EM structure of $A = 2, 3$ nuclei in $\chi$EFT

———For Sergio’s 80th birthday———

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June 27, 2014
EM structure of few-nucleon systems in $\chi$EFT

Outline:

- General considerations
- EM current and charge operators up to one loop
- Elastic form factors of $A=2$ and 3 systems
- Summary and outlook

In collaboration with:

L. Girlanda    L. Marcucci    M. Piarulli
A. Kievsky    S. Pastore    M. Viviani

References:

Pastore et al., PRC80, 034004 (2009); Girlanda et al., PRL105, 232502 (2010);
Pastore et al., PRC84, 024001 (2011); Piarulli et al., PRC87, 014006 (2013)
Nuclear $\chi$EFT approach

Weinberg, PLB 251, 288 (1990); NPB 363, 3 (1991); PLB 295, 114 (1992)

- $\chi$EFT exploits the $\chi$-symmetry of QCD to restrict the form of $\pi$ interactions with other $\pi$’s, and with $N$’s, . . .
- The pion couples by powers of its momentum $Q$, and $\mathcal{L}_{\text{eff}}$ can be systematically expanded in powers of $Q/\Lambda_{\chi}$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

- $\chi$EFT allows for a perturbative treatment in terms of a $Q$—as opposed to a coupling constant—expansion
- The unknown coefficients in this expansion—the LEC’s—are fixed by comparison with experimental data
Formalism

- **Time-ordered perturbation theory (TOPT):**
  \[ \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left( \frac{1}{E_i - H_0 + i \eta} H_1 \right)^{n-1} \mid i \rangle \]

- A contribution with \( N \) interaction vertices and \( L \) loops scales as
  \[
  \left( \prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right) \times Q^{-(N - NK - 1)} Q^{-2NK} \times Q^{3L} \]

- Each of the \( NK \) energy denominators involving only nucleons is of order \( Q^{-2} \)

- Each of the other \( N - NK - 1 \) energy denominators involving also pion energies is expanded as
  \[
  \frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[ 1 + \frac{E_i - E_I}{\omega_\pi} + \frac{(E_i - E_I)^2}{\omega_\pi^2} + \ldots \right]
  \]

- **Power counting:**
  \[ T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots , \text{ and } T^{N^nLO} \sim (Q/\Lambda_\chi)^n T^{LO} \]
$T^{(n)}$ up to order $n = 1$

Time-ordered diagrams contributing to the $\chi$EFT $T$-matrix up to order $Q^1$:
From amplitudes to potentials

- Construct $v$ such that when inserted in LS equation
  \[ v + v G_0 v + v G_0 v G_0 v + \ldots \quad G_0 = 1/(E_i - E_I + i\eta) \]
  leads to $T$-matrix order by order in the power counting

- Assume
  \[ v = v^{(0)} + v^{(1)} + v^{(2)} + \ldots \quad v^{(n)} \sim Q^n \]

- Determine $v^{(n)}$ from
  \[
  \begin{align*}
  v^{(0)} & = T^{(0)} \\
  v^{(1)} & = T^{(1)} - \left[ v^{(0)} G_0 v^{(0)} \right] \\
  v^{(2)} & = T^{(2)} - \left[ v^{(0)} G_0 v^{(0)} G_0 v^{(0)} \right] - \left[ v^{(1)} G_0 v^{(0)} + v^{(0)} G_0 v^{(1)} \right]
  \end{align*}
  \]

where
  \[ v^{(m)} G_0 v^{(n)} \sim Q^{m+n+1} \]
\( \nu(n) \) up to order \( n = 1 \)

\[ \nu^{(0)} = T^{(0)} \text{ consists of (static) OPE and contact terms} \]

\[ \nu^{(1)} = T^{(1)} - [\nu^{(0)} G_0 \nu^{(0)}] \text{ vanishes} \]
Including EM interactions

- Power counting of EM interactions (treated in first order)

\[ T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \ldots \quad T_\gamma^{(n)} \sim e Q^n \]

- For \( v_\gamma = A^0 \rho - A \cdot j \) to match \( T_\gamma \) order by order

\[
\begin{align*}
  v_\gamma^{(-3)} &= T_\gamma^{(-3)} \\
  v_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-3)} \right] \\
  v_\gamma^{(-1)} &= T_\gamma^{(-1)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} + \text{permutations} \right] \\
  &\quad - \left[ v_\gamma^{(-2)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-2)} \right]
\end{align*}
\]

and so on up to \( n = 1 \ (e Q) \)
EM operators up to one loop

NN potential:

and consistent EM current (and charge) operators:

