Recent developments in Pythia 8

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EIC software meeting

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Outline

- Motivation / introduction
- Parton showering, higher-order calculations, combining PS and multiple higher-order calculations.
- New parton showers for PYTHIA and SHERPA
- Summary

A biased view of particle physics

We are in the privileged position that we have experiments covering vastly different phenomena:

- Low-energy experiments
- Collider experiments
- Dark matter searches
- Cosmic ray experiments

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  Hadron structure, non-perturbative or "soft" physics
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  Jets, perturbative QCD with small non-pert. corrections
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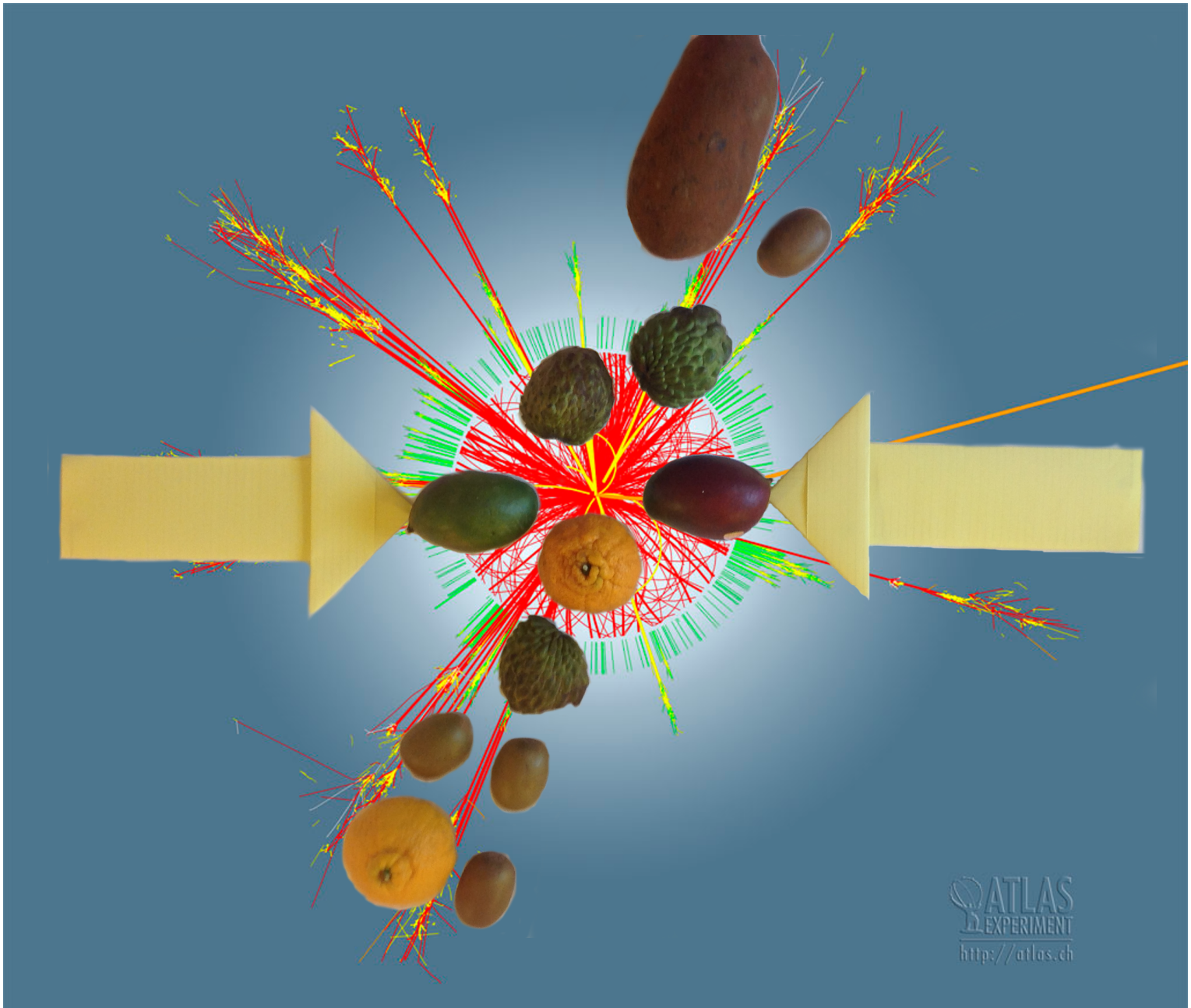
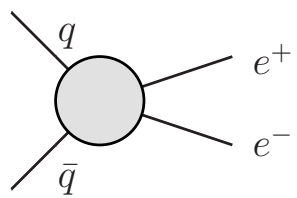
The phenomena at these experiments can be quite different!

$\Rightarrow$ To interpret the results, and design new experiments, we need
$\Rightarrow$ models that mimic the Standard Model as well as possible.
$\Rightarrow$ General Purpose Event Generators

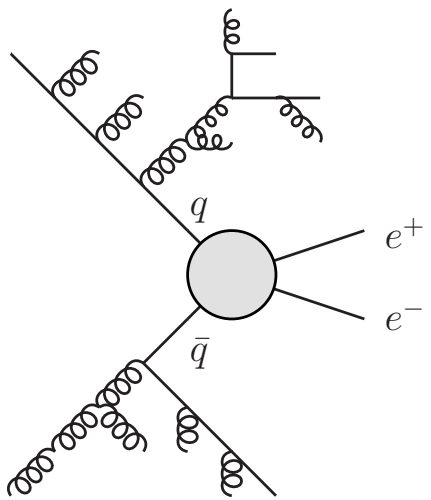
Modelling collider events



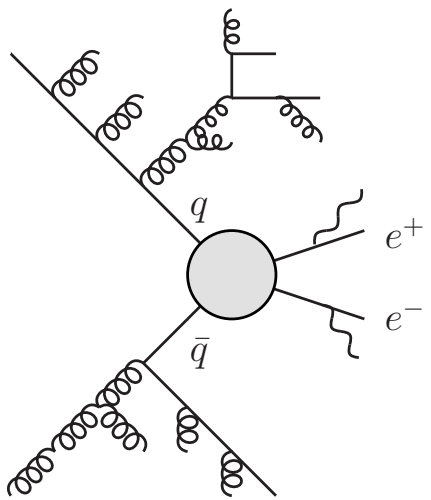
High-energy scattering at the "core" of the interaction



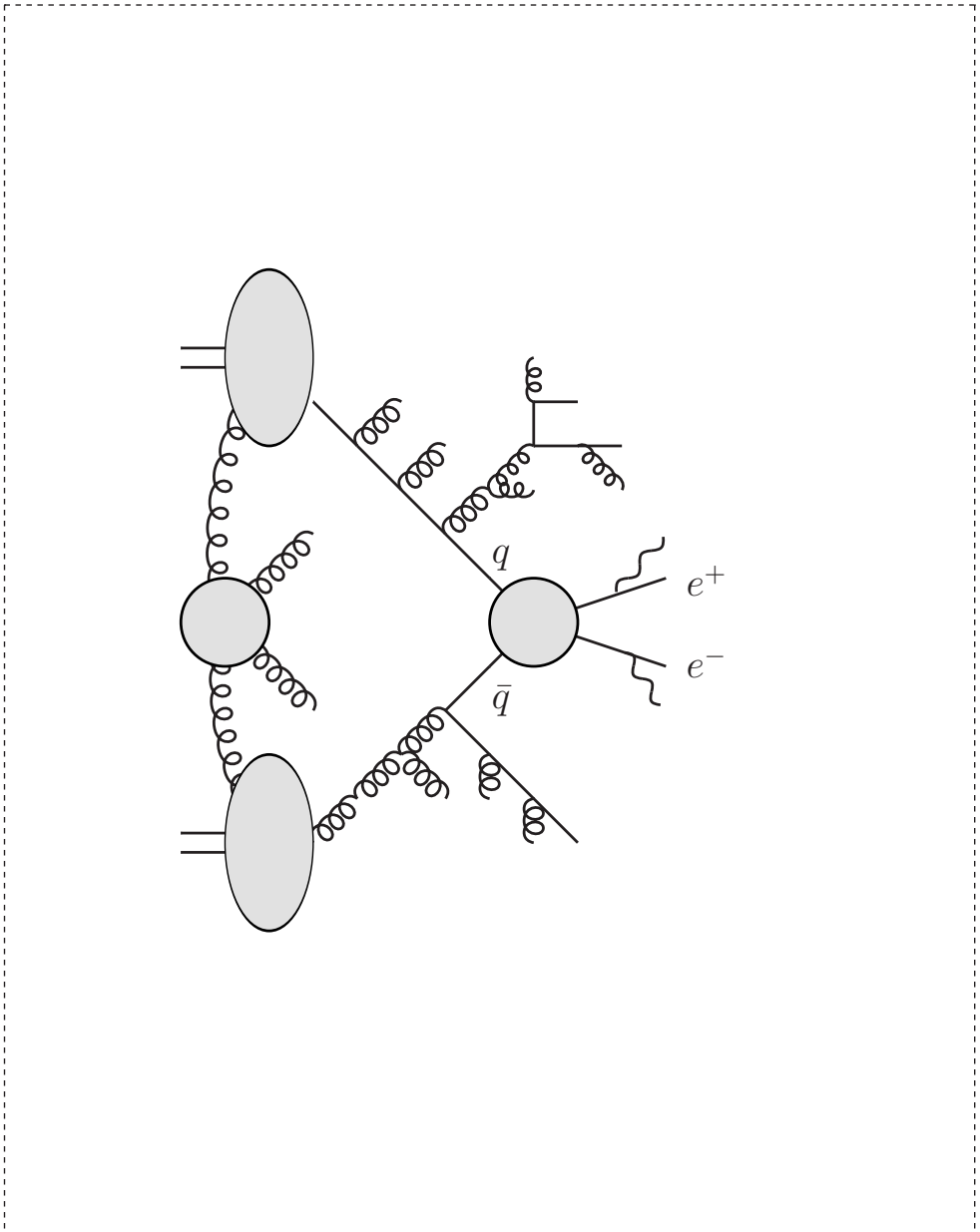
Decellerate by emitting gluons quarks arbitrarily often,



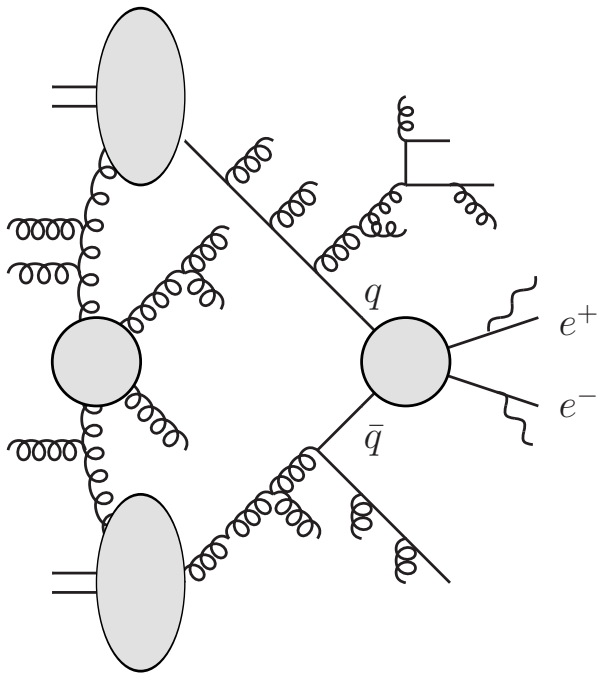
...and emit photons (or even W- or Z-bosons) from charged fermions.



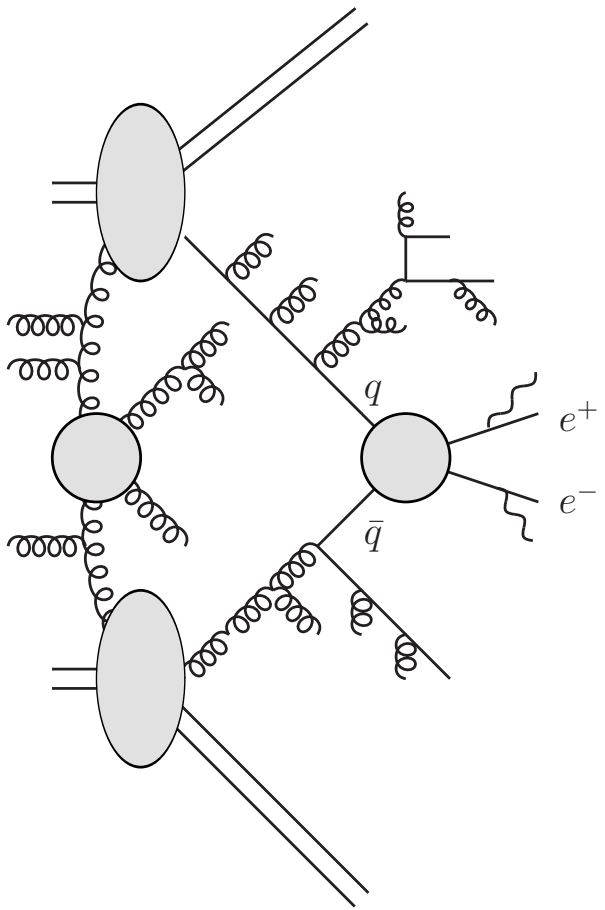
Multiple constituents of the composite protons $\Longrightarrow$ multiple interactions



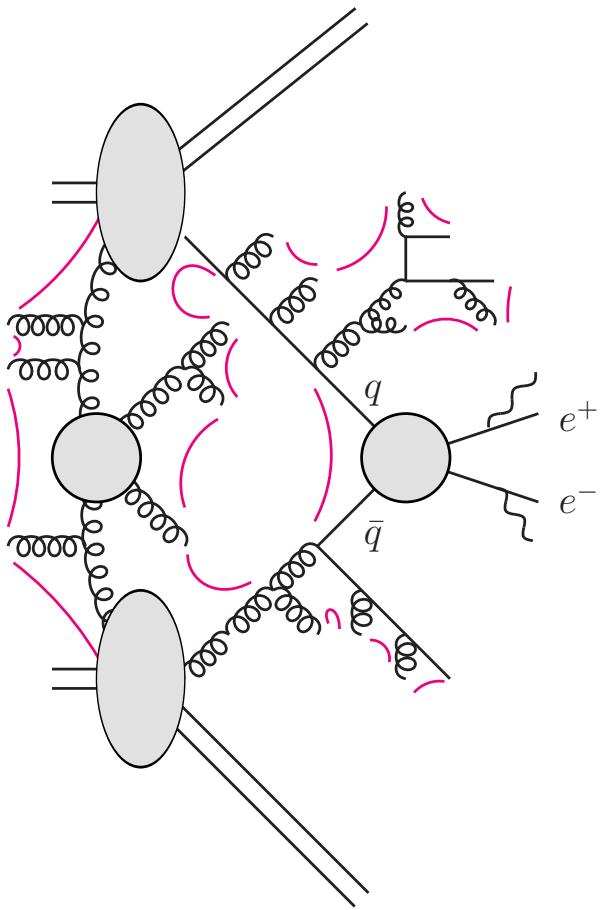
...which again produce more radiation.



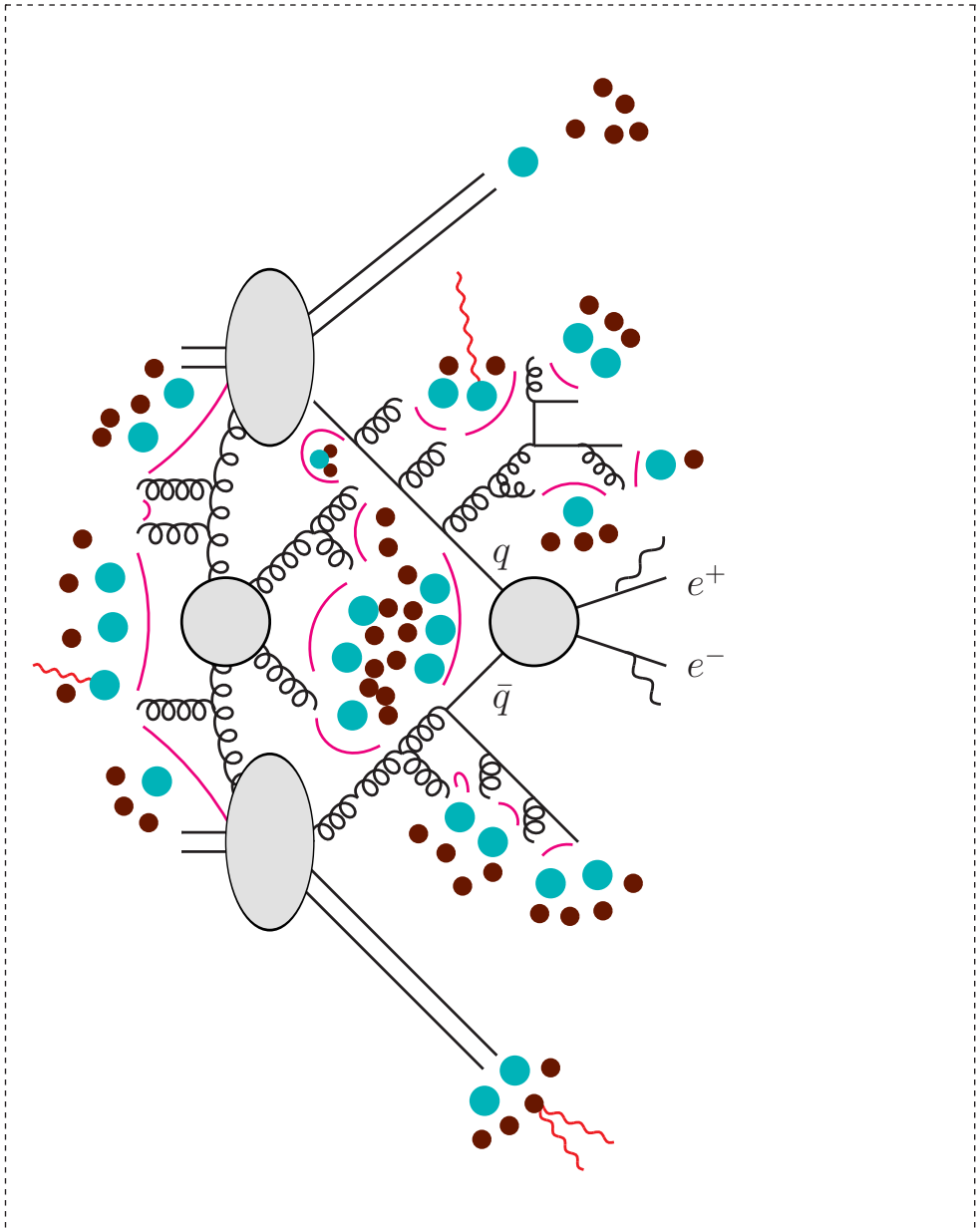
A colourless state is formed by adding beam remnants,



...forming strings (colour flux tubes),



...forming hadrons from strings / remnants, decaying excited hadrons



Our goal

Why PYTHIA8, and not PYTHIA6?

- No ep, γp or $\gamma\gamma$ incoming beams.
- Fewer in-built processes.

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- \- No ep, γp or $\gamma\gamma$ incoming beams.
- \- Fewer in-built processes.
- \+ Evolved MPI model, sophisticated diffractive machinery.
- \+ τ polarisation in production and decay.
- \+ More perturbative physics: Matching and Merging!
- \+ Simple card files: Should be better match for software frameworks. Compatible with modern in- and outputs
- \+ Simple and extensive online documentation

  `http://home.thep.lu.se/~torbjorn/pythia82html/Welcome.html`

Why PYTHIA8, and not PYTHIA6?

Sophisticated showers for ep available. $\gamma p/\gamma\gamma$ under investigation.

