$N \rightarrow N^*$ Transitions from Holographic QCD

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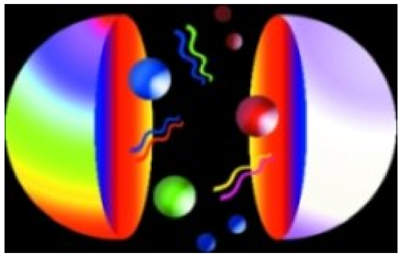
Nucleon Resonance Structure Electroproduction at High Photon Virtualities with the Class 12 Detector Workshop

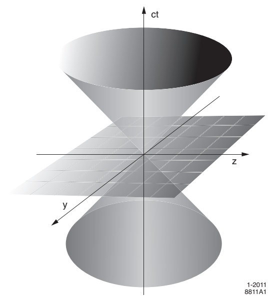
Jefferson Lab, May 16, 2011



From Nick Evans

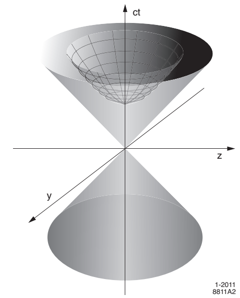
1 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical *vs.* kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



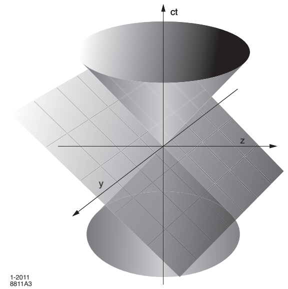
$$H, \, \mathbf{K} \;\; \text{dynamical}, \qquad \mathbf{L}, \, \mathbf{P} \;\; \text{kinematical}$$

- *Point form*: hypersurface is an hyperboloid



$$P^{\mu} \;\; \text{dynamical}, \qquad M^{\mu\nu} \;\; \text{kinematical}$$

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$



$$P^{-}, \, L^{x}, \, L^{y} \;\; \text{dynamical}, \qquad P^{+}, \, \mathbf{P}_{\perp}, \, L^{z}, \, \mathbf{K} \;\; \text{kinematical}$$

- LF quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of partonic content of a hadron

- Calculation of a matrix elements $\langle P+q|J|P\rangle$ requires boosting the hadronic bound state from $|P\rangle$ to $|P+q\rangle$: extremely complicated in the instant form, whereas $\mathbf{K}$ is trivial in the LF

- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible

- LF coordinates

  $x^+ = x^0 + x^3$ light-front time

  $x^- = x^0 - x^3$ longitudinal space variable

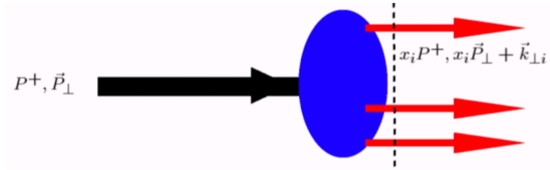
  $k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

  $k^- = k^0 - k^3$ light-front energy

  $$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

  On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Light-Front Fock Representation



- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \qquad\qquad |n\rangle = \{\, |uud\rangle, |uudg\rangle, |uud\overline{q}q\rangle, \; \cdots \}$$

with $k_i^2 = m_i^2, \;\; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each constituent i in state n

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

Semiclassical Approximation to QCD in the Light Front

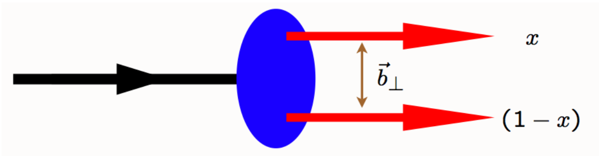
[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$
- Relevant variable in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x,\zeta,\varphi) = e^{iL^z\varphi} X(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$
(P^+, $\mathbf{P}_\perp$ and J_z commute with P^-)

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = |L^z|)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

