Coherent electron cooling*

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Coherent Electron Cooling

Comprehensive option

High gain FEL (for electrons)

Dispersion section (for hadrons)

Kicker

Electrons

Hadrons

Modulator

I₁

I₂

E < Eₜ

E > Eₜ

E < Eₜ

E > Eₜ

Economic option

High gain FEL (for electrons) / Dispersion section (for hadrons)

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Transverse cooling

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the wave-fronts of the charged planes in electron beam

\[ \delta(ct) = -R_{26} \cdot x \]

\[ \Delta E = -eZ^2 \cdot E_o \cdot l_2 \cdot \sin\left\{ k \left( D \frac{E - E_o}{E_o} + R_{16}x' - R_{26}x + R_{36}y' + R_{46}y \right) \right\} ; \]

\[ \Delta x = -D_x \cdot eZ^2 \cdot E_o \cdot L_2 \cdot kR_{26}x + ... \]

\[ \xi_{\perp} = J_{\perp} \xi_{cC}; \quad \xi_{//} = (1 - 2J_{\perp}) \xi_{cC}; \]

\[ \frac{d\xi_x}{dt} = -\frac{\xi_x}{\tau_{cC\perp}}; \quad \frac{d\sigma^2_x}{dt} = -\frac{\sigma^2_x}{\tau_{cC\perp}}; \]

\[ \tau_{cC\perp} = \frac{1}{2J_{\perp} \xi_{cC}}; \quad \tau_{cC\parallel} = \frac{1}{2(1 - 2J_{\perp}) \xi_{cC}}; \]

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, i.e. decrement of longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: \( J'_s + J'_h + J'_v = 1 \)
- Vertical (better to say the second eigen mode) cooling is coming from transverse coupling

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e-Density modulation caused by a hadron (co-moving frame)

Induces charge: \( q = -Ze \cdot (1 - \cos \omega_p t) \)


\[
\tilde{n}(\mathbf{r},t) = \frac{Zn_0 \omega_p^3}{\pi^2 \sigma_{x_1} \sigma_{y_1} \sigma_{z_1}} \int_0^\infty d\tau \sin \tau \left( \frac{\mathbf{r} - \mathbf{v}_h \tau / \omega_p}{r_{Dx}} \right)^2 + \left( \frac{\mathbf{r} - \mathbf{v}_h \tau / \omega_p}{r_{Dy}} \right)^2 + \left( \frac{\mathbf{r} - \mathbf{v}_h \tau / \omega_p}{r_{Dz}} \right)^2 \right)^{-2} d\tau
\]

Density plots for a quarter of plasma oscillation

Ion moves in c.m. with \( v_{hz} = 10 \sigma_{vze} \)

(0,0) is the location of the ion

**Parameters of the problem**

- \( R_{Dz} \propto (|V_0| + \sigma_{V_z}) / \omega_p; \alpha = x, y, z \)
- \( t = \tau / \omega_p; \mathbf{r} = \mathbf{v} \tau / \omega_p; \mathbf{r} = \rho \sigma_{V_z} / \omega_p; \omega_p = \sqrt{4 \pi \varepsilon_0 n_e m} \)
- \( s = r_{Dz} = \frac{\sigma_{V_z}}{\omega_p} \)

\[ R = \frac{\sigma_{V_1}}{\sigma_{V_2}}; T = \frac{V_h}{\sigma_{V_2}}; L = \frac{V_h}{\sigma_{V_2}}; \zeta = \frac{Z}{4 \pi m_e R^2 s^3}; \]

\[ A = \frac{a}{s}; X = \frac{x_{ho}}{a}; Y = \frac{y_{ho}}{a}. \]

Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

Figure 4: A longitudinal cross section of the wake behind a gold ion, with the color denoting density enhancement.