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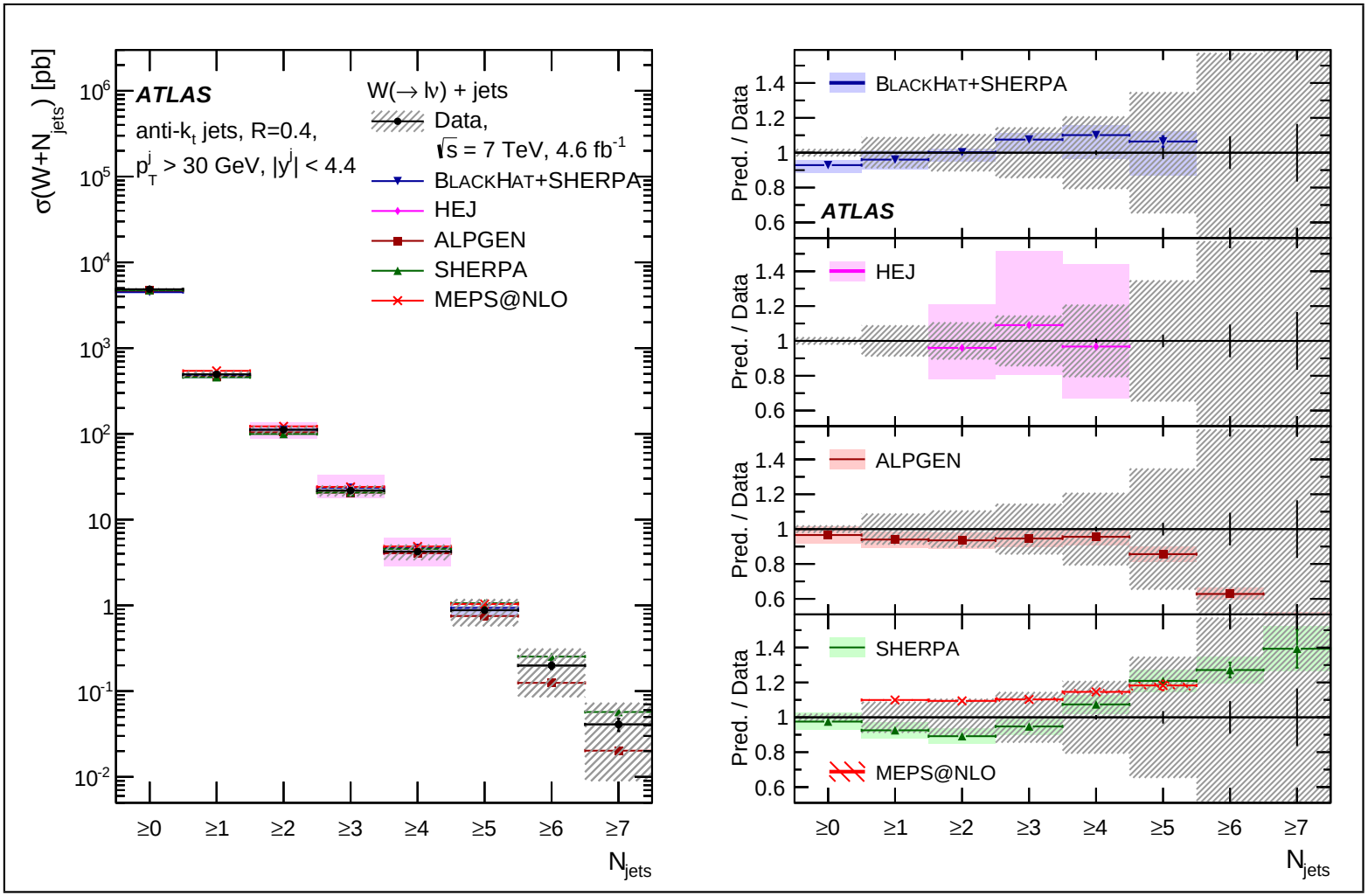
This is a plus!

Only for pp

PYTHIA6 development has stopped. PYTHIA6 not (really) supported by the developers any longer!

Event generators should describe multijet observables



(Figures taken from EPJC 75 (2015) 82)

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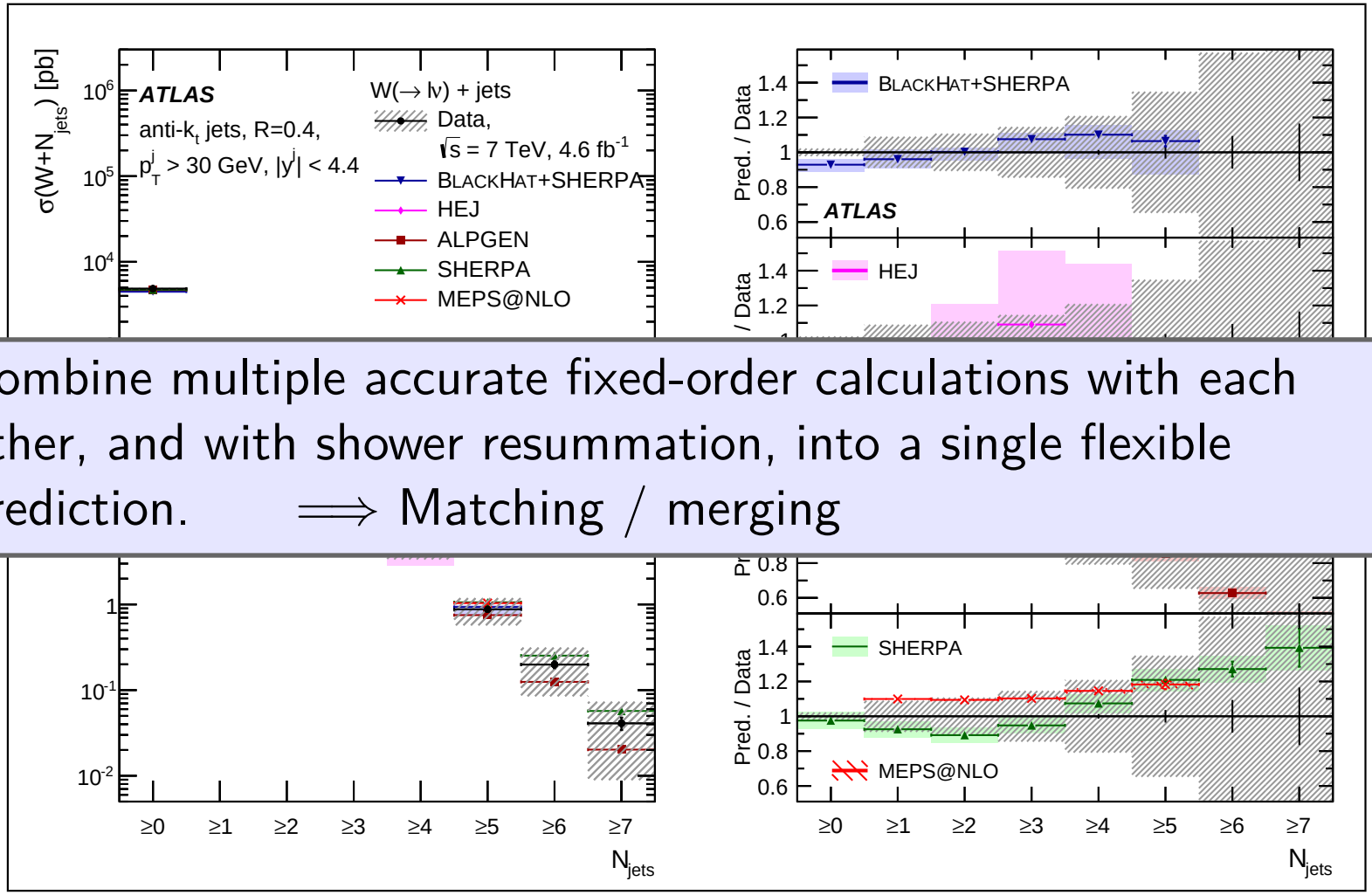

Combine multiple accurate fixed-order calculations with each other, and with shower resummation, into a single flexible prediction. $\Longrightarrow$ Matching / merging

(Figures taken from EPJC 75 (2015) 82)

Fixed-order calculations

NLO calculations on paper: Add Born + Virtual + Real.

$$\langle \mathcal{O} \rangle^{\text{NLO}} = \int \mathrm{B}_n \mathcal{O}(\Phi_n) d\Phi_n + \int \mathrm{V}_n \mathcal{O}_n(\Phi_n) d\Phi_n + \int \mathrm{B}_{n+1} \mathcal{O}(\Phi_n) d\Phi_{n+1}$$

Reality: Phase space integral separately divergent $\Rightarrow$ Add zero!

$$\langle \mathcal{O} \rangle^{\text{NLO}} = \int \left[\mathrm{B}_n + \mathrm{V}_n + \int \mathrm{D}_{n+1} \right] \mathcal{O}(\Phi_n) d\Phi_n + \int \left[\mathrm{B}_{n+1} \mathcal{O}(\Phi_{n+1}) - \mathrm{D}_{n+1} \mathcal{O}(\Phi'_n) \right] d\Phi_{n+1}$$

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Real reality: States Φ_{n+1} and Φ'_n are correlated. $\Rightarrow$ Problematic, since further manipulations (e.g. hadronisation) can spoil the cancellations

$$\begin{aligned} \langle \mathcal{O} \rangle^{\text{NLO}} &= \int \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\text{rad}} \left(\qquad - \mathrm{D}_{n+1} \right) \right] \mathcal{O}(\Phi_n) d\Phi_n \\ &+ \int \left(\mathrm{B}_{n+1} \qquad \right) \mathcal{O}(\Phi_{n+1}) \\ &+ \int \left(\qquad \right) \end{aligned}$$

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&+ \int \left(\mathrm{B}_{n+1} - \mathrm{B}'_{n+1} \right) \mathcal{O}(\Phi_{n+1}) \\
&+ \int \left(\mathrm{B}'_{n+1} \mathcal{O}(\Phi_{n+1}) - \mathrm{B}'_{n+1} \mathcal{O}(\Phi_n) \right) \longleftarrow \text{That's the } \mathcal{O}(\alpha_s) \text{ of a PS step!}
\end{aligned}$$

Parton Shower basics I

Start with the splitting kernels and Sudakov factors

$$\int d\Phi_1 \frac{\mathrm{B}'_{n+1}}{\mathrm{B}_n} = \int \frac{d\bar{t}}{\bar{t}} \int dz \frac{\alpha_s(\bar{t})}{2\pi} P(z) \qquad \Delta_a(\mu, t) = \exp\left(- \int_t^\mu \frac{d\bar{t}}{\bar{t}} \int_{z_-}^{z_+} dz \frac{\alpha_s(\bar{t})}{2\pi} P(z) \right)$$

and a generating function

$$\mathcal{F}_n(x, \mu, t) = f_n(x, t) \Delta_n(\mu, t) = f_n(x, \mu)\, \Pi_n(x, \mu, t) \ .$$

The terms P and Π are probabilities:

$$\int_t^\mu \frac{d\bar{t}}{\bar{t}} \int_{z_-}^{z_+} dz \frac{\alpha_s}{2\pi} P(z) \quad \equiv$$

Probability of a resolvable emission with "hardness" in the range $[\mu, t]$.

$$\exp\left(- \int_{\rho_{min}}^{\rho_0} \frac{d\bar{t}}{\bar{t}} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \right) \equiv$$

Probability of no resolvable emission with "hardness" in the range $[\mu, t]$.

The PS prediction $\mathcal{F}$ for an observable O is then

$$\mathcal{F}_n(\Phi_n, t_c, t_0; O) = \mathcal{F}_n(\Phi_n, t_c, t_0)\, O(\Phi_n) + \int_{t_c}^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d \ln \mathcal{F}_n(\Phi_n, \bar{t}, t_0)}{d \ln \bar{t}} \mathcal{F}_{n+1}(\Phi_{n+1}, \bar{t}, t_c; O)$$

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No emission

$\Rightarrow$ Exclusive (resummed) zero-additional-jet cross section

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$\Rightarrow$ Contains approximate virtual corrections

One (or more) emissions
$\Rightarrow$ Inclusive (resummed) one-additional-jet cross section
$\Rightarrow$ Product of splitting kernel, low-scale PDFs and Sudakov factors
$\Rightarrow$ Contains approximate virtual and real corrections

NLO matching

For NLO matching, we start out with a shower-dependent cross section and a Sudakov factor

$$\overline{\mathrm{B}}_n = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_1 \left(\mathrm{B}'_{n+1} - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_n$$

$$\Delta(t_0, t_c) = \exp\left(-\int_{t_c}^{t_0} d\Phi_1 \frac{\mathrm{B}'_{n+1}}{\mathrm{B}_n}\right)$$

and perform a PS step on $\overline{\mathrm{B}}_n$[1] and add the remainder. This gives

$$\overline{\mathrm{B}}_n \Delta(t_0, t_c) \mathcal{F}_n(\Phi_n) + \int_{t_c}^{t_0} d\Phi_1 \overline{\mathrm{B}}_n \frac{\mathrm{B}'_{n+1}}{\mathrm{B}_n} \Delta(t_0, t) \mathcal{F}_{n+1}(\Phi_{n+1}) + \left(\mathrm{B}_{n+1} - \mathrm{B}'_{n+1}\right) \mathcal{O}_{n+1}$$

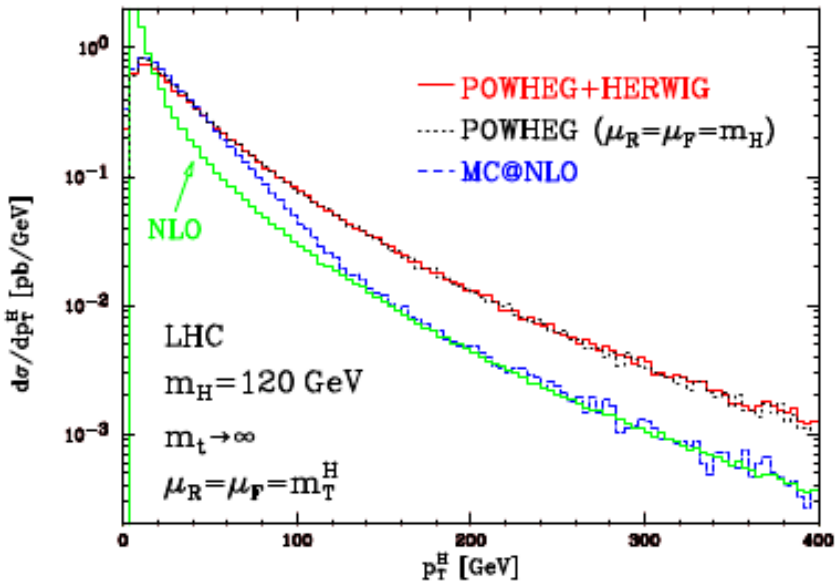
At $\mathcal{O}(\alpha_s^{n+1})$, this gives back the NLO cross section. Common schemes are

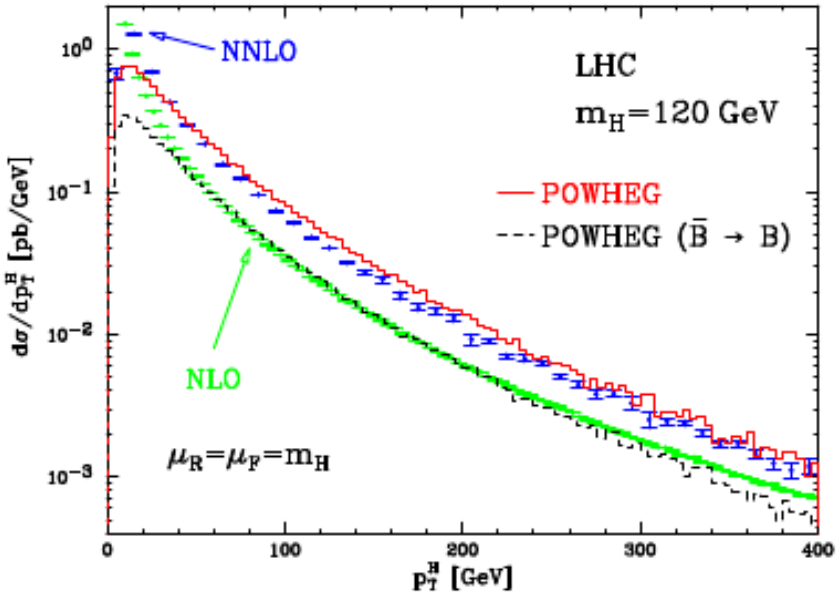
POWHEG: $\mathrm{B}'_{n+1} = \mathrm{B}_{n+1} \frac{h^2}{h^2 + p_\perp^2}$, $t_0 = s$

MC@NLO: $\mathrm{B}'_{n+1} = \mathrm{B}_n \, \alpha_s P(z) \, \Theta(\mu_Q - t(S_{+1}))$, $\mu_Q = kQ^2$

[1] Glossing over subtleties with the PS interface here.

NLO+PS matched predictions





NLO+PS methods mostly behave well, but may sometimes show large differences

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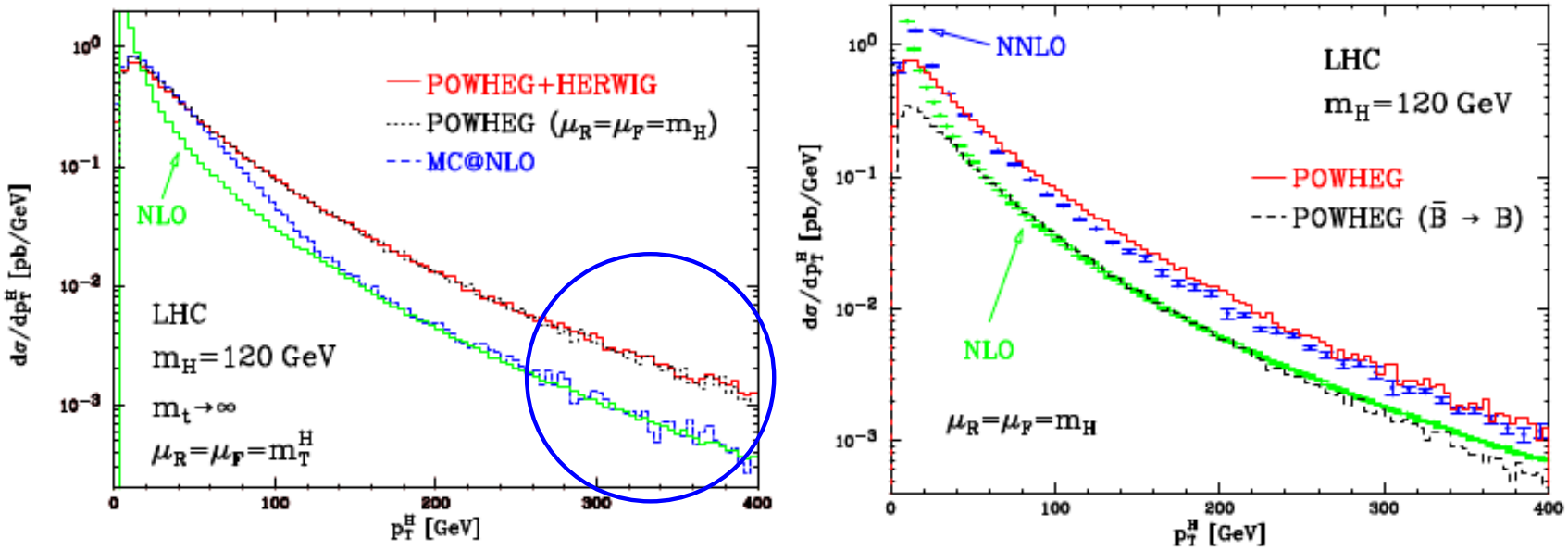

NLO+PS methods mostly behave well, but may sometimes show large differences

◇ in the real-emission dominated region, since (smearing of NLO "seed" cross section into) real emission treated differently;
⇒ Merging with further accurate multi-jet calculations helps.

