WIMP-Nucleus scattering in chiral EFT

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Outline

• Dark Matter and its direct detection
• From WIMP-quark to WIMP-nucleus using EFT
• Phenomenology:
  • Scalar couplings**
  • Axial-vector couplings
• Conclusions

** Based on:
Dark Matter and its “direct detection”
Dark Matter

- It’s out there:

- Evidence also from gravitational lensing, CMB anisotropy, structure formation

- Combined observations → $\Omega_{dm} \sim 0.21 \pm 0.01$, mostly cold
Particle candidates

- Basic requirements:
  - Stable on cosmological time scales
  - Very weakly interacting with EM radiation
  - Correct density
Particle candidates

- Basic requirements:
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  - Correct density
Handles on WIMPs

- Direct detection
- Indirect detection
- Production at colliders

\[ \implies \text{Infer properties of WIMP (mass, spin, interactions, ...)} \]
Direct detection (I)

- Observe WIMP-nucleus scattering via nuclear recoil

- Typical scales involved

\[ |\vec{q}|_{\text{max}} = 2 \mu v_{\text{rel}} < 2 m_A v_0 \sim 200 \text{MeV} \]

\[ E_R < \frac{|\vec{q}|_{\text{max}}^2}{2m_A} \sim 200 \text{KeV} \]

\[ m_A \sim 100 \text{GeV} \quad v_0 \sim 10^{-3} \]
Direct detection (II)

- Event rate

\[ \frac{dN}{dE_R} \propto \Phi \otimes \frac{d\sigma}{dE_R} \]

- Flux factor: DM local density and velocity distribution

- WIMP-nucleus cross section: particle physics $\times$ hadronic & nuclear physics

Engel-Pittel-Vogel 1992
Direct detection (II)

- Event rate

\[
\frac{dN}{dE_R} \propto \Phi \otimes \frac{d\sigma}{dE_R}
\]

\[\Phi \sim n_v f(v)\]

Flux factor:
DM local density and velocity distribution

\[d\sigma \sim \sigma_p \otimes R_{\text{nucl}}\]

WIMP-nucleus cross section:
particle physics \times \text{hadronic & nuclear physics}

Engel-Pittel-Vogel 1992

- Here focus on hadronic / nuclear physics effects, using chiral EFT power counting as organizing principle
From WIMP-quark to WIMP-nucleus
EFT framework

$\mathcal{L}_{\text{BSM}}$

$\mathcal{L}_{\text{SM}} + \sum_{k,i} \frac{c_i^{(k)}}{\Lambda_k} O_i^{(k)} [X^{(i)}; X; q, G, ...]$

BSM dynamics includes WIMP candidate “X”, and other particles “Y”, eg:

$X, G, ..$

$X, G, ..$

$X, G, ..$

$X, G, ..$
EFT framework

\[ \mathcal{L}_{BSM} \]

\[ \mathcal{L}_{SM} + \sum_{k,i} \frac{c_i^{(k)}}{\Lambda_k} O^{(k)}_i [X^{(i)}; X; q, G, ...] \]

Non-perturbative matching

q \ll M_N : use chiral EFT

\[ \mathcal{L}_{ChPT} [X; N, \pi, ...] \rightarrow H_I = V_{XN} + V_{XNN} + ... \]

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\[ \mathcal{L}_{\text{ChPT}} \left[ X; N, \pi, \ldots \right] \rightarrow H_I = V_{XN} + V_{XNN} + \ldots \]

\[ \langle \Phi'_{(A,Z)} X' \mid H_I \mid \Phi_{(A,Z)} X \rangle \]
BSM dynamics includes WIMP candidate “X”, and other particles “Y”, eg:

\[ \mathcal{L}_{\text{BSM}} \]

\[ \mathcal{L}_{\text{SM}} + \sum_{k,i} \frac{c_i^{(k)}}{\Lambda_k} O_i^{(k)} [X^{(i)}; X; q, G, ...] \]

Non-perturbative matching
q << M_N : use chiral EFT

\[ \mathcal{L}_{\text{ChPT}} [X; N, \pi, ...] \rightarrow H_I = V_{XN} + V_{XNN} + ... \]