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Central Section of CeC

$$D = D_{\text{free}} + D_{\text{chicane}}; \quad D_{\text{free}} = \frac{L}{\gamma^2}; \quad D_{\text{chicane}} = l_{\text{chicane}} \cdot \theta^2$$

Electron density modulation is amplified in the FEL and made into a train with duration of $$N_c \sim L_{\text{gain}}/\lambda_w$$ alternating hills (high density) and valleys (low density) with period of FEL wavelength $$\lambda$$. Maximum gain for the electron density of High Gain FEL is $$\sim 10^3$$.

$$\lambda_{\text{fel}} = \lambda_w \left(1 + \langle r_w^2 \rangle \right) / 2 \gamma_0^2$$

$$L_{Go} = \frac{\lambda_w}{4 \pi \rho \sqrt{3}}$$

$$L_G = L_{Go} (1 + \Lambda)$$

Electron group velocity is $$v_{\text{group}} = (c + 2v_{//})/3 = c \left(1 - \frac{1}{2 \gamma^2} + \frac{c}{3 \gamma^2} (1 - 2a_w^2)\right) = v_{\text{hadrons}} + \frac{c}{3 \gamma^2} (1 - 2a_w^2)$$

**Economic option requires:** $$2a_w^2 < 1$$ !!!
The amplitude (blue line) and the phase (red line, in the units of $\pi$) of the FEL gain envelope after 7.5 gain-lengths (300 period). Total slippage in the FEL is $300\lambda$, $\lambda=0.5 \, \mu$m. A clip shows the central part of the full gain function for the range of $\zeta=\{50\lambda, 60\lambda\}$.
Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis. The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory, i.e. electrons carry ~75% for the "information"

\[ v_g \approx \frac{c + 3\langle v_z \rangle}{4} = c \left( 1 - \frac{3}{8} \frac{1 + a_w^2}{\gamma_o^2} \right) \]

Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis. Gain length for the optical power is 1 m (20 periods) and for the amplitude/modulation is 2 m (40 periods)

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The Kicker

A hadron with central energy \( (E_o) \) phased with the hill where longitudinal electric field is zero, a hadron with higher energy \( (E > E_o) \) arrives earlier and is decelerated, while hadron with lower energy \( (E < E_o) \) arrives later and is accelerated by the collective field of electrons.

**Analytical estimation**

\[
\Delta \varphi = 4 \pi \rho \Rightarrow \varphi = -\frac{8G \cdot Ze}{\pi \beta v_k cm} \cdot \cos(k cm z), \quad r \frac{dr}{\varphi} = -\frac{8G \cdot Ze}{\pi \beta v_k} \cdot \sin(k cm z)
\]

\( E < E_o \)

\( \lambda_{FEL} \)

\( E > E_o \)

**Periodical longitudinal electric field**

\[
dE = -eE_{peak} \cdot \sin \left\{ kD \frac{E - E_o}{E_o} \right\};
\]

\[
kD \sigma_\delta \sim 1
\]

\[
\sigma_\delta = \frac{\sigma_E}{E_o}
\]

\[
\zeta_{CEC} = -\frac{\Delta E}{E - E_o} \approx \frac{e \cdot E_o \cdot l_2}{\gamma \sigma m_p c^2 \cdot \sigma_e} \cdot \frac{Z^2}{A}
\]

Step 1: use 3D FEL code out output + tracking

First simulation indicate that equations on the left significantly underestimate the kick, i.e. the density modulation continues to grow after beam leaves the FEL.

**Output from Genesis propagated for 25 m**

©I.Ben Zvi

**Simulations: only started**

Step 2: use VORPAL with input from Genesis, in preparation
Note that damping decrement

\[ \langle \zeta_{CeC} \rangle = \zeta \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}} = \kappa \cdot 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p \cdot \sigma_{\tau,e}}{\varepsilon_{\perp n} \left( \sigma_\delta \cdot \sigma_{\tau,h} \right)}; \kappa \sim 1 \]

\[ \langle \zeta_{CeC} \rangle \sim \frac{1}{\varepsilon_{\text{long},h} \varepsilon_{\text{trans},h}} \]

Note that damping decrement

a) Does not depend on the energy of particles!
b) Improves as cooling goes on

It makes it realistic to think about cooling intense proton beam in RHIC & LHC at 100s of GeV and 7 TeV energies

Even though LHC needs one more trick (back up slides)
Possible layout in RHIC IP of CeC driven by a single linac – to boost polarized pp- luminosity

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<th>$E_p$, GeV</th>
<th>$\gamma$</th>
<th>$E_\gamma$, MeV</th>
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<td>250</td>
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<td>136.15</td>
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<tr>
<td>325</td>
<td>346.38</td>
<td>177.00</td>
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</table>

