Pion Loop Corrections to Parton Distribution Functions*

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Outline

- **Brief Motivation**
- **Nonanalytic Behavior of Pion Loop Corrections**
- **PV vs. PS Coupling**
- **Resolution of Discrepancies**
- **Summary and Outlook**

PDFs using ChPT

- *Motivation*: can one understand flavor asymmetries in the nucleon ($e.g.\ \bar{d} - \bar{u}$) from QCD?

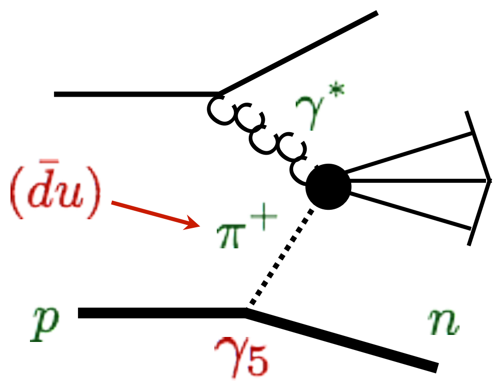
$\longrightarrow$ origin of 5-quark Fock components $|qqq\,\bar{q}q\rangle$ of nucleon

$$\int_0^1 dx\ (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

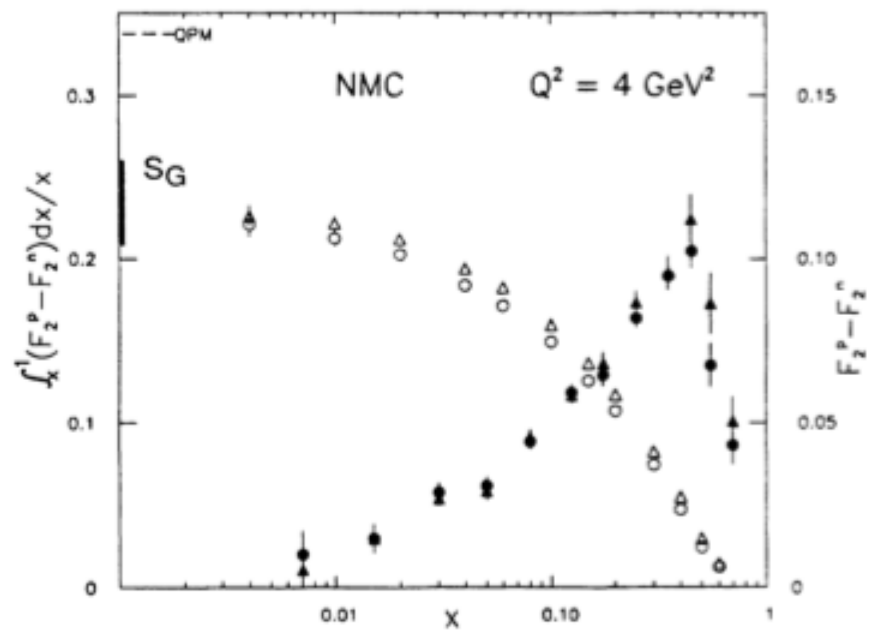
E866 (Fermilab), PRD ***64****, 052002 (2001)*

Sullivan process

$\bar{d} > \bar{u}$ $\longleftrightarrow$ $\pi^+ n > \pi^- \Delta^{++}$



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u}) \;= 0.235(26)$$

New Muon Collaboration, PRD ***50****, 1 (1994)*

Connection with QCD

- $(\bar{d}-\bar{u})(x) = \frac{2}{3}\int_x^1 \frac{dy}{y}\, f_\pi(y)\, \bar{q}^\pi(x/y)$

$$f_\pi(y) = \frac{3g_{\pi NN}^2}{16\pi^2}\, y \int dt \frac{-t\, \mathcal{F}_{\pi NN}^2(t)}{(t-m_\pi^2)^2}$$

→ *model-independent* leading nonanalytic (LNA) behavior *consistent with Chiral Symmetry of QCD*.

