Quark-Hadron Duality in Electron-Pion Scattering

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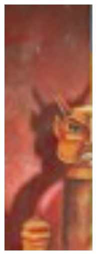
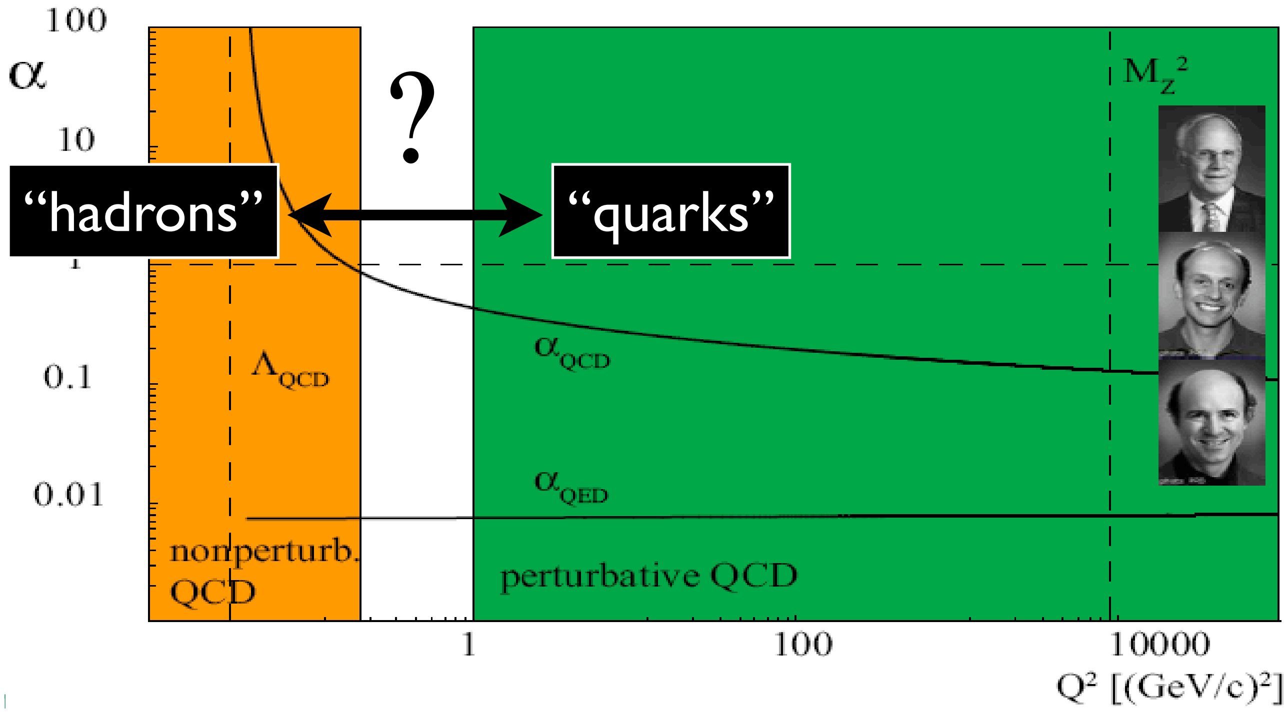
Jefferson Lab



low
energy
long
distance



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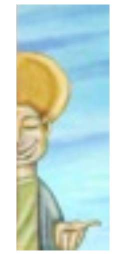
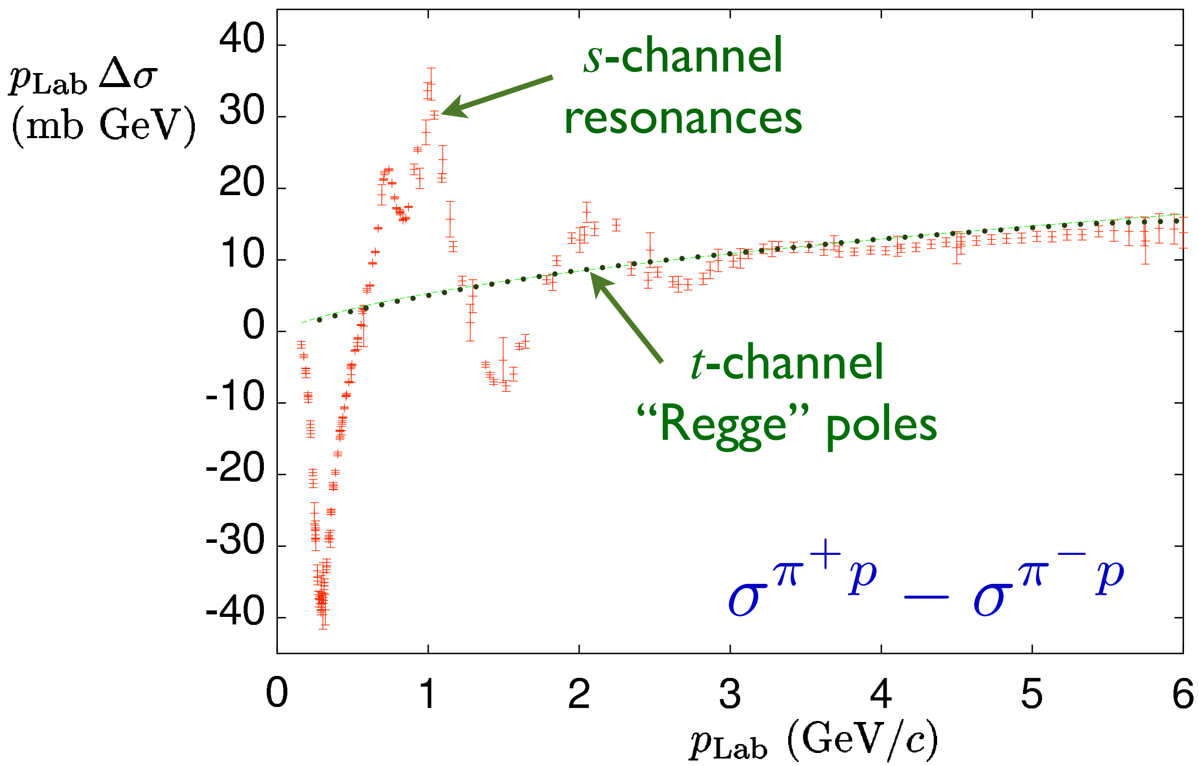


- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

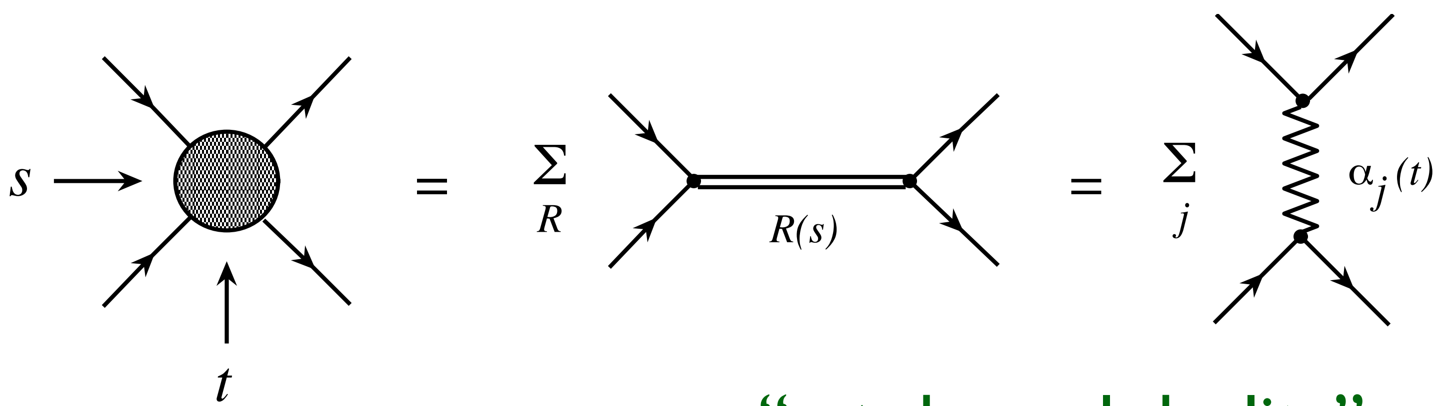
$$\sum_{hadrons} = \sum_{quarks}$$

→ can use either set of *complete* basis states to describe physical phenomena

Duality in hadron-hadron scattering

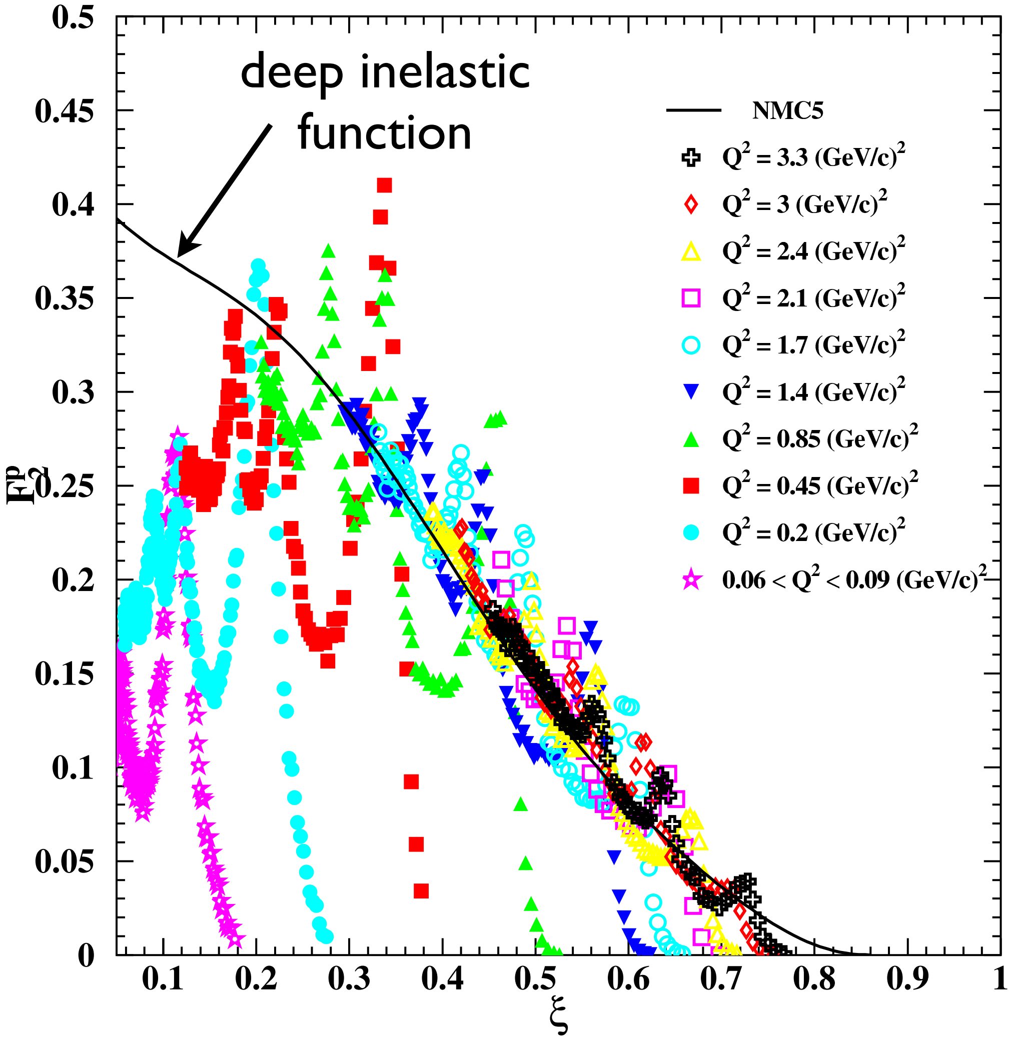


Igi (1962)
Dolen, Horn, Schmidt (1968)



"s-t channel duality"

Duality in electron-nucleon scattering



average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

finite-energy
sum rule

$$\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \; \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \; \nu W_2(\omega')$$

"hadrons" "quarks"

Bloom, Gilman
*PRL **85**, 1185 (1970)*

Duality and QCD

- **Operator product expansion**

→ expand *moments* of structure functions in powers of $1/Q^2$

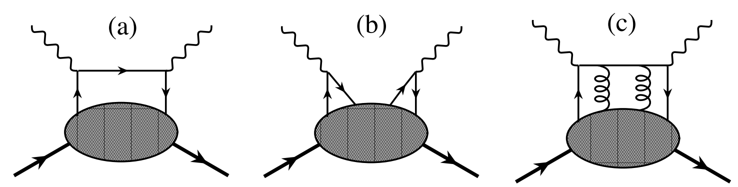
$x \to 1 \iff W \to M$

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2}\, F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