V.N. Litvinenko, EIC AC meeting, TJNAF, November 2-3, 2009
Example: CeC vs. IBS at RHIC


\[ \frac{\sigma_s^2}{\tau_{IBS//}} = \frac{N_r c^2}{2^5 \pi^2 / 2^{3/2}} \frac{f(x_m)}{\beta_y v} \] \[ \epsilon_x = \frac{N_r c^2}{2^5 \pi^2 / 2^{3/2}} \frac{H}{\beta_y} f(x_m) ; \ k = 1 \]

\[ f(x_m) = \int_{x_m}^\infty \frac{d\chi}{\chi} \ln \left( \frac{x}{\chi_m} \right) e^{-\chi} ; \ x_m = \frac{r m^2 c^4}{b_{max} \sigma_E^2} ; b_{max} \approx n^{-1/3} \]

\[ \tau_{IBS//} = 4.6 \text{ hrs} ; \tau_{IBS/\perp} = 1.6 \text{ hrs} , \]

Stationary solution:

\[ X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS//} / \tau_{IBS/\perp}}} \frac{1}{\sqrt{\xi_\perp (1-2\xi_\perp)}} ; \ S = \tau_{CeC} \cdot \sqrt{\frac{\tau_{IBS//}}{\tau_{IBS/\perp}}} \cdot \sqrt{\frac{\xi_\perp}{(1-2\xi_\perp)^3}} \]

\[ \epsilon_{x0} = 2 \mu m ; \sigma_s = 13 \text{ cm} ; \sigma_d = 4 \cdot 10^{-4} \]

Reference value was provided by A. Fedotov using Beta-cool code © Dubna

\[ \sigma_s = 4.9 \text{ cm} \]

This may allow

a) RHIC pp - keep the luminosity at beam-beam limit all the time
b) RHIC pp - reduce bunch length to few cm (from present 1 m)
   1. to reduce hourglass effect
   2. To concentrate event in short vertexes of the detectors
c) eRHIC - reduce polarized beam current down to 50 mA while keeping the same luminosity
d) eRHIC - increase electron beam energy to 20 GeV
e) Both - increase luminosity by reducing \( \beta^* \) to 5-10 cm from present 0.5m
Possible layout for Coherent Electron Cooling proof-of-principle experiment in RHIC IR

19.6 m

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Species in RHIC</td>
<td>Au ions, 40 GeV/u</td>
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<tr>
<td>Electron energy</td>
<td>21.8 MeV</td>
</tr>
<tr>
<td>Charge per bunch</td>
<td>1 nC</td>
</tr>
<tr>
<td>Train</td>
<td>5 bunches</td>
</tr>
<tr>
<td>Rep-rate</td>
<td>78.3 kHz</td>
</tr>
<tr>
<td>e-beam current</td>
<td>0.39 mA</td>
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<tr>
<td>e-beam power</td>
<td>8.5 kW</td>
</tr>
</tbody>
</table>

V.N. Litvinenko, EIC AC meeting, TJNAF, November 2-3, 2009
Content

• A bit of history
• Principles of Coherent Electron Cooling (CeC)
• Analytical estimations, Simulations
• Proof of Principle test using R&D ERL
• Conclusions
History

possibility of coherent electron cooling was discussed qualitatively by Yaroslav Derbenev about 28 years ago

- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY, Hamburg, Germany, 1995 ...........

UM HE 91-28
August 7, 1991

CONCLUSION

The method considered above combines principles of electron and stochastic cooling and microwave amplification. Such an unification promises to frequently increase the cooling rate and stacking of high-temperature, intensive heavy particle beams. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.
Q: What's new in today's presentation?

- It is a new CeC is the scheme and the first with complete analytical and quantitative evaluation
- The spirit of amplifying the interaction remains the same as in 80's, but the underlying physics of interaction is different and also specific
- ERLs and FEL did advanced in last 30 years - hence, the practicality of the scheme
- Now we can analytically estimate and numerically calculate CeC cooling decrements for a wide variety of cases