LO : eQ^{-2}

NLO : eQ^{-1}

N^2LO : eQ^0

N^3LO : eQ

unknown LEC's
The LEC’s characterizing $\rho$ and $j$

- Pion loop corrections known ($g_A$ and $F_\pi$)
- No unknown LEC’s in $\rho$
- Five unknown LEC’s in $j$: $d$’s could be determined by $(\gamma, \pi)$ data on the nucleon

- $d^S, d^V_1, d^V_2$ fixed by assuming $\Delta$ dominance, $d^S_1$, $c^S$, and $c^V$ fixed by fitting $A = 2$ and 3 EM observables
- Three-body currents at N$^3$LO vanish:
Determining the unknown LEC’s in $j$

- After (perturbative) renormalization, resulting operators still need to be regularized:

$$C_\Lambda(k) = e^{-(k/\Lambda)^4}$$

- For each $\Lambda$:
  - Isovector $(d^V_1, d^V_2)$ from $\Delta$-resonance saturation, isoscalar $(d^S, c^S)$ by reproducing $\mu_d$ and $\mu^S$
  - Isovector $c^V$ fixed by reproducing either $\sigma_{np}$ or $\mu^V$

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$c^S$</th>
<th>$d^S \times 10$</th>
<th>$c^V(\sigma_{np})$</th>
<th>$c^V(\mu^V)$</th>
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</thead>
<tbody>
<tr>
<td>500</td>
<td>4.072</td>
<td>2.190</td>
<td>-13.3</td>
<td>-7.98</td>
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<tr>
<td>600</td>
<td>11.38</td>
<td>3.231</td>
<td>-22.3</td>
<td>-11.7</td>
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Deuteron static properties

- $r_d^{\text{EXP}} = 1.9734(44) \text{ fm}$ versus N3LO (AV18)

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<thead>
<tr>
<th>$\Lambda$</th>
<th>500</th>
<th>600</th>
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<tbody>
<tr>
<td>LO</td>
<td>1.976 (1.969)</td>
<td>1.968 (1.969)</td>
</tr>
<tr>
<td>N3LO</td>
<td>1.976 (1.969)</td>
<td>1.968 (1.969)</td>
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</table>

- $Q_d^{\text{EXP}} = 0.2859(3) \text{ fm}^2$ versus N3LO (AV18)

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<th>$\Lambda$</th>
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<tbody>
<tr>
<td>LO</td>
<td>0.275 (0.270)</td>
<td>0.2711 (0.270)</td>
</tr>
<tr>
<td>N2LO</td>
<td>0.273 (0.268)</td>
<td>0.2692 (0.268)</td>
</tr>
<tr>
<td>N3LO</td>
<td>0.285 (0.281)</td>
<td>0.2820 (0.280)</td>
</tr>
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Deuteron waves
Deuteron $A$ and $T_{20}$ structure functions
Deuteron $B$ structure function
$r(^3\text{He}) = 1.959(30) \text{ fm}$ and $r(^3\text{H}) = 1.755(86) \text{ fm}$ versus N3LO/N2LO

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<th>$^3\text{He}$</th>
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<tbody>
<tr>
<td>$\Lambda$</td>
<td>500</td>
<td>600</td>
<td>500</td>
</tr>
<tr>
<td>LO</td>
<td>1.966</td>
<td>1.958</td>
<td>1.762</td>
</tr>
<tr>
<td>N4LO</td>
<td>1.966</td>
<td>1.958</td>
<td>1.762</td>
</tr>
</tbody>
</table>

and versus AV18/UIX values of $r(^3\text{He}) = 1.950 \text{ fm}$ and $r(^3\text{H}) = 1.743 \text{ fm}$
$^3$He/$^3$H charge form factors
$^3\text{He}/^3\text{H}$ magnetic form factors

Summary

$^3\text{He}/^3\text{H}$ results

$^2\text{H}$ results

EM operators

EM structure

Nuclear $\chi$EFT

Formalism

$^2\text{H}$ results

Summary
Sensitivity to LEC’s in $A=3$ magnetic structure

- Set II: $c^V$ fixed by $\sigma_{np}$; $\mu^V$ overestimated by $\approx 3\%$
- Set III: $c^V$ fixed by $\mu^V$; $\sigma_{np}$ underestimated by $\approx 1\%$
Summary

- Charge operators at one loop only depend on $g_A$ and $F_\pi$; current operators depend on five additional LEC's.
- Various strategies to fix these LEC's investigated.
- $Q_d^{\text{EXP}}$ reproduced (under-predicted) by N3LO (AV18) potential; differences traced back to deuteron w.f.'s.
- EM structure of $A=2$–$3$ nuclei well reproduced with chiral charge and current operators for $q \lesssim 3 m_\pi$. 