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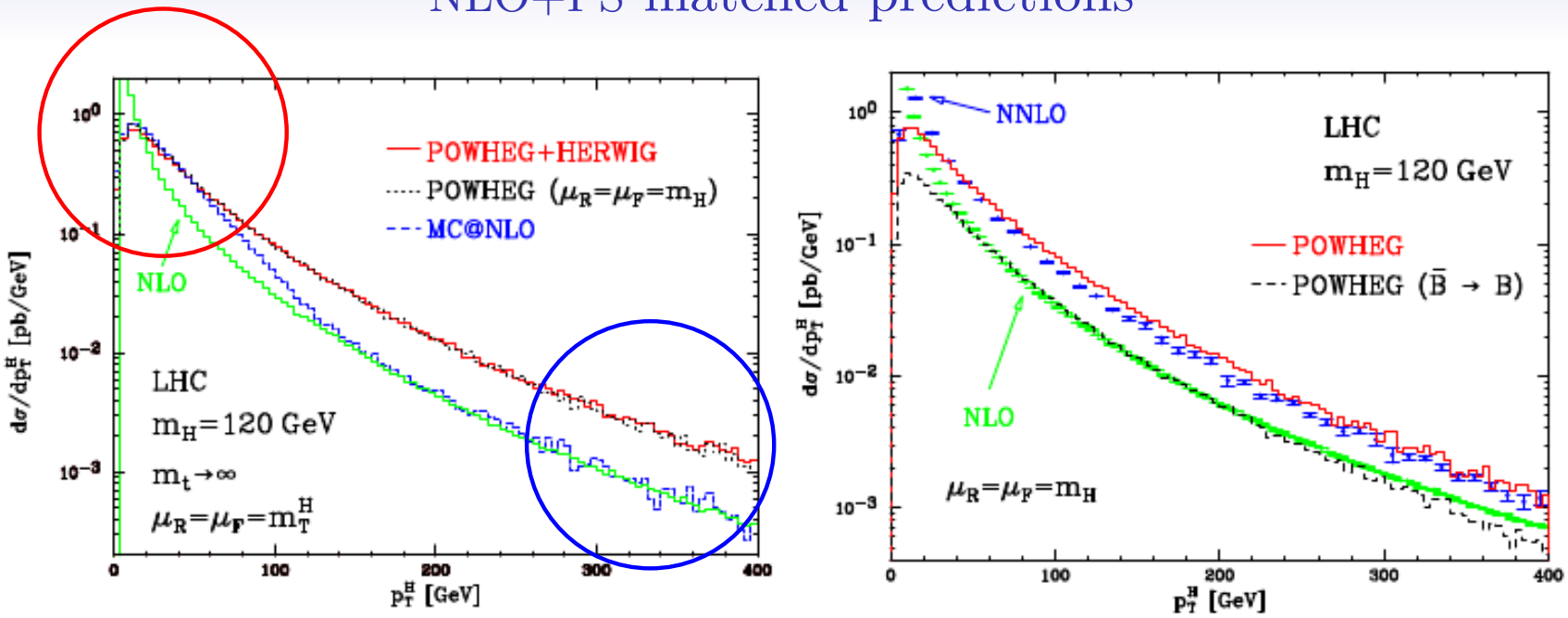

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◇ in the real-emission dominated region, since (smearing of NLO "seed" cross section into) real emission treated differently;
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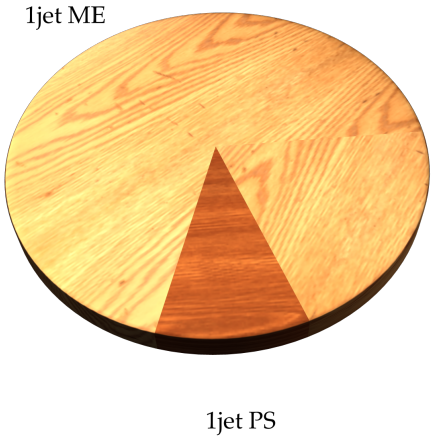
◇ in the resummation region, since exponentiation different.
⇒ Compare to inclusive resummation, improve shower accuracy.

Merging: Iterative improvements by slicing

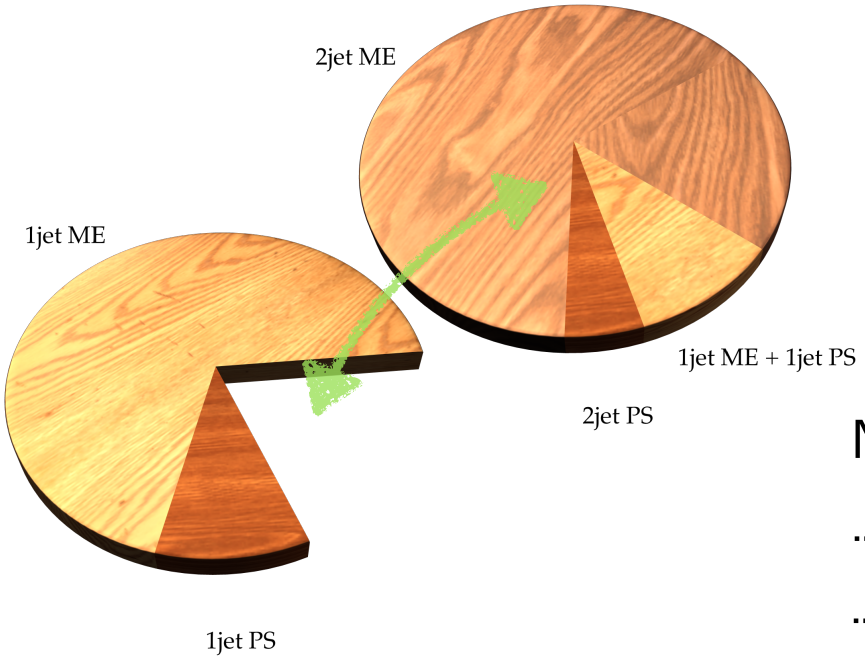


Look at one-jet states:

...use PS for soft emissions

...use ME for hard emissions

Merging: Iterative improvements by slicing

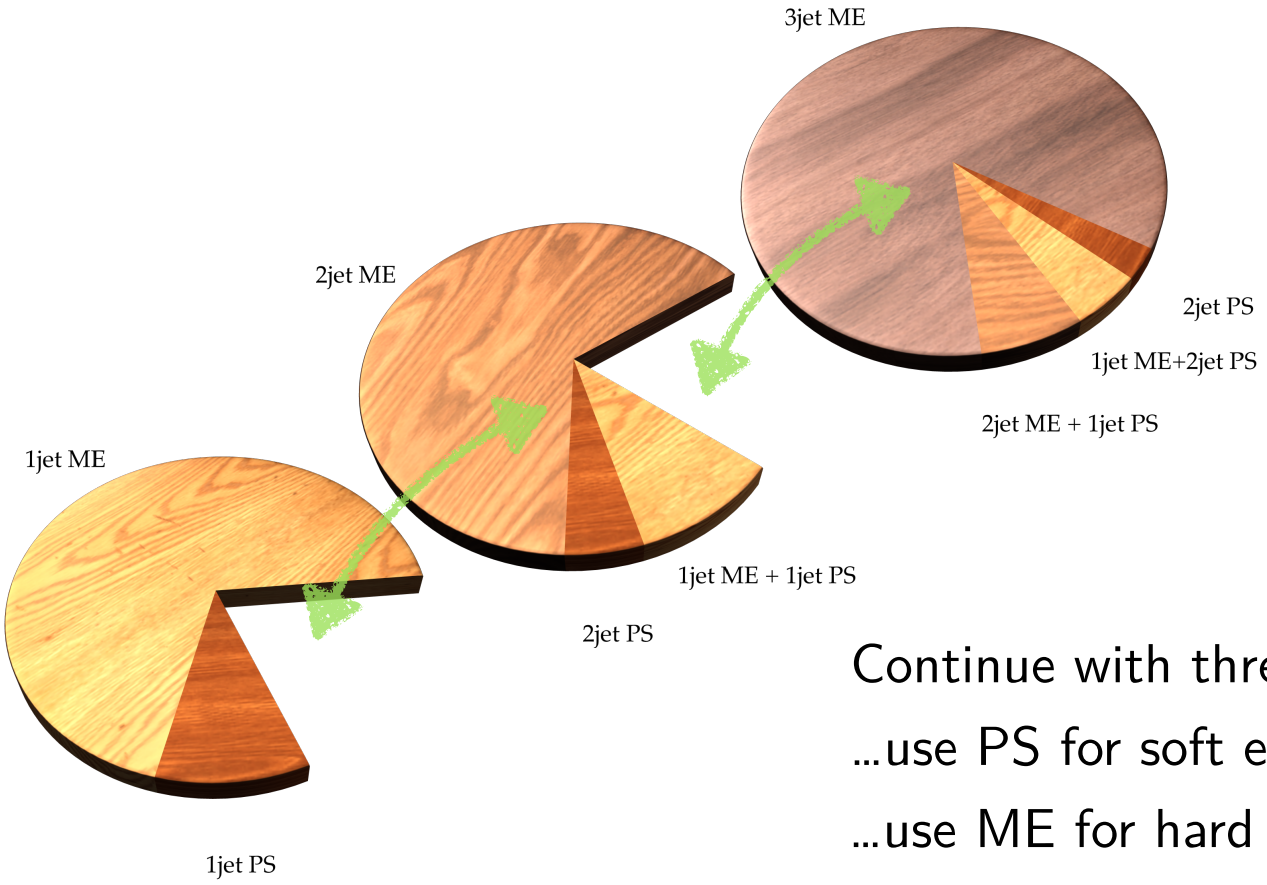


Now improve two-jet states:

...use PS for soft emissions

...use ME for hard emissions

Merging: Iterative improvements by slicing

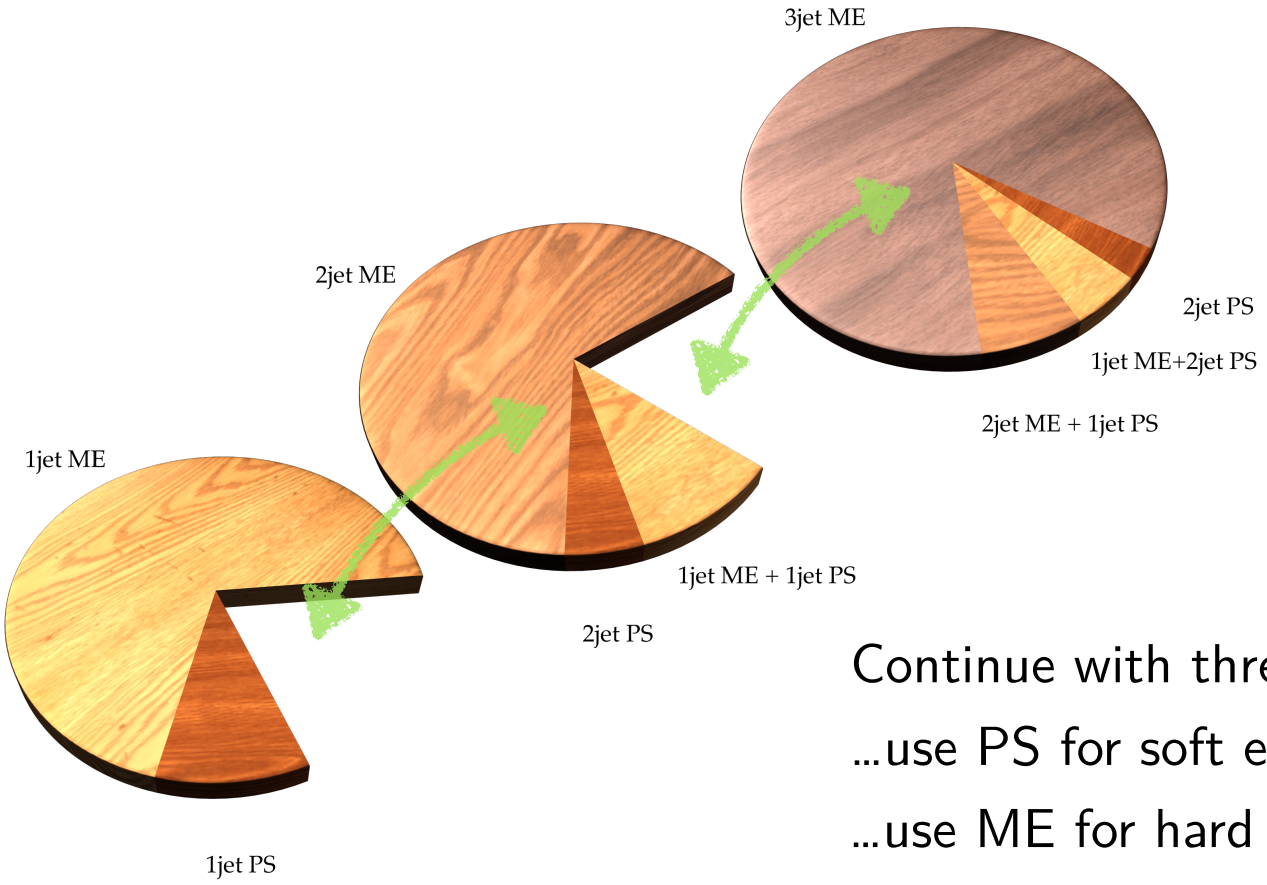


Continue with three-jet states:

...use PS for soft emissions

...use ME for hard emissions

Merging: Iterative improvements by slicing



Continue with three-jet states:

...use PS for soft emissions

...use ME for hard emissions

The dependence on the hard-soft separation (merging scale) is removed by resummation, i.e. by including Sudakov form factors.

Leading-order merging

Start with seed cross sections and Sudakov factors

$$\mathrm{B}_n\mathcal{O}_n \qquad \Delta(t_0,t) = \exp\left(-\int_t^{t_0} d\Phi_R \alpha_s(\Phi_R)P(\Phi_R)\right)$$

$$\Delta^u(t_n,t_{\mathrm{MS}}) = 1 - \int_{t_{\mathrm{MS}}}^{t_n} \mathrm{B}_{n+1}F_{n+1}(\Phi_{n+1})\Delta(t_n,t) \qquad (\text{"unitarised Sudakov"})$$

where $F_m(\Phi_m) = \Theta(t(\Phi_m) - t_{\mathrm{MS}})$, $t_0 \sim Q$. Two multiplicities merged by

$$\mathrm{B}_n\Delta^u(t_0,t_{\mathrm{MS}})\Delta(t_{\mathrm{MS}},t_{cut})\mathcal{O}_n$$

$$+\int_{t_{\mathrm{MS}}}^{t_0} d\Phi_R \left[\mathrm{B}_n\alpha_s(\Phi_R)P(\Phi_R)(1-F_{n+1})\Delta^u(t_0,t_{\mathrm{MS}})\Delta(t_{\mathrm{MS}},t_{cut}) + \mathrm{B}_{n+1}F_{n+1}\Delta(t_0,t_1)\Delta(t_1,t_2)\right]\mathcal{O}_{n+1}$$

$$+\int_{t_{\mathrm{MS}}}^{t_0} d\Phi_{RR}\ \mathrm{B}_{n+1}F_{n+1}\Delta(t_0,t_1)\Delta(t_1,t_2)\alpha_s(\Phi_R)P(\Phi_R)\mathcal{O}_{n+2} + \text{further emissions in } (t_2,t_{cut}]$$

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where $F_m(\Phi_m) = \Theta(t(\Phi_m) - t_{\mathrm{MS}})$, $t_0 \sim Q$. Three multiplicities merged by

$$\mathrm{B}_n \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \mathcal{O}_n$$

$$+ \int_{t_{\mathrm{MS}}}^{t_0} d\Phi_R \left[\mathrm{B}_n \alpha_s(\Phi_R) P(\Phi_R)(1 - F_{n+1}) \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) + \mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1) \Delta(t_1, t_2)\right] \mathcal{O}_{n+1}$$

$$+ \int_{t_{\mathrm{MS}}}^{t_0} d\Phi_{RR} \Big[\mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1)(1 - F_{n+2}) \Delta^u(t_1, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \alpha_s(\Phi_R) P(\Phi_R)$$

$$+ \mathrm{B}_{n+2} F_{n+2} \Delta(t_0, t_1) \Delta(t_1, t_2) \Delta(t_2, t_3)\Big] \mathcal{O}_{n+2} + \text{further emissions in } (t_3, t_{cut}]$$

Leading-order merging

Start with seed cross sections and Sudakov factors probabilities

$$\mathrm{B}_n\mathcal{O}_n \qquad \Delta(t_0,t) \;=\; \exp\left(-\int_t^{t_0} d\Phi_{\mathrm{rad}}\,\alpha_s(\Phi_R)P(\Phi_R)\right)$$

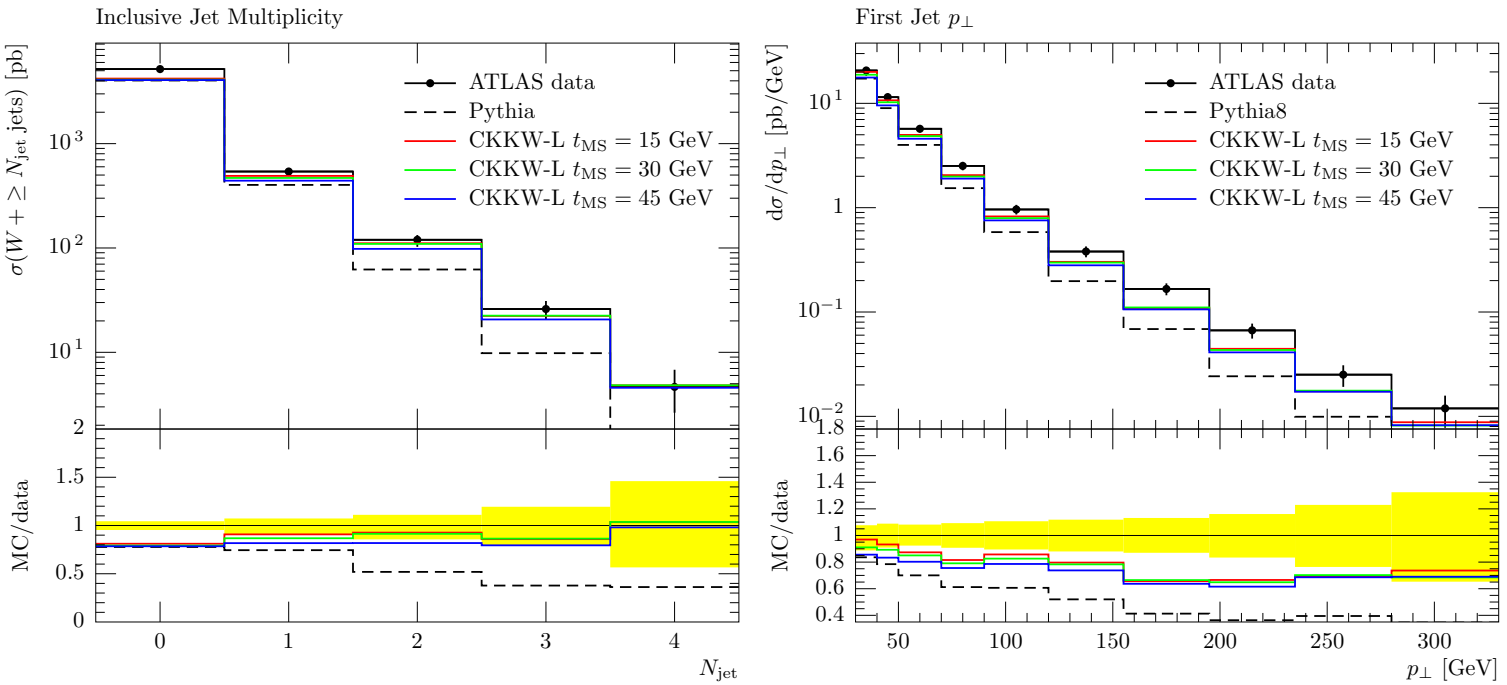
Merging methods valid for any multiplicity.
Merging methods use shower Sudakov factors $\Rightarrow$ Improve when showers improve.
NB: Usual CKKW approximation $\Delta^u(t_n,t_{\mathrm{MS}}) \to \Delta(t_n,t_{\mathrm{MS}})$ calls for $t_{\mathrm{MS}} \gg t_{cut}$ or $\mathrm{B}_n P(\Phi) \to \mathrm{B}_{n+1}$. We use $t_{\mathrm{MS}} = \mathcal{O}(1\mathrm{GeV})$ for NNLO.

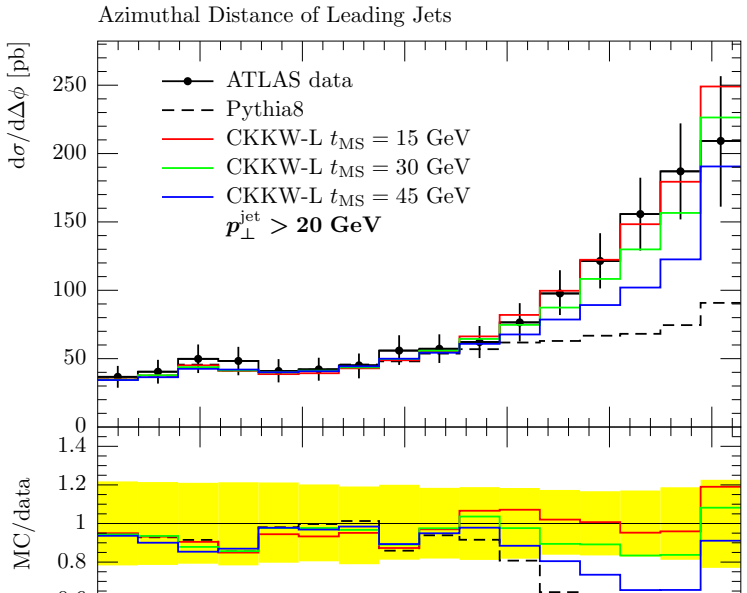
$$+\int_{t_0}^{t_n} d\Phi_R \left[\mathrm{B}_n\alpha_s(\Phi_R)P(\Phi_R)(1-F_{n+1})\Delta^u(t_0,t_{\mathrm{MS}})\Delta(t_{\mathrm{MS}},t_{cut}) \;+\; \mathrm{B}_{n+1}F_{n+1}\Delta(t_0,t_1)\Delta(t_1,t_2)\right]\mathcal{O}_{n+1}$$

$$+\int_{t_0}^{t_0} d\Phi_{RR}\Big[\mathrm{B}_{n+1}F_{n+1}\Delta(t_0,t_1)(1-F_{n+2})\Delta^u(t_1,t_{\mathrm{MS}})\Delta(t_{\mathrm{MS}},t_{cut})\alpha_s(\Phi_R)P(\Phi_R)$$

$$+\;\; \mathrm{B}_{n+2}F_{n+2}\Delta(t_0,t_1)\Delta(t_1,t_2)\Delta(t_2,t_3)\Big]\mathcal{O}_{n+2} \;+\; \text{further emissions in } (t_3,t_{cut}]$$

LO merged results





Predictions from leading-order merged PYTHIA8 for W+jets at LHC Run I, compared to ATLAS data.