Fitzpatrick-Haxton-Katz-Lubbers-Xu 2012

\[ \langle \Phi'_{(A,Z)}X' | H_I | \Phi_{(A,Z)}X \rangle \]
A concrete example (SUSY)

• Higgs and squark exchange mediate **scalar interactions**

• Z and squark exchange mediate **axial-vector interactions**
Strategy

- For a given quark-level operator:

1. Chiral symmetry $\rightarrow$ WIMP couplings to $\pi, N$ at small $q$
2. Chiral power counting for $X_{N_1...N_A} \rightarrow X_{N_1...N_A}$ amplitudes $\Rightarrow V_{X_{N_1}, V_{X_{N_2}}}, ...$ to a given order in $p \sim q/m_N \sim m_\pi/m_N$
3. Nuclear matrix elements

Same strategy as in electron-nucleus or neutrino-nucleus scattering
(difference is in the operator structure, kinematic regime, target nuclei)
Strategy

- For a given quark-level operator:

1. Chiral symmetry $\rightarrow$ WIMP couplings to $\pi, N$ at small $q$
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Same strategy as in electron-nucleus or neutrino-nucleus scattering
(differece is in the operator structure, kinematic regime, target nuclei)

Next, some details on the scalar interaction
Scalar interactions

- GeV-scale effective Lagrangian* involves 4 short-distance couplings

\[
\mathcal{L}_{SM} + \frac{1}{v} \frac{XX}{\Lambda^{XX-1}} \left[ \sum_{q=u,d,s} \lambda_q \, m_q \, \bar{q}q + \lambda_\theta \, \theta_\mu^\mu \right] + \ldots
\]

* After integrating out heavy quarks and trading GG operator for trace of energy-mom. tensor:

\[
\theta_\mu^\mu = \sum_q m_q \bar{q}q - \frac{9 \alpha_s}{8 \pi} G^{\mu}_a G^{a\mu
u}
\]
1. WIMP couplings to $\pi, N$

- Hadronic realization of

\[ \mathcal{L}_{\text{QCD}} + \sum_{q=u,d,s} s_q \bar{q}q + s_\theta \theta^\mu_\mu \]

\[ s_q = \lambda_q \frac{m_q}{v} \frac{XX}{\Lambda[XX]^{-1}} \]

- Scalar source appears in chiral effective Lagrangian in the same way as quark mass matrix

\[ B_0 F^2 = -\langle 0|\bar{q}q|0 \rangle \]

\[ \langle N|\bar{q}q|N \rangle \]
1. WIMP couplings to $\pi$, N

- Hadronic realization of

$$\mathcal{L}_{QCD} + \sum_{q=u,d,s} s_q \bar{q}q + s_\theta \theta_\mu^\mu$$

$$s_\theta = \lambda_\theta \frac{1}{v \Lambda [XX]^{-1}}$$

- Energy-momentum tensor operator identified by coupling chiral EFT to external metric

Donoghue-Leutwyler 1991
2. Power counting and $V_{XN}, V_{XNN}, ...$

- Key object: A-nucleon irreducible amplitude with one insertion of $s_q, \theta$
  (Fourier transform of potential)

\[ M_{A,X} = \]
• NLO amplitude: 1-nucleon

\[
M_{1,X} = f_N(q^2) \quad XX \quad \bar{NN} \quad N = p, n
\]

\[
f_{p/n}(q^2) = \frac{1}{v \Lambda_{np}^2} \left[ \sum_{q=u,d,s} \lambda_q \sigma_q^{(p/n)} + \lambda_\Theta m_{p/n} - \frac{g_A^2}{64\pi F_\pi^2} \left( A(q^2) \pm B(q^2) \right) \right]
\]

Usual q=0 term, controlled by sigma terms:

\[
\langle i | m_q q \bar{q} | i \rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i
\]

Giedt-Thomas-Young 0907.4177  Kronfeld 1203.1204
• NLO amplitude: 1-nucleon

$$M_{1,X} = f_N(q^2) \ XX \ \bar{N}N$$

$$f_{p/n}(q^2) = \frac{1}{\Lambda_{np}^2} \left[ \sum_{q=u,d,s} \lambda_q \sigma_{q}^{(p/n)} + \lambda_{\Theta} m_{p/n} - \frac{g_A^2}{64\pi F_\pi^2} \left( A(q^2) \pm B(q^2) \right) \right]$$

