π & K

Distribution Amplitudes & Distribution Functions

Craig Roberts

Physics Division
Collaborators: 2012-Present

1. Rocio BERMUDEZ (U Michoácan);
2. Fei GAO (PKU);
3. S. HERNÁNDEZ (U Michoácan);
4. Trang NGUYEN (KSU);
5. Khépani RAYA (U Michoácan);
6. Hannes ROBERTS (ANL, FZJ, UBerkeley);
7. Chien-Yeah SENG (UW-Mad);
8. Kun-lun WANG (PKU);
9. Chen CHEN (USTC);
10. J. Javier COBOS-MARTINEZ (U.Sonora);
11. Mario PITSCHMANN (Vienna);
12. Si-xue QIN (U. Frankfurt am Main, PKU);
13. Jorge SEGOVIA (ANL);
14. David WILSON (ODU);
15. Lei CHANG (U.Adelaide, PKU);
16. Ian CLOËT (ANL);
17. Bruno EL-BENNICH (São Paulo);
18. Adnan BASHIR (U Michoácan);
19. Stan BRODSKY (SLAC);
20. Gastão KREIN (São Paulo);
21. Roy HOLT (ANL);
22. Mikhail IVANOV (Dubna);
23. Yu-xin LIU (PKU);
24. Michael RAMSEY-MUSOLF (UW-Mad);
25. Alfredo RAYA (U Michoácan);
26. Sebastian SCHMIDT (IAS-FZJ & JARA);
27. Robert SHROCK (Stony Brook);
28. Peter TANDY (KSU);
29. Tony THOMAS (U.Adelaide);
30. Shaolong WAN (USTC)

Craig Roberts: pion and Kaon PDAs & PDFs (24p)
Enigma of Mass
Dynamical Chiral Symmetry Breaking

**DCSB is a fact in QCD**

- **Dynamical**, not spontaneous
  - Add nothing to QCD, no Higgs field, nothing! Effect achieved purely through quark+gluon dynamics.
  - It’s the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for ≈98% of the proton’s mass.
    - Higgs mechanism is (almost) irrelevant to light-quarks.
- Just like gluons and quarks, and for the same reasons, condensates are confined within hadrons.
  - There are no vacuum condensates.

Pion’s Goldberger-Treiman relation

- Pion’s Bethe-Salpeter amplitude
  Solution of the Bethe-Salpeter equation
  \[
  \Gamma_{\pi^j}(k; P) = \tau_{\pi^j} \mathbf{\gamma}_5 \left[ iE_\pi(k; P) + \mathbf{\gamma} \cdot PF_\pi(k; P) + \mathbf{\gamma} \cdot k \cdot k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right]
  \]

- Dressed-quark propagator
  \[
  S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}
  \]

- Axial-vector Ward-Takahashi identity entails
  \[
  f_\pi E_\pi(k; P = 0) = B(k^2)
  \]

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known.
The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz., Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

This emphasises that Goldstone's theorem has a pointwise expression in QCD.

Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Thus, enigmatically, the properties of the massless pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.
Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it’s how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC

- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - Computation is critical
  - Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics
Need for calculation is emphasised by Saga of pion’s valence-quark distribution:

- **1989**: $u_v^\pi \sim (1-x)^1$ – inferred from LO-Drell-Yan & disagrees with QCD;
- **2001**: DSE- QCD predicts $u_v^\pi \sim (1-x)^2$ argues that distribution inferred from data can’t be correct;
Need for calculation is emphasised by Saga of pion’s valence-quark distribution:

- 1989: $u_\nu^\pi \sim (1-x)^1$ – inferred from LO-Drell-Yan & disagrees with QCD;
- 2001: DSE- QCD predicts $u_\nu^\pi \sim (1-x)^2$ argues that distribution inferred from data can’t be correct;
- 2010: NLO reanalysis including soft-gluon resummation, inferred distribution agrees with DSE and QCD.
\( q_v \pi(x) \) \& \( q_v K(x) \)

- \( m_s \approx 24 m_u \) & \( M_s \approx 1.25 M_u \)

Expect the \( s \)-quark to carry more of the kaon’s momentum than the \( u \)-quark, so that \( x s_K(x) \) peaks at larger value of \( x \) than \( xu_K(x) \)

- Expectation confirmed in computations, with \( s \)-quark distribution peaking at 15% larger value of \( x \)

- Even though deep inelastic scattering is a high-\( Q^2 \) process, constituent-like mass-scale explains the shift
Drell-Yan experiments at CERN (1980 & 1983) provide the only extant measurement of this ratio.