Differences merging/matching

NLO matching is NLO-correct.

$\Longrightarrow$ Good uncertainty estimate, limited applicability.

Merging can be used to combine "any number" of LO calculations.

$\Longrightarrow$ Questionable uncertainty, broad applicability.

We can be lucky if

...NLO matched calculation describes very exclusive data.

...merged calculations describe normalisations.

It would be unreasonable to expect

Luck in one process = Luck in another process

$\Rightarrow$ Both strategies are incomplete and need to be combined for a satisfactory result.

NLO merging

Use the full NLO results for any multi-jet state!

NLO multi-jet merging from LO scheme **X**:

- ◇ Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add multiple NLO calculations.
- ◇ Make sure fixed-order calculations do not overlap careful by cutting, vetoing events and/or vetoing emissions (e.g. remove real contribution to n-jet in favour of $n+1$-jet NLO).
- ◇ Adjust higher orders to suit other needs (e.g. to preserve the inclusive cross section).

$\Rightarrow$ **X**@NLO

Comparison of NLO merging schemes

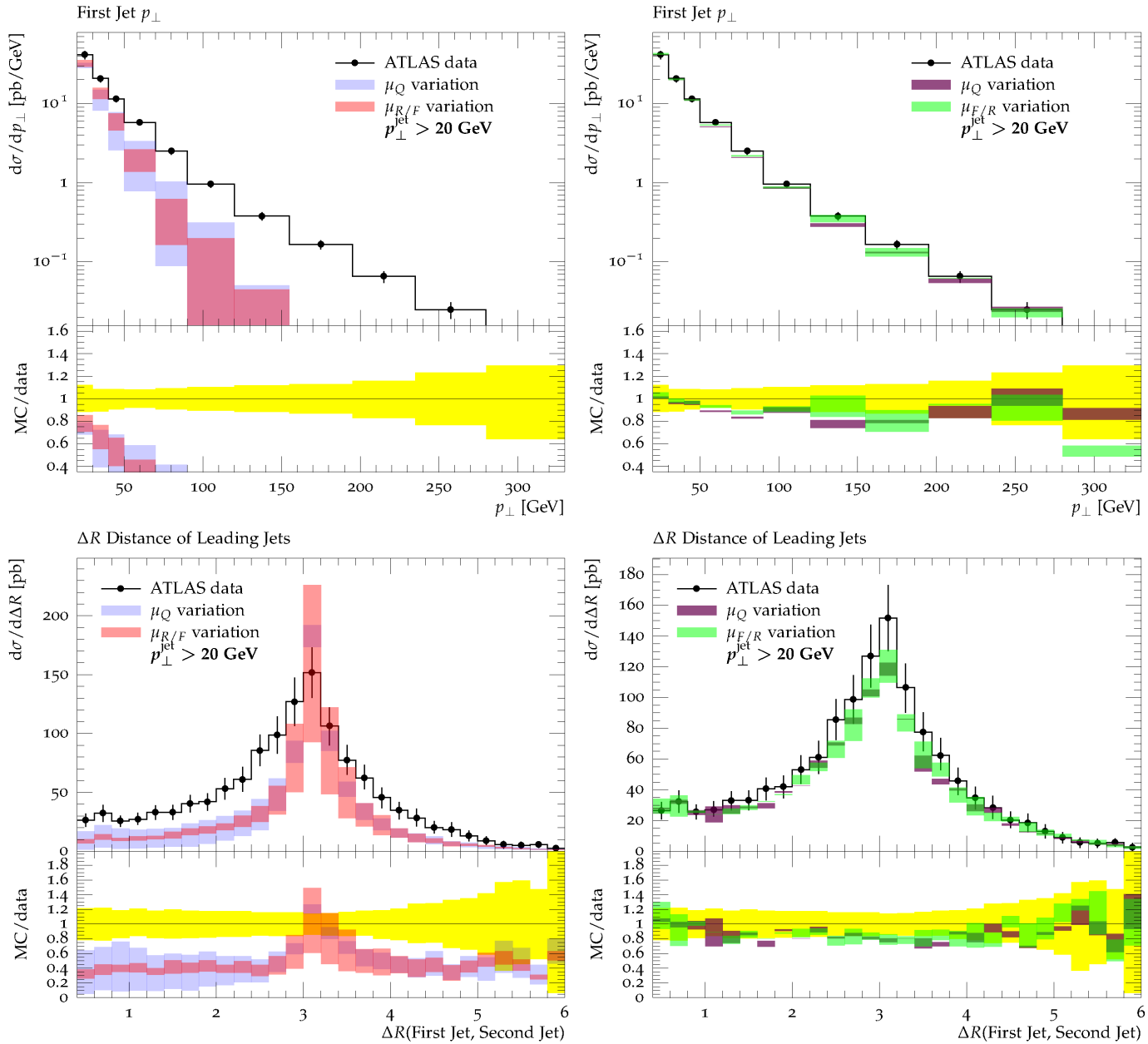
FxFx: Restricted range of merging scales. Accuracy unclear.
Probably fewest counter events.

MEPS@NLO: Larger t_{MS} range due to colour-correct Sudakov of MC@NLO for the first emission. Small cross section changes.

UNLOPS: Inclusive observables strictly NLO correct. Further shower improvements also directly improve the results.
Many counter events if done naively.
Was moulded into an NNLO matching.

MiNLO: applies analytical (N)NLL Sudakov factors, which cancel problematic logs, only merging two multiplicities.
Was moulded into an NNLO matching.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO-merged predictions for Higgs + jets

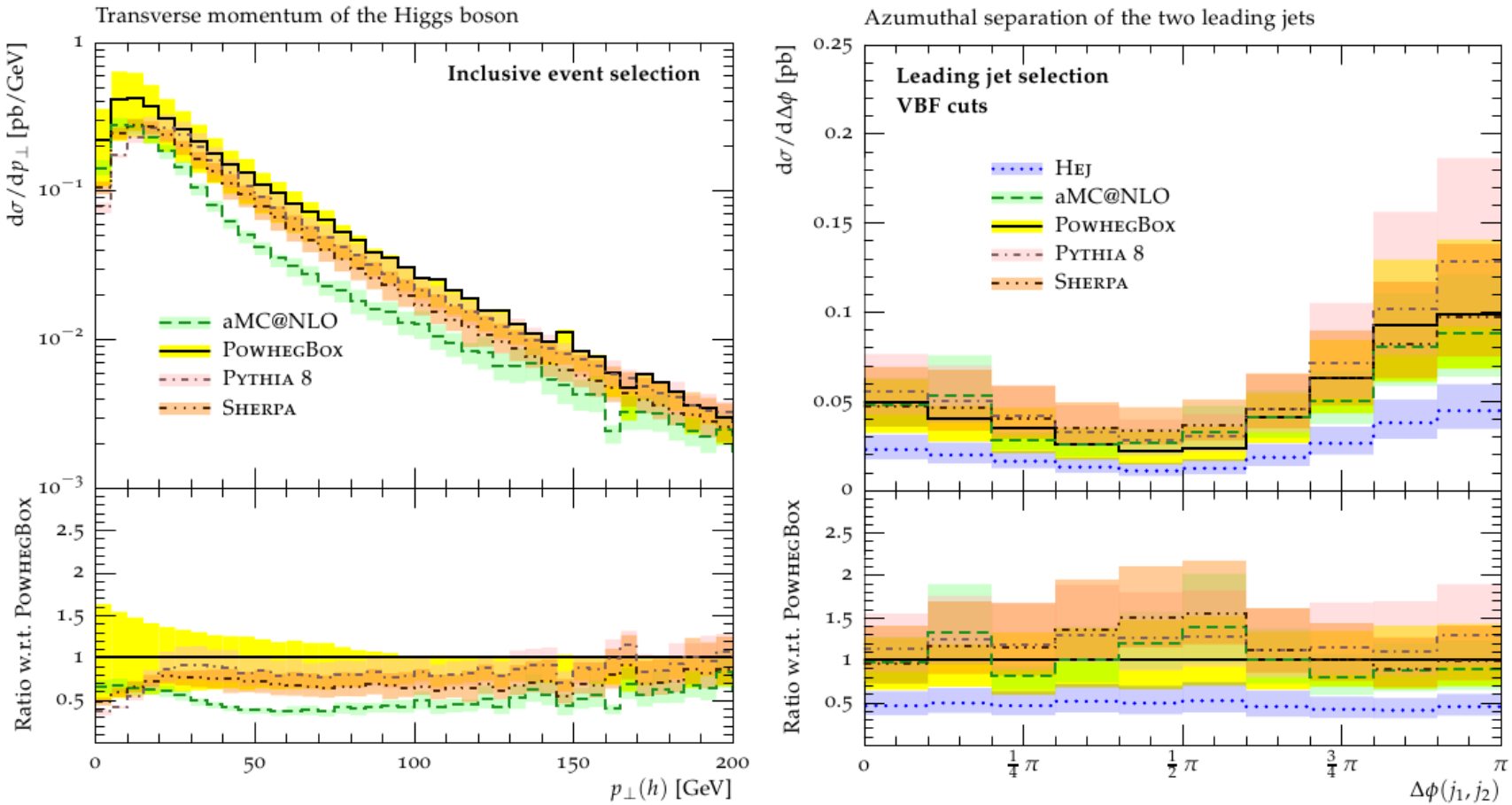
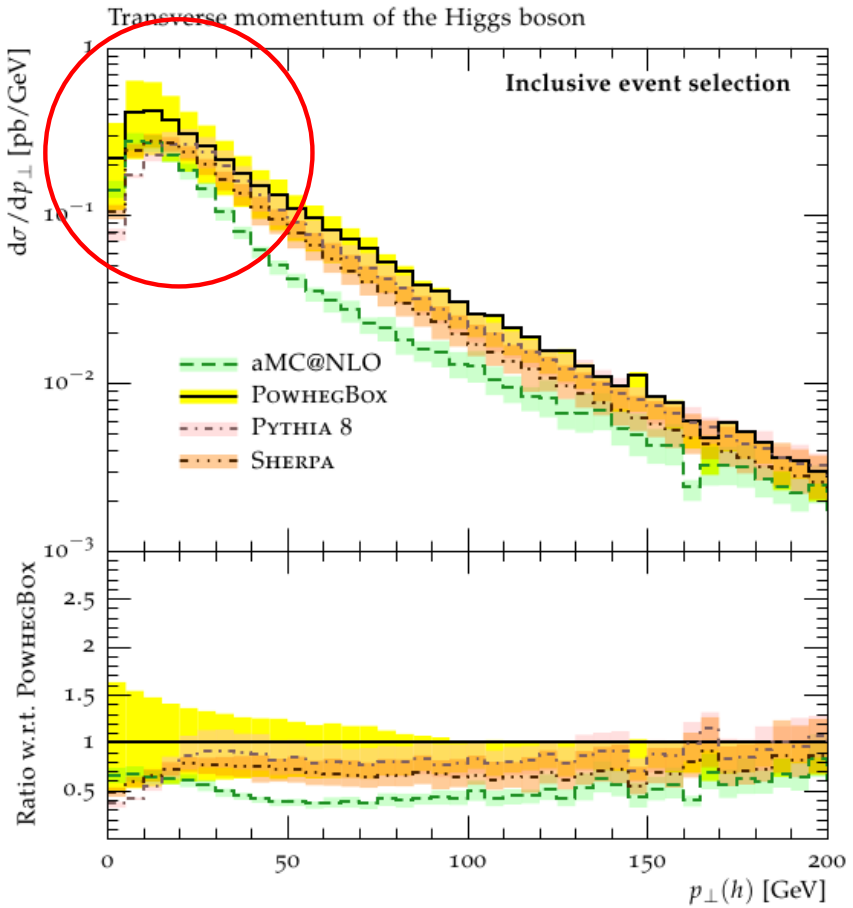
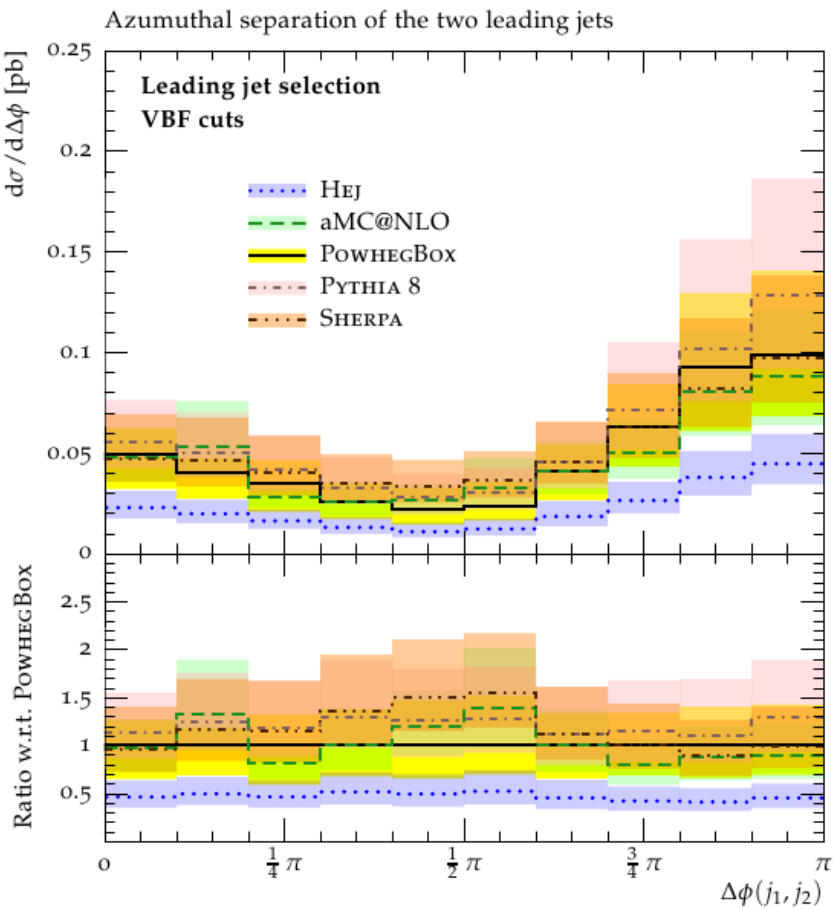


Figure: $p_{\perp,H}$ and $\Delta\phi_{12}$ for gg→H after merging (H+0)@NLO, (H+1)@NLO, (H+2)@NLO, (H+3)@LO, compared to other generators.

⇒ LH study: The generators come closer together if NLO merging is employed.

NLO-merged predictions for Higgs + jets





Still $\mathcal{O}(50\%)$ differences in resummation region – important uncertainty for analyses that use jet binning!

$\Rightarrow$ LH study: The generators come closer together if NLO merging is employed.

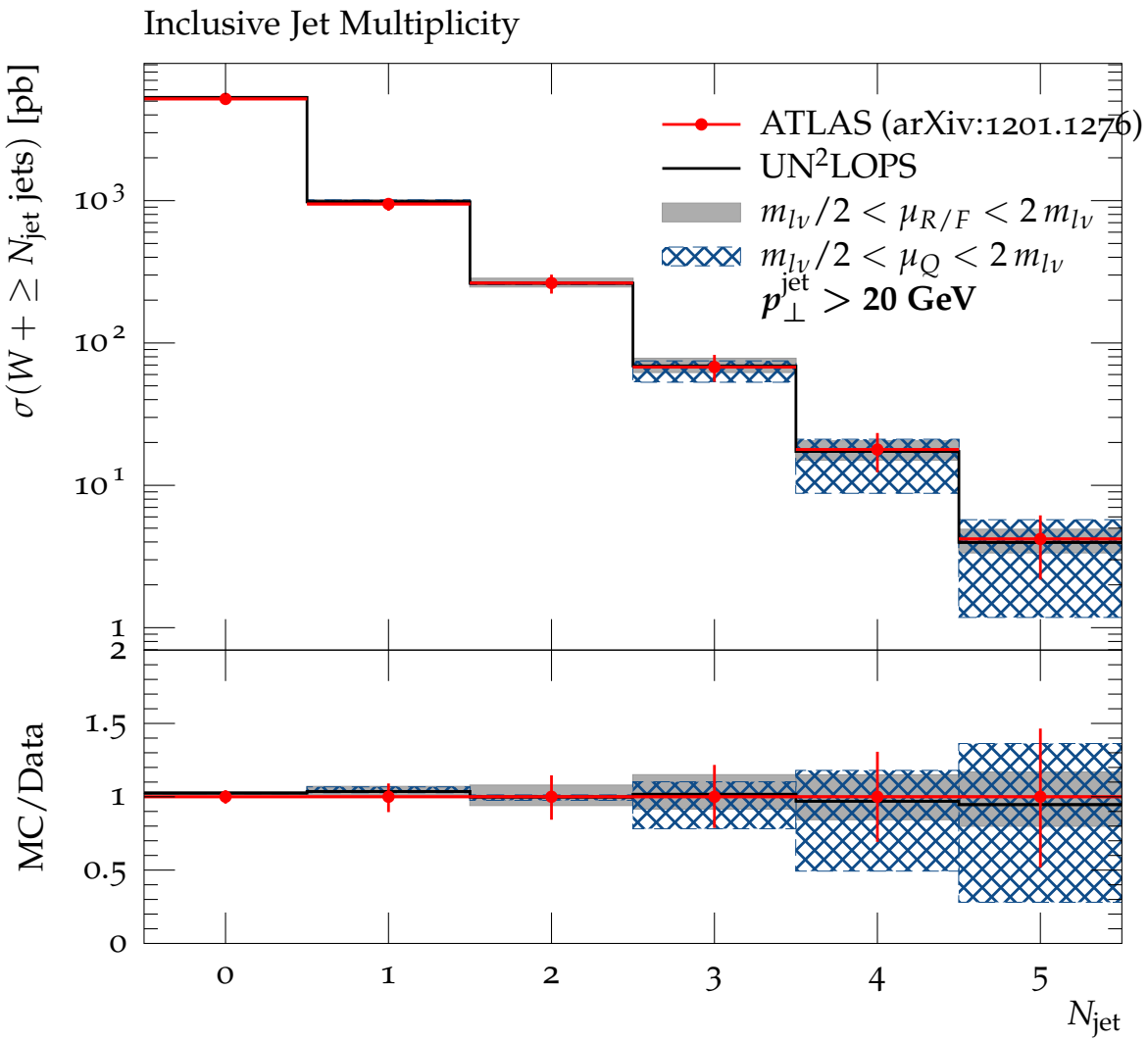
The next step(s): Matching @ NNLO

Aim: For important processes – lumi monitors like Drell-Yan, precision studies (ggH, ZH, WBF,...) – reduce uncertainties and remove personal bias. But make sure all other improvements stay intact!

Observation: If an NLO merged calculation leads to a well-defined zero-jet inclusive cross section, it is easy to upgrade this cross section to NNLO.