Usual q=0 term, controlled by sigma terms:

$$\langle i | m_q q \bar{q} | i \rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i$$

Slope term: no new couplings

$$A, B \sim c(\lambda_q) m_M q^2$$

$$M = \pi, K, \eta$$

Giedt-Thomas-Young 0907.4177    Kronfeld 1203.1204
• NLO amplitude: 2-nucleons

\[ M_{2,X} = M_{\pi\pi} + M_{\eta\eta} \]

\[ M_{\pi\pi} = -\frac{1}{v} \frac{g_A^2}{\Lambda_{np}^2} \frac{m_{\pi}^2 \lambda_+}{F_{\pi}^2 (q_1^2 - m_{\pi}^2) (q_2^2 - m_{\pi}^2)} \tilde{N} q_1 \cdot S \tau_1^k N \tilde{N} q_2 \cdot S \tau_2^k N \tilde{\chi} \]

\[ M_{\eta\eta} = -\frac{1}{v} \frac{g_A^2}{\Lambda_{np}^2} \frac{m_{\pi}^2 \lambda_+ + 4 (M_K^2 - \frac{1}{2} m_{\pi}^2) \lambda_s}{3 F_{\pi}^2 (q_1^2 - m_{\eta}^2) (q_2^2 - m_{\eta}^2)} \tilde{N} q_1 \cdot S N \tilde{N} q_2 \cdot S N \tilde{\chi} \]

\[ \lambda_{\pm} \equiv (m_u \lambda_u \pm m_d \lambda_d) / (m_u + m_d) \]

\[ g_A = D + F = 1.27 \]

\[ \alpha = F / (D + F) \approx 0.4 \]

\(\pi\pi\) contribution first considered in Kamionkoski-Kurylov-Prezeau-Vogel, 2003
3. Nuclear matrix elements

\[ \langle f|\hat{T}|i \rangle = (2\pi)^3 \delta^{(3)}(\vec{q}_X + \vec{q}_A) T(\vec{q}_X) \]

\[ T(\vec{q}_X) = T_1 + T_2 \]

\[ T_1 = \sum_{i=1,A} \int d\vec{x}_i \left( \rho_1(\vec{x}_i) \otimes \tilde{V}_1(\vec{q}_X; \vec{x}_i) \right) \]

\[ T_2 = \sum_{i<j} \int d\vec{x}_i \, d\vec{x}_j \left( \rho_2(\vec{x}_i, \vec{x}_j) \otimes \tilde{V}_2(\vec{q}_X; \vec{x}_i, \vec{x}_j) \right) \]

Nuclear structure input: one- and two-body densities in the ground state

q-dependent “potentials”, related to momentum-space amplitudes
• **One-body**: factorization of nucleon and nuclear effects

\[ T_1 = -F(|\vec{q}_x|^2) \left( Z f_p(|\vec{q}_x|^2) + (A - Z) f_n(|\vec{q}_x|^2) \right) \]

- **Nuclear form factor** (Fourier transform of one-body density)
- **Nucleon form factors**
- **One-body**: factorization of nucleon and nuclear effects

\[
T_1 = -F(|\vec{q}_X|^2) \left( Z f_p(|\vec{q}_X|^2) + (A - Z) f_n(|\vec{q}_X|^2) \right)
\]

Nuclear form factor (Fourier transform of one-body density) 
Nucleon form factors

- **Two-body**: use shell model to get first rough estimate

\[
T^{(\pi\pi)}_2 = -\frac{\lambda_+}{v \Lambda_{np}^2} \frac{g_A^2 m_\pi^3}{96 \pi F_\pi^2} N_{\pi\pi} \times \exp \left( -\frac{|\vec{q}_X|^2 R^2(A)}{6} \right)
\]

\[N_{\pi\pi} = -1.19 A\]
Phenomenology: scalar interactions
Differential rate

\[
\frac{dR}{dE_R} = \frac{\kappa_X \rho_X}{\pi m_X} \left| \left[ Z f_p(E_R) + (A-Z) f_n(E_R) \right] F(E_R) - T_2(E_R, A, Z) \right|^2 \eta(E_R)
\]