Examples of hadron beams cooling

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<tbody>
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<td>Au</td>
<td>40</td>
<td>-</td>
<td>-</td>
<td>~ 1</td>
<td>0.02/0.06</td>
</tr>
<tr>
<td>eRHIC</td>
<td>Au</td>
<td>130</td>
<td>~1</td>
<td>20,961 ∞</td>
<td>~ 1</td>
<td>0.015/0.05</td>
</tr>
<tr>
<td>eRHIC</td>
<td>p</td>
<td>325</td>
<td>~100</td>
<td>40,246 ∞</td>
<td>&gt; 30</td>
<td>0.1/0.3</td>
</tr>
<tr>
<td>LHC</td>
<td>p</td>
<td>7,000</td>
<td>~ 1,000</td>
<td>13/26</td>
<td>∞ ∞</td>
<td>0.3/&lt;1</td>
</tr>
</tbody>
</table>

Potential increases in luminosities:

RHIC polarized pp ~ 2–4 fold, eRHIC ~ 5–10 fold, LHC ~ 2 fold
At a half of plasma oscillation

\[ q_{\lambda_{\text{FEL}}} \approx \int_0^{\lambda_{\text{FEL}}} \rho(z) \cos(k_{\text{FEL}} z) \, dz \]

\[ \rho_k = k q(\phi_k); \quad n_k = \frac{\rho_k}{2 \pi \beta \epsilon_k}; \]

\[ \chi_0 \approx \frac{1}{\sqrt{2}} \left( \frac{L_{\text{Go}}}{L_{\text{FEL}}} \right)^{\frac{1}{2}} \]

\[ \Delta \phi = 4 \pi n \Rightarrow \varphi = -\phi_0 \cdot \cos(k_{\text{cm}} z) \]

\[ \Delta \varphi = \frac{L_{\text{FEL}}}{\sqrt{3} L_G} \]

\[ \lambda_{\text{FEL}} = \lambda_w \left( 1 + \left( \frac{r_a}{a_w} \right)^2 \right) / \gamma_0^2 \]

\[ r_a = e A_w / mc^2 \]

\[ L_G = L_{Go} \left( 1 + \Lambda \right) \]

\[ L_{Go} = \frac{\lambda_w}{4 \pi \rho \sqrt{3}} \]

\[ G_{\text{FEL}} = e^{L_{\text{FEL}} / L_G} \]

\[ q = -Ze \cdot (1 - \cos \phi_k) \]

\[ \phi_k = \omega_p k / c \gamma \]

\[ q_{\text{peak}} = -2Ze \]

\[ \omega_p = \sqrt{\frac{4 m_e e^2}{\gamma_0 m_c}} \]

\[ q_{\text{peak}} = -Ze \cdot (1 - \cos \phi_k) \]

\[ \rho_k = k q(\phi_k); \quad n_k = \frac{\rho_k}{2 \pi \beta \epsilon_k}; \]

\[ \lambda_{\text{FEL}} \approx \frac{1}{\sqrt{2}} \left( \frac{L_{\text{Go}}}{L_{\text{FEL}}} \right)^{\frac{1}{2}} \]

\[ \Delta \phi = 4 \pi n \Rightarrow \varphi = -\phi_0 \cdot \cos(k_{\text{cm}} z) \]

\[ E = -\nabla \varphi = -2E_0 \cdot X \sin(k_{\text{cm}} z) \]

\[ E_0 = 2G \rho_0 \frac{e}{\beta \epsilon_{\text{cm}}} \]

\[ X = q / e \approx Z(1 - \cos \phi_k) \approx Z \]
Analytical formula for damping decrement

- 1/2 of plasma oscillation in the modulator creates a pancake of electrons with the charge \(-2Ze\)
- electron clamp is well within \(\Delta z \sim \lambda_{\text{FEL}} / 2\pi\)
- gain in SASE FEL is \(G \sim 10^2 \cdot 10^3\)
- electron beam is wider than \(2\gamma_0 \lambda_{\text{FEL}}\) - it is 1D field
- Length of the kicker is \(\sim \beta\)-function

\[
\zeta = -\frac{\Delta E_i}{E - E_o} = A \cdot \frac{L_2}{\beta} \cdot \chi \cdot \frac{\sin \varphi_3}{\varphi_3} \cdot \frac{\sin \varphi_2}{\varphi_2} \left( \frac{\sin \varphi_1}{2} \right)^2
\]

\[
A = 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p}{\varepsilon_{\perp n} \sigma_\delta}; \quad \chi = k_{\text{FEL}} D \cdot \sigma_\delta;
\]

\[
\varphi_3 = k_{\text{FEL}} D \delta; \quad \delta = \frac{E - E_o}{E_o}
\]

\[
\frac{L_2}{\beta} \cdot \chi \cdot \text{sinc}(\varphi_3) \cdot \text{sinc}\varphi_2 \cdot \left( \frac{\sin \varphi_1}{2} \right)^2 \sim 1
\]