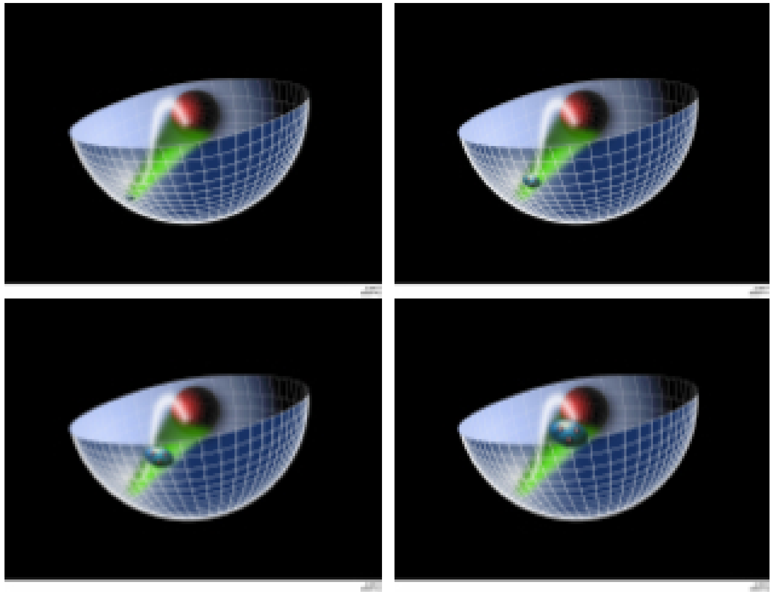
$$\boxed{\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption

2 Light-Front Holographic Mapping



- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space
- Short distances $x_\mu x^\mu \to 0$ map to UV conformal AdS$_5$ boundary $z \to 0$
- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\quad \left(x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z^2)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'}$$
$$- \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

$$[D_N, D_K] \Phi_{M_1 \cdots M_J} = - R^L_{M_1 N K} \Phi_{L \cdots M_J} - \cdots - R^L_{M_J N K} \Phi_{M_1 \cdots L}$$

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu \!=\! M^2$

- Construct effective action in terms of spin-J modes Φ_J with only physical degrees of freedom

  [H. G. Dosch, S. J. Brodsky and GdT (in preparation)]

- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \cdots A_J} = e^{M_1}_{A_1} e^{M_2}_{A_2} \cdots e^{M_J}_{A_J} \, \Phi_{M_1 M_2 \cdots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \cdots A_J}$$

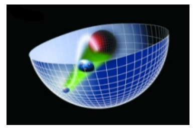
- Find effective action

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{N N'} \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \partial_N \hat{\Phi}_{\mu_1 \cdots \mu_J} \partial_{N'} \hat{\Phi}_{\mu_1' \cdots \mu_J'} - \mu^2 \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \hat{\Phi}_{\mu_1 \cdots \mu_J} \hat{\Phi}_{\mu_1' \cdots \mu_J'} \Big)$$

  upon μ-rescaling

- Variation of the action gives AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



  with $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$ and all indices along 3+1

Dual QCD Light-Front Wave Equation

$\Phi_P(z) \quad \Leftrightarrow \quad |\psi(P)\rangle$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = \mathcal{M}^2\Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