No-parameter DSE prediction in complete accord with the measurement.

New experiments at modern facilities are capable of validating this comparison.

Necessary ... so that complete understanding can be claimed.

Value of ratio at x=0 will approach “1” under evolution to higher resolving scales. This is a feature of perturbative dynamics.

Using DSEs in QCD, one derives that the x=1 value is \( \approx (f_\pi/f_K)^2 (M_u/M_s)^4 = 0.3 \).

Value of ratio at x=1 is a fixed point of the evolution equations. Hence, x≈1 domain is always a direct measure of nonperturbative dynamics.
Pion’s valence-quark Distribution Amplitude

- Developed DSE methods to compute $\varphi_\pi(x) = \text{projection of the pion's Poincaré-covariant wave-function onto the light-front}$

\[
\varphi_\pi(x) = Z_2 \text{tr}_{CD} \int \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k - xn \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k; P) S(k - P)
\]

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

\[x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5\]
Pion’s valence-quark Distribution Amplitude

- Both kernels agree: marked broadening of $\phi_\pi(x)$, which owes to DCSB

- This may be claimed because PDA is computed at a low renormalisation scale in the chiral limit, whereat the quark mass function owes entirely to DCSB.

- Difference between RL and DB results is readily understood: $B(p^2)$ is more slowly varying with DB kernel and hence a more balanced result

PDAs of ground-state mesons are concave functions at all scales

When is asymptotic PDA valid?

- PDA is a wave function \( \therefore \) not directly observable
  
  But \( \ldots \) PDF is.

- \( \varphi_\pi^{asy}(x) \) can only be a good approximation to the pion's PDA when it is accurate to write

  \[
  u_\nu^\pi(x) \approx \delta(x)
  \]

  for the pion's valence-quark distribution function.

- This is far from valid at currently accessible scales.
When is asymptotic PDA valid?

- When is asymptopia reached?
- When is \( \langle x \rangle \) small?

\[ \langle x \rangle = \int_0^1 dx \ x \ u_v^\pi(x) = 0; \]

i.e., the light-front momentum fraction carried by valence-quarks is ZERO.

\[ \therefore \text{Asymptopia is reached when } \langle x \rangle \text{ is “small”} \]

- As usual, the computed valence-quark distribution produces \((\pi = u+d_{\text{bar}})\)

\[ 2\langle x \rangle_{2\text{GeV}} = 44\% \]

- NLO evolution of PDF, computation of \( \langle x \rangle \).

- Even at LHC energies, light-front fraction of the \( \pi \) momentum:

\[ \langle x \rangle_{\text{dressed valence-quarks}} = 25\% \]
\[ \langle x \rangle_{\text{glue}} = 54\%, \langle x \rangle_{\text{sea-quarks}} = 21\% \]
When is asymptotic PDA valid?

- When is asymptopia reached?
  - If \( u_\pi \pi(x) \approx \delta(x) \), then

\[
\langle x \rangle = \int_0^1 dx \, x \, u_\pi \pi(x) = 0.
\]

i.e., the light-front momentum fraction carried by valence-quarks is zero.

\[ \therefore \text{Asymptopia is reached when } \langle x \rangle \text{ is "small".} \]

- As usual, the computed valence-quark distribution produces \( (\pi = u + d_{\text{bar}}) \)

\[ \langle x \rangle_{\text{dressed valence-quarks}} = 25\% \]

\[ \langle x \rangle_{\text{glue}} = 54\%, \langle x \rangle_{\text{sea-quarks}} = 21\% \]

