Microscopic Calculations of Neutron Matter

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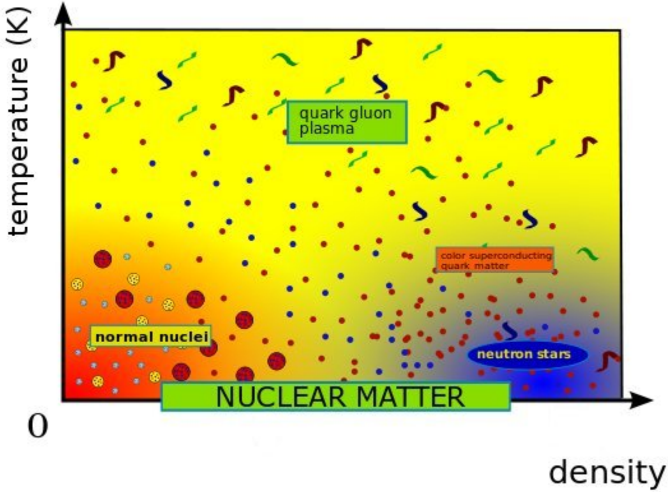
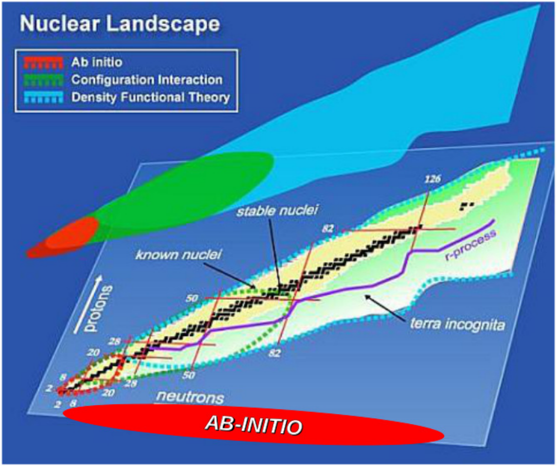
Homogeneous neutron matter



Inhomogeneous neutron matter



W. Nazarewicz – UNEDF

Outline

- The model and the method
- **Homogeneous neutron matter**
  - Three-neutron force and the equation of state of neutron matter
  - Symmetry energy
  - Neutron star structure
- **Neutron stars observations**
- Conclusions

Quantum Monte Carlo

Evolution of Schrodinger equation in imaginary time **t**:

$$\psi(R,t) = e^{-(H-E_T)t}\psi(R,0)$$

In the limit of $t \to \infty$ it approaches to the lowest energy eigenstate (not orthogonal to $\psi(R,0)$).

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR' G(R,R',t)\psi(R',0)$$

$G(R,R',t)$ is an approximate propagator (small-time limit). We iterate the above integral equation many times in the small time-step limit.
→ parallel codes and supercomputers.

For a given microscopic Hamiltonian, this method solves the ground–state within a systematic uncertainty of **1–2%** in a **non-perturbative way.**

Nuclear Hamiltonian

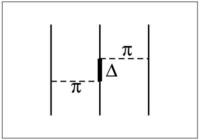
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

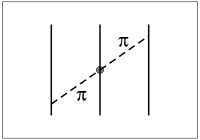
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

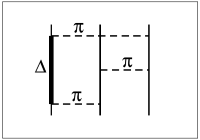
v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

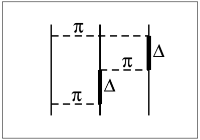
$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}) \,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

Urbana–Illinois V_{ijk} models processes like





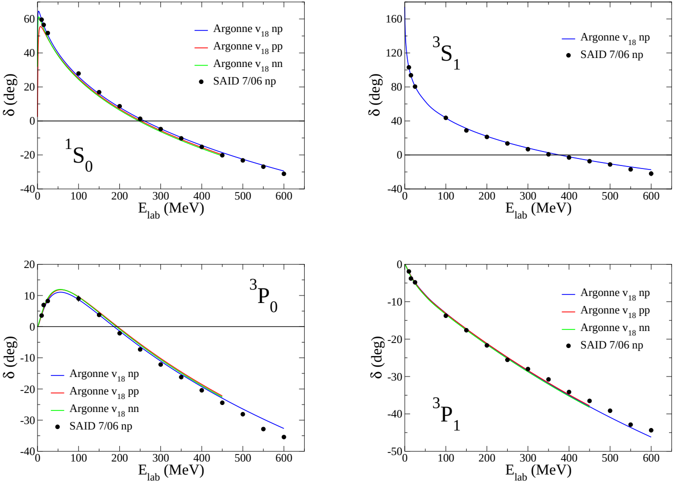




$+$ Phenomenological term.