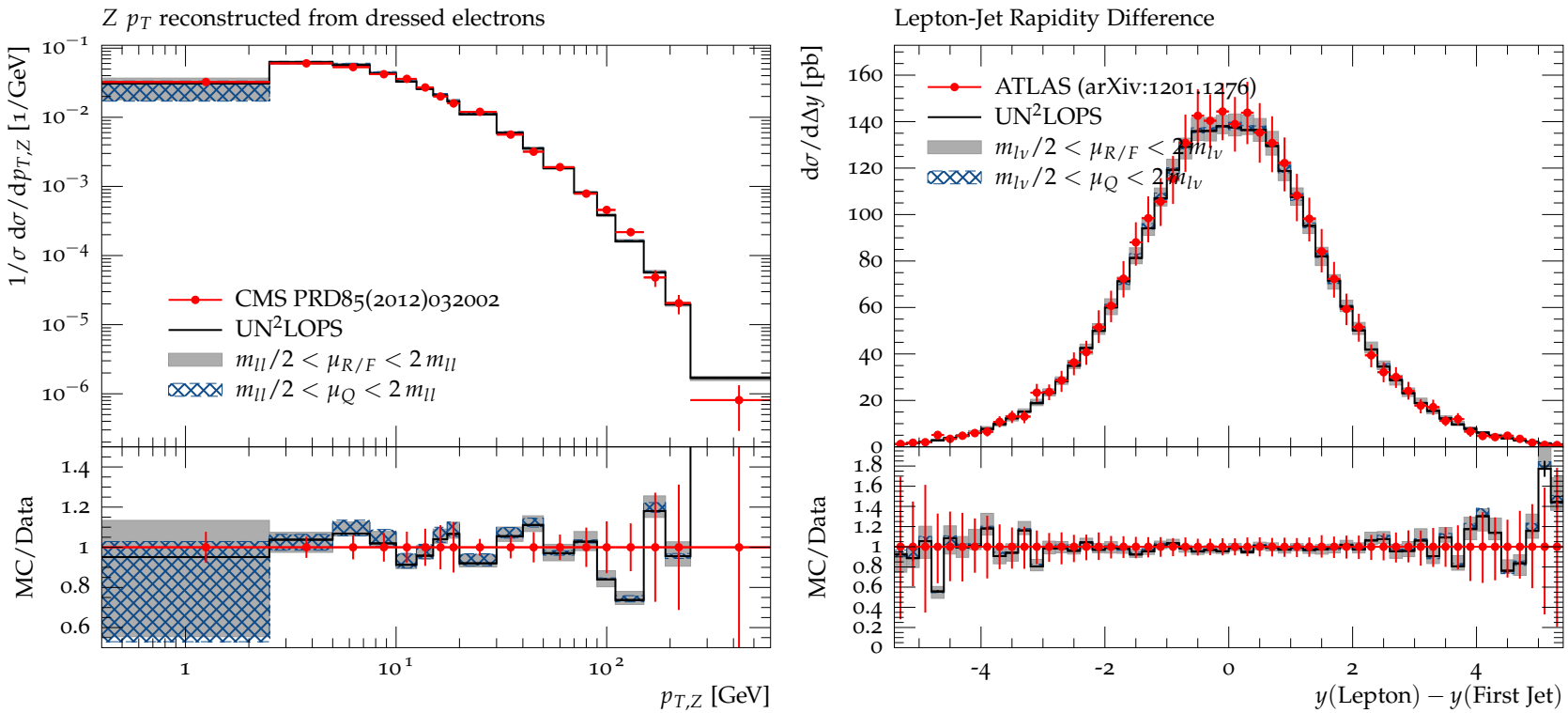
$\Longrightarrow$ Fulfilled by two NLO merging schemes: MiNLO and UNLOPS

UN^{2}LOPS (neutral current Drell-Yan)



n_{jets} in association with a Z-boson. UN^{2}LOPS has (very) small uncertainty in zero/one-jet rate. Shower uncertainty larger for higher multiplicities.

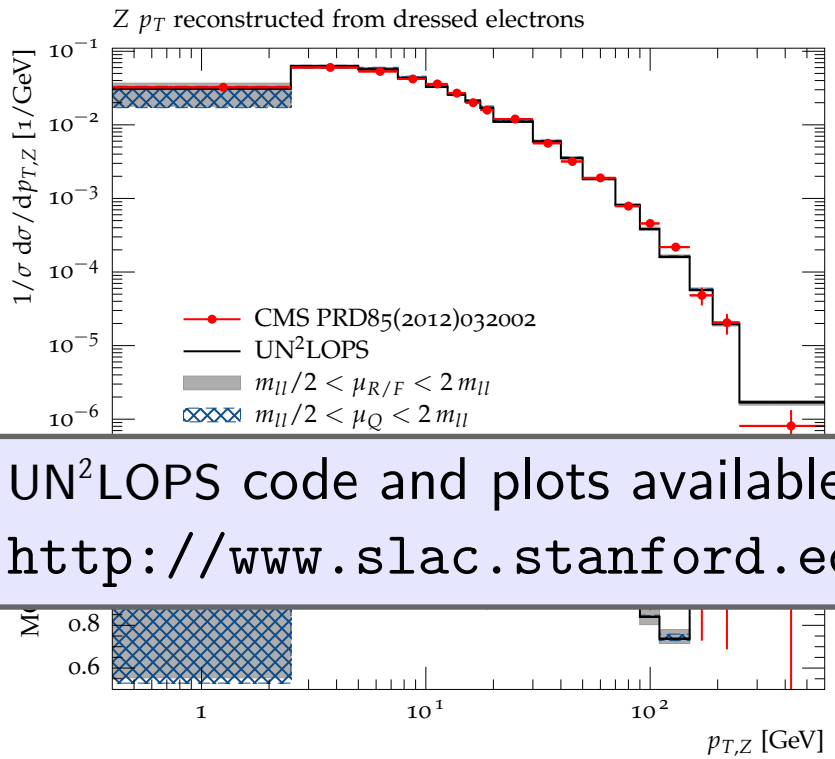
UN^{2}LOPS (neutral current Drell-Yan)

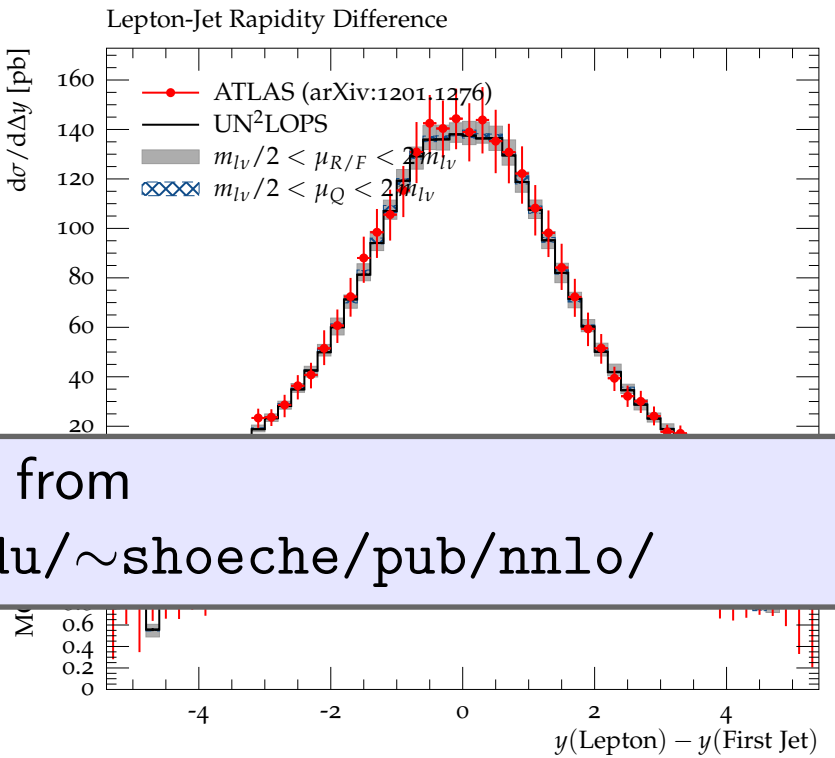


CMS data for Z-boson $p_\perp$. UN^{2}LOPS does quite well. Large band at low $p_\perp$ reflects log scale and shower modelling. ATLAS data for charged current well described.

(Work with Stefan Höche and Ye Li)

UN^{2}LOPS (neutral current Drell-Yan)





UN^{2}LOPS code and plots available from

`http://www.slac.stanford.edu/~shoeche/pub/nnlo/`

CMS data for Z-boson $p_\perp$. UN^{2}LOPS does quite well. Large band at low $p_\perp$ reflects log scale and shower modelling. ATLAS data for charged current well described.

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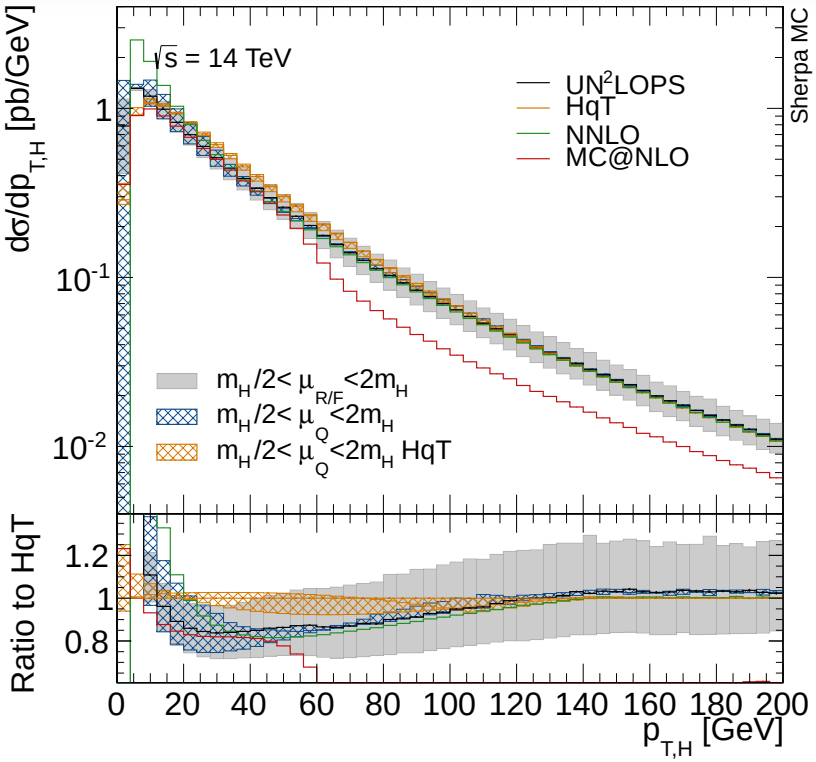
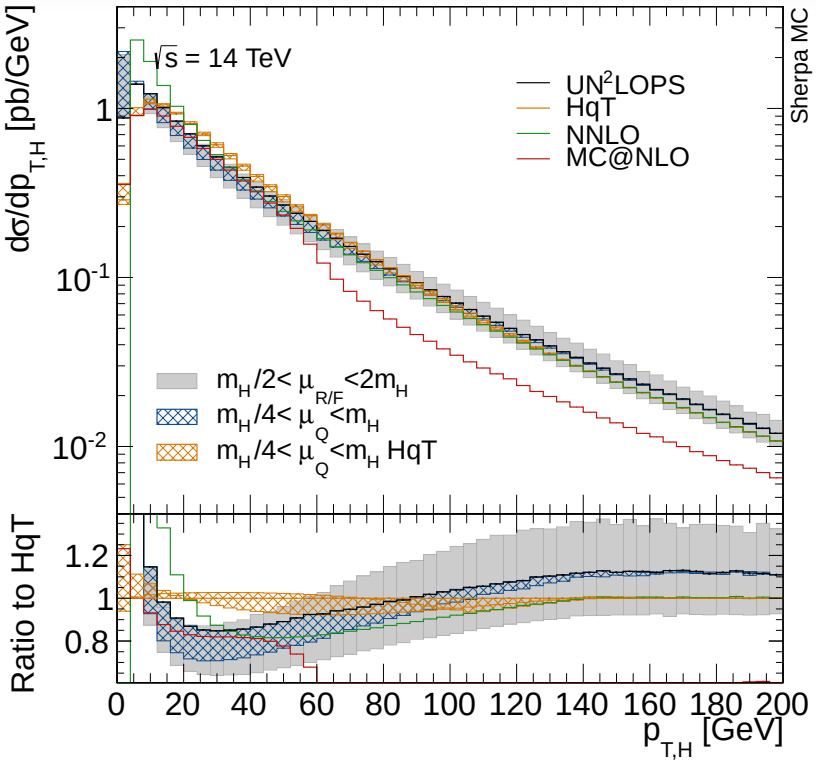
UN^{2}LOPS (Higgs production)

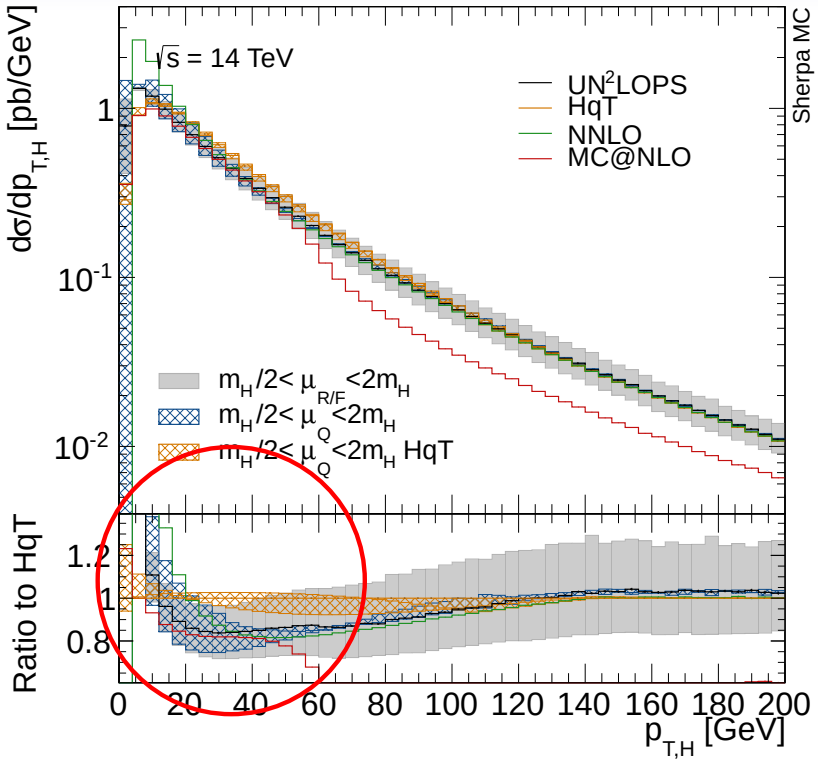


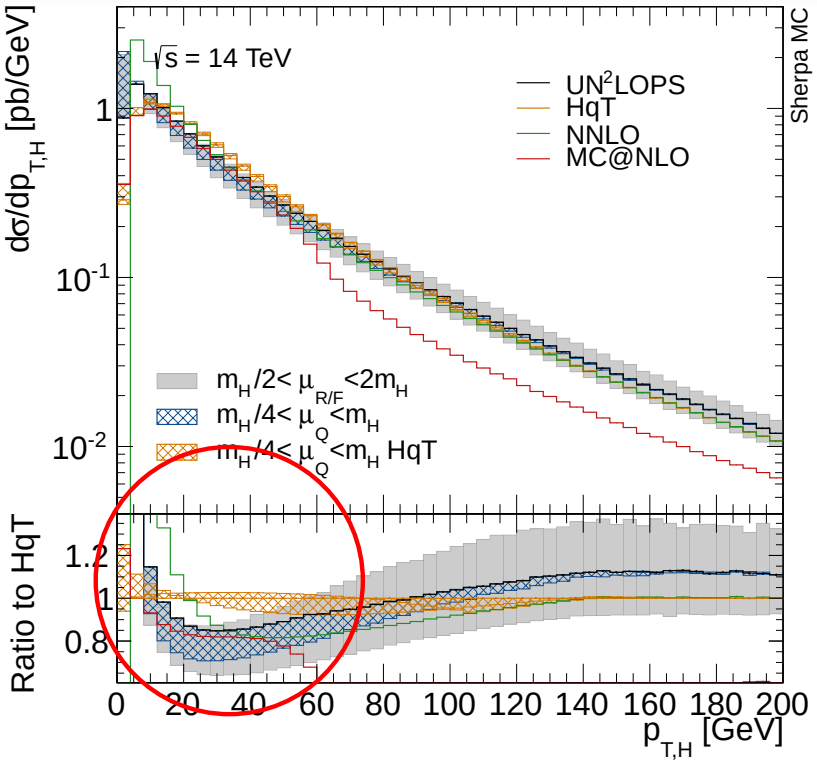


$p_\perp$ of the Higgs-boson for two different matching schemes in UN^{2}LOPS – mimicing the philosophical differences between common NLO matching schemes.

(Work with Stefan Höche and Ye Li)

UN^{2}LOPS (Higgs production)





We have NLO matching, NLO merging and NNLO matching. Still, predictions differ appreciably because of limited shower accuracy.

Reliable parton showering

We have hints that current parton showers (e.g. PYTHIA, SHERPA) are not ideal in the Sudakov region. This can mean

- Problems in the (matching of shower and) fixed-order parts
- Problems in the parton showers
- Problems in the non-perturbative modelling
- Bugs

$\Rightarrow$ We want a simple, theoretically clean, extendable parton shower, to so that comparison to observable-based resummation becomes possible.

...and we want to face data from ee, ep and pp experiments to find out what the "QCD structure" of the parton shower should be.

Reliable parton showering

We want a

Simple

(Simple splitting functions, simple phase space boundaries…)

Theoretically clean

(Recover eikonal in soft limit, AP kernels in collinear limit, "collinear" anomalous dimensions as in analytic resummation, flavour/momentum sum rules, no choices introducing iffy subleading logs…)

Extentable

(Updating splitting functions, for QCD, QED, EW…)

parton shower.

$\Rightarrow$ Need exact, massive phase space factorisation and full phase space coverage, need QCD coherence, need sum rules.

Reliable parton showering

We have defined a new dipole-parton shower, and implemented independently in the PYTHIA and SHERPA to minimise bug contamination (arXiv:1506.05057). Codes will become PYTHIA and SHERPA plugins.

The PS prediction $\mathcal{F}$ for an observable O is then

$$\mathcal{F}_n(\Phi_n, t_c, t_0; O) = \mathcal{F}_n(\Phi_n, t_c, t_0)\, O(\Phi_n) + \int_{t_c}^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d \ln \mathcal{F}_n(\Phi_n, \bar{t}, t_0)}{d \ln \bar{t}} \mathcal{F}_{n+1}(\Phi_{n+1}, \bar{t}, t_c; O)$$

From this, we find

$$\begin{aligned}
\mathcal{F}_n(\Phi_n, t_0, t_c; O) \quad &\xrightarrow[t_c \to 0]{} \quad \int^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d \ln \mathcal{F}_n(\Phi_n, t_0, \bar{t})}{d \ln \bar{t}} \mathcal{F}_{n+1}(\Phi'_{n+1}, \bar{t}, t_c; O) \\
&\xrightarrow[\text{inclusive}]{} \quad \int^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d \ln \mathcal{F}_n(\Phi_n, t_0, \bar{t})}{d \ln \bar{t}} \mathcal{O}_{n+1} \\
&= \quad \int^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d\sigma^{\text{resummed}}}{dt dz d\Phi_0} \mathcal{O}_{n+1}(\Phi_{n+1})
\end{aligned}$$

$\Rightarrow$ The PS emission rate is a product of perturbative TMDs and perturbative jet functions, matched onto collinear PDFs.
$\Rightarrow$ Carefully defined shower can be compared to analytic resummation.

Deriving a DIRE shower

Catani-Seymour dipoles are a nice starting point because

A capture correct divergences at $\mathcal{O}(\alpha_s)$ (including coherence)

B correct single log for $q_\perp$ (even with trivial phase space boundaries).

C come with exact phase space factorisation

Deriving a DIRE shower

Catani-Seymour dipoles are a bad starting point because

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   $\leftarrow$ unified by good phase space parametrization + combining with Jacobian

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3. not immediately applicable for evolution, because not normalized by momentum / flavour sum rules ← demand sum rules exactly.