- **E-dependent nucleon scalar form factors**
  \(-q^2 \simeq |\vec{q}|^2 = 2m_A E_R\)

- **Nuclear form factor**

- **Two-body term**
  \(\sim A \text{ at } E_R = 0\)

- **Integral over local DM velocity distribution**
Differential rate

\[ \frac{dR}{dE_R} = \frac{\kappa_X \rho_X}{\pi m_X} \left| \left[ Z f_p(E_R) + (A-Z) f_n(E_R) \right] F(E_R) - T_2(E_R, A, Z) \right|^2 \eta(E_R) \]

- Anatomy of recoil spectrum: “astrophysics” factor

\[ \eta(E_R) = \int_{u_{\text{min}}}^{u_{\text{esc}}} \frac{f(u)}{u} d^3u \]

\[ u_{\text{min}} = \sqrt{\frac{m_A E_R}{2 \mu_{XA}^2}} \quad \mu_{XA} = \frac{m_X m_A}{m_A + m_A} \]
Differential rate

\[
\frac{dR}{dE_R} = \frac{\kappa X \rho_X}{\pi m_X} \left[ Z f_p(E_R) + (A-Z) f_n(E_R) \right]^2 F(E_R) T_2(E_R, A, Z) \eta(E_R)
\]

- Anatomy of recoil spectrum: hadronic / nuclear effects at LO

\[ f_{p,n} (E_R) = f_{p,n} \, [\lambda_q, \theta] \]

sigma-terms \( \otimes \) short distance \( \lambda \)'s

![Graph showing differential rate as a function of energy with different nuclei A=20, A=76, A=133]
Differential rate

\[
\frac{dR}{dE_R} = \frac{\kappa_X \rho_X}{\pi m_X} \left| \left[ Z f_p(E_R) + (A-Z) f_n(E_R) \right] F(E_R) - T_2(E_R, A, Z) \right|^2 \eta(E_R)
\]

- Anatomy of recoil spectrum: hadronic / nuclear effects at NLO

\[ f_{p,n}(E_R) = f_{p,n}[\lambda_q, \theta] + k_{n,p}[\lambda_q] \ A \ E_R \]

Fractional correction depends on short distance \( \lambda \)'s

A=133, slope is proportional to A

\( \lambda_u \neq 0 \)

\( \lambda_s \neq 0 \)

\( \lambda_d \neq 0 \)
Differential rate

\[
\frac{dR}{dE_R} = \frac{\kappa_X \rho_X}{\pi m_X} \left| \left[ Z f_p(E_R) + (A-Z) f_n(E_R) \right] F(E_R) - T_2(E_R, A, Z) \right|^2 \eta(E_R)
\]

- Anatomy of recoil spectrum: hadronic / nuclear effects at NLO

\[- \quad f_{p,n}(E_R) = f_{p,n}[\lambda_{q,\theta}] + k_{n,p}[\lambda_q] \cdot A \cdot E_R \]

Fractional correction depends on short distance \(\lambda\)'s

\[- \quad T_2(0,A,Z)/(A f_p(0)) \sim 5\%

Similar to LQCD* estimate 1306.6939
NLO vs LO: integrated rates

- Huge effects along “singular” lines where LO is suppressed: $f_p \to 0$ (at finite $r$) and $Z + (A-Z) r = 0$
- Similar features for different targets and $\lambda_s/\lambda_\Theta$, $m_X$
NLO vs LO: spectra

\( m_X = 10 \) GeV

Model B

Model A

\[ \frac{dR}{dE_R}(\text{events/kg/year/keV}) \]

\[ E_R \text{ (keV)} \]

\[ \frac{dR}{dE_R}(\text{events/kg/year/keV}) \]

\[ E_R \text{ (keV)} \]
NLO vs LO: spectra

- Large distortion:
  \[ f_{n,p}(E_R) \text{ vs } \eta(E_R) \]
  \[ E_R \sim (2\mu_{XA^2}v_0^2)/m_A \]
Impact of NLO corrections

• While consistent with power counting, NLO effects can have significant impact

• LO and NLO terms depend differently on \( \lambda \)'s → corrections to amplitude can be larger than what expected by power counting, for certain choices of \( \lambda \)'s