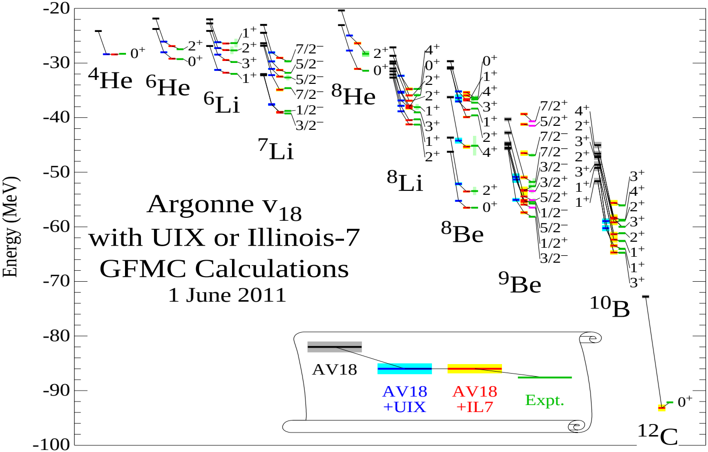
Nuclear Hamiltonian

Nucleon-Nucleon interaction fit scattering data:



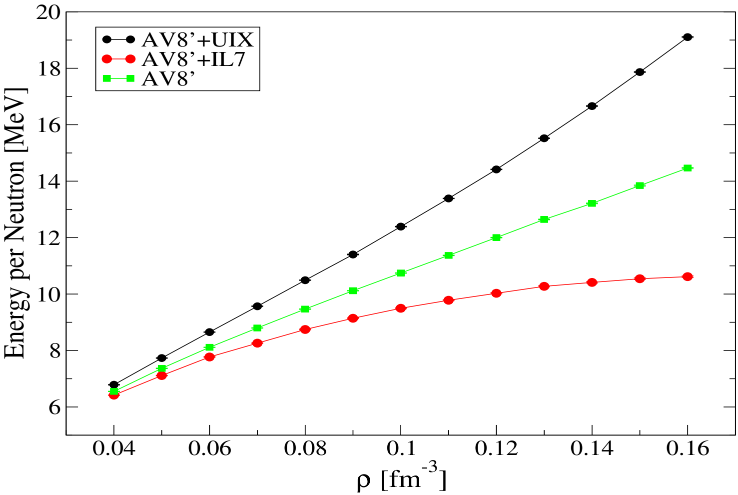
Wiringa, Stoks, Schiavilla (1995)

Light nuclei spectrum computed with GFMC



Carlson, Pieper, Wiringa, many papers

Neutron matter and the puzzle of the three-body force



Note: AV8'+UIX and (almost) AV8' are stiff enough to support observed neutron stars. → How to reconcile with nuclei???

Neutron matter

Assumptions:

- The two-nucleon interaction reproduces well (elastic) *pp*, *np* and *nn* scattering data up to high energies ($E_{lab} \sim 600$MeV).
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part (but zero in neutron matter). **Difficult to study in light nuclei.**

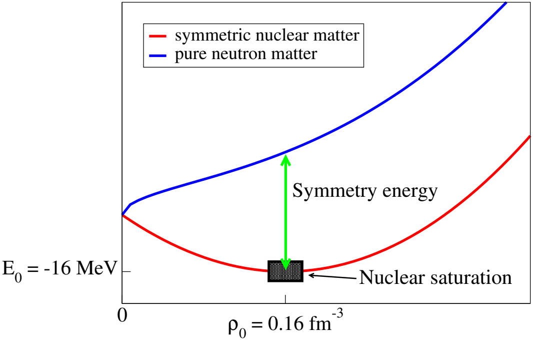
Symmetry energy

Nuclear matter EOS:

$$E(\rho, x) = E_{SNM}(\rho) + E^{(2)}_{sym}(\rho)(1-2x)^2 + \cdots$$

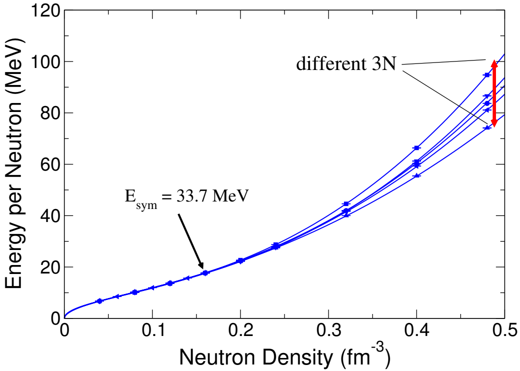
where

$$\rho = \rho_n + \rho_p \,, \quad x = \frac{\rho_p}{\rho}$$



Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.

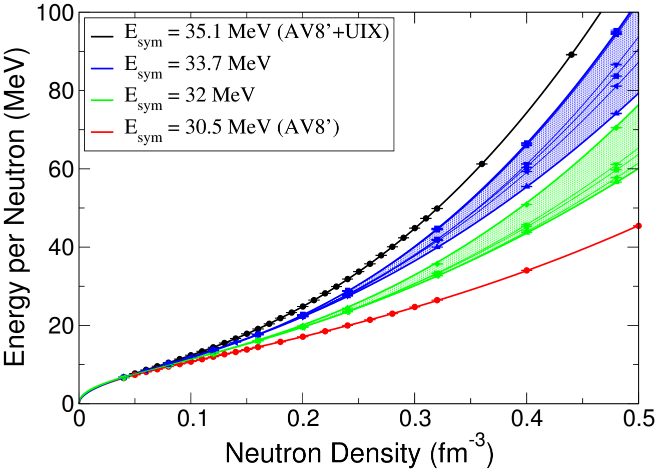


different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Neutron matter and symmetry energy

We then try to change the neutron matter energy at saturation:

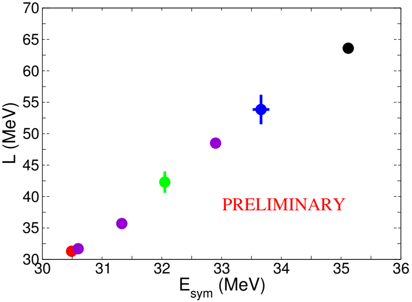


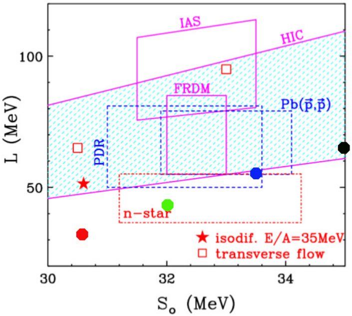
Gandolfi, Carlson, Reddy, PRC (2012).

Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



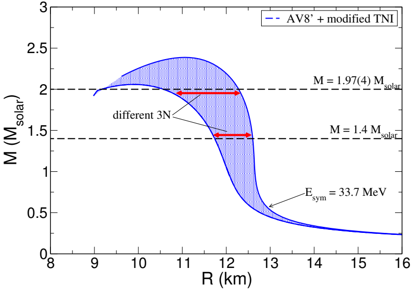


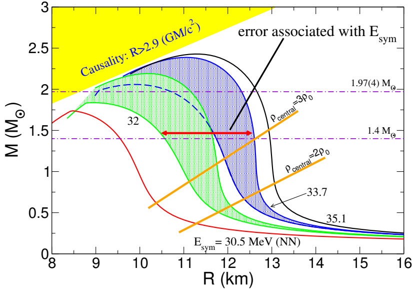
Tsang *et al.*, PRC (2012)

Very weak dependence to the model of 3N force for a given E_{sym}.
Role of NN will be investigated next.

Neutron star structure

EOS used to solve the TOV equations.