Addressing these points completely fixes *all* (II, IF, FI, FF) dipoles to

$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \frac{3}{2}\,C_F\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \delta(1-z)\left(\frac{11}{6}C_A - \frac{2}{3}n_f T_R\right)$$

$$P_{gq}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{qg}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

Exact phase space factorisation

$\Rightarrow$ Borrow PSP factorisation of Catani and Seymour NLO subtraction:

$$d\Phi(p_i, p_j, Q + p_a) = \int dx d\Phi(\tilde{p}_{ij}, Q + x p_a) dp_i \qquad dp_i = \frac{2\tilde{p}_k p_a}{16\pi^2} \frac{d\phi}{2\pi} dx_{ij,a} d\tilde{z}_i$$

$$\text{where} \quad x_{ij,a} = \frac{s_{ia} + s_{ja} - s_{ij}}{s_{ia} + s_{ja}} \quad \text{and} \quad \tilde{z}_i = \frac{s_{ia}}{s_{ia} + s_{ja}}$$

for final-initial splittings. For initial-final splittings, the PSP factorises as

$$d\Phi(p_i, p_k, Q + p_a) = \int dx d\Phi(\tilde{p}_k, Q + x p_a) dp_i \qquad dp_i = \frac{2\tilde{p}_k p_a}{16\pi^2} \frac{d\phi}{2\pi} dx_{ik,a} du_i$$

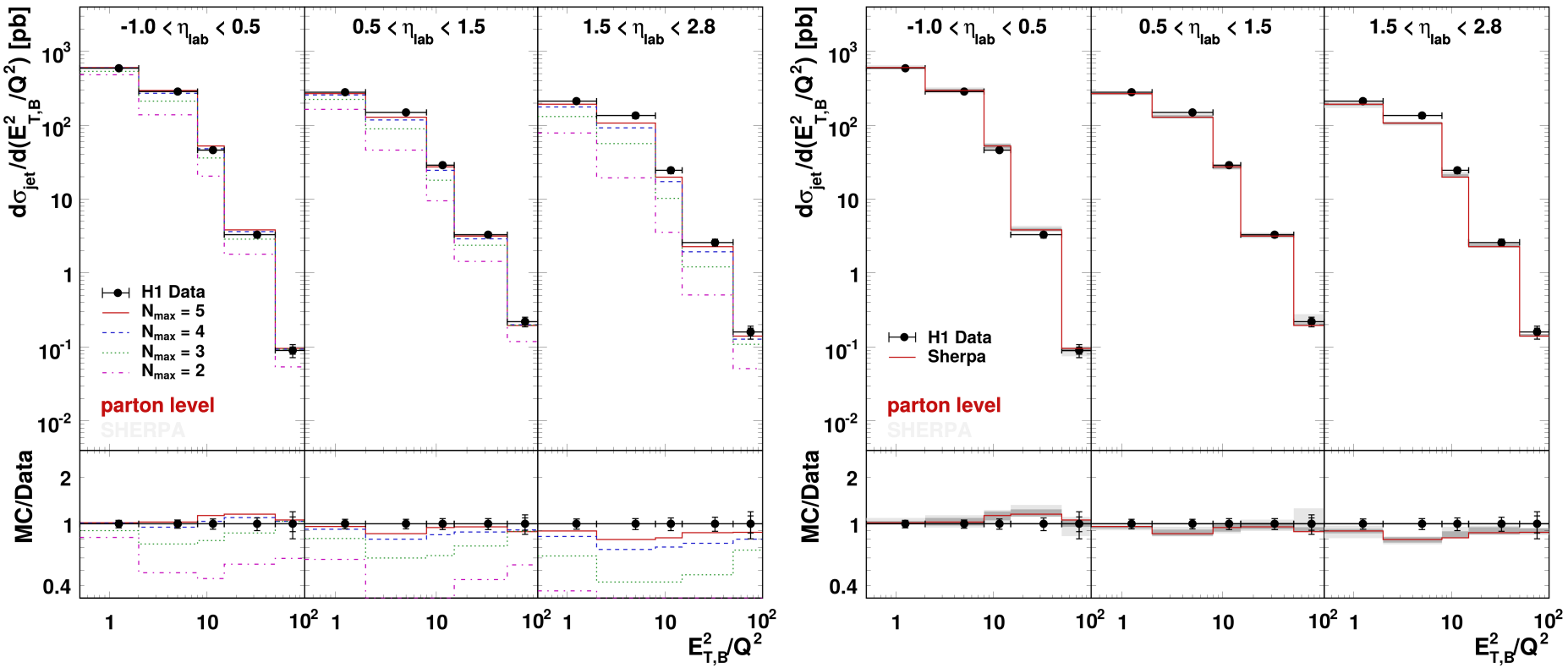
$$\text{where} \quad x_{ik,a} = \frac{s_{ai} + s_{ak} - s_{ik}}{s_{ai} + s_{ak}} \quad \text{and} \quad u_i = \frac{s_{ai}}{s_{ai} + s_{ak}}$$

PSP factorisation is exact and invertible, no approximations are involved.

$\Longrightarrow$ Matching / merging become (directly) accessible for ep scattering!

(NB: Bjorken-x or Q^2 values of the hard process not changed by shower)

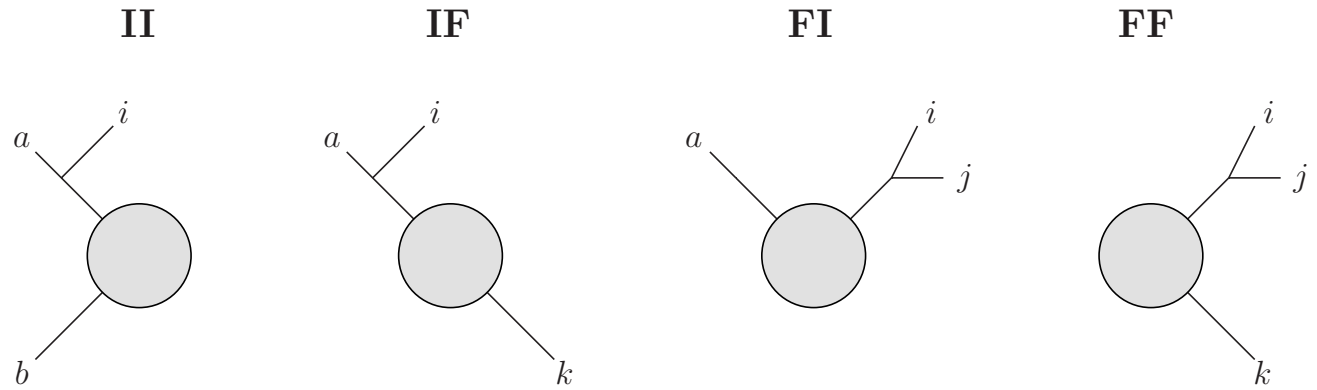
Merged predictions for deep inelastic scattering



SHERPA predictions for the inclusive jet cross section as a function of $E^2_{T,B}/Q^2$ in bins of η_{lab}, as measured by H1. Plot taken arXiv:1006.5696

$\Rightarrow$ Merging a "DGLAP" PS with LO calculations captures low-x effects.

DIRE ordering variables



More concretely, we generate phase space in the variables

$$\rho_{\mathrm{II}} = \frac{s_{ai}s_{bi}}{s_{ab}}\frac{s_{aib}}{s_{ab}} \qquad z_{\mathrm{II}} = 1 - \frac{s_{bi}}{s_{ab}}$$

$$\rho_{\mathrm{IF}} = \frac{s_{ai}s_{ik}}{s_{ai}+s_{ak}}\frac{s_{ai}+s_{ik}+s_{ak}}{s_{ai}+s_{ak}} \qquad z_{\mathrm{IF}} = 1 - \frac{s_{ik}}{s_{ai}+s_{ak}}$$

$$\rho_{\mathrm{FI}} = \frac{s_{aj}s_{ij}}{s_{ai}+s_{aj}}\frac{s_{ij}+s_{aj}+s_{ai}}{s_{ai}+s_{aj}} \qquad z_{\mathrm{FI}} = \frac{s_{ai}}{s_{ai}+s_{aj}}$$

$$\rho_{\mathrm{FF}} = \frac{s_{ij}s_{jk}}{s_{ij}+s_{ik}+s_{jk}} \qquad z_{\mathrm{FF}} = \frac{s_{ij}+s_{ik}}{s_{ij}+s_{ik}+s_{jk}}$$

and pick Φ randomly in $[0, 2\pi]$. Note that the evolution variables ρ are inverse eikonal factors!

Improvements

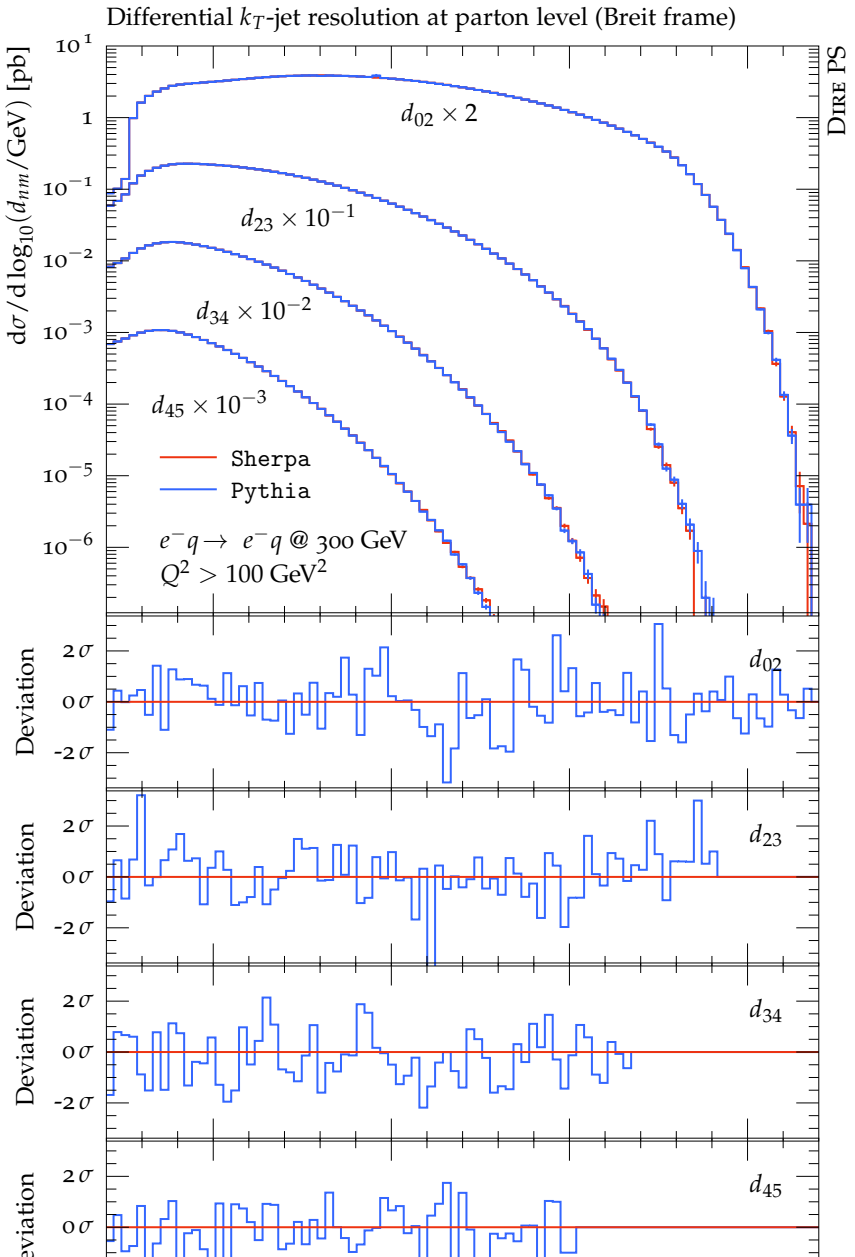
Ordering variables were chosen to obtain very simple phase space boundaries and splitting functions, so that a comparison with analytic results is possible.

$\Longrightarrow$ The soft terms can directly be rescaled with higher-order cusp anomalous dimensions to get A_2 and A_3:

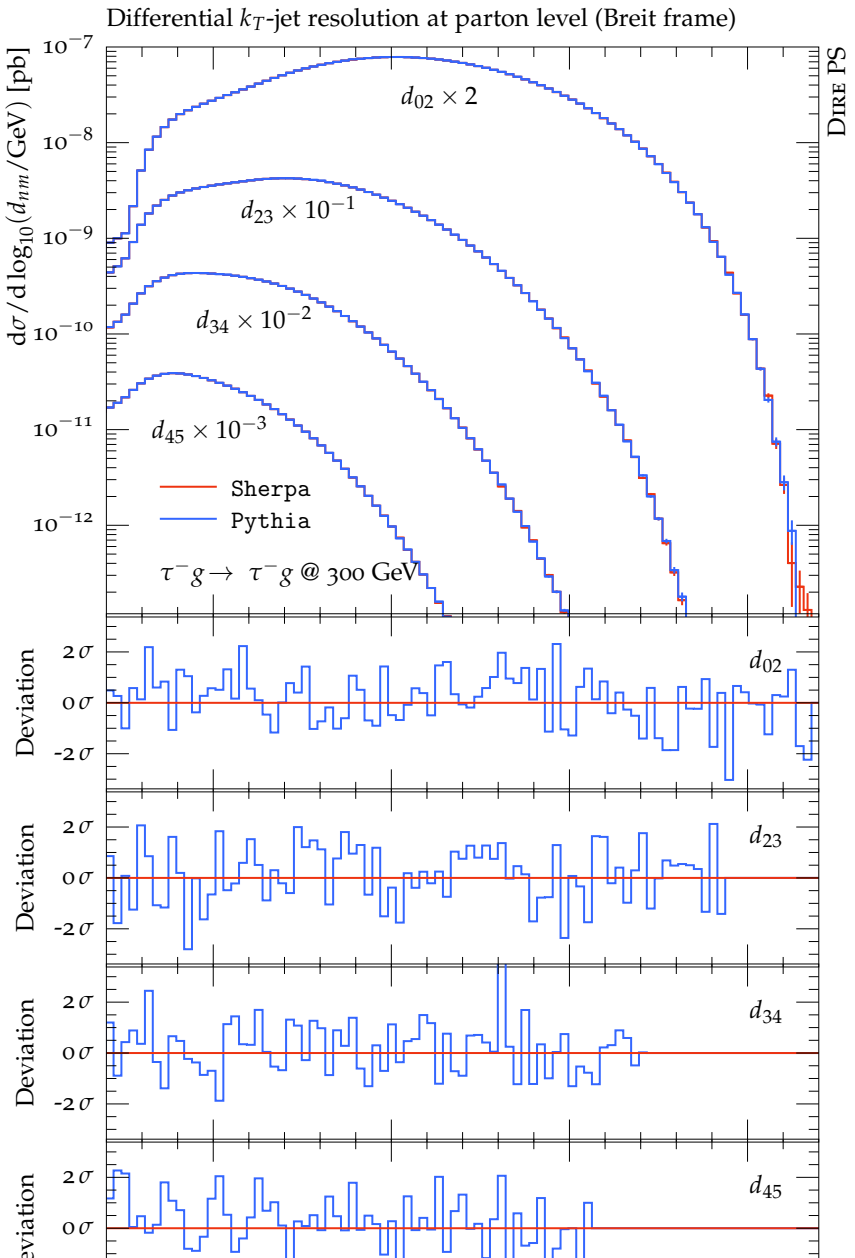
$$\frac{2(1-z)}{(1-z)^2+\kappa^2} \longrightarrow \frac{2(1-z)}{(1-z)^2+\kappa^2}\left(1+\frac{\alpha_s}{2\pi}\gamma^{(1)}_{cusp}+\frac{\alpha_s^2}{4\pi^2}\gamma^{(2)}_{cusp}\right)$$

Note: α_s not in the CMW scheme (two-loop cusp is absorbed via redefinition of Λ_{QCD}), as this can introduce problematic collinear anomalous dimensions beyond NLL (depending on shower details).

DIS cross-validation: Jet scales



DIS validation: Jet scales



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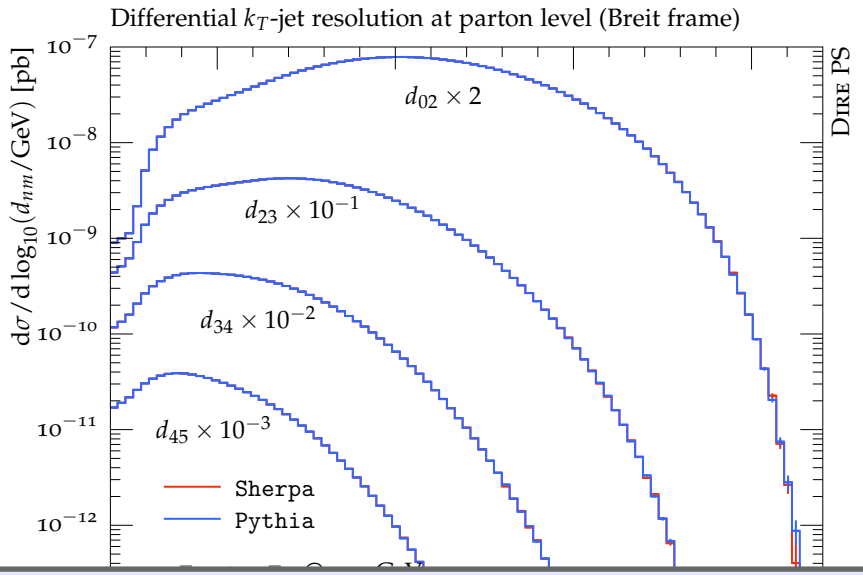
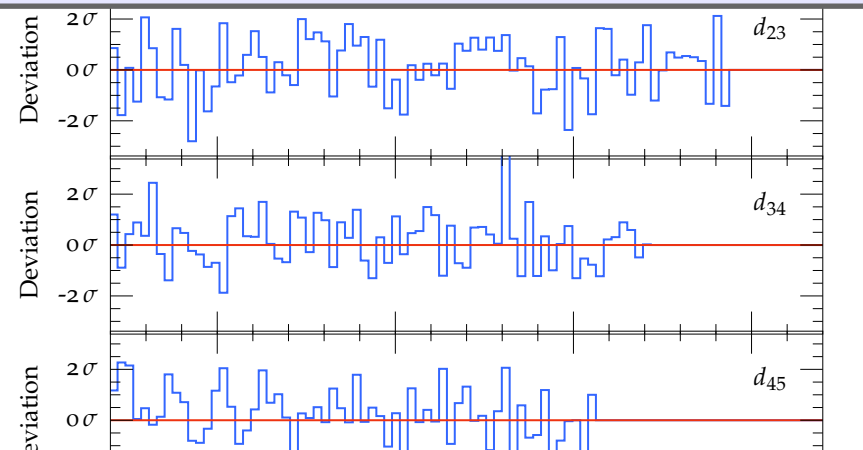
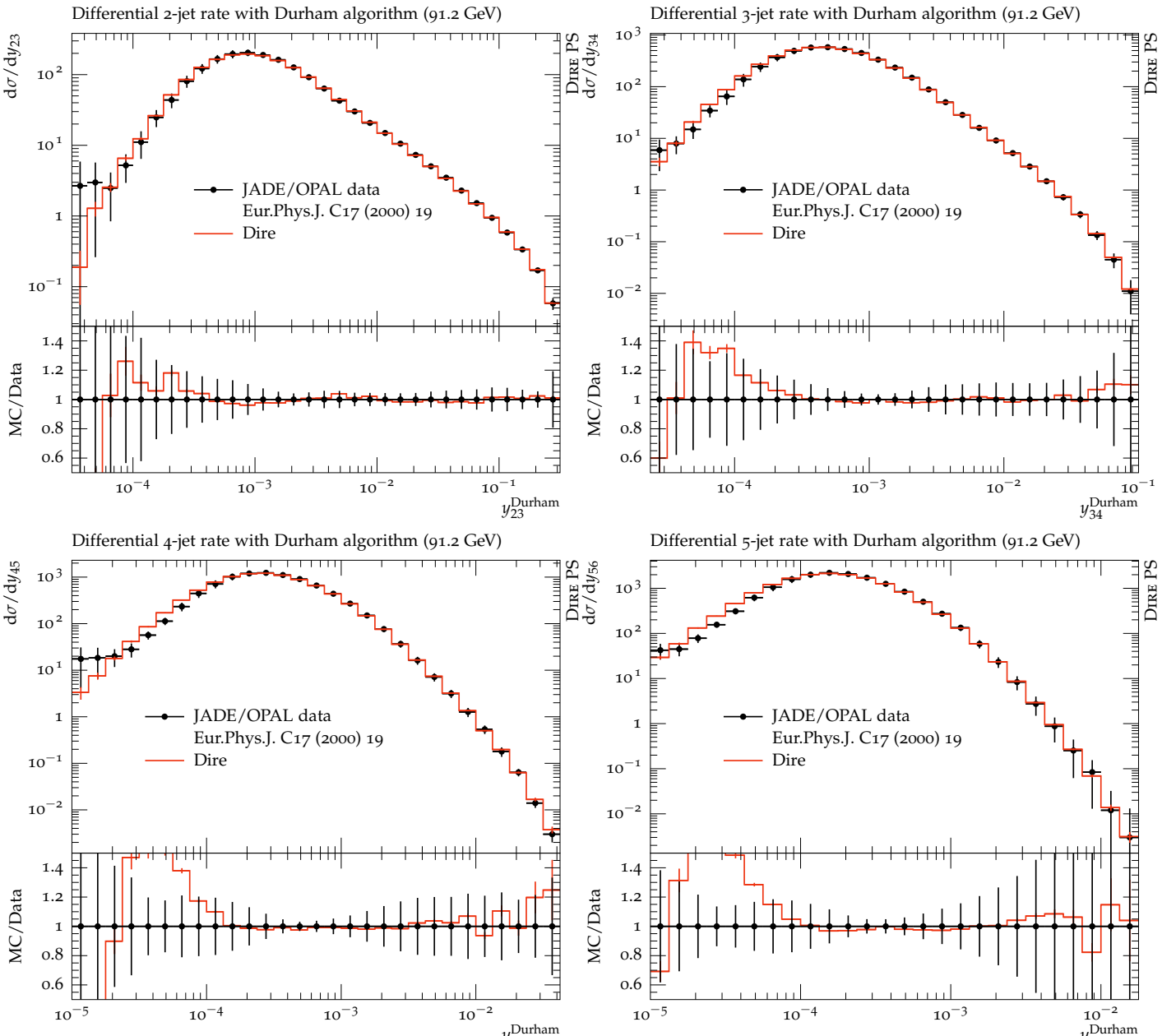


First ever DIS prediction with PYTHIA8.
Note that we used "Higgs–DIS" as a test case.
$\rightarrow$ It's good to have external tools for the hard process!