Beam-Average decrement

\[
\int \frac{2J_1(x)}{x} e^{-x^2/2} \, dx = 0.889
\]

\(\langle \zeta_{CeC} \rangle = \zeta \cdot \frac{\sigma_{z,e}}{\sigma_{z,h}} = 2 \frac{G_o \cdot \sigma_{z,e}}{\sigma_\delta \cdot \sigma_{z,h}} \frac{Z^2}{A} \cdot \frac{r_p}{\varepsilon_{\perp n}} \cdot \kappa; \quad \kappa \sim 1
\]

Electron bunches are usually much shorter and cooling time for the entire bunch is proportional to the bunch-lengths ratios

\(\chi = 1\)

\(a = 3.8317\sigma_\delta\)

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Effects of the surrounding particles

Each charged particle causes generation of an electric field wave-packet proportional to its charge and synchronized with its initial position in the bunch

\[ E_{total}(\zeta) = E_o \cdot \text{Im} \left( \sum_{i, \text{hadrons}} K(\zeta - \zeta_i) e^{ik(\zeta - \zeta_i)} - \sum_{j, \text{electrons}} K(\zeta - \zeta_j) e^{ik(\zeta - \zeta_j)} \right) \]

\[ E_o = 2G_o \cdot \gamma_o \cdot \frac{e}{\beta \varepsilon_{\perp n}} \]

\[ X = q/e \cong Z(1 - \cos \varphi_1) \sim Z \]

Evolution of the RMS value resembles stochastic cooling!

Best cooling rate achievable is \( \sim 1/N_{\text{eff}} \), \( N_{\text{eff}} \) is effective number of hadrons in coherent sample (\( \Lambda_k = N_c \lambda \))

\[ \langle \delta^2 \rangle' = -2\xi \langle \delta^2 \rangle + D \]

\[ \xi = -g \delta_i \text{Im} \left( K(\Delta \zeta_i) e^{ik\Delta \zeta_i} \right) / \langle \delta^2 \rangle; \quad D = g^2 N_{\text{eff}} / 2; \]

\[ g = G_o \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \left\{ 2f(\varphi_2)(1 - \cos \varphi_1) \frac{l_2}{\beta} \right\} \]

\[ \xi_{\text{CeC}}(\text{max}) = \frac{\Delta}{2\sigma_\gamma} = \frac{2}{N_{\text{eff}}} \left( kD\sigma_\varepsilon \right) \propto \frac{1}{N_{\text{eff}}} \]

Fortunately, the bandwidth of FELs \( \Delta f \sim 10^{13} - 10^{15} \text{ Hz} \) is so large that this limitation does not play any practical role in most HE cases.
Conclusions

• Coherent electron cooling has potential of cooling high intensity TeV scale proton and ion beams with reasonable (under an hour) cooling time

• Electron accelerator of choice for such cooler is energy recovery linac (ERL)

• ERL seems to be capable of providing required beam quality for such coolers

• Majority of the technical limitation and requirements on the beam and magnets stability are well within limit of current technology, even though satisfying all of them in nontrivial fit

• We plan a proof of principle experiment of coherent electron cooling with Au ions in RHIC at ~ 40 GeV/n and existing R&D ERL as part of eRHIC R&D
Conclusions

• Coherent electron cooling is very promising method for significant luminosity increases in hadron colliders from RHIC to LHC
• Initial studies did not find any phenomena, which challenges the concept of CeC
• Our CeC estimations passed a number of tests
• At the same time, we found a number of new and interesting details to pursue further
• Future studies will refine the model and improve the quality of predictions
• We plan to test validity of the concept experimentally in Proof-of-Principle experiment using BNL’s R&D ERL installed in one of available IPs at RHIC

Supported by the Office on Nuclear Physics, US DoE
**Coherent electron cooling, ultra-relativistic case ($\gamma >> 1$)**