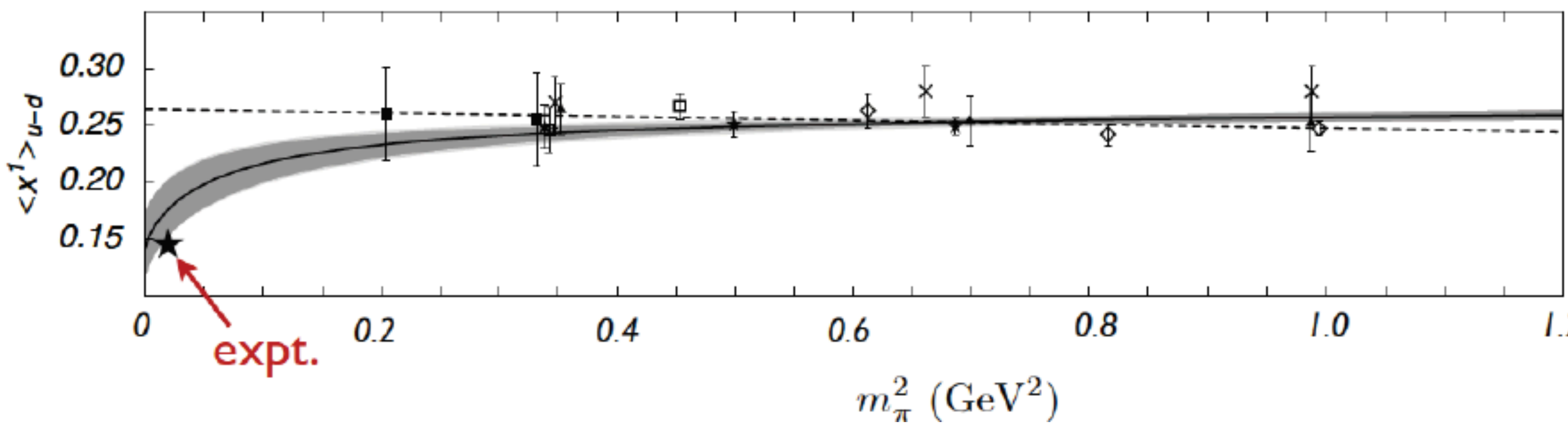
$$\langle x^0 \rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u})$$

$$m_\pi^2 f_\pi^2 = -2m_q < \bar{q}q >$$

$$= \frac{2}{3}\int_0^1 dy\, f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \ + \ \text{analytic terms}$$

- Nonanalytic behavior vital for chiral extrapolation of lattice data

Thomas, Melnitchouk, Steffens PRL ***85****, 2892 (2000)*



- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

cf. $4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$

*Chen, X. Ji, PLB **523**, 107 (2001)*
*Arndt, Savage, NPA **692**, 429 (2002)*

*Thomas,Melnitchouk,Steffens,PRL **85**, 2892 (2000)*

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

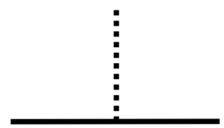
→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

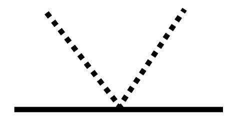
C. Ji, W. Melnitchouk & A. Thomas, PRD80, 054018 (2009); PRL110, 179101 (2013); PRD88, 076005 (2013)
M. Burkardt, K. Hendricks, C. Ji, W. Melnitchouk & A. Thomas, PRD87, 056009 (2013)

πN Lagrangian

- Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau}\cdot\partial_\mu \vec{\pi}\,\psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$





$g_A = 1.267$

$f_\pi = 93$ MeV

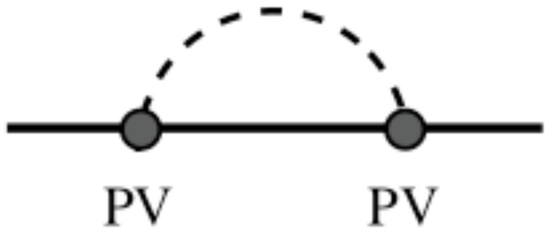
⟶ lowest order approximation of chiral perturbation theory Lagrangian

- Pseudoscalar Lagrangian (à la "Sullivan")

$$\mathcal{L}^{\rm PS}_{\pi N} = -g_{\pi NN}\,\bar{\psi}_N\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi_N$$

$$\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M}$$ (Goldberger-Treiman Relation)