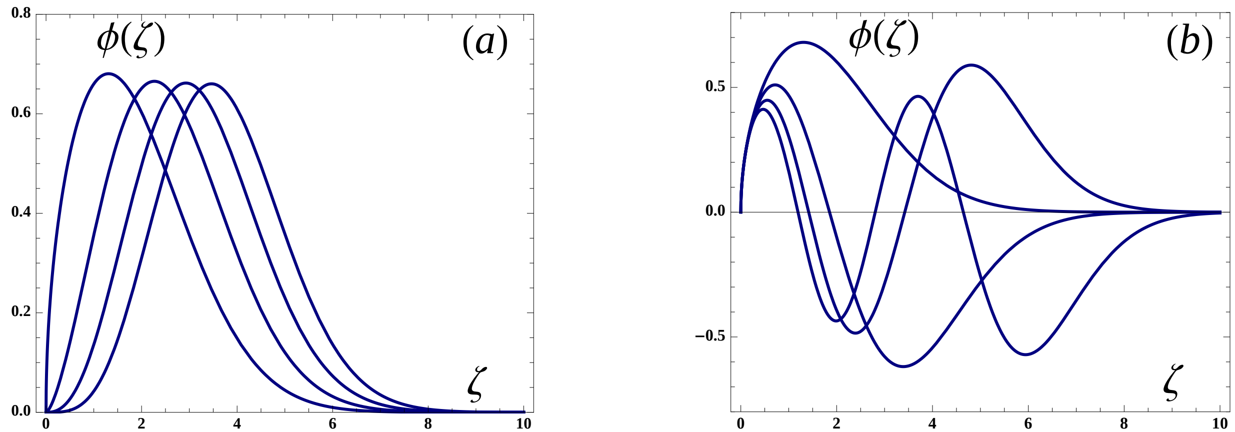
Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

- Dilaton $\varphi(z) = +\kappa^2 z^2$ (Minkowski metrics), $\varphi(z) = -\kappa^2 z^2$ (Euclidean metrics)
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, |\phi(z)^2| = 1$

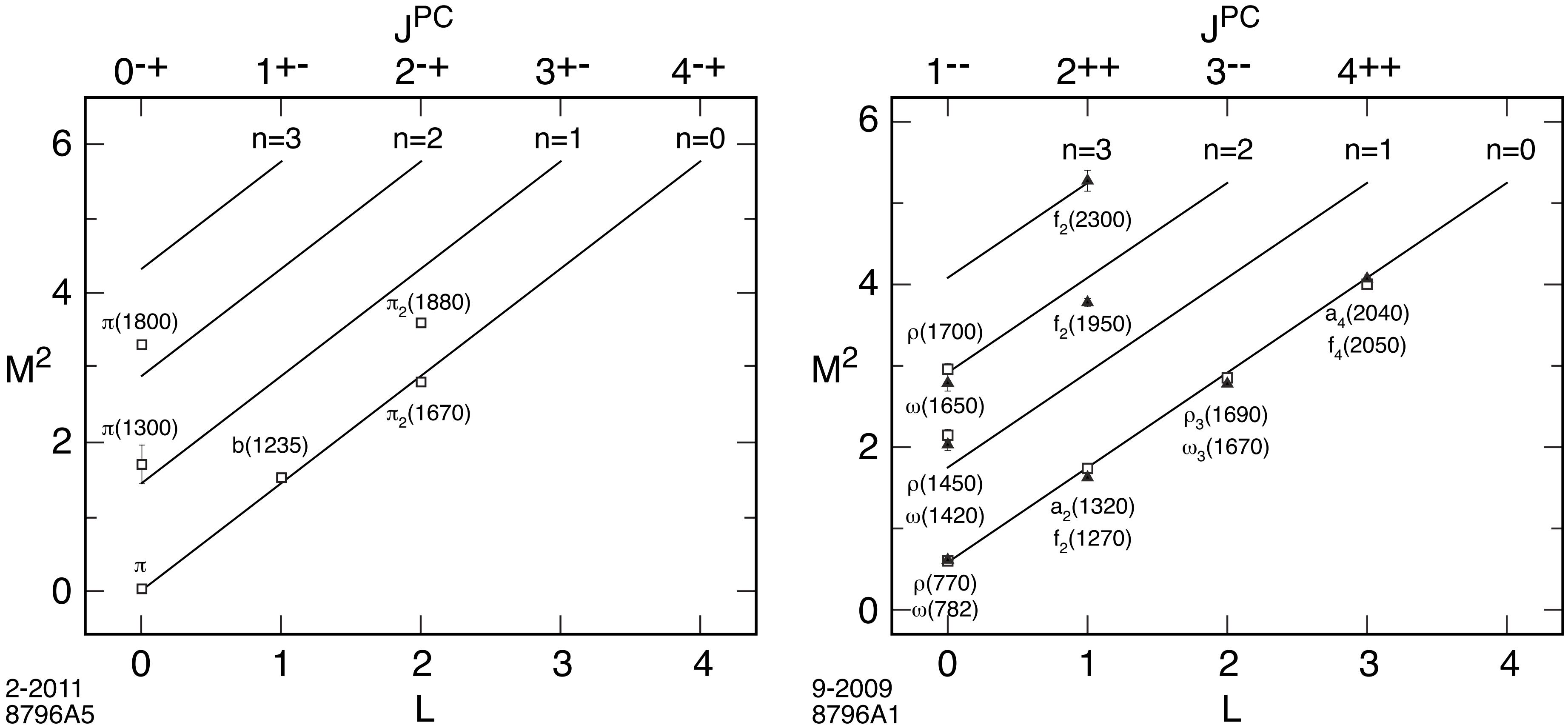
$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$