**Economic option**

Electrons → **Modulator:** region 1  
A quarter to a half of plasma oscillation

**High gain FEL (for electrons) / Dispersion section (for hadrons)**

**Kicker:** region 2

**Hadrons**

Electrons

Amplifier of the e-beam modulation via High Gain FEL and Longitudinal dispersion for hadrons

Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength $\lambda$. Maximum gain for the electron density of HG FEL is $\sim 10^3$.

\[
 v_{\text{group}} = (c + 2v_{//})/3 = c \left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c \left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2} (1 - 2a_w^2) = v_{\text{hadrons}} + \frac{c}{3\gamma^2} (1 - 2a_w^2)
\]

**Economic option requires:** $2a_w^2 < 1$ !!
Response - 1D FEL after 10 gain lengths

\[ \left\{ x \rightarrow \frac{2 \pm \sqrt{c}}{3} \right\} \frac{2^{1/3} \sqrt{c^3}}{27x - 2 \pm c^3 + 3 \sqrt{3} \sqrt{-27 + 4 c^3}}^{1/3} \]

\[ \left\{ x \rightarrow \frac{2 \pm \sqrt{c}}{3} \right\} \frac{(1 + \sqrt{3}) c^3}{3^{2/3} \left[ 27x - 2 \pm c^3 + 3 \sqrt{3} \sqrt{-27 + 4 c^3} \right]^{1/3}} \]

\[ \left\{ x \rightarrow \frac{2 \pm \sqrt{c}}{3} \right\} \frac{(1 - \sqrt{3}) c^3}{3^{2/3} \left[ 27x - 2 \pm c^3 + 3 \sqrt{3} \sqrt{-27 + 4 c^3} \right]^{1/3}} \]

Green-function envelope (Abs, Re and Im)

Maximum located at 3.744 slippage units, (i.e. just a bit further that expected 3 and 1/3)

The Green function (with oscillations) had effective RMS length of 1.48 slippage units.

\[ v_g = \frac{c + 2 \langle v_z \rangle}{3} = c \left( 1 - \frac{1 + a_w^2}{3 \gamma_o^2} \right) \]
FEL's Green Function

1D - analytical approach

$G(\tau, z) = \text{Re}(\tilde{G}_z(\tau)e^{i\omega_0 \tau})$

3D - 3D FEL codes RON and Genesis 1.3

FEL parameters for Genesis 1.3 and RON simulations

FEL gain length: 1 m (power), 2m (amplitude)

Main FEL parameters for eRHIC with 250 GeV protons

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Energy, MeV</td>
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<td>$a_w$</td>
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<td>Helical</td>
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Modulation after 20m of FEL

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CeC: FEL response

\[ f_{\text{input}}(\vec{r}_\perp, \vec{p}, t) = f_{\text{o input}}(\vec{r}_\perp, \vec{p}) + \delta f(\vec{r}_\perp, \vec{p}, t) \]

\[ f_{\text{exit}}(\vec{r}_\perp, \vec{p}, t) = f_{\text{o exit}}(\vec{r}_\perp, \vec{p}) + \int K(\vec{r}_\perp, \vec{p}, \vec{r}_\perp, \vec{p}_1, t - t_1) \cdot \delta f(\vec{r}_1, \vec{p}_1, t_1) \cdot d\vec{r}_\perp d\vec{p}_1 dt_1 \]

1D FEL response

\[ \rho_{\text{exit}}(t; z) = \rho_o + \int G(\tau; z) \cdot \delta \rho(t - \tau; 0) \cdot d\tau \]

\[ G(\tau; z) = \text{Re}\left( \tilde{G}_z(\tau) e^{i\omega_o \tau} \right) \quad \omega_o = \frac{2\pi c}{\lambda_o} \]
Modulator