$\tau = 2$ $\qquad$ $\tau > 2$

Duality and QCD

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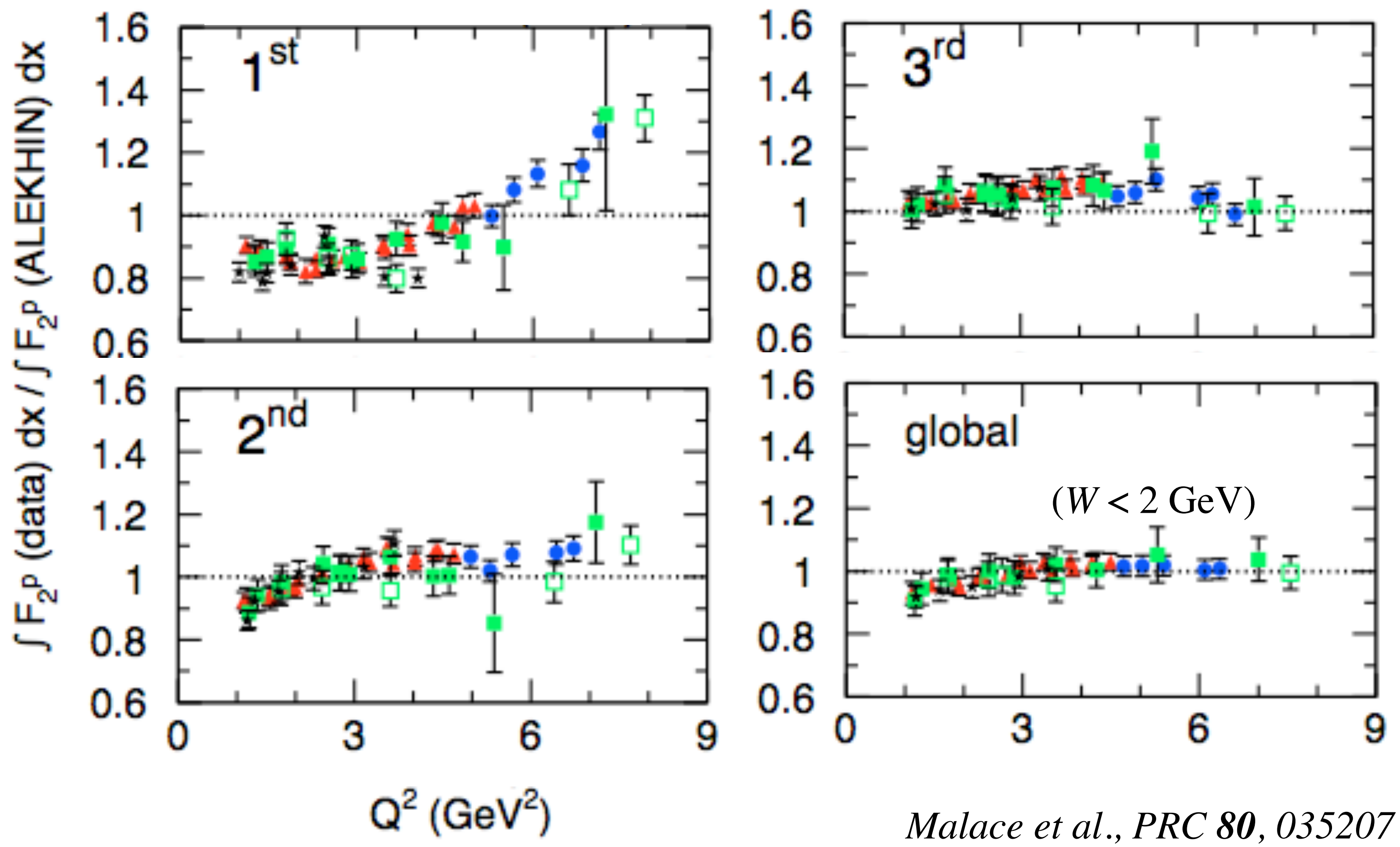
de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

- If moment $\approx$ independent of Q^2

  → "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality ⟷ suppression of higher twists

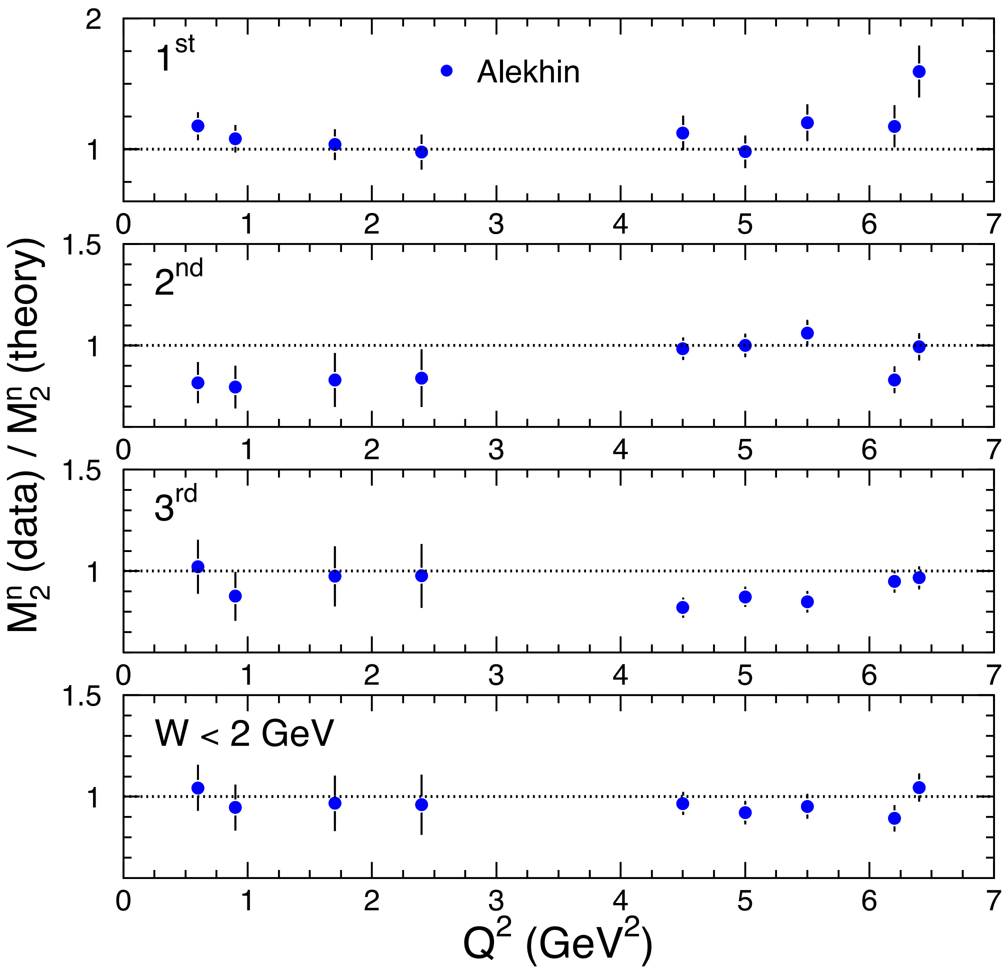
Duality in electron-proton scattering



*Malace et al., PRC **80**, 035207 (2009)*
[JLab Hall C]

→ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Duality in electron-neutron scattering



→ "theory": fit to $W > 2$ GeV data

→ *locally*, violations of duality in resonance regions < 15–20% (largest in Δ region)