- Even at LHC energies, light-front fraction of the \( \pi \) momentum:

\[ \langle x \rangle_{\text{dressed valence-quarks}} = 25\% \]

\[ \langle x \rangle_{\text{glue}} = 54\%, \langle x \rangle_{\text{sea-quarks}} = 21\% \]
Distribution amplitudes of light-quark mesons from lattice QCD, Jorge Segovia, et al.
arXiv:1311.1390 [nucl-th]

Distribution Amplitudes of light-quark mesons from lattice-QCD

- PDA Moments from lattice-QCD
- Best available results show no distinction between pseudoscalar and vector, nor between light-front parallel and perpendicular polarisations
- DSE studies underway, reveal there are certainly differences:

\[ \varphi^V_{\parallel} \text{ narrower-than } \varphi^V_{\perp} \text{ narrower-than } \varphi^P \]

Table 1: Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with \( N_f = 2 + 1 \) domain-wall fermions and nonperturbative renormalisation of lattice operators [17]: linear extrapolation to physical pion mass, \( \overline{\text{MS}} \)-scheme at \( \zeta = 2 \) GeV, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the \( s \)-quark mass, discretisation and renormalisation.

<table>
<thead>
<tr>
<th>meson</th>
<th>( \langle (2x - 1)^n \rangle )</th>
<th>( 16^3 \times 32 )</th>
<th>( 24^3 \times 64 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.28(1)(2)</td>
</tr>
<tr>
<td>( \rho_{\parallel} )</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.27(1)(2)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.25(2)(1)</td>
</tr>
<tr>
<td>( K )</td>
<td>n=1</td>
<td>0.035(2)(2)</td>
<td>0.036(1)(2)</td>
</tr>
<tr>
<td>( K_{\parallel} )</td>
<td>n=1</td>
<td>0.037(1)(2)</td>
<td>0.043(2)(3)</td>
</tr>
<tr>
<td>( K )</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.26(1)(2)</td>
</tr>
<tr>
<td>( K_{\parallel} )</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.25(2)(2)</td>
</tr>
</tbody>
</table>
Distribution amplitudes of light-quark mesons from lattice QCD, Jorge Segovia, et al.
arXiv:1311.1390 [nucl-th]

Distribution Amplitudes of light-quark mesons from lattice-QCD


- SU(3) breaking is measured by shift in peak: ratio =1.10
  \[ x_{max} = 0.55 \text{ for } \phi_K(x_{max}) \text{ cf. } x_{max} = \frac{1}{2} \text{ for } \phi_\pi(x) \]
  cf. ratio of 1.15 for peak of \[ \frac{x s_v^K(x_{max})}{x u_v^K(x_{max})} \]

- Lattice-based estimate
  \[ Q^2=4\text{GeV}^2 \]
  \[ \frac{F_K(\zeta^2_2)}{F_\pi(\zeta^2_2)} \]
  \[ DSE \text{ ratio}=1.13 \]

Craig Roberts: pion and Kaon PDAs & PDFs (24p)

- \( \phi_\pi \sim x^\alpha (1-x)^\alpha \)
  \[ \alpha = 0.50^{+0.20}_{-0.16} = 0.70 \quad 0.34 \]

- \( \phi_K \sim x^\alpha (1-x)^\beta \)
  \[ \alpha = 0.39\pm0.10 \quad \beta = 0.47 -/+ 0.16 \]

- DCSB-driven

- Appearance of precision in lattice moments is misleading
  Large-lattice result for pion is unreliable – systematic error
Pseudoscalar projection of pion’s Bethe-Salpeter amplitude onto the light-front:

\[ \rho_\pi^\xi \omega_\pi(x) = \text{tr}_{CD} Z_4 \int_{dk} \delta(n \cdot k - x n \cdot K) \gamma_5 \chi_\pi(k; K) \]

\( \rho_\pi^\xi \) is the “in-pion condensate”