• Description of scalar-mediated WIMP-nucleus scattering involves 4 parameters \((\lambda_{u,d,s,\theta}/\Lambda^2)\) rather than 2 \((\sigma_p \text{ and } r = f_n/f_p)\)
“Isospin-violating dark matter”

- $n$ and $p$ do not have to couple to DM in the same way

  Kurylov-Kamionkowski 2003
  Giuliani 2005
  Chang-Liu-Pierce-Weiner-Yavin 2010
  Feng-Kumar-Marfatia-Sanford 2011
“Isospin-violating dark matter”

- $n$ and $p$ do not have to couple to DM in the same way.
- Idea revived by conflicts in searches using different nuclei, when interpreted at LO with $r = f_n/f_p = 1$.

PDG review on Dark Matter
“Isospin-violating dark matter”

- n and p do not have to couple to DM in the same way.

- Idea revived by conflicts in searches using different nuclei, when interpreted at LO with $r = f_n/f_p = 1$.

- Can reconcile experiments by having $f_n \neq f_p$? (i.e. destructive interference in Xe but not-so-destructive for other nuclei).
Degradation factors: LO

\[ D_{LO}(r) = \frac{R_{LO}(r, \sigma_p)}{R_{LO}(1, \sigma_p)} \]

\[ D_{LO}(r) = \frac{(Z + (A - Z)r)^2}{A^2} \]

Singles out “Xe-phobic” region \( r = -0.7 \)
• LO fit with $r = -0.7$ shows compatibility between CMDS-Si and LUX
• But it relies on large suppression of LO Xe cross-section
• What about NLO chiral corrections?
Degradation factors: NLO

\[
D_{NLO}^{NLO}(r, \bar{\lambda}_s, \bar{\lambda}_\theta) = \frac{R_{NLO}(r, \sigma_p, \bar{\lambda}_s, \bar{\lambda}_\theta)}{R_{LO}(1, \sigma_p)}
\]

- \(D_{NLO}^{NLO}\) still quadratic in \(r\)
- Location of the minimum shifts, depending on \(\bar{\lambda}_s, \bar{\lambda}_\theta\)

\[
\bar{\lambda}_{s,\theta} \equiv \frac{\lambda_{s,\theta}}{\lambda_u}
\]

(couplings to heavy quarks)
• Location and depth of the “dip” vs heavy-quark coupling:

- This points to a manifold of “Xe-phobic” couplings (beyond $r=-0.7$)

- $r_{\text{MIN}}$ can take any value
- In most cases “degradation”
NLO fit to data (I)

- \( r=-0.7 \) leads to good compatibility or full exclusion, depending on the values of \( \lambda_s, \theta \)
NLO fit to data (II)

- New compatibility regions at \( r \neq -0.7 \)

- Characterized by quite different values of \( \lambda_u,d,s,\theta \)

- Different collider and indirect-detection signatures: richer structure!
What about $N^2$LO, ...?

- At small recoil energy, to all orders in $\chi$EFT

\[
dR \sim \left( Z + \Delta x + r (A - Z - \Delta x) \right)^2
\]
What about $N^2$LO, ...?

- At small recoil energy, to all orders in $\chi$EFT

$$r_{\text{min}} = \frac{\bar{Z}}{1 - \bar{Z}} \cdot \frac{1 + \frac{\Delta_\chi}{Z}}{1 - \frac{\Delta_\chi}{1 - Z}}$$

$$\bar{Z} = \frac{Z}{A}$$

- As long as $\Delta_\chi$ has well behaved expansion, $r_{\text{min}}$ and $dR(r)$ are stable against higher order corrections

$$dR \sim \left( Z + \Delta_\chi + r (A - Z - \Delta_\chi) \right)^2$$

$\Delta_\chi = 0.15, 0.17, \ldots$

Xe, $\bar{Z} \sim 0.4$
Phenomenology: axial-vector interactions
Axial-vector interaction

- GeV-scale effective Lagrangian involves 3 short-distance couplings

\[ \mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{\chi} \gamma_5 \chi \cdot \sum_q A_q \bar{\psi}_q \gamma \gamma_5 \psi_q \]