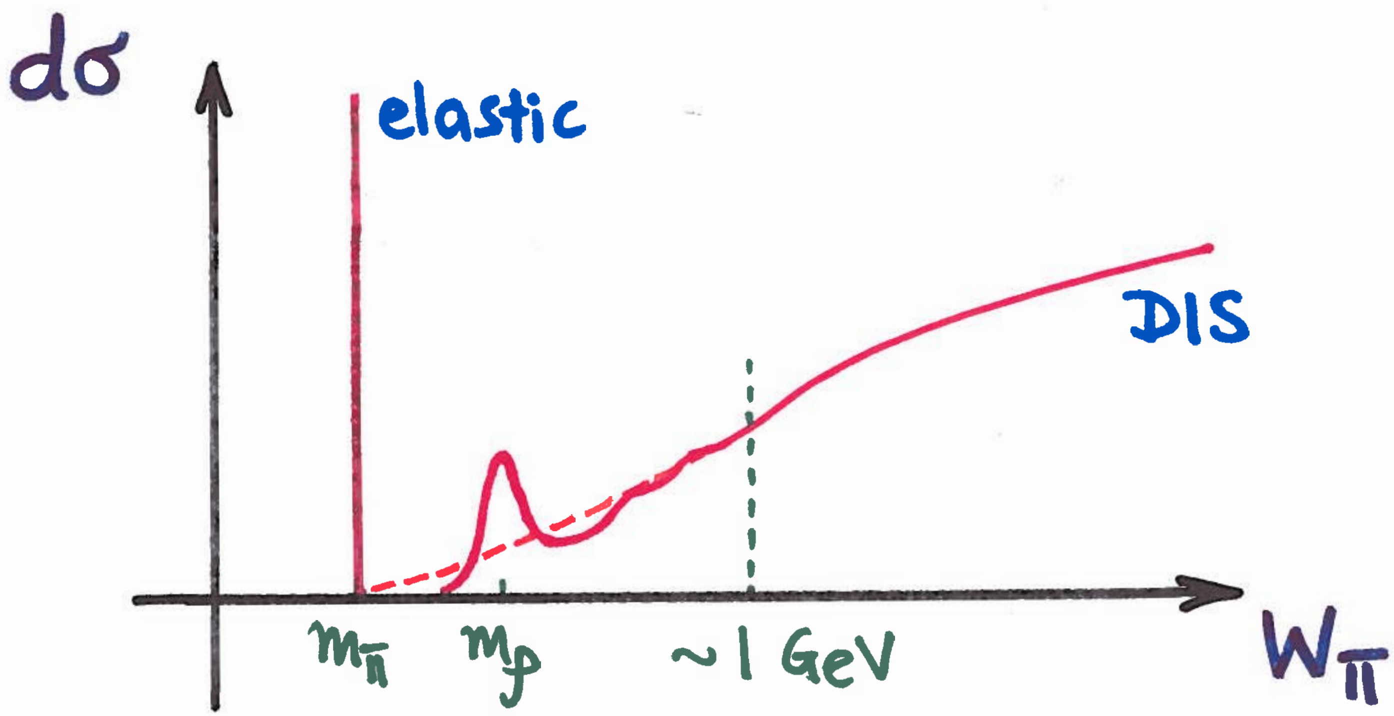
→ *globally*, violations $< 10\%$

Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

→ suggests resonance region data could be used to learn about *leading twist* structure!

Duality in electron-pion scattering

- How might duality work for pion?



- Is average over pion resonances dual to DIS function, extrapolated to low W ?

Duality in electron-pion scattering

- Inclusive pion spectrum known for elastic and DIS, but not known in intermediate region
  - → assume low-W spectrum dominated by elastic and $\pi \to \rho$ transitions
  - → elastic contribution to structure function

$$F_2^{\pi(\mathrm{el})}(x=1,Q^2) = 2m_\pi \nu \left[F_\pi(Q^2)\right]^2 \delta(W^2 - m_\pi^2)$$

  - → pion form factor parametrization

$$F_\pi(Q^2) = \frac{1}{1+Q^2/m_\rho^2}\left(\frac{1+c_1 Z + c_2 Z^2}{1+c_3 Z + c_4 Z^2 + c_5 Z^3}\right)$$

$$Z = \log(1+Q^2/\Lambda^2)$$

$$\Lambda \approx 0.25 \text{ GeV}$$

*WM, EPJA **17**, 223 (2003)*

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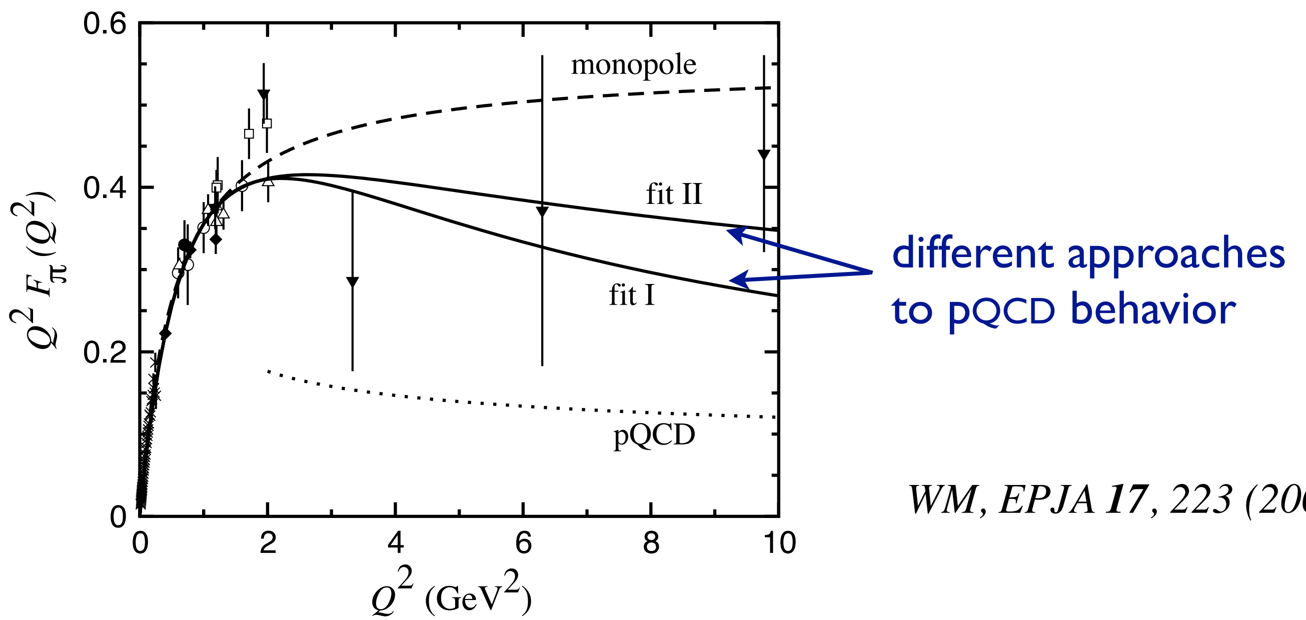


*WM, EPJA **17**, 223 (2003)*

Duality in electron-pion scattering

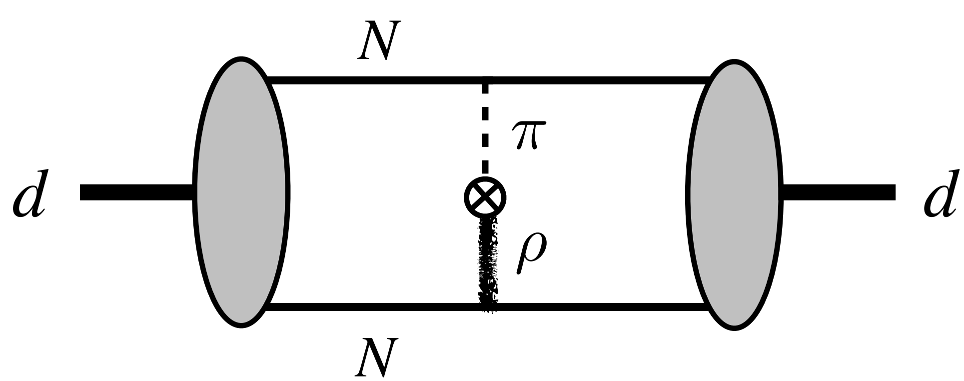
- Inclusive pion spectrum known for elastic and DIS, but not known in intermediate region