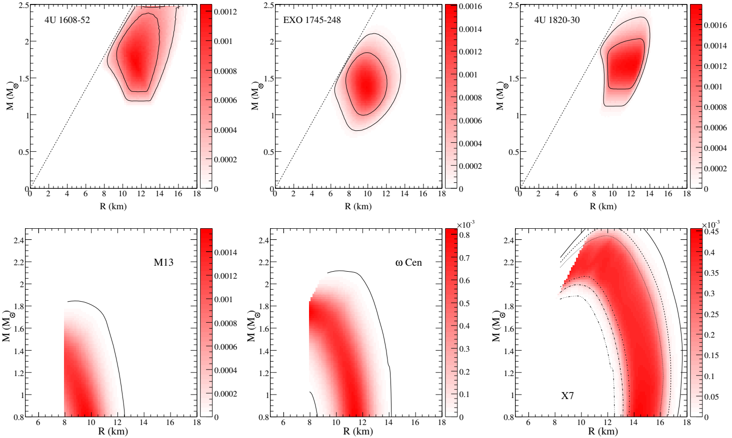


Accurate measurement of E_{sym} would put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

$M = 1.97\ M_{solar}$ observed – Demorest et al., Nature (2010).

Neutron stars

Observations of the mass-radius relation are available:



Steiner, Lattimer, Brown, ApJ (2010)

We can use neutron star observations to 'measure' the EOS and constrain E_{sym} and L.

Neutron star matter

We model neutron star matter as

$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta} , \qquad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model, Alford et al., ApJ (2005).

By changing ρ_t and the high density model we can understand systematic errors in E_{NSM} parametrization.

We also add a correction to account for the proton fraction present in neutron stars.

Observations

What can we learn by fitting our model to observations?

- Symmetry energy and its slope:

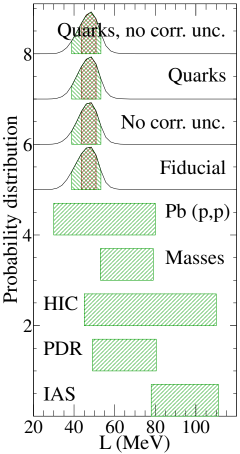
$$E_{sym} = a + b + 16\,, \qquad L = 3(a\alpha + b\beta)$$

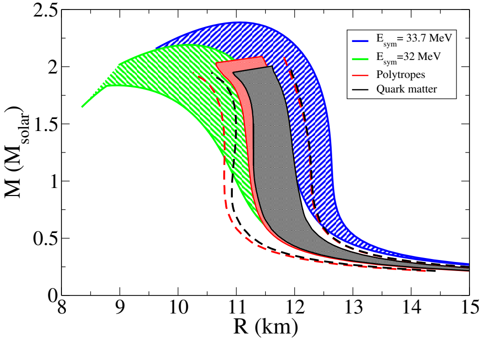
- Strength of 3N:

3N force	E_{sym} (MeV)	L (MeV)	a (MeV)	α	b (MeV)	β
none	30.5	31.3	12.7	0.49	1.78	2.26
$V_{2\pi} + V^R_{\mu=300}$	32.0	40.6	12.8	0.488	3.19	2.20
$V_{2\pi} + V^R_{\mu=600}$	32.0	41.3	12.8	0.488	3.19	2.20
$V_{2\pi} + V_R$	32.1	41.3	12.7	0.476	3.34	2.22
$V_{3\pi} + V_R$	32.0	44.0	13.0	0.49	3.21	2.47
$V_{2\pi} + V_R$	33.7	52.9	13.3	0.512	4.38	2.39
$V_{3\pi} + V_R$	33.8	56.2	13.0	0.50	4.71	2.49
UIX	35.1	63.6	13.4	0.514	5.62	2.436

Note: a and α don't depend too much to the model of 3N!

Neutron star observations





$32 < E_{sym} < 34\,MeV,\ 43 < L < 52\,MeV$

Steiner, Gandolfi, PRL (2012).

Conclusions

QMC used to study neutron matter:

- Effect of three–neutron forces to high-density neutron matter; the systematic uncertainty due to 3N is relatively small.
- E_{sym} strongly constrain L. Weak dependence to the model of 3N.
- Uncertainty of the radius of neutron stars mainly due E_{sym} rather than 3N.
- Neutron star observations becoming competitive with terrestrial experiments.

Thanks for the attention