- Chiral realization up to $O(p^3)$: one- and two-body currents

Axial-vector interaction

- GeV-scale effective Lagrangian involves 3 short-distance couplings
  \[ \mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{\chi} \gamma_5 \chi \cdot \sum_q A_q \bar{\psi}_q \gamma_5 \psi_q \]

- Chiral realization up to $O(p^3)$: one- and two-body currents

- Two-body treated by summing 2nd nucleon over occupied states: density-dependent 1-body currents

- Compute within shell model spin response of $^{129,131}$Xe, $^{127}$I, $^{73}$Ge, $^{20}$Si...
• Cross-section

\[ \frac{d\sigma}{dp^2} = \frac{8G_F^2}{(2J + 1)v^2} \, S_A(p) \]

\[ S_A(p) = a_0^2 S_{00}(p) + a_0 a_1 S_{01}(p) + a_1^2 S_{11}(p) \]

\[ a_1 = (A_u - A_d)g_A \]

\[ a_0 = (A_u + A_d)(\Delta u + \Delta d) + 2A_s \Delta s. \]

• 25-55% (!) reduction of isovector structure factors $S_{11}$ at low $p^2$
• Limits on spin-dependent WIMP-nucleus cross section
Conclusions

• Chiral EFT: systematic tool to analyze WIMP-Nucleus scattering and reconstruct or bound WIMP-quark couplings

• Impact of chiral corrections
  • **Quantitative**: precision DM phenomenology
  • **Qualitative**: sizable effects possible through interplay of short-distance parameters and hadronic effects (dramatic effects for IVDM)
Future directions

- Operator matching beyond Scalar and Axial-vector
- Nuclear structure input:
  - One body response functions beyond shell model?
  - Two-body currents (keep in mind that a calculation with 30% error is better than ignoring them)
Extra slides
WIMPs

- Species $X$ freezes out with
  \[ \Omega_X \propto \sigma_A^{-1} \]
  ($\sim$ independent of mass)

- Correct DM density if $\sigma_A \sim$ EW strength

- Candidates in most TeV-scale extensions of the Standard Model

- Interaction strength such that several probes are possible
Galactic halo properties
(important input for non-gravitational detection)

- DM density profiles.
  Local density: $\sim 0.4 \text{ GeV/cm}^3$

- Velocity distribution: nearly Maxwellian with $v_0 \sim 220-270$ Km/s and $v_{\text{esc}} \sim 500-600$ Km/s
1. WIMP couplings to $\pi$, $N$

- Hadronic realization of

$$\mathcal{L}_{\text{QCD}} + \sum_{q=u,d,s} s_q \overline{q} q + s_\theta \theta^\mu\_\mu$$

- Usual chiral Lagrangian, organized as expansion in derivatives $\partial \sim \mathcal{O}(p)$ and chiral symmetry breaking $m_q \sim \mathcal{O}(p^2)$

\[\mathcal{L}_M = \mathcal{L}_M^{(2)} + \mathcal{L}_M^{(4)} + \ldots\]

\[\sim \frac{p_i \cdot p_j}{F_\pi^2}\]

\[\mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \ldots\]

\[\sim \frac{g_A}{F_\pi} p_\pi \cdot S_N\]
2. Power counting and $V_{XN}, V_{XNN}, \ldots$

- Non-perturbative amplitude $T$: sum of ladder diagrams

- One insertion of $A$-nucleon irreducible amplitude with external probe attached
  - Has well defined power counting (no IR enhancements $\sim M_n/q$)

- Arbitrary many insertions of $A$-nucleon irreducible amplitude with only strong vertices

\[ M_{A,X} \]
2. Power counting and $V_{XN}, V_{XNN}, \ldots$

- Non-perturbative amplitude $T$: sum of ladder diagrams

- Match $T$ to non-rel. Lippmann-Schwinger description

\[ M_{A,X}(\vec{q}_1, \ldots, \vec{q}_A, \vec{q}_X) \leftrightarrow H_I(\vec{x}_1, \ldots, \vec{x}_A, \vec{x}_X) = V_{XN} + V_{XNN} + \ldots \]

Leading terms in $H_I$ (potentials) determined by power counting of $M_{A,X}$
• Power counting for $M_{A,X} \sim p^\nu$

\[
\nu = 4 - A - 2C + 2L + \sum_i V_i \epsilon_i + \epsilon_W
\]

Weinberg 1979, 1991

- connected parts
- loops
- $\#$ of vertices of type “i”
- chiral dimension of vertex
- $\#$ of nucleon legs

• Leading terms:
  • maximize $C$
  • minimize $L$
  • minimize $\epsilon_{i,W}$ (use low order Lagrangians)
• Scalar density:

• Leading order:

• NLO:

• NNLO: ....