→ $\pi \to \rho$ transition form factor $F_{\pi\rho}(Q^2)$ not known empirically → take from models

- relativistic Bethe-Salpeter vertex function *Ito, Gross, PRL **71**, 2555 (1993)*
- covariant Dyson-Schwinger equations *Maris, Tandy, PRC **65**, 045211 (2002)*
- light-cone QCD sum rules *Khodjamirian, EPJC **6**, 477 (1999)*

→ assume 100% uncertainty

→ $F_{\pi\rho}(Q^2)$ an important ingredient in meson-exchange current contributions to deuteron form factors



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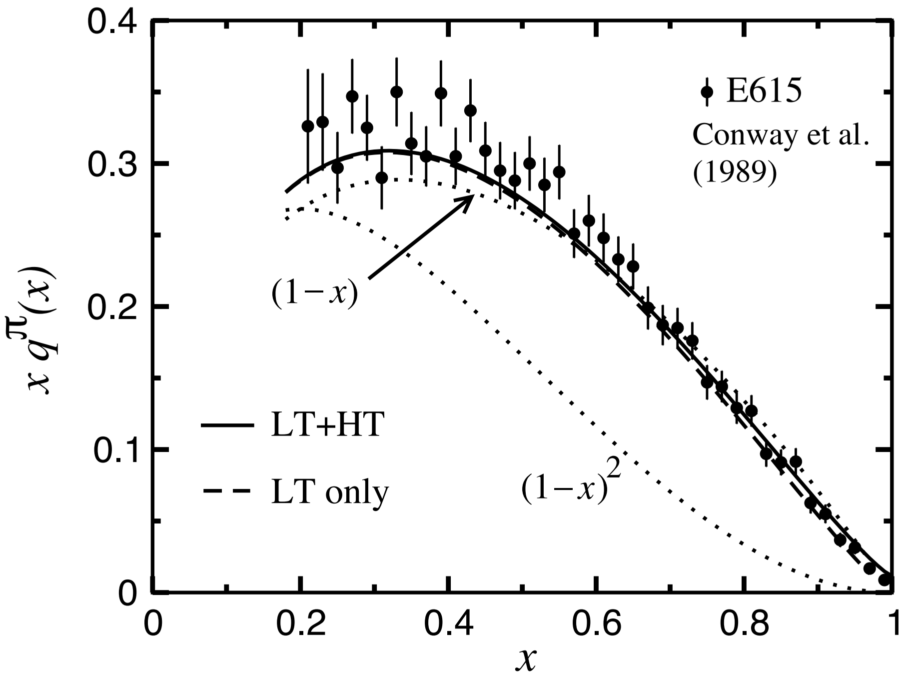
→ at high Q^2, $F_{\pi\rho} \sim 1/Q^4$ $\quad$ (*cf.* $F_\pi \sim 1/Q^2$)
$\longrightarrow$ contribution to structure function suppressed

Duality in electron-pion scattering

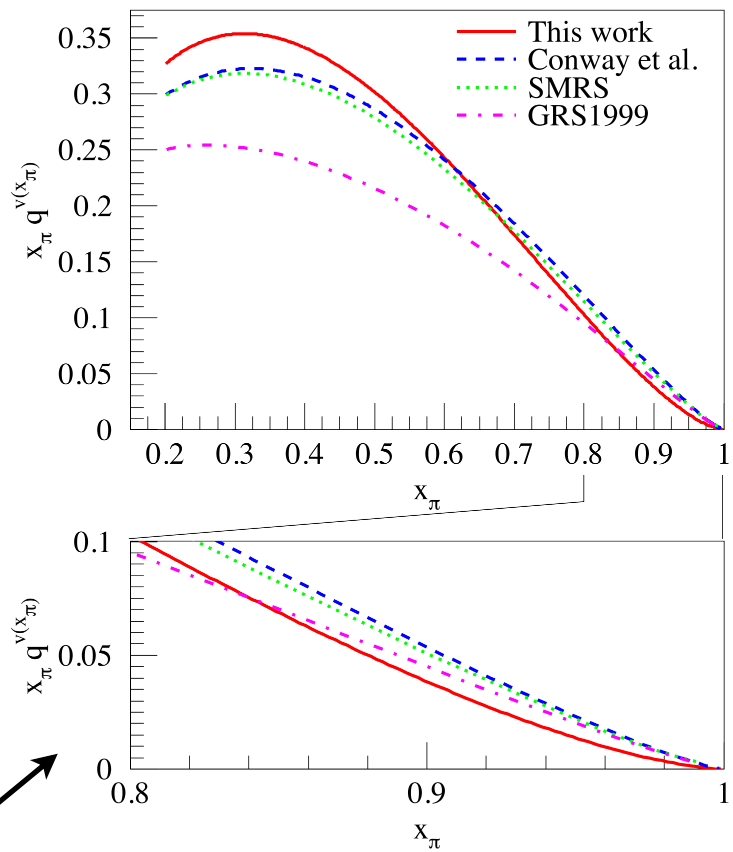
- Inclusive pion spectrum known for elastic and DIS, but not known in intermediate region

  → in DIS region strongest constraints from pion Drell-Yan measurements (FNAL E615 experiment)