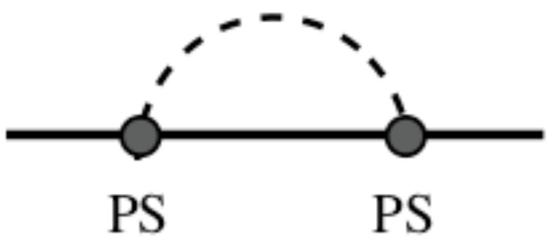
πN Interaction with PS vs. PV Coupling
(No tad pole in self-energy)



$$\Sigma^{PV} = \frac{1}{2}\sum_s \bar{u}(p,s)\,\hat{\Sigma}^{PV}\,u(p,s)$$

$$\hat{\Sigma}^{PV} = -i\left(\frac{2g_A}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}\gamma_5(\not{p}-\not{k}+M)\gamma_5\not{k}}{D_\pi D_N}$$

$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$



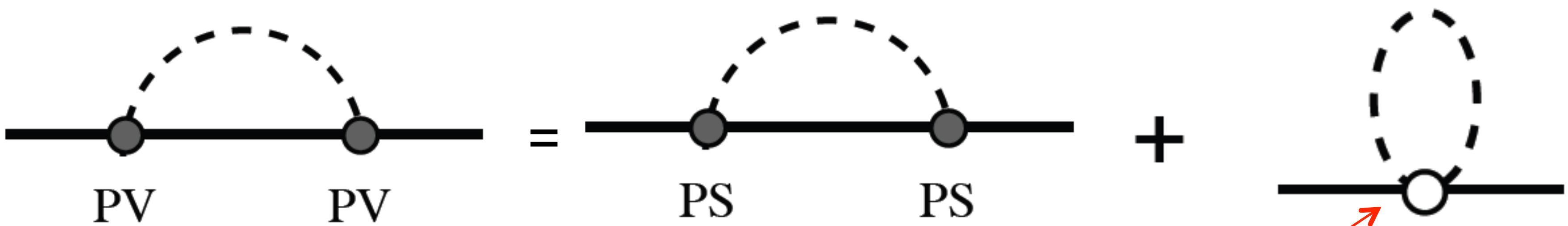
$$\Sigma^{PS} = \frac{1}{2}\sum_s \bar{u}(p,s)\,\hat{\Sigma}^{PS}\,u(p,s)$$

$$\hat{\Sigma}^{PS} = -i\left(\frac{g_A M}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int \frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not{p}-\not{k}+M)\gamma_5}{D_\pi D_N}$$

Note the relation between PV and PS coupling amplitudes.

$$\bar{u}(p)\not{k}\gamma^5 \frac{1}{\not{p}-\not{k}-M}\gamma^5\not{k}\,u(p) = \bar{u}(p)[\not{k}-\not{p}+M]\gamma^5 \frac{1}{\not{p}-\not{k}-M}\gamma^5[\not{k}-\not{p}+M]u(p)$$

$$= 4M^2\,\bar{u}(p)\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5 u(p) + 2M\,\bar{u}(p)u(p) + \bar{u}(p)\not{k}u(p)$$



"treacherous" $k^+ = 0$ (end-point) term

$$\frac{1}{\not{p}-\not{k}-M} = \frac{\sum_s u(p-k,s)\bar{u}(p-k,s)}{(p-k)^2-M^2} + \frac{\gamma^+}{2(p^+-k^+)}$$

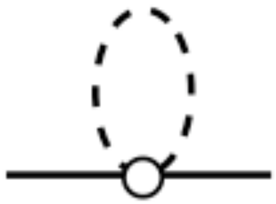
Chang & Yan, PRD **7**, *1147 (1973)*

→ $(\Sigma^{\mathrm{PS}} = \Sigma^{\mathrm{PV}})_{\text{on-energy-shell}}$: *Alberg & Miller, PRL* **108**, *172001 (2012)*

→ LNA behavior of the end-point term is intact even with the form factors.

Ji, Melnitchouk, Thomas, PRL **110**, *179191 (2013)*

Treacherous Amplitudes in LFD



$$I = \int d^2k \frac{1}{k^2 - m^2 + i\varepsilon}$$

Manifestly Covariant Calculation

$$I = -i\pi\left(\frac{2}{2-n} - \log\pi - \gamma + O(2-n) - \log\frac{m^2}{\mu^2}\right)_{n\to 2}$$