Dimensionless equations of motion

\[ \frac{\partial \vec{f}_e}{\partial t} + \frac{\partial \vec{f}_e}{\partial \vec{n}} \cdot \frac{e\vec{E}}{m} + \frac{\partial \vec{f}_e}{\partial \vec{r}} \cdot \vec{v} = 0; \quad \vec{r}_h(t) \equiv \vec{r}_0 + \vec{v}_h t; \]

\[ \nabla \cdot \vec{E} = 4\pi e_n \left( \frac{Z}{n_e} \delta(\vec{r} - \vec{r}_h(t)) - \int f_e d\vec{v}^3 \right). \]

\[ \frac{\partial \vec{f}_e}{\partial \tau} + \frac{\partial \vec{f}_e}{\partial \vec{v}} \cdot \vec{g} + \frac{\partial \vec{f}_e}{\partial \vec{p}} \cdot \vec{v} = 0; \quad \vec{g} = \frac{e\vec{E}}{m\omega_p^2 s}; \]

\[ \nabla \cdot \vec{g} = \frac{Z}{s^3 n_e} \delta(\vec{p} - \vec{p}_i(t)) - \int f_e d\vec{v}^3; \quad \nabla \equiv \partial_p. \]

\[ t = \tau / \omega_p; \quad \vec{v} = \vec{v}_{o}; \quad \vec{r} = \vec{p}_{o} / \omega_p; \quad \omega_p^2 = \frac{4\pi e^2 n_e}{m}; \quad s = r_{D_z} = \sigma_{v_z} / \omega_p \]

Parameters of the problem

\[ R = \frac{\sigma_{v_{\perp}}}{\sigma_{v_z}}; \quad T = \frac{V_{hx}}{\sigma_{v_z}}; \quad L = \frac{V_{hz}}{\sigma_{v_z}}; \quad \xi = \frac{Z}{4\pi n_e R^2 s^3}; \]

\[ A = \frac{a}{s}; \quad X = \frac{x_{ho}}{a}; \quad Y = \frac{y_{ho}}{a}. \]
Velocity map & buncher \((\gamma > 1000)\)

\[
\frac{\delta E}{E}(z,r) = -Z r_e \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c \Delta t
\]

\[
\langle \frac{\delta E}{E} \rangle \approx -2 Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \left( \frac{z}{|z|} - \frac{z}{\sqrt{a^2 / \gamma^2 + z^2}} \right)
\]
Exact calculations: solving Vlasov equation

For 7 TeV p in LHC CeC case: simple "gut-feeling" estimate gave 22.9 boost in the induced charge by a buncher, while exact calculations gave 21.7.
Comprehensive studies
Analytical, Numerical and Computer Tools to:

1. find reaction \((\text{distortion of the distribution function of electrons})\) on a presence of moving hadron inside an electron beam

\[
\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{v}} \cdot \frac{e\vec{E}}{m} + \frac{\partial f_e}{\partial \vec{r}} \cdot \vec{v} = 0; \quad \vec{r}_h(t) \equiv \vec{r}_o + \vec{v}_h t;
\]
\[
(\vec{\nabla} \cdot \vec{E}) = 4\pi e n_e \left( \frac{Z}{n_e} \delta(\vec{r} - \vec{r}_h(t)) - \int f_e d\vec{v} \right).
\]

2a. Find how an arbitrary \(\delta f\) is amplified in high-gain FEL

\[
f_{\text{exit}}(\vec{r}_\bot, \vec{p}, t) = f_{o \text{ exit}}(\vec{r}_\bot, \vec{p}) + \int K(\vec{r}_\bot, \vec{p}, \vec{r}_\bot, \vec{p}_1, t - t_1) \cdot \delta f(\vec{r}_1, \vec{p}_1, t_1) \cdot d\vec{r}_\bot d\vec{p}_1 dt_1
\]

2b. Design cost effective lattice for hadrons + coupling

3. Find how the amplified reaction of the e-beam acts on the hadron (including coupling to transverse motion)
Genesis: 3D FEL

Evolution of the normalized bunching envelope

The Green function (with oscillations) after 10 gain-lengths had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns)

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Evolution of the bunching and optical power envelopes (vertical scale is logarithmic)
PoP test using BNL R&D ERL:  
Au ions in RHIC with 40 GeV/n, $L_{\text{cooler}} = 14$ m