→ *see talks of Reimer, Roberts*

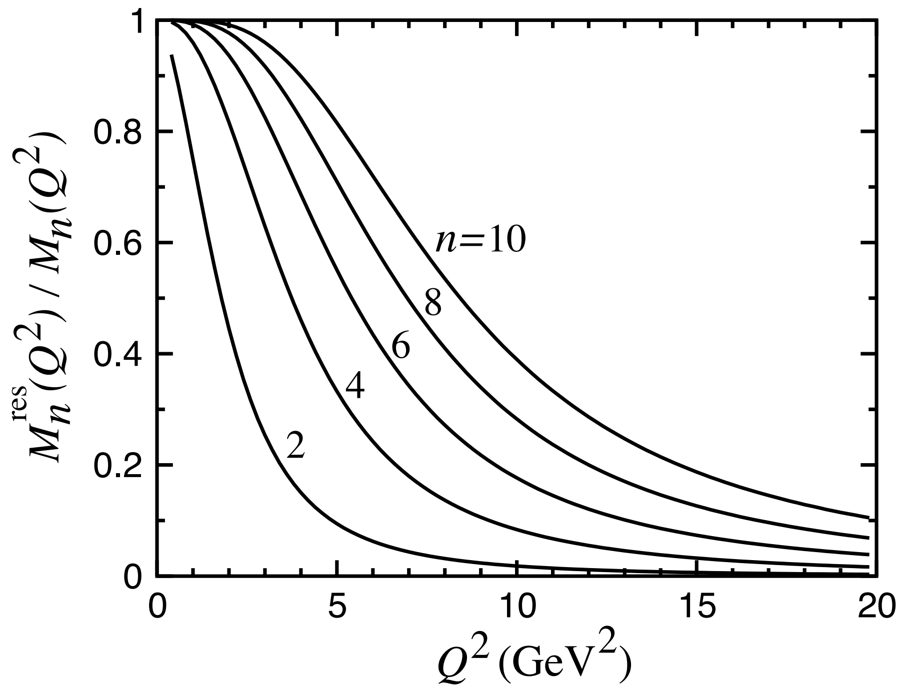
NLO reanalysis

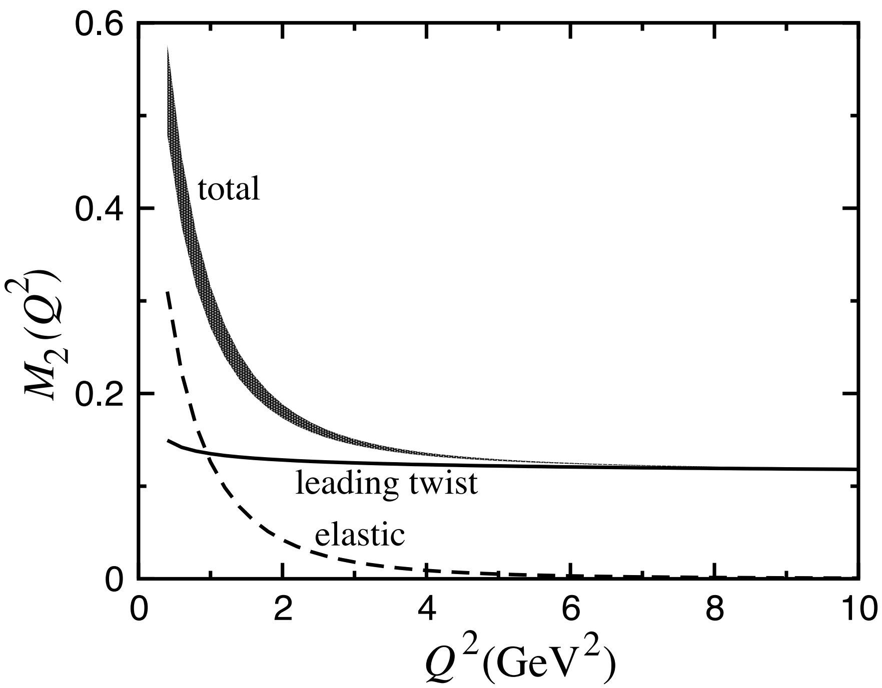


Wijesooriya, Reimer, Holt
PRC ***72****, 065203 (2005)*

Duality in electron-pion scattering

QCD moment analysis





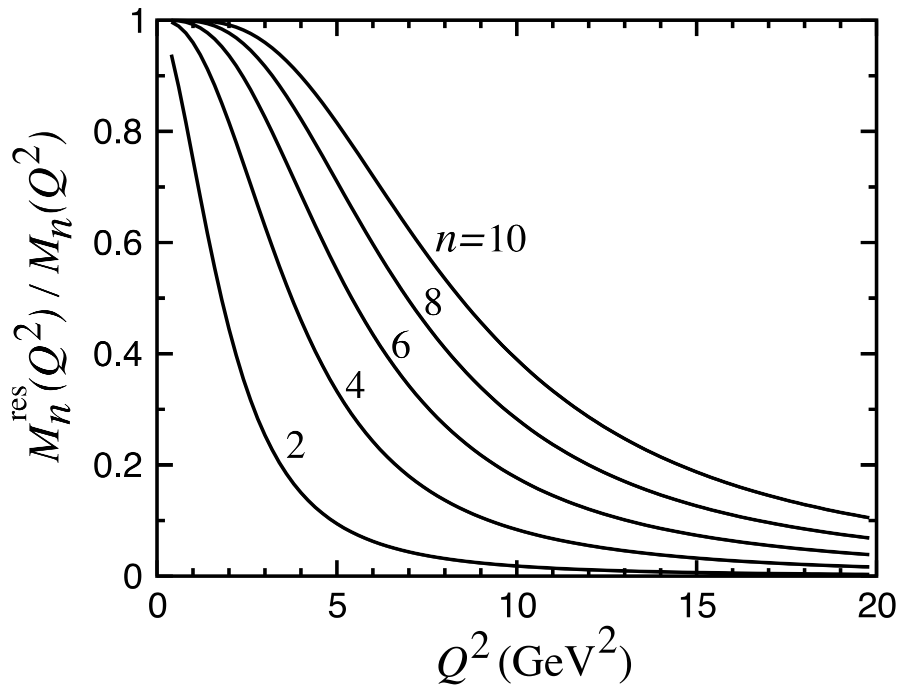
→ resonance contribution defined for $W < 1$ GeV

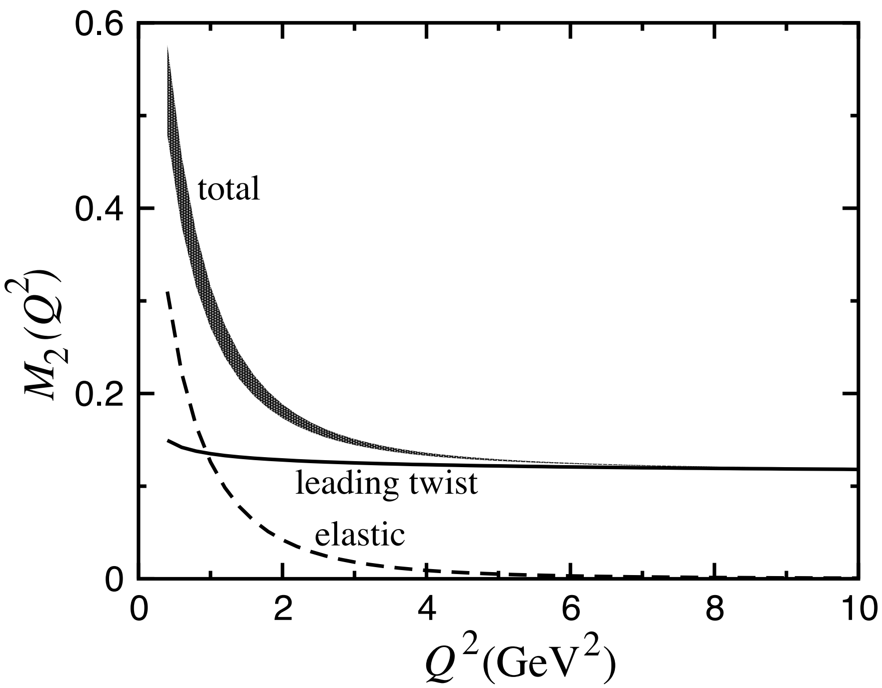
→ at $Q^2 = 1\,\mathrm{GeV}^2$, ~75% of $n = 2$ moment from low W