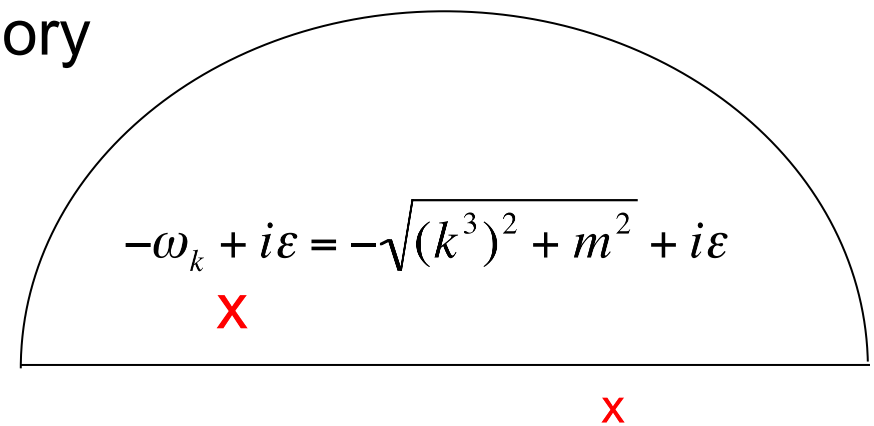
$$I_{LNA} = i\pi \log m^2$$

Time-Ordered Perturbation Theory

$$I = \int dk^3 dk^0 \frac{1}{(k^0 + \omega_k - i\varepsilon)(k^0 - \omega_k + i\varepsilon)}$$

$$= -2i\pi \int_0^{\Lambda\to\infty} dk^3 \frac{1}{\sqrt{(k^3)^2 + m^2}}$$

$$= -2i\pi \log\left(\frac{2\Lambda}{m}\right)_{\Lambda\to\infty}$$

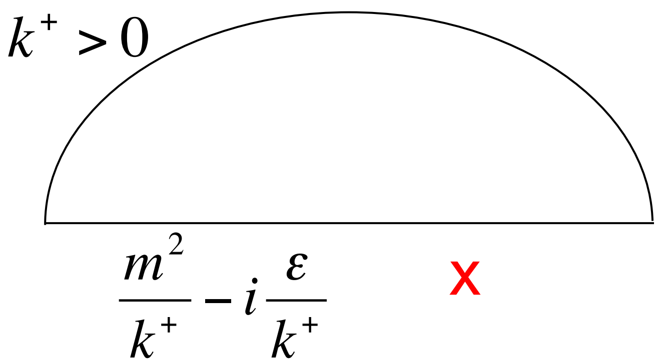


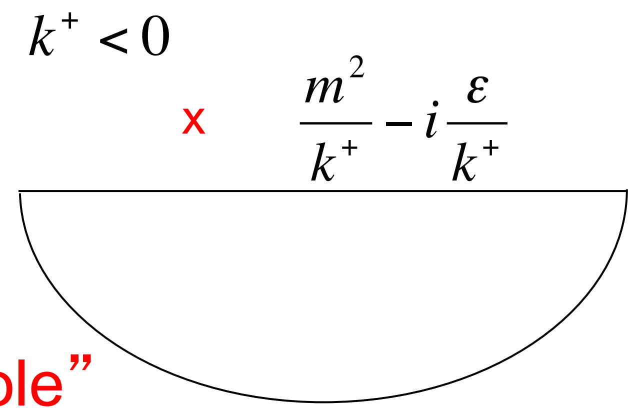
$$-\omega_k + i\varepsilon = -\sqrt{(k^3)^2 + m^2} + i\varepsilon$$

$$\omega_k - i\varepsilon = \sqrt{(k^3)^2 + m^2} - i\varepsilon$$

LFD

$$I = \frac{1}{2}\int dk^+ dk^- \frac{1}{k^+k^- - m^2 + i\varepsilon} = \frac{1}{2}\int \frac{dk^+}{k^+}\int dk^- \frac{1}{k^- - \frac{m^2}{k^+} + i\frac{\varepsilon}{k^+}}$$





``Moving Pole”