LFWFs $\phi_{n,L}(\zeta)$ in physical space time for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

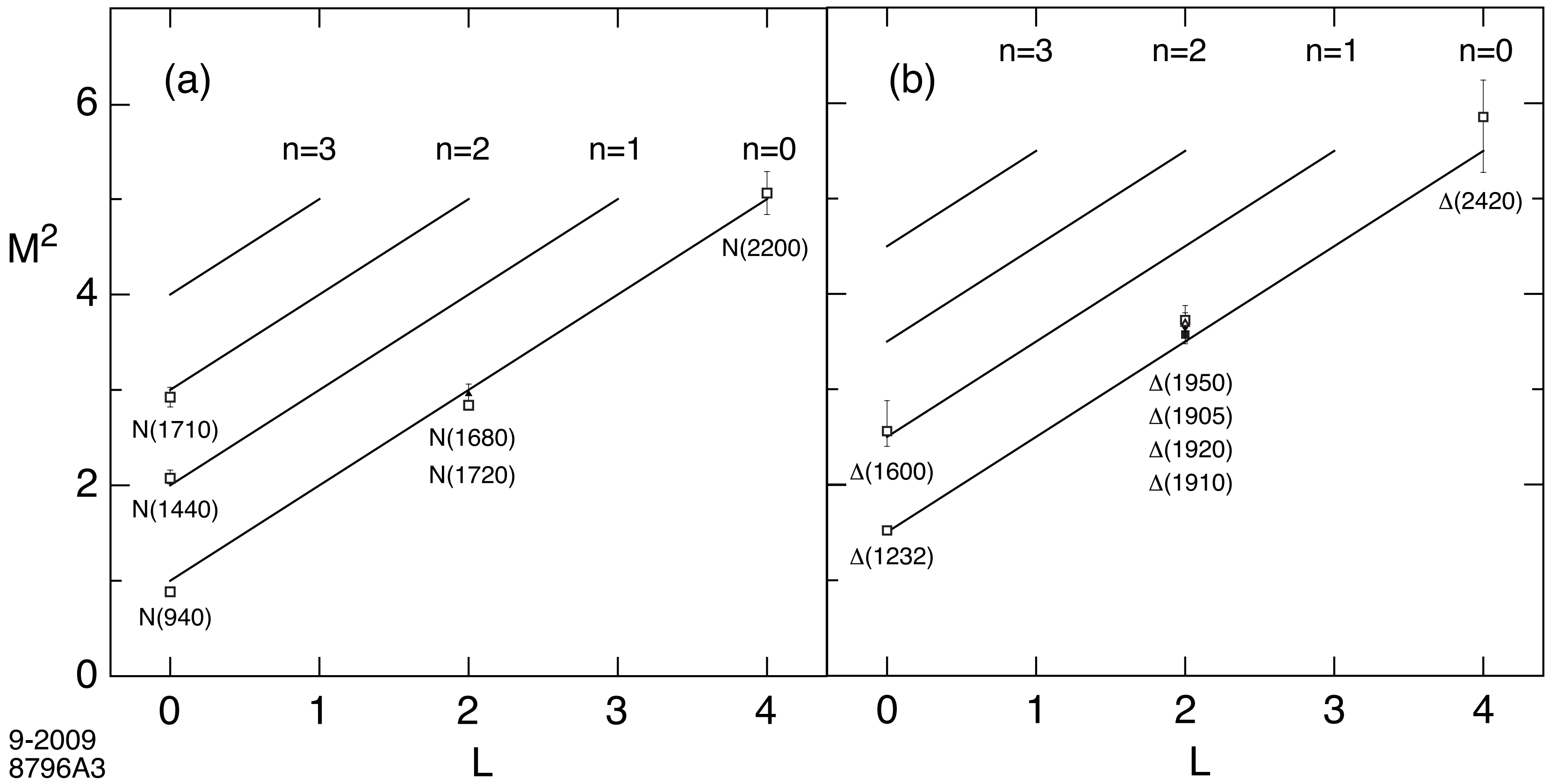


Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

Fermionic Modes and Baryon Spectrum

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same multiplicity of states for mesons and baryons!



Regge trajectories for positive parity N and Δ baryon families ($\kappa = 0.5$ GeV)

3 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)], PRD **77**, 056007 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} A^M(x, z) \Phi^*_{P'}(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \left(P + P'\right)^\mu F(Q^2)$$

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

  [Polchinski and Strassler (2002)]

- Mapping of J^+ elements at fixed light-front time: $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} V(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z\,\partial_z - z^2 Q^2\right] V(Q,z) = 0$$