→ higher twist contributes ~50% of total moment

Duality in electron-pion scattering

QCD moment analysis





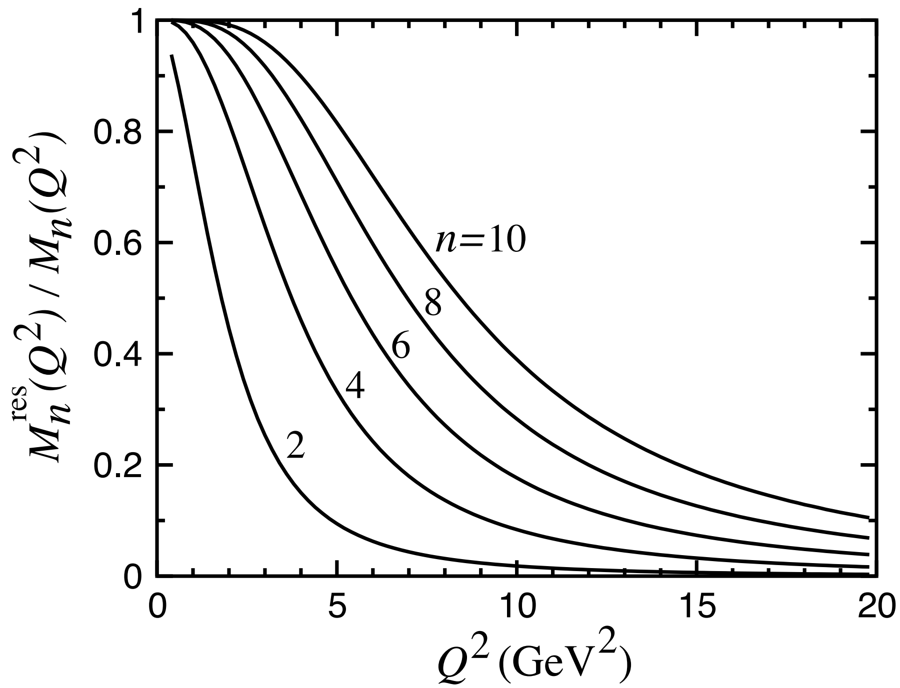
→ resonance contribution defined for $W < 1$ GeV

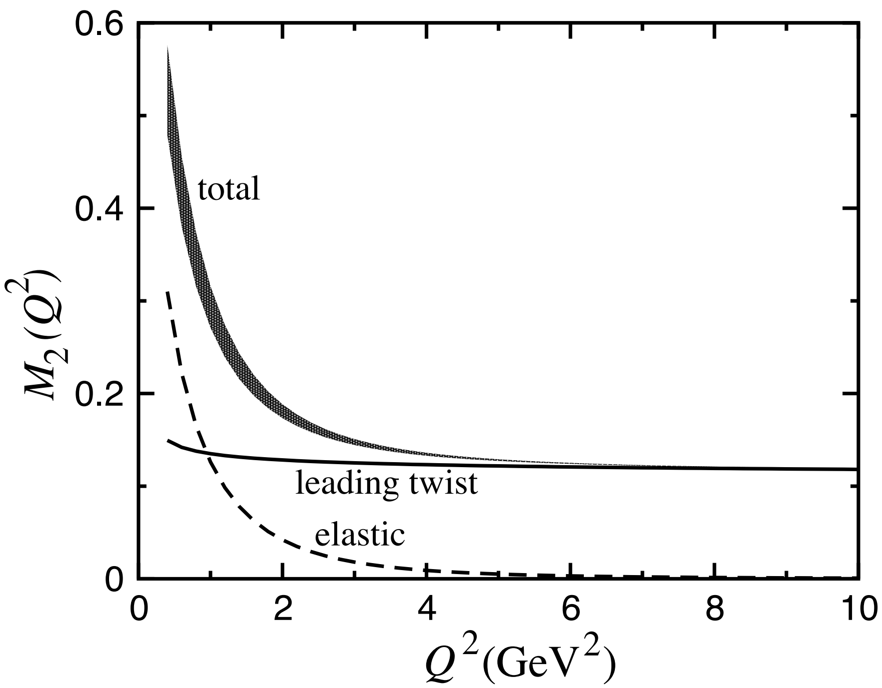
→ at $Q^2 = 2\,\mathrm{GeV}^2$, ~50% of $n = 2$ moment from low W

→ higher twist contributes ~30% of total moment

Duality in electron-pion scattering

QCD moment analysis





→ scale of higher twists set by average quark intrinsic transverse momentum $\langle k_T^2 \rangle$

→ expected to scale with confinement radius $\langle k_T^2 \rangle \sim 1/R^2$

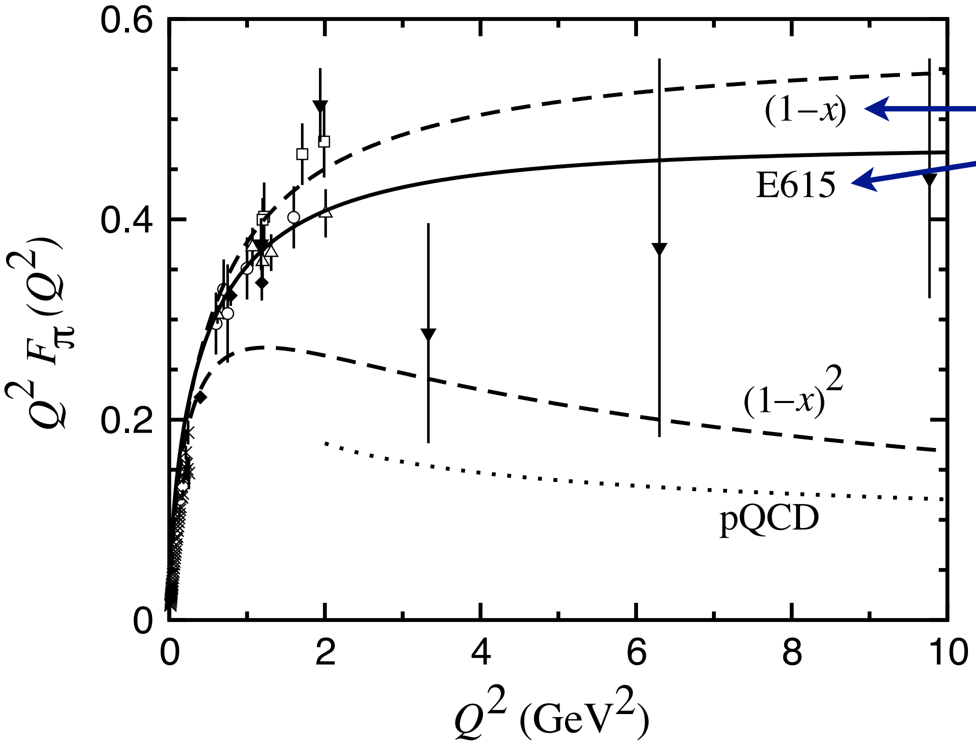
$$\langle k_T^2 \rangle_{\rm E615} = (0.8 \pm 0.3)\,{\rm GeV}^2 \quad \gg \quad \langle k_T^2 \rangle_{\rm nucleon}$$