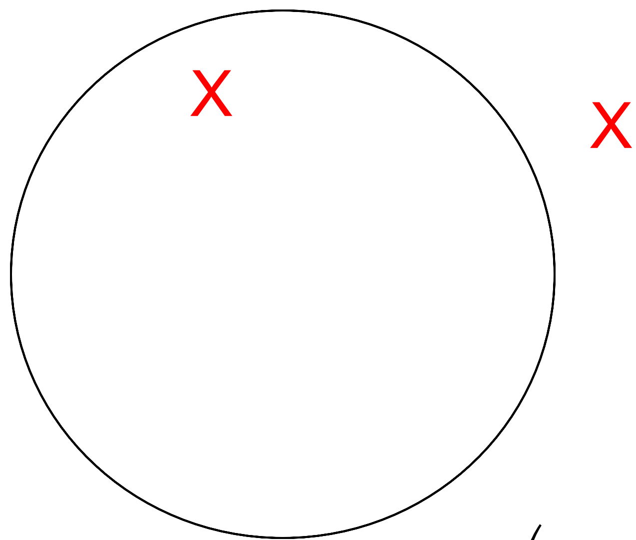
LF Polar Coordinate $\quad k^+ = r\cos\phi \quad k^- = r\sin\phi$

$$I = \int_0^\infty dr\, r\int_0^{2\pi} d\phi \frac{1}{r^2\sin\phi\cos\phi - m^2 + i\varepsilon} = -i\pi\left(\log\frac{R^2 e^{-i\pi/2}}{2m^2} + O(1/R^4)\right)_{R\to\infty}$$

$$I_{LNA}^{LF} = i\pi\log m^2$$

Capture the pole inside a circle with the change of variable: $z = e^{2i\phi}$

$$I = \int_0^\infty \frac{dr}{r} \oint dz \frac{2}{\left[z - (i\alpha + \sqrt{1-\alpha^2} + \varepsilon)\right]\left[z - (i\alpha - \sqrt{1-\alpha^2} + \varepsilon)\right]}$$



$$\alpha = \frac{2m^2}{r^2}$$

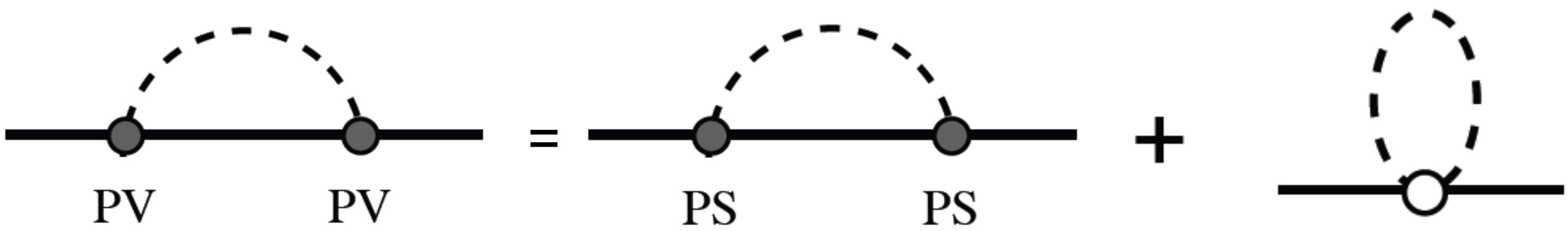
$$I = -2\pi i \int_0^{R\to\infty} dr \frac{r}{\sqrt{r^4 - 4m^4}} = -i\pi \tanh^{-1}\left(\frac{1}{\sqrt{1-(4m^4/R^4)}}\right)_{R\to\infty}$$

$$= -i\pi\left(\log\frac{R^2 e^{-i\pi/2}}{2m^2} + O(1/R^4)\right)_{R\to\infty}$$

All agree!

- Covariant (dimensional regularization)
- Equal time (rest frame)
- Equal time (infinite momentum frame)
- Light-front

$$\Sigma^{\rm PS}_{\rm end-pt} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2}\int_0^\infty dt\, \frac{\sqrt{t}\, F^2(m_\pi^2, -t)}{\sqrt{t+m_\pi^2}} \xrightarrow{\rm LNA} \frac{3g_A^2}{32\pi f_\pi^2}\frac{M}{\pi} m_\pi^2 \log m_\pi^2$$