LEP data comparisons (plain showering)



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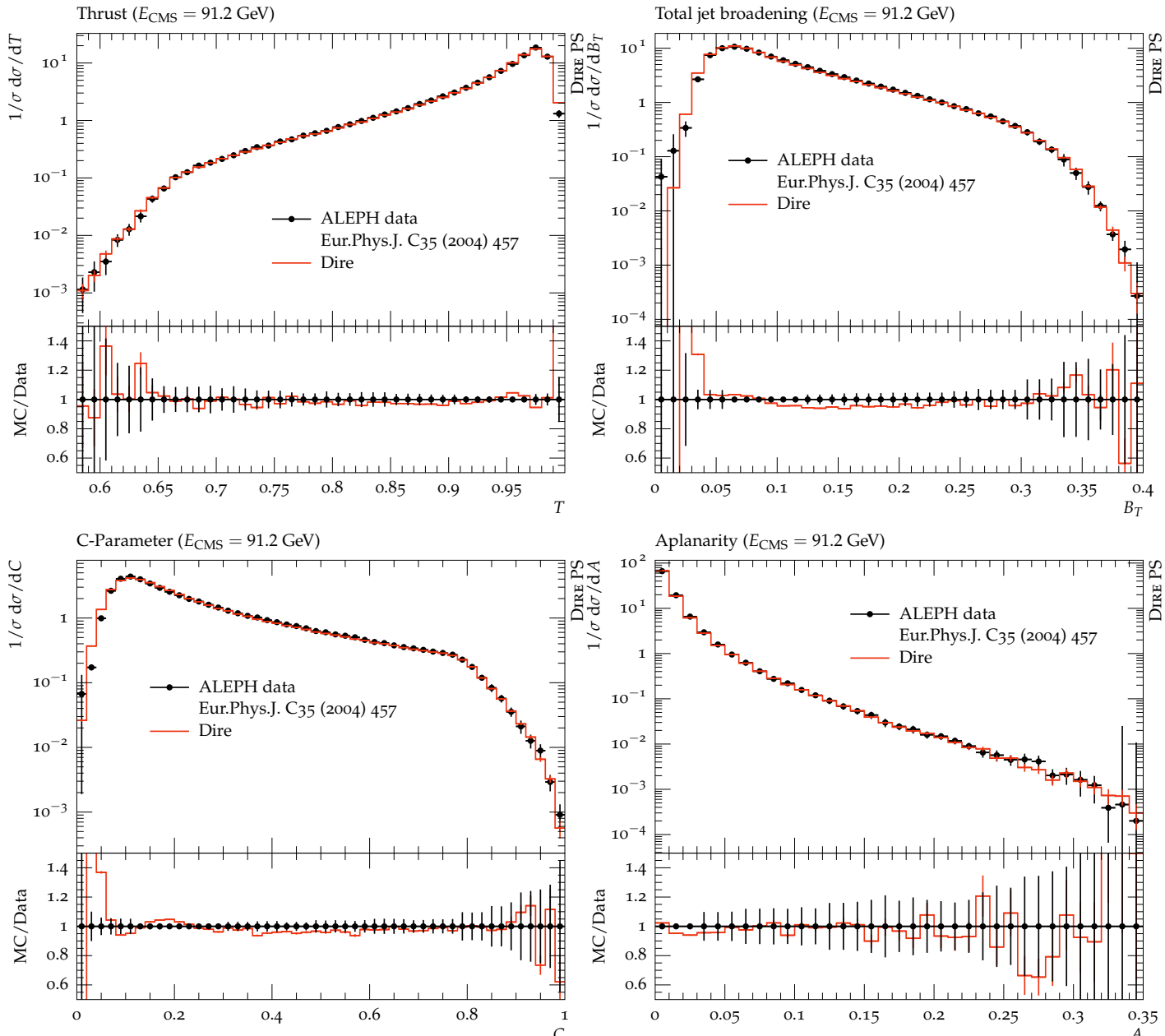

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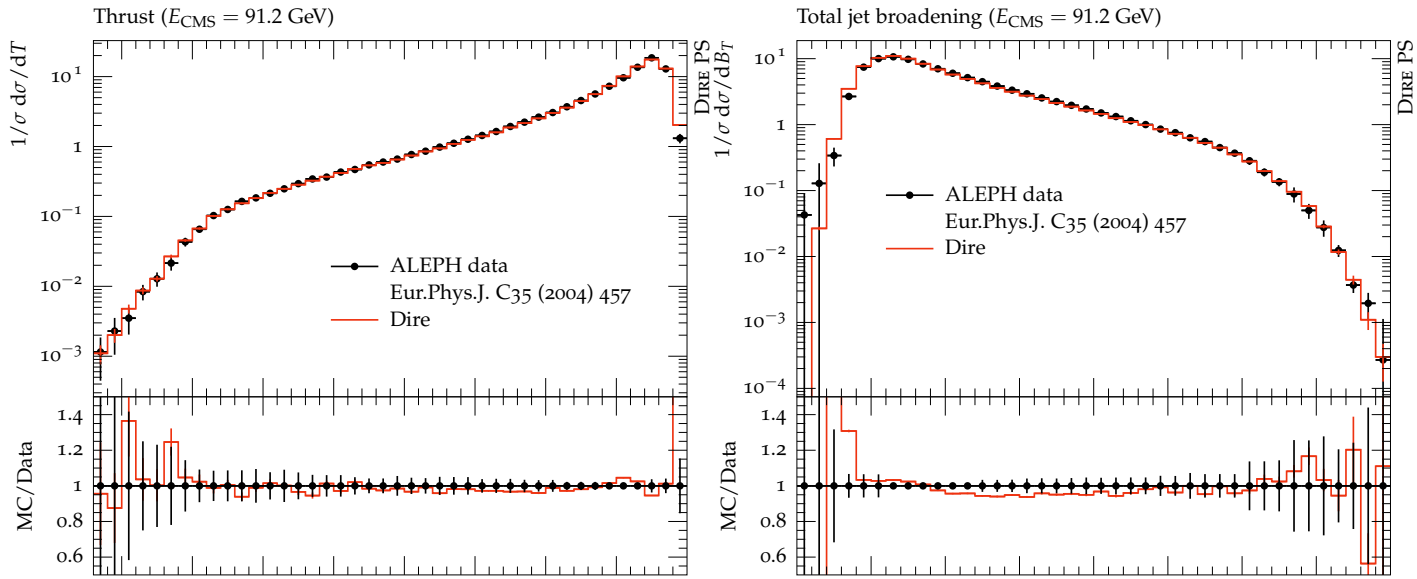
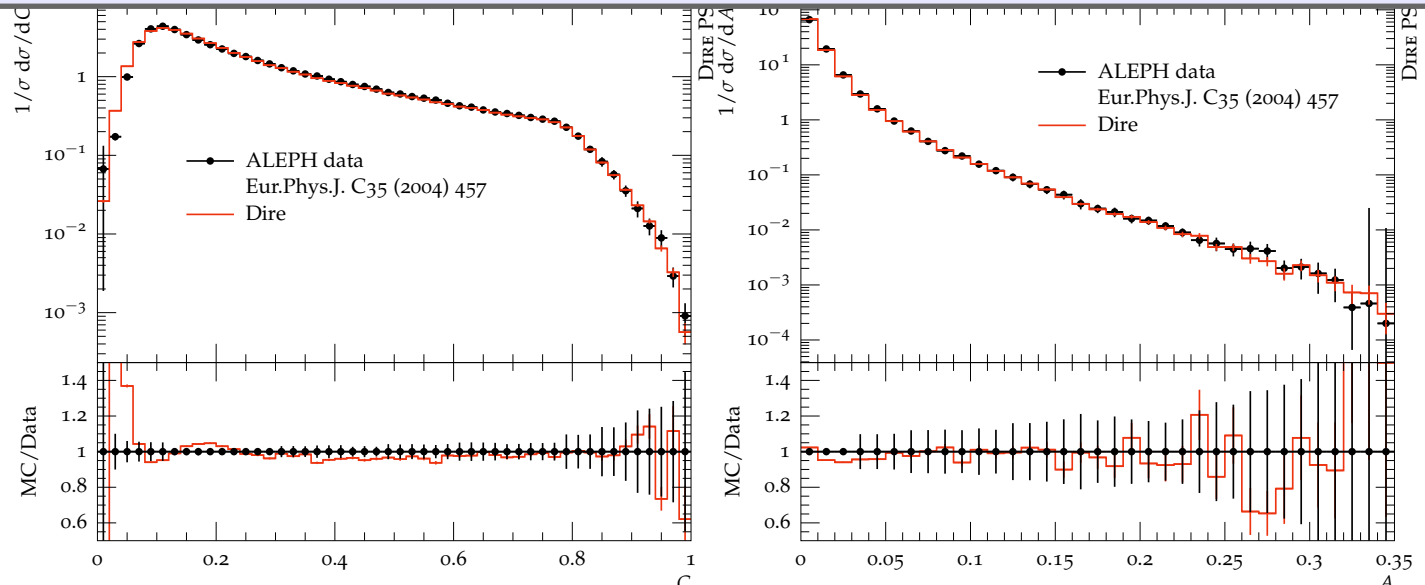
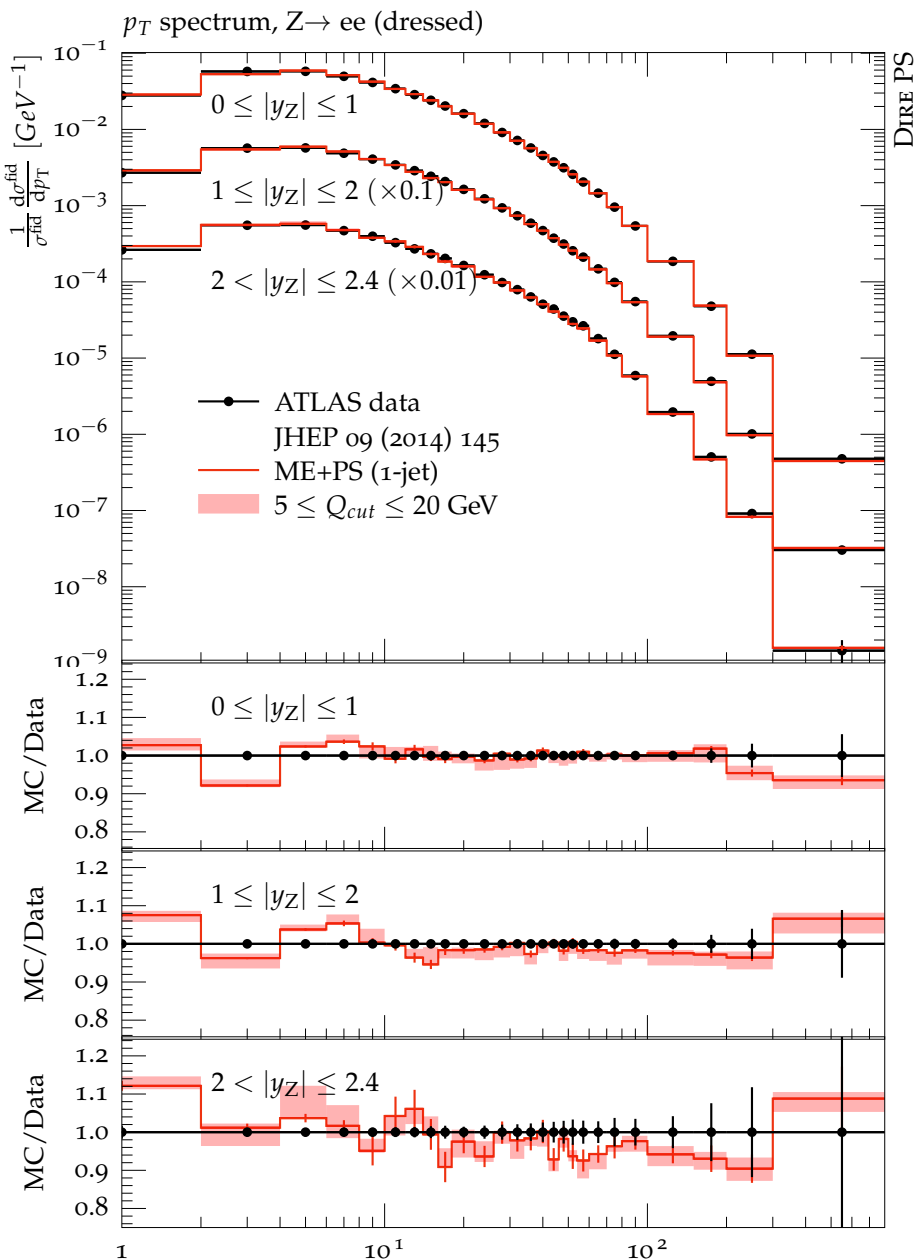


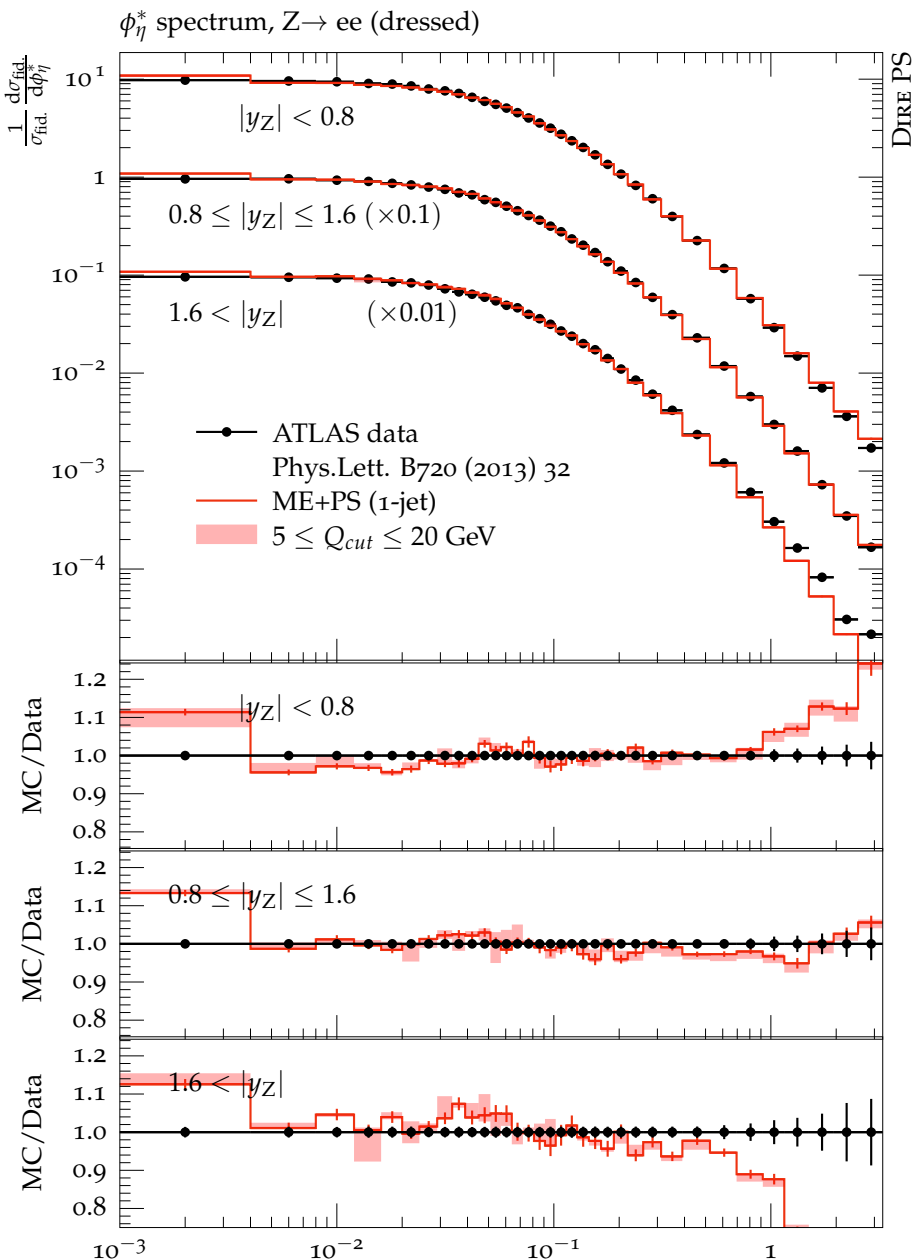
Nice description of LEP data.



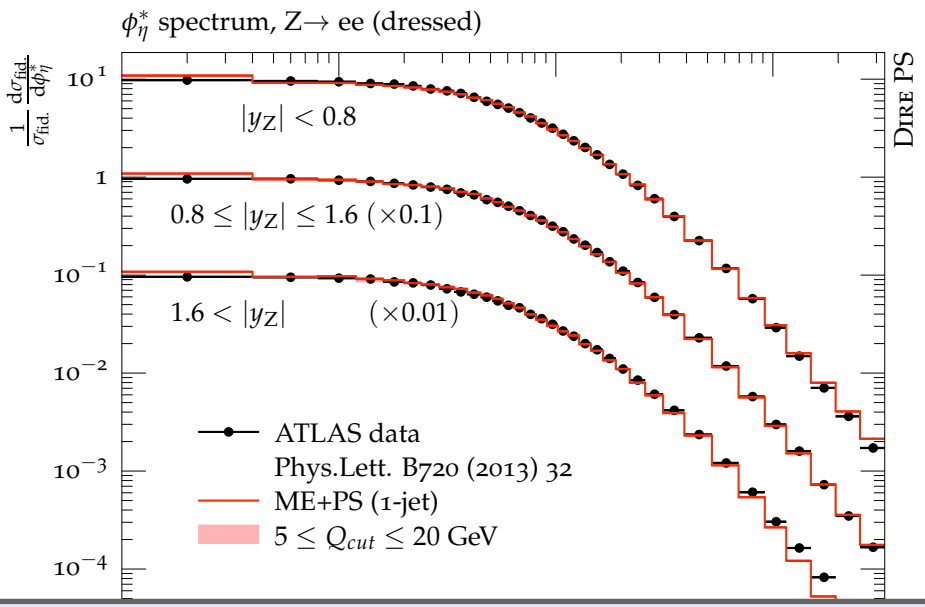
LHC data comparisons (1-jet CKKW-L merged)



LHC data comparisons (1-jet CKKW-L merged)