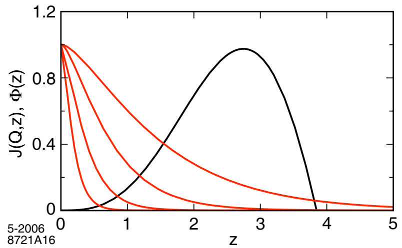
- Solution $\;V(Q,z) = zQK_1(zQ)\;$ for "free current"

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \qquad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{\varphi(z)} V(Q,z)\,\Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

Electromagnetic Form-Factor

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Consider a two-quark π^+ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q\!=\!0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{\varphi(z)} V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx\, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3}\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \Phi^2_{\pi^+}(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

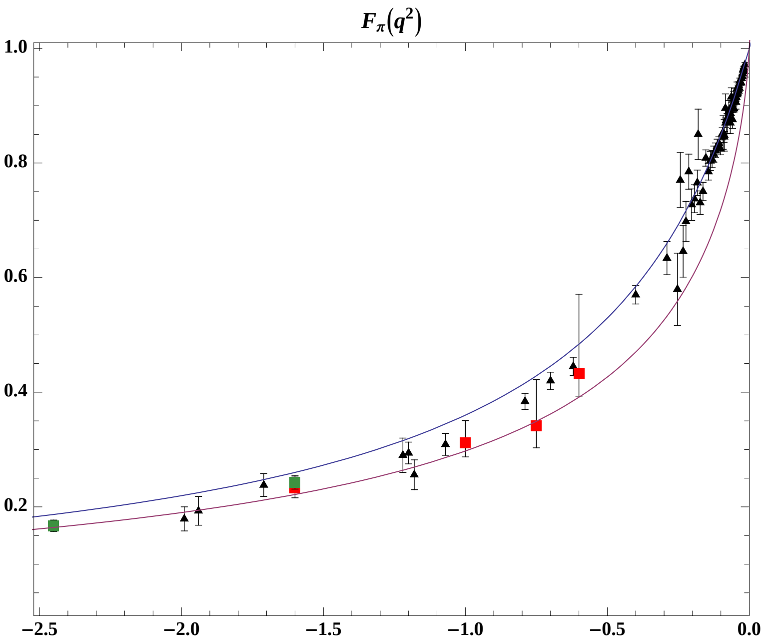
$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} e^{\varphi(z)/2}\, \Phi(\zeta), \qquad z \to \zeta$$