Local duality

- Extend finite-energy sum rule relation to elastic (threshold) region

$$\left[F_\pi(Q^2)\right]^2 \approx \int_1^{\omega_{\max}} d\omega\ \nu W_2^\pi(\omega)$$

$$\omega = 1/x\,, \quad \omega_{\max} = 1 + (W^2_{\max} - m^2_\pi)/Q^2$$



different input pion structure functions

→ Q^2 dependence of form factor as $Q^2 \to \infty$ correlated with $x \to 1$ behavior of structure function

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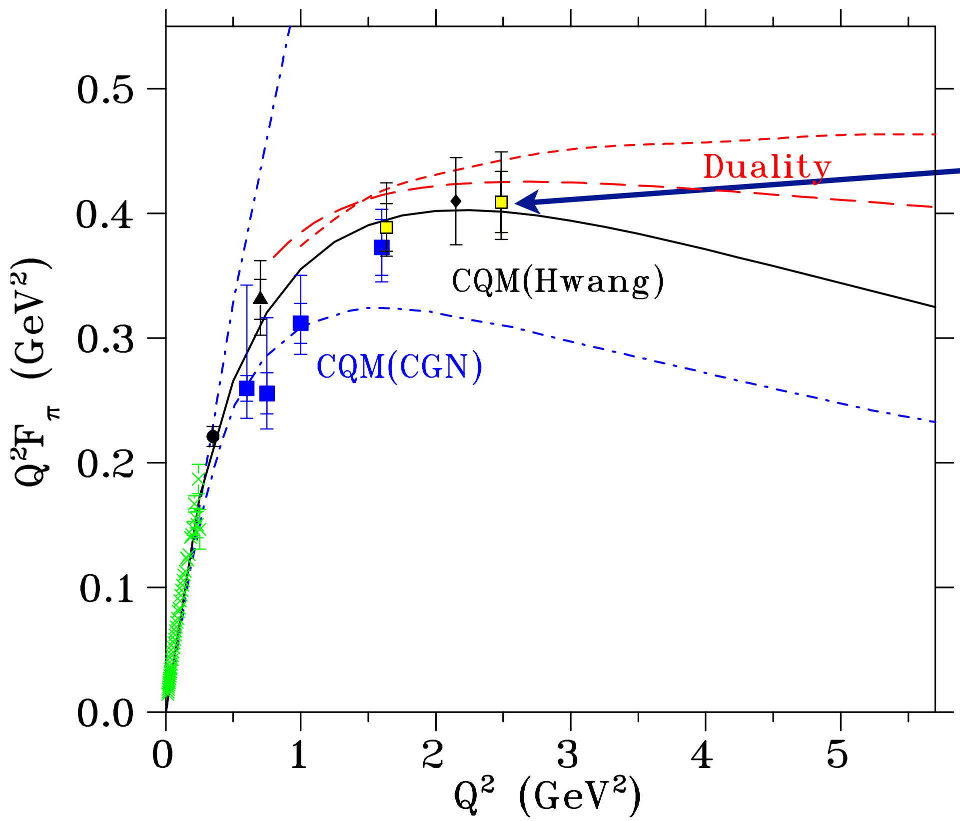

Huber et al., PRC **78**, 045203 (2008)

cf. more recent JLab (Hall C) data

→ Q^2 dependence of form factor as $Q^2 \to \infty$ correlated with $x \to 1$ behavior of structure function

Local duality

- But ... elastic scattering contributes only to *longitudinal* cross section, while DIS dominated by *transverse* scattering

  → non-trivial relation between L and T cross sections?

  → $\pi \to \rho$ transition purely transverse, so suggestion that *average* of elastic and $\pi \to \rho$ contributions dual to DIS structure function at $x \sim 1$

de Rujula, Georgi, Politzer
Ann. Phys. ***103***, *315 (1975)*

$$\left[F_\pi(Q^2)\right]^2 \longrightarrow \left[F_\pi(Q^2)\right]^2 + \omega_\rho \left[F_{\pi\rho}(Q^2)\right]^2$$

$$\omega_\rho = 1 + (m_\rho^2 - m_\pi^2)/Q^2$$

Local duality

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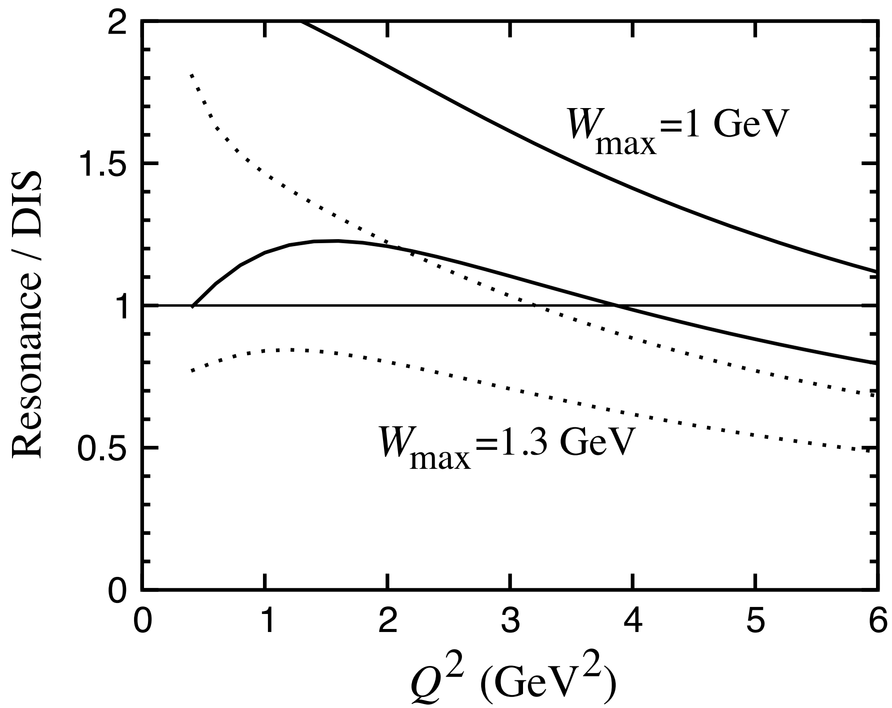
elastic $+$ $\pi \to \rho$



→ qualitative agreement between π resonance and DIS structure functions, within large uncertainties on excitation spectrum beyond $Q^2 \sim 2\ \mathrm{GeV}^2$

Conclusion

- Intriguing connection between inclusive and exclusive pion observables through quark-hadron duality

- Assume low-W inclusive pion spectrum dominated by elastic and $\pi \to \rho$ transitions

  → indicates higher twists in π larger than in N

- Local duality relates large-x structure functions with high-Q^2 form factors

  → tested using pion PDFs from Drell-Yan data

- Motivates pion resonance spectrum measurements in the region $m_\pi < W < 2\ \mathrm{GeV}$