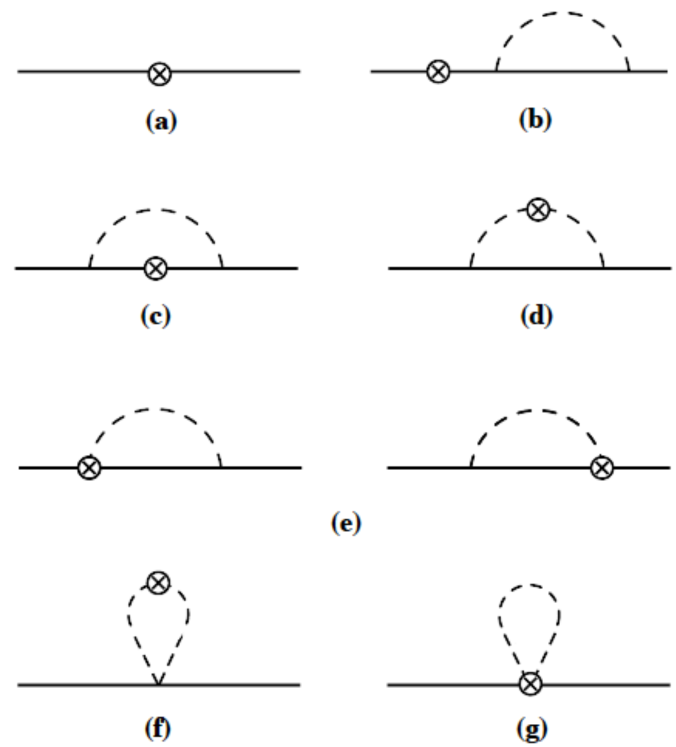


Chiral partner (σ=2π) is crucial for the restoration of equivalence between PV and PS coupling theories. In LFD, it is the "treacherous" $k^+ = 0$ (end-point) term.

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

→ N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\,\bar{u}(p)\,\gamma^\mu\,u(p) = \bar{u}(p)\,\Lambda^\mu\,u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\,\bar{u}(p)\,\Lambda^+\,u(p)$

Ward-Takahashi identity $\Lambda^\mu = -\dfrac{\partial\hat{\Sigma}}{\partial p_\mu} \longrightarrow Z_1 = Z_2$

$$\frac{1}{\not p - M - \hat{\Sigma}} = \frac{Z_2}{\not p - (M + \delta M)} \quad ; \quad \hat{\Sigma} = -(Z_2^{-1} - 1)(\not p - M - \delta M) + \delta M$$

- Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

for isovector (p−n) distribution

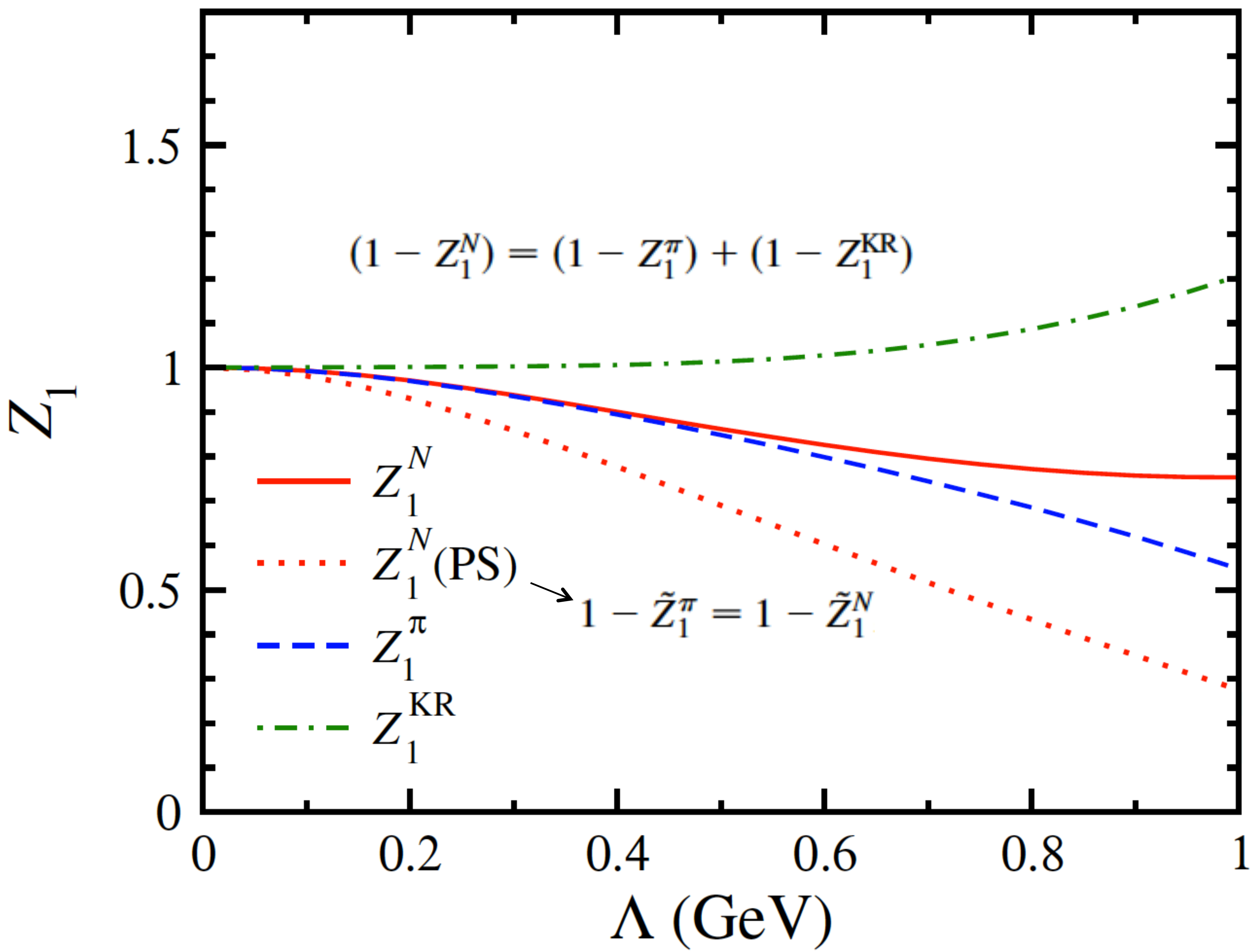
where

$$f_\pi(y) = 4f^{(\mathrm{on})}(y) + 4f^{(\delta)}(y)$$
$$f_N(y) = -f^{(\mathrm{on})}(y) - f^{(\mathrm{off})}(y) + f^{(\delta)}(y)$$
$$f_{\mathrm{KR}}(y) = 4f^{(\mathrm{off})}(y) - 8f^{(\delta)}(y)$$
$$f_{\pi(\mathrm{tad})}(y) = -f_{N(\mathrm{tad})}(y) \;=\; 2f^{(\mathrm{tad})}(y)$$