- (A) DSE result at \( \xi = 2 \text{GeV} \)
- (B) DSE result at \( \xi = 19 \text{GeV} \)
- (C) conformal QCD
pion PDAs

Twist-3, 2-particle ($\sigma_{\mu\nu}$)

- Pseudotensor projection of pion’s Bethe-Salpeter amplitude onto the light-front

\[
\frac{1}{4} \rho_\pi^\zeta (n \cdot P) \tau_\pi (x) = \text{tr}_{CD} \int_{dq} \delta(n \cdot q/n \cdot P - x) \gamma_5 \sigma_{\mu\nu} n_\mu P_\nu \chi_\pi (q; P)
\]

$\rho_\pi^\zeta$ is the “in-pion condensate”

- DSE result at $\zeta = 2\text{GeV}$
- DSE result at $\zeta = 19\text{GeV}$
- conformal QCD

$\tau_\pi \text{asy}(x) = 6x(1-x) = \varphi_\pi \text{asy}(x)$

- This PDA is slightly narrower at accessible scales
Far valence domain \( x \approx 1 \)

- Endpoint of the far valence domain: \( x \approx 1 \), is especially significant
  - All familiar PDFs vanish at \( x = 1 \); but ratios of any two need not
  - Under DGLAP evolution, the value of such a ratio is invariant.

- Thus, e.g.,
  - \( \lim_{x \to 1} \frac{d_\nu(x)}{u_\nu(x)} \)
    is unambiguous, scale invariant, nonperturbative feature of QCD.
    ∴ keen discriminator between frameworks
    that claim to explain nucleon structure.

- Furthermore, Bjorken-\( x = 1 \) corresponds strictly to the situation in
  which the invariant mass of the hadronic final state is precisely that
  of the target; viz., elastic scattering.

  ∴ Structure functions inferred experimentally on \( x \approx 1 \)
  are determined theoretically by target's elastic form factors.
Faddeev equation analyses of nucleon structure show that this and similar ratios measure the relative strength of scalar- and axial-vector diquark correlations.
Nucleon spin structure at very high-x
Craig D. Roberts, Roy J. Holt and Sebastian M. Schmidt

- Similar formulae for nucleon longitudinal structure functions.
- Plainly, existing data cannot distinguish between modern pictures of nucleon structure.
- Empirical results for nucleon longitudinal spin asymmetries on \( x \approx 1 \) promise to add greatly to our capacity for discriminating between contemporary pictures of nucleon structure.

NB. pQCD is actually model-dependent: assumes \( SU(6) \) spin-flavour wave function for the proton's valence-quarks and the corollary that a hard photon may interact only with a quark that possesses the same helicity as the target.
DCSB → equivalence between 1-body and 0$^+$ 2-body problem  
- Complete understanding of pion ... (almost) no model dependence

Prediction of valence-quark PDF in pion is confirmed  
- Predictions for kaon are verifiable in foreseeable future

Leading-twist PDAs of ground-state mesons are concave  
- They are broader than asymptotic distribution owing to DCSB  
- I.C. Cloët ... this is measurable in pion’s electromagnetic form factor

Leading-twist asymptotic distribution, $\varphi^{\text{asy}}$, is poor approximation to $\varphi_{\pi,K}$ at all mass-scales accessible terrestrially.

Using Bayesian analysis, it’s possible to infer pointwise form of meson PDAs from lattice-QCD  
- SU(3) breaking in meson PDAs is a measure of DCSB

Predictions for twist-3 pion PDAs  
- Model-independent results  
- They are almost indistinguishable from respective asymptotic forms

Combination of unpolarised and polarised structure function data will discriminate between pictures of nucleon structure
This is not the end
# Table of Contents

I. Introduction  
II. Enigma of mass  
III. Pion valence-quark distribution  
IV. Pion valence-quark parton distribution amplitude  
V. When is the asymptotic PDA a good approximation?  
VI. Kaon PDA  
VII. Pion twist-3 2-particle  
VIII. Far valence domain $x \approx 1$  
IX. Epilogue