• Energy-momentum tensor: first corrections arise to NNLO ($\varepsilon_W = -1$ for nucleon vertex and $\varepsilon_W = 0$ for pion vertex)
3. Nuclear matrix elements

\[ \langle f | \hat{T} | i \rangle = (2\pi)^3 \delta^{(3)}(\vec{q}_X + \vec{q}_A) \, T(\vec{q}_X) \quad \text{for} \quad T(\vec{q}_X) = T_1 + T_2 \]

\[ T_1 = \sum_{i=1,A} \int d\vec{x}_i \, \rho_1(\vec{x}_i) \otimes \tilde{V}_1(\vec{q}_X; \vec{x}_i) \]

\[ T_2 = \sum_{i<j} \int d\vec{x}_i \, d\vec{x}_j \, \rho_2(\vec{x}_i, \vec{x}_j) \otimes \tilde{V}_2(\vec{q}_X; \vec{x}_i, \vec{x}_j) \]

q-dependent “potentials”, related to momentum-space amplitudes

\[ \tilde{V}_1(\vec{q}_X; \vec{x}_i) = -\int \frac{d\vec{q}_i}{(2\pi)^3} e^{-i\vec{q}_i \cdot \vec{x}_i} (2\pi)^3 \delta^{(3)}(\vec{q}_i + \vec{q}_X) \, M_{1,\chi}(\vec{q}_i, \vec{q}_X) \]

\[ \tilde{V}_2(\vec{q}_X; \vec{x}_i, \vec{x}_j) = -\int \frac{d\vec{q}_i}{(2\pi)^3} \frac{d\vec{q}_j}{(2\pi)^3} e^{-i\vec{q}_i \cdot \vec{x}_i} e^{-i\vec{q}_j \cdot \vec{x}_j} (2\pi)^3 \delta^{(3)}(\vec{q}_i + \vec{q}_j + \vec{q}_X) \, M_{2,\chi}(\vec{q}_i, \vec{q}_j, \vec{q}_X) \]
3. Nuclear matrix elements

\[ \langle f | \hat{T} | i \rangle = (2\pi)^3 \delta^{(3)}(\bar{q}_X + \bar{q}_A) \ T(\bar{q}_X) \quad \text{with} \quad T(\bar{q}_X) = T_1 + T_2 \]

\[
T_1 = \sum_{i=1,A} \int d\vec{x}_i \ \rho_1(\vec{x}_i) \otimes \tilde{V}_1(\bar{q}_X; \vec{x}_i) \\
T_2 = \sum_{i<j} \int d\vec{x}_i \ d\vec{x}_j \ \rho_2(\vec{x}_i, \vec{x}_j) \otimes \tilde{V}_2(\bar{q}_X; \vec{x}_i, \vec{x}_j)
\]

Nuclear structure input: one- and two-body densities in the ground state

\[
\rho_1(\vec{x}) = \int d\vec{x}_1 \ldots d\vec{x}_{A-1} \ |\psi_0(\vec{x}_1, \ldots, \vec{x}_{A-1}, \vec{x})|^2 \\
\rho_2(\vec{x}, \vec{y}) = \int d\vec{x}_1 \ldots d\vec{x}_{A-2} \ |\psi_0(\vec{x}_1, \ldots, \vec{x}_{A-2}, \vec{x}, \vec{y})|^2
\]
Ratios of Xe to Si, Ge rates

\[ m_X = 10 \text{ GeV} \]

\[ (\lambda_s, \lambda_\theta, \sigma_{\pi N}/\text{MeV}) = (0, 0, 45), (0, 0.1, 45), (0, -0.1, 45), (0, -0.025, 45), (1, 1, 60) \]