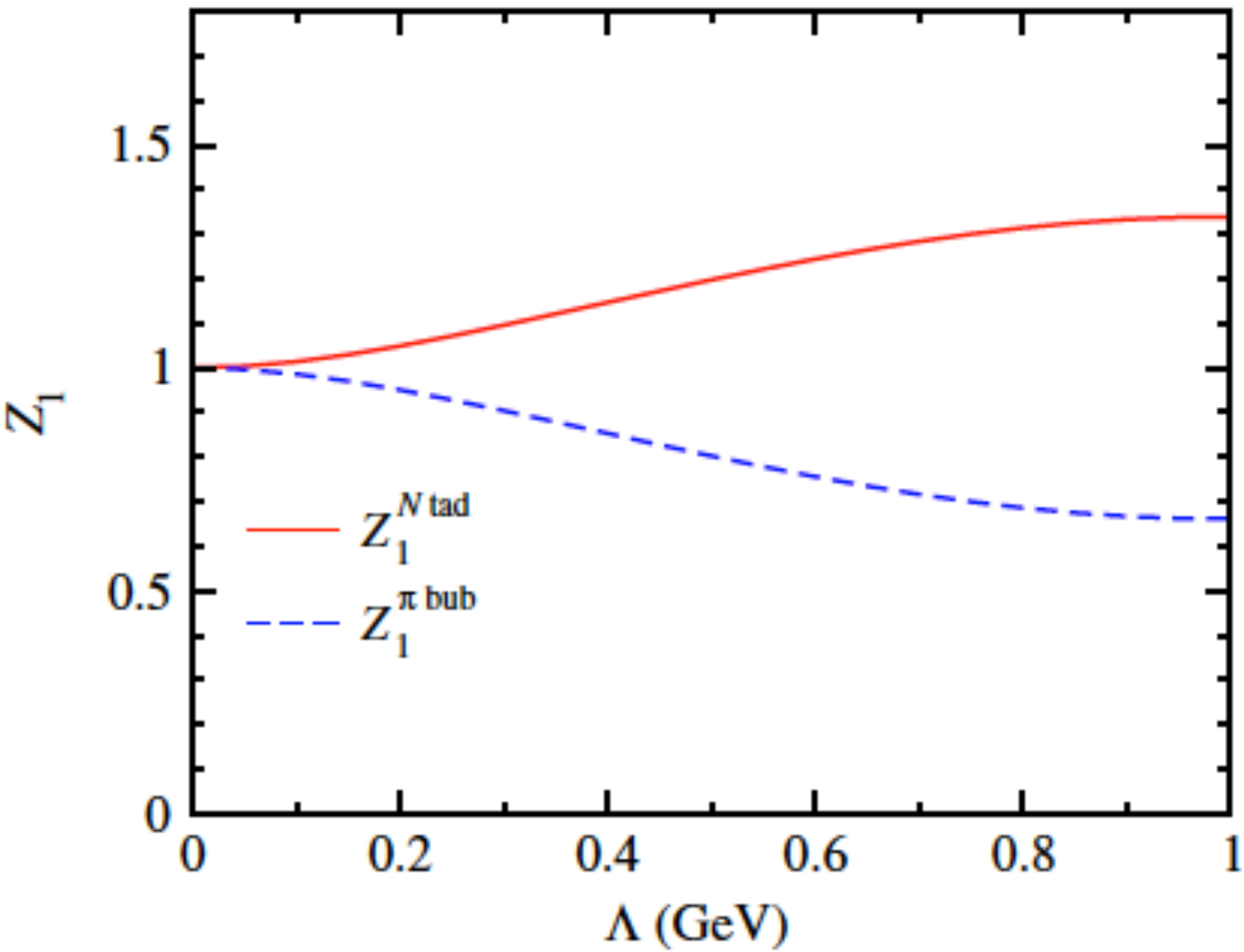
with components

$$f^{(\mathrm{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$
$$f^{(\mathrm{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$
$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$
$$f^{(\mathrm{tad})}(y) = -\frac{1}{(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

Burkardt, Hendricks, Ji, Melintchouk, Thomas, PRD ***87****, 056009 (2013)*



$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{KR})$
$1 - \tilde{Z}_1^\pi = 1 - \tilde{Z}_1^N$
Z_1^N
Z_1^N(PS)
Z_1^π
Z_1^{KR}
Z_1
Λ (GeV)
0
0.2
0.4
0.6
0.8
1
0.5
1.5



1.5
1
0.5
0
Z_1
$Z_1^{N\,\text{tad}}$
$Z_1^{\pi\,\text{bub}}$
0
0.2
0.4
0.6
0.8
1
Λ (GeV)

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^{\rm KR}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1 - Z_1^{N\,\rm tad}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi\,\rm tad}$	0	0	0	$1/2$	$1/2$	0

* also in PS

in units of $\dfrac{1}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$

→ origin of ChPT *vs*. Sullivan process difference clear!

$$\left(1 - Z_1^{N\,({\rm PV})}\right)_{\rm LNA} = \frac{3}{4}\left(1 - Z_1^{N\,({\rm PS})}\right)_{\rm LNA}$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N|\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n}|N\rangle \;=\; 2\,\langle x^{n-1}\rangle_q\; p^{\{\mu_1}\cdots p^{\mu_n\}}$$

→ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1}\rangle_q \;=\; \int_0^1 dx\, x^{n-1}\left(q(x)+(-1)^n\bar{q}(x)\right)$$

→ operator is $\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n} \;=\; \bar{\psi}\gamma^{\{\mu_1}\,iD^{\mu_2}\cdots iD^{\mu_n\}}\psi \;-\; \text{traces}$

- Lowest $(n{=}1)$ moment $\langle x^0\rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

$4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$

Chen, X. Ji, PLB ***523****, 107 (2001)*
Arndt, Savage, NPA ***692****, 429 (2002)*

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

C. Ji, W. Melnitchouk & A. Thomas, PRD88, 076005 (2013)

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

PS ("on-shell") contribution

δ-function contribution

Summary and Outlook

- Gauge invariance relations for vertex corrections verified to all orders in m_π

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{KR}})$$

→ difference between PDF moments in ChPT (PV) & "Sullivan" process (PS)

→ model-independent constraints on LC distributions $f_i(y)$

→ impact on $\bar{d} - \bar{u}$ data analysis in progress

- Application to nucleon GPDs in ChPT is plausible as we have already shown the equivalence of our work to the leading chiral correction to the nucleon GPDs at zero skewness.

A. M. Moiseeva and A. A. Vladimirov, Eur. Phys. J. A 49,23 (2013).