- “Free current” $V(Q,z) = zQK_1(zQ) \to$ infinite radius (mauve), no pole structure in time-like region
- “Dressed current” non-perturbative sum of an infinite number of terms $\longrightarrow$ finite radius (blue)
- Form factor in soft-wall model expressed as $N-1$ product of poles along vector radial trajectory [Brodsky and GdT (2008)] $\left(\mathcal{M}_\rho{}^2 \to 4\kappa^2(n+1/2)\right)$

$$F(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\cdots\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}$$



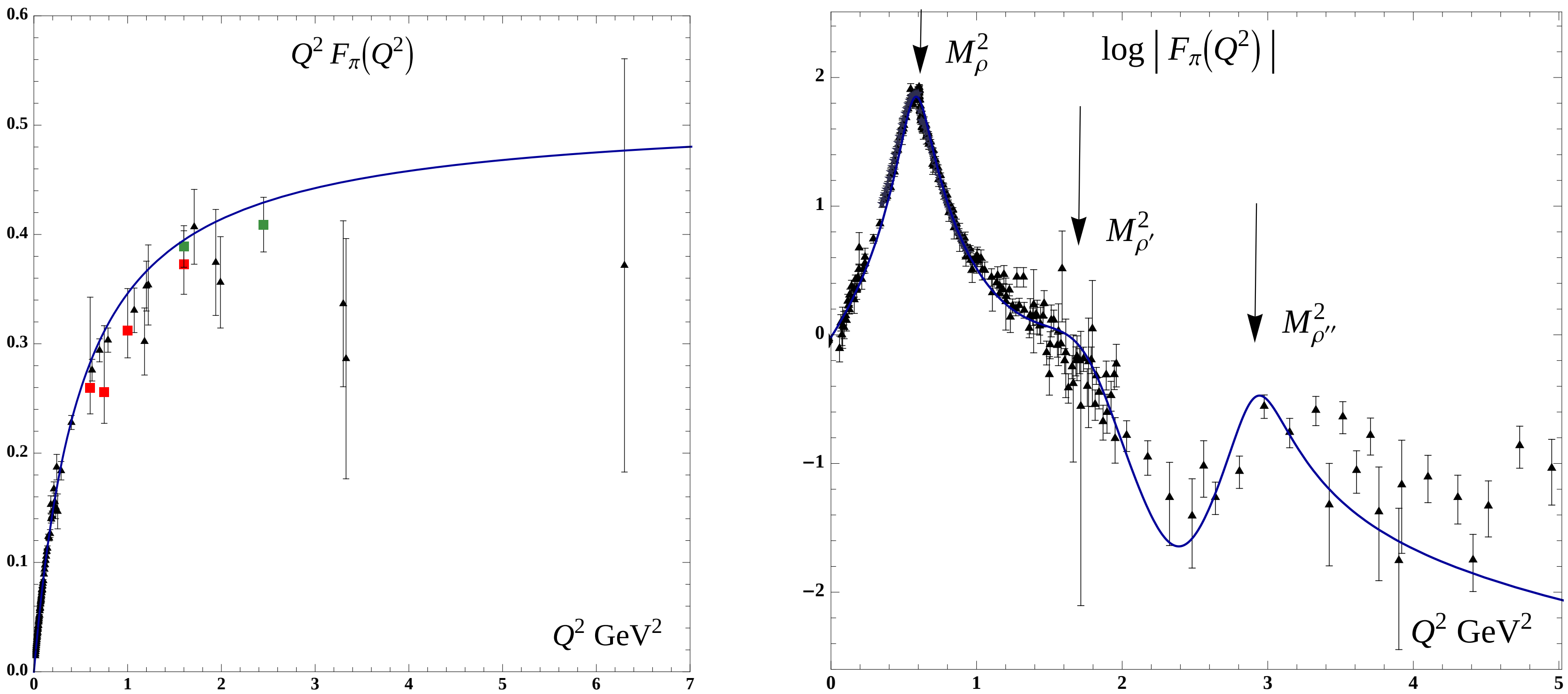
Pion form factor (lowest mode)

- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\overline{q}/\pi}|q\overline{q}\rangle_{\tau=2} + \psi_{q\overline{q}q\overline{q}/\pi}|q\overline{q}q\overline{q}\rangle_{\tau=4} + \cdots$$

- Expansion of LFWF up to twist 4 (monopole + tripole)

$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130, \; \Gamma_{\rho'} = 400, \; \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\overline{q}q\overline{q}} = 13\%$



- Only interaction in LF holographic semiclassical approx is the confinement potential: create Fock states with extra quark-antiquark pairs, no dynamical gluons

Nucleon Form Factors

- Light Front Holographic Approach [Brodsky and GdT]
- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode $\Psi_P(x,z)$

$$\int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \overline{\Psi}_P(x,z) \, e^M_A \Gamma^A A_M(x,z) \Psi_P(x,z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \langle \psi(P'), \sigma' | J^\mu | \psi(P), \sigma \rangle$$

- Effective AdS/QCD model [Abidin and Carlson, Phys. Rev. D79, 115003 (2009)]

  Additional term in the 5-dim action:

$$\eta \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \overline{\Psi} \, e^M_A e^N_B \left[\Gamma^A, \Gamma^B\right] F_{MN} \Psi$$

  Couplings η determined by static quantities

- Compute Dirac form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function in soft-wall model

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right)$$

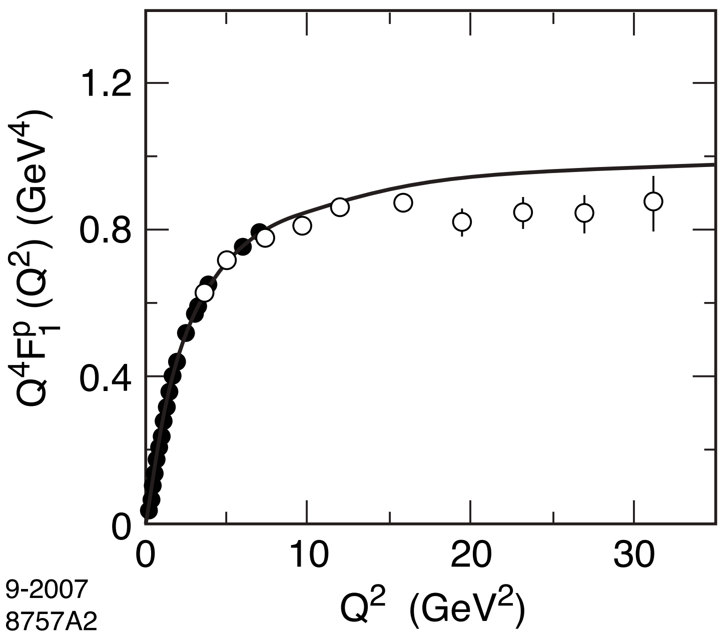
$$\Psi_-(z) = \frac{\kappa^{3+L}}{R^2} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} z^{9/2+L} L_n^{L+2}\left(\kappa^2 z^2\right)$$

- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} \, e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\Big(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\Big)\Big(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\Big)}$$



Data compilation from M. Diehl (2006)

Nucleon Transition Form Factors