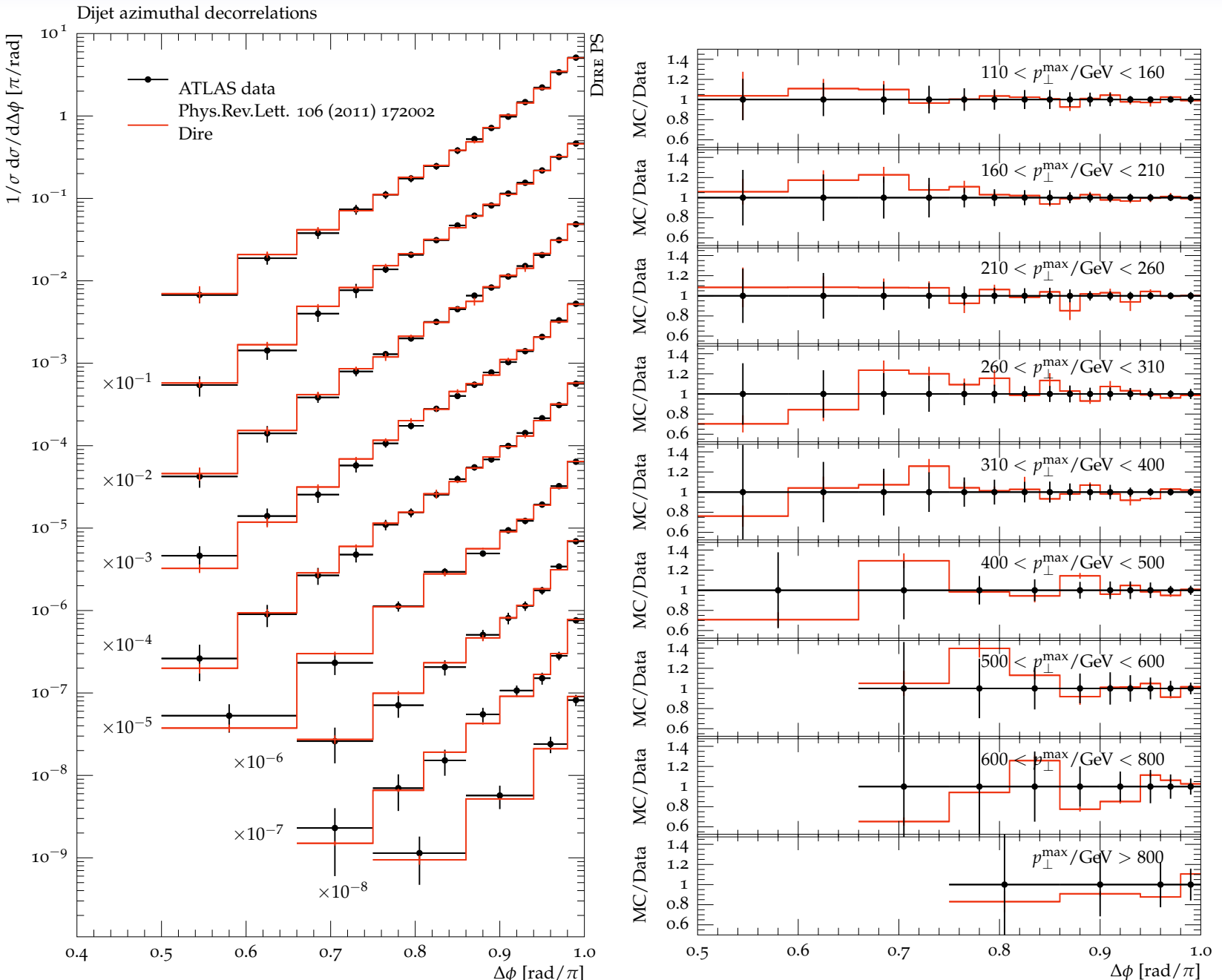


LHC data comparisons (1-jet CKKW-L merged)

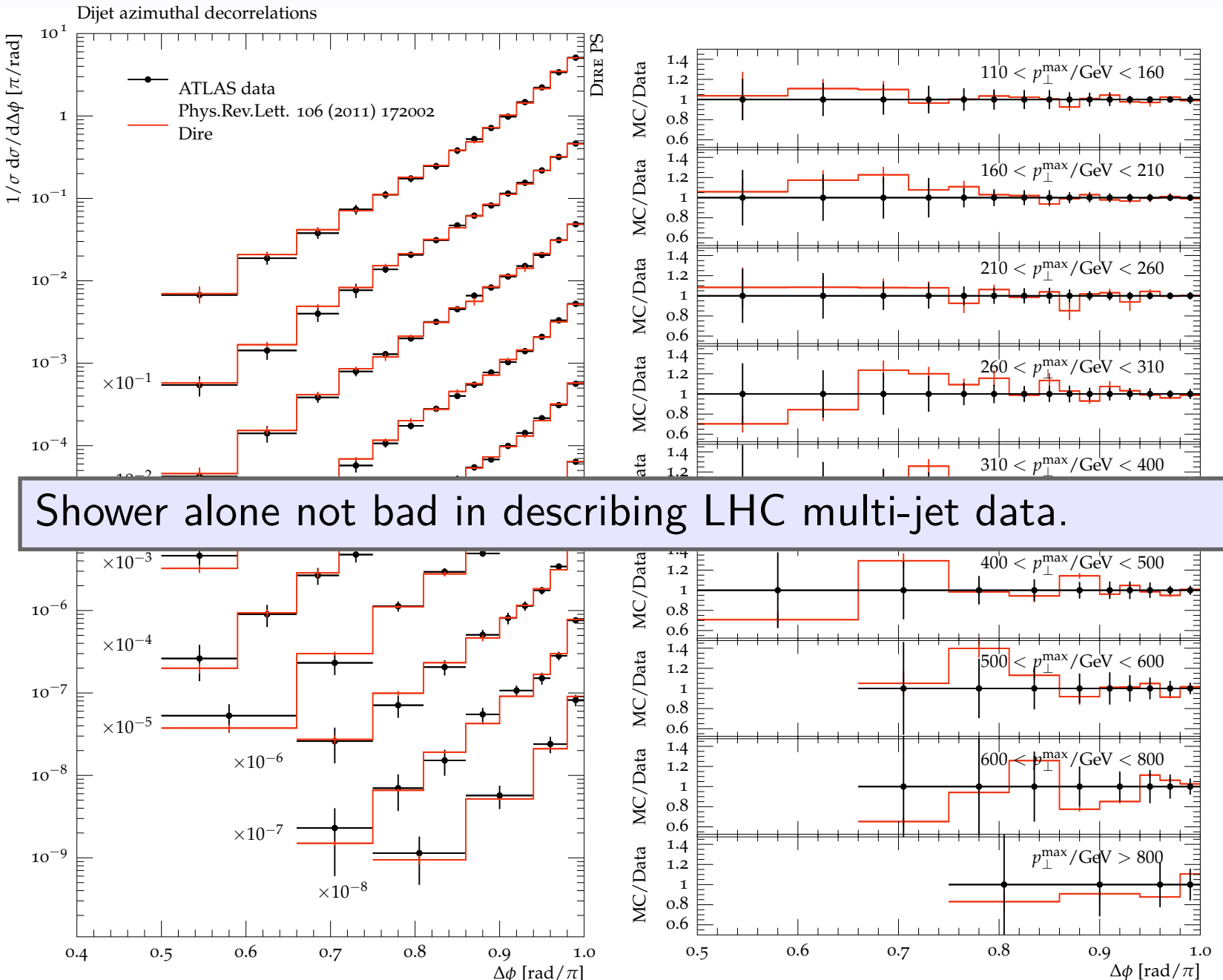


Not too terrible description of Drell-Yan data.

LHC data comparisons (plain showering)



LHC data comparisons (plain showering)



Shower alone not bad in describing LHC multi-jet data.

Imagine we can combine multiple NLO calculations with each other and with accurate parton showering

...we still have some work to do.

Summary

- Fixed-order + parton shower continues to be an active field.
  For ee or pp initiated processes, we can combine
  ...multiple LO calculations,
  ...multiple NLO calculations, or
  ...match $pp \to$ *colour singlett* at NNLO +PS.
- One main issue remains to construct more reliable showers.
- For simple processes, we can hope to match terms in the shower with analytic results
  ...but we need cleanly defined shower for that.
- Results of the new DIRE shower in PYTHIA and SHERPA look promising.
- DIRE makes PYTHIA usable for leptoproduction.
- The new PYTHIA showers are available at
  `slac.stanford.edu/~prestel/DIRE`
  ...and are compatible with PYTHIA8.212
- Next steps: Matching and merging for DIRE, also for ep.

Back-up

References

POWHEG
JHEP 0411 (2004) 040 JHEP 0711 (2007) 070 POWHEG-BOX (JHEP 1006 (2010) 043)

MC@NLO
Original (JHEP 0206 (2002) 029) Herwig++ (Eur.Phys.J. C72 (2012) 2187)
Sherpa (JHEP 1209 (2012) 049) aMC@NLO (arXiv:1405.0301)

NLO matching results and comparisons
Plots taken from Ann.Rev.Nucl.Part.Sci. 62 (2012) 187 Plots taken from JHEP 0904 (2009) 002

Tree-level merging MLM (Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002, Mangano et al. JHEP 0701 (2007) 013)
Pseudoshower (JHEP 0405 (2004) 040)
CKKW (JHEP 0111 (2001) 063, JHEP 0208 (2002) 015)
CKKW-L (JHEP 0205 (2002) 046, JHEP 0507 (2005) 054, JHEP 1203 (2012) 019)
METS (JHEP 0911 (2009) 038, JHEP 0905 (2009) 053)

Unitarised merging
Pythia (JHEP 1302 (2013) 094) Herwig (JHEP 1308 (2013) 114) Sherpa (arXiv:1405.3607)

FxFx: Jet matching @ NLO: JHEP 1212 (2012) 061

MEPS@NLO
JHEP 1304 (2013) 027 JHEP 1301 (2013) 144 Plots taken from arXiv:1401.7971

UNLOPS JHEP 1303 (2013) 166 arXiv:1405.1067

MiNLO: Original (JHEP 1210 (2012) 155) Improved (JHEP 1305 (2013) 082)

MiNLO-NNLOPS: JHEP 1310 (2013) 222 arXiv:1407.2940
arXiv:1501.04637

UN^{2}LOPS: arXiv:1405.3607 arXiv:1407.3773

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int \widehat{\mathrm{B}}_{1\to 0} \qquad - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\qquad \widehat{\mathrm{B}}_1 \qquad - \int \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \int\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$- and $\mathcal{O}(\alpha_s^{n+1})$-terms:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\qquad\qquad - \Big[\int \widehat{\mathrm{B}}_{1\to 0} \Big]_{-1,2} \qquad - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\quad \Big[\widehat{\mathrm{B}}_1 \Big]_{-1,2} - \Big[\int \widehat{\mathrm{B}}_{2\to 1} \Big]_{-2} \Bigg) + \int\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Unitarise:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(s_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(s_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \iint \mathcal{O}(s_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Deriving an UN^{2}LOPS matching

We basically follow a "merging strategy":

- Pick calculations to combine (two MC@NLOs) with each other *and* with the PS resummation.
- Remove kinematic overlaps between the two MC@NLOs by dividing the one-jet phase space.
- Reweight one-jet MC@NLO (*to make it exclusive* $\leftrightarrow$ *want to describe hardest jet with this*),
  remove all undesired terms at $\mathcal{O}(\alpha_s^{1+1})$
  and make sure that the whole thing is numerically stable.
  Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with virtuals.
- Add and subtract reweighted one-jet MC@NLO, ($\rightarrow$ unitarise) to ensure inclusive zero-jet cross section is unchanged w.r.t. NLO.
- Remove all terms up to $\mathcal{O}(\alpha_s^2)$ in the zero-jet contribution, replace by NNLO jet-vetoed cross section.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, then upgrade to NNLO directly.

Work with Stefan Höche and Ye Li.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, then upgrade to NNLO directly.

Start over again, now combining MC@NLO's because those are resonably stable. Thus:

- ◇ Use 0-jet matched (MC@NLO $_0$) and 1-jet matched calculation (MC@NLO $_1$).
- ◇ Remove hard ($q_T > \rho_{\text{MS}}$) reals in MC@NLO $_0$.
- ◇ Reweight B_1 of MC@NLO $_1$ with "zero-jet Sudakov" factor $\Pi_{S_{+0}}/\alpha_s$ running.
- ◇ Reweight NLO part $\widetilde{\mathrm{B}}_1^{\mathrm{R}}$ of MC@NLO $_1$ with "zero-jet Sudakov" factor.
- ◇ Subtract erroneous $\mathcal{O}(\alpha_s^{+1})$ terms multiplying B_1.
- ◇ Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with $\widetilde{\mathrm{B}}_1^{\mathrm{R}}$.
- ◇ Put $\rho_{\text{MS}} \to \rho_c < 1\text{GeV}$. ($\to$ MC@NLO $_0$ becomes exclusive NLO)
- ◇ Unitarise by subtracting the processed MC@NLO $'_1$ from the "zero-q_T bin".
- ◇ Remove all terms up to α_s^2 from the "zero-q_T bin" and add the q_T-vetoed NNLO cross section.

$\Rightarrow \sigma_{inclusive}$ @ NNLO, resummation as accurate as Sudakov, stats fine.

NNLO logarithmic parts from q_T-vetoed TMDs (EFT calculation),
hard coefficients from q_T-subtraction (i.e. DYNNLO, HNNLO),
power corrections from MC@NLO $_1$.

Work with Stefan Höche and Ye Li.

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \; \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,0)
\end{aligned}
$$

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,0)
\end{aligned}
$$

Note that this is just an extention of the old Sudakov veto algorithm:
Run trial shower on the reconstructed zero-jet state,
If trial shower produces an emission, keep zero-jet kinematics and stop;
else start PS off one-jet state.

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,0)
\end{aligned}
$$

Note: $\left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}$ etc. comes from using q_T-vetoed cross sections.

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \; \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,0)
\end{aligned}
$$

$\bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}} + \tilde{\mathrm{B}}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{E}} = \mathrm{B}_{\mathrm{NNLO}}$

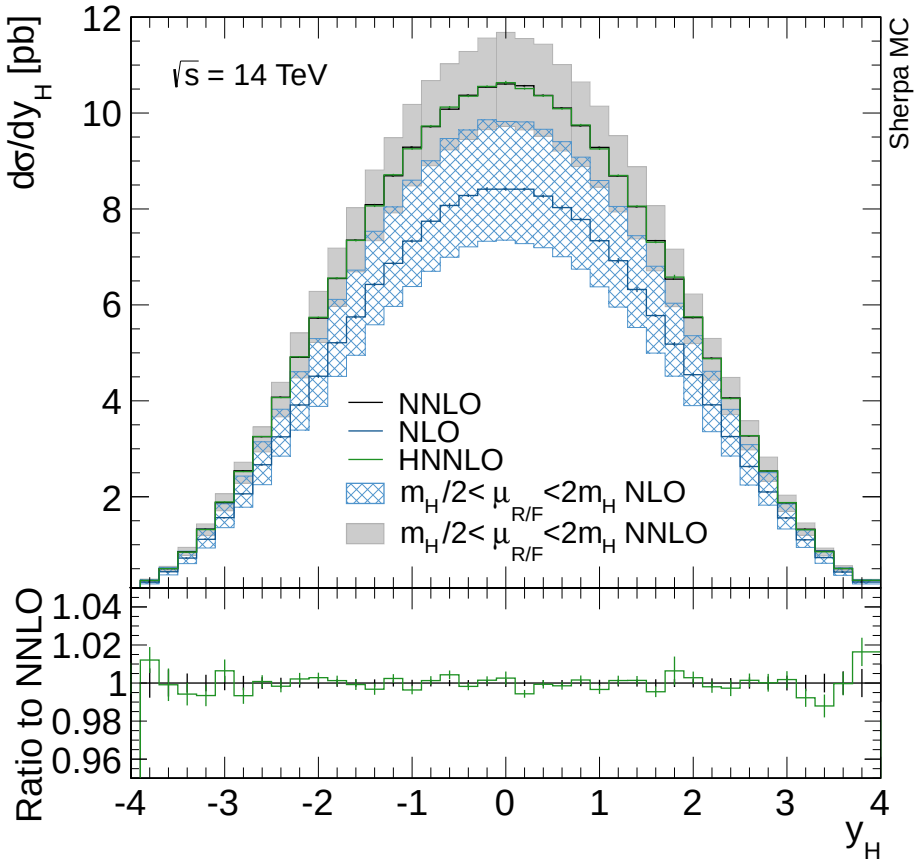
Other terms drop out in inclusive observables.

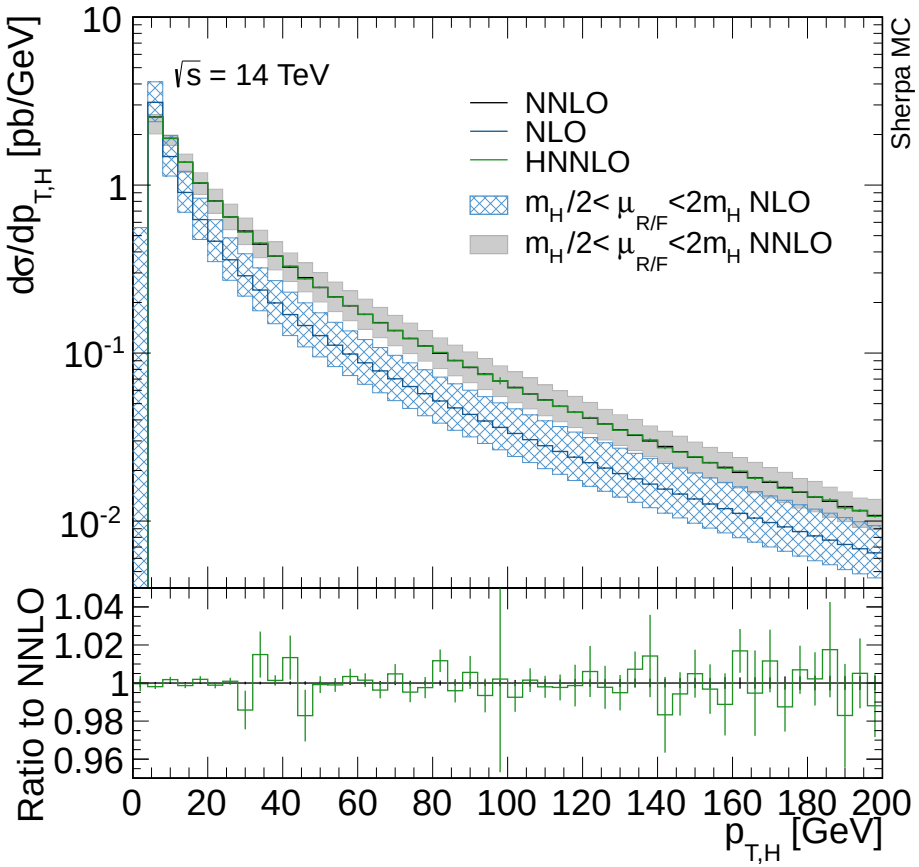
UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_T,\mathrm{cut}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,0) \\
& + \int_{q_T,\mathrm{cut}} d\Phi_2 \, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,0)
\end{aligned}
$$

Orange terms do not contain any universal α_s corrections present in the PS.
H_1 do not contribute in the soft/collinear limit.
$\Longrightarrow$ PS accuracy is preserved.

UN^{2}LOPS (Higgs production)





Rapidity and $p_\perp$ of the Higgs-boson, comparing SHERPA-NNLO and HNNLO. Our independent NNLO calculation works nicely.

Shower tricks: Weighted shower (JHEP 1209 (2012) 049)

For frequently negative contributions, or if we do not want to find an ideal overestimate, we can use a weighted shower instead:

- For each phase space point, store the full splitting probability f, the differential overestimate g, and an auxiliary weight $h = \mathrm{sgn}\,(f) \cdot g$.
- Pick/discard splitting as usual according to $\mathrm{abs}\left(\frac{f}{g}\right)$. If the emission...
  - ...was picked, multiply $w_{picked} = \frac{h}{g}\frac{g-f}{h-f}$ to event weight
  - ...was discarded, multiply $w_{discarded} = \frac{h}{g}$ to event weight

If the differential overestimate is not ideal, i.e. $\frac{f}{g} = a > 1$, then use

$$\frac{f}{g} \to \frac{f}{(ka)\cdot g} \quad \text{and} \quad h \to (ka)\cdot h$$

with an additional factor $k \gtrsim 1$, and continue as before.

DIRE Anomalous Dimensions

The anomalous dimensions

$$\gamma_{ab}(N,\kappa^2) = \int_0^1 z^N P_{ab}(z,\kappa^2) \qquad \text{are given by} \tag{1}$$

$$\begin{aligned}
\gamma_{qq}(N,\kappa^2) &= 2C_F\,\Gamma(N,\kappa^2) - \frac{C_F\,(2N+3)}{(N+1)(N+2)} + \gamma_q \\
\gamma_{gq}(N,\kappa^2) &= 2C_F\,\mathrm{K}(N,\kappa^2) - \frac{C_F\,(N+3)}{(N+1)(N+2)} \\
\gamma_{gg}(N,\kappa^2) &= 2C_A\,\Gamma(N,\kappa^2) + 2C_A\,\mathrm{K}(N,\kappa^2) - \frac{2C_A\,(N+3)}{(N+1)(N+2)} - \frac{2C_A}{N+3} + \gamma_g \\
\gamma_{qg}(N,\kappa^2) &= -\frac{T_R\,N}{(N+1)(N+2)} + \frac{2T_R}{N+3}
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
2\Gamma(N,\kappa^2) &= \frac{2\,{}_3F_2\left(1,1,\frac{3}{2};\frac{N+3}{2},\frac{N+4}{2};-\kappa^{-2}\right)}{(N+1)(N+2)\,\kappa^2} - \ln\frac{1+\kappa^2}{\kappa^2}\,, \\
2\mathrm{K}(N,\kappa^2) &= \frac{2\,{}_2F_1\left(1,\frac{N+2}{2};\frac{N+4}{2};-\kappa^{-2}\right)}{(N+2)\,\kappa^2}\,.
\end{aligned} \tag{3}$$