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N per bunch</td>
<td>$1 \times 10^9$</td>
</tr>
<tr>
<td>Energy Au, GeV/n</td>
<td>40</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>3.2</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>2.5</td>
</tr>
<tr>
<td>Energy $e^-$, MeV</td>
<td>21.79</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5 (or 4 x 1.4)</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>5 (4)</td>
</tr>
<tr>
<td>$\beta_{\perp}, m$</td>
<td>5</td>
</tr>
<tr>
<td>$\omega_{\text{pe}}, CM, Hz$</td>
<td>$5.03 \times 10^9$</td>
</tr>
<tr>
<td>$\lambda_{\text{D} \perp}, \mu m$</td>
<td>611</td>
</tr>
<tr>
<td>$\lambda_{\text{FEL}}, \mu m$</td>
<td>18</td>
</tr>
<tr>
<td>$a_w$</td>
<td>0.555</td>
</tr>
<tr>
<td>Amplitude gain =150, $L_w$, m</td>
<td>6.75 (7)</td>
</tr>
<tr>
<td>$L_2$ (lab frame), m</td>
<td>3</td>
</tr>
<tr>
<td>$N_{\text{turns}}, \tilde{N}$, 5% BW</td>
<td>$8 \times 10^6 &gt; 6 \times 10^4$</td>
</tr>
<tr>
<td>$L_{\text{Go}}$, m</td>
<td>0.67</td>
</tr>
<tr>
<td>$L_{\text{G3D}}$, m</td>
<td>1.35</td>
</tr>
<tr>
<td>Cooling time, local, minimum</td>
<td>0.05 minutes</td>
</tr>
<tr>
<td>Cooling time, beam, min</td>
<td>2.6 minutes</td>
</tr>
</tbody>
</table>
### 325 GeV polarized protons in RHIC, $L_{\text{cooler}}$ fits in IR

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$Z, A$</th>
<th>1, 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N per bunch</strong></td>
<td>$2 \times 10^{11}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Energy Au, GeV/n</strong></td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RMS bunch length, nsec</strong></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emittance norm, μm</strong></td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Energy $e^-$, MeV</strong></td>
<td>136.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Charge per bunch, nC</strong></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emittance norm, μm</strong></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$\beta_\perp, \text{m}$</strong></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$\omega_{pe}, \text{CM, Hz}$</strong></td>
<td>$4.19 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$\lambda_{\text{DL}}, \mu m$</strong></td>
<td>1004</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$\lambda_{\text{FEL}}, \mu m$</strong></td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$a_w$</strong></td>
<td>0.648</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Amplitude gain =100, $L_w, \text{m}$</strong></td>
<td>13 ($\sim 15$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$L_2, \text{(lab frame)}, \text{m}$</strong></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$\bar{N}_{\text{min, turns or }\bar{N} \text{ in 10% BW}}$</strong></td>
<td>$6.7 \times 10^6 &gt; 5.9 \times 10^6$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Not optimized!**
### Au ions in RHIC with 100 GeV/n, $L_{\text{cooler}} \sim 20$ m

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ per bunch</td>
<td>$2 \times 10^9$</td>
</tr>
<tr>
<td>Energy Au, GeV/n</td>
<td>100</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>1</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>2.5</td>
</tr>
<tr>
<td>Energy $e^-$, MeV</td>
<td>54.5</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5</td>
</tr>
<tr>
<td>Bunch length, nsec</td>
<td>0.1</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>3</td>
</tr>
<tr>
<td>$\beta_{\perp}$, m</td>
<td>10</td>
</tr>
<tr>
<td>$\omega_{pe}$, CM, Hz</td>
<td>$5.9 \times 10^9$</td>
</tr>
<tr>
<td>$\lambda_{D\perp}$, $\mu$m</td>
<td>78</td>
</tr>
<tr>
<td>$\lambda_{D\parallel}$, $\mu$m</td>
<td>0.75</td>
</tr>
<tr>
<td>$\lambda_{\text{FEL}}$, $\mu$m</td>
<td>3</td>
</tr>
<tr>
<td>$a_w$</td>
<td>0.603</td>
</tr>
<tr>
<td>$L_w$, m</td>
<td>8.11 ($\rightarrow$ 9)</td>
</tr>
<tr>
<td>$L_{G_{3D}}$, m</td>
<td>0.77</td>
</tr>
<tr>
<td>$L_2$ (lab frame), m</td>
<td>5</td>
</tr>
<tr>
<td>$N_{\min \text{ turns or } \tilde{N}}$ in 5% BW</td>
<td>$6 \times 10^5 \rightarrow 2 \times 10^5$</td>
</tr>
</tbody>
</table>

**Cooling times:**
- Cooling time, local, minimum: 0.08 minutes
- Cooling time, beam, min: 1.93 minutes