>
<td>Antiproton abs.</td>
<td>0.18 ± 0.03</td>
</tr>
</tbody>
</table>

S. Abrahamyan et al., PRL. 108, 112502 (2012).  
A. Tamii et al., PRL 107, 062502 (2011).  
Pierson product-moment correlation coefficient for various dynamic quantities of dipole excitations versus neutron skin thickness ($r_{np}$):

Strength-related quantities ($B(E1)$, energy weighted strength (EWS), $\alpha_D$) are better correlated with $r_{np}$ than excitation energies.

Both the PDR and overall strength properties are correlated with the neutron skin thickness.
CORRELATIONS WITH THE SYMMETRY ENERGY AT SATURATION DENSITY (J)

- SV-min
- SLy5
- DDME -min1
- PC-min1

- J - rnp
- J - αD
- J - PDR
- J - GDR
CORRELATIONS WITH DIPOLE POLARIZABILITY

- $\alpha_D - J$
- $\alpha_D - r_{np}$
- $\alpha_D - K$
- $\alpha_D - GDR$
CORRELATIONS WITH THE PYGMY DIPOLE STRENGTH

SV-min
SLy5
DDME-min1
PC-min1

PDR - J

SV-min
SLy5
DDME-min1
PC-min1

PDR - GDR

SV-min
SLy5
DDME-min1
PC-min1

PDR - K

SV-min
SLy5
DDME-min1
PC-min1

PDR - r_{np}
Weak charge form factors ($^{48}$Ca) reasonably correlate with the neutron radii ($^{208}$Pb), and neutron skin thicknesses ($^{48}$Ca, $^{132}$Sn, $^{208}$Pb).
ANTI-ANALOG GDR AND NEUTRON-SKIN THICKNESS

\[ T = T_0 - 1 \]
\[ \Delta L = 1 \]
\[ 1^-, T_0^- \]
\[ 0^-, 1^-, 2^-, T_0^- \]
\[ \Delta L = 1, \Delta S = 1 \]
\[ 0^+, T_0^- \]
\[ \Delta S = 1 \]
\[ 1^+, T_0^- \]

Strong \((p,n)\)

\[ T_Z = T_0 - 1 \]

Daughter nucleus

\[ T_Z = T_0 \]

Target nucleus

\[ 0^+, T_0 \]

\[ GTR \]

\[ IAS \]

\[ E1 \]

\[ T = T_0 \]

\[ \text{Sn isotopes} \]

\[ E_{\text{DR}} - E_{\text{IAS}} \text{ (MeV)} \]

\[ \text{Mass number (A)} \]

\[ \text{AGDR} (S^*-S^+) \]

\[ \text{DDME-min1} \]

\[ r_{np} \]
TEST CASE: $^{124}$Sn

ANTI-ANALOG GDR AND NEUTRON-SKIN THICKNESS

METHOD | $\Delta R_{pn}$ (fm)  
--- | ---  
(p,p) 0.8 GeV | 0.25 ± 0.05  
($\alpha,\alpha'$) IVGDR 120 MeV | 0.21 ± 0.11  
Antiproton absorption | 0.19 ± 0.09  
($^3$He,t) IVSGDR | 0.27 ± 0.07  
Pygmy dipole resonance | 0.19 ± 0.05  
(p,p) 295 MeV | 0.185 ± 0.05  
AGDR - present result | 0.21 ± 0.05

• Dipole excitations, charge exchange excitations, parity violating electron scattering (PREX II, CREX) provide valuable constraints on the neutron skin thickness and nuclear matter symmetry energy.

• Statistical covariance analysis based on various effective interactions (SV-min, SLy5, PC-min1, DDME-min1) indicates important correlations between various quantities and observables and provides the insight into the features of model dependence.

• Possible extensions and improvements of the isovector part of the EDFs? E.g. point coupling interactions with four isovector terms (isovector-vector + isovector-scalar, 4 parameters), revised exp. data sets?

• Constraining theoretical models using PREX, CREX, (p,p), (p,n) data?
The pygmy dipole mode provides possible way to constrain the nuclear matter properties and neutron skin thickness in finite nuclei. Why?

1) PDR is dominated by neutron transitions, its transition densities at the surface are dominated by neutrons.

2) PDR strength is strongly sensitive on the neutron excess (more than GDR)

3) Covariance analysis with PC-min1, DDME-min1, SLy5 effective interactions indicates correlation between the PDR transition strength and symmetry energy at saturation density:

J. Piekarewicz, PRC 83, 034319 (2011)
What can we learn from comparison between the isovector and isoscalar dipole transition strength in neutron rich nuclei?

Both in the isoscalar and isovector channel, pronounced low-energy dipole transition strengths are peaked at exactly the same energy. Their structure is dominated by identical neutron transitions with similar relative contributions in the transition strength (apart from the decoherence in the isovector channel).
1. Relativistic point coupling model

\[ \mathcal{L} = \mathcal{L}^\text{free} + \mathcal{L}^\text{4f} + \mathcal{L}^\text{hot} + \mathcal{L}^\text{der} + \mathcal{L}^\text{em}, \]

\[ \mathcal{L}^\text{free} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi, \]

\[ \mathcal{L}^\text{4f} = -\frac{1}{2} \alpha_S(\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi) - \frac{1}{2} \alpha_V(\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi) - \frac{1}{2} \alpha_{TS}(\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi), \]

\[ \mathcal{L}^\text{hot} = -\frac{1}{3} \beta_S(\bar{\psi}\psi)^3 - \frac{1}{4} \gamma_S(\bar{\psi}\psi)^4 - \frac{1}{4} \gamma_V[(\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi)]^2, \]

\[ \mathcal{L}^\text{der} = -\frac{1}{2} \delta_S(\partial_\mu \bar{\psi}\psi)(\partial^\nu \bar{\psi}\psi) - \frac{1}{2} \delta_V(\partial_\mu \bar{\psi}\gamma_\mu \psi)(\partial^\nu \bar{\psi}\gamma_\mu \psi) - \frac{1}{2} \delta_{TS}(\partial_\mu \bar{\psi}\gamma_\mu \psi)(\partial^\nu \bar{\psi}\gamma_\mu \psi), \]

\[ \mathcal{L}^\text{em} = -eA_\mu \bar{\psi}[(1 - \tau_3)/2] \gamma^\mu \psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}. \]
2. Relativistic model with density-dependent meson-nucleon couplings

\[
\mathcal{L} = \bar{\psi} \left( i \gamma \cdot \partial - m \right) \psi + \frac{1}{2} (\partial \sigma)^2 - \frac{1}{2} m_\sigma \sigma^2
- \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} + \frac{1}{2} m_\omega \omega^2 - \frac{1}{4} R_{\mu \nu} R^{\mu \nu} + \frac{1}{2} m_\rho \rho^2
- \frac{1}{4} F_{\mu \nu} F^{\mu \nu}
- g_\sigma \bar{\psi} \sigma \psi
- g_\omega \bar{\psi} \gamma \cdot \omega \psi
- g_\rho \bar{\psi} \gamma \cdot \rho \tau \psi
- e \bar{\psi} \gamma \cdot A \frac{(1 - \tau_3)}{2} \psi
\]

The density dependence of the vertex functions

\[
g_i(\rho) = g_i(\rho_{sat}) f_i(x) \quad \text{for} \quad i = \sigma, \omega
\]

\[
f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}
\]

\[
g_\rho(\rho) = g_\rho(\rho_{sat}) \exp \left[ -a_\rho(x - 1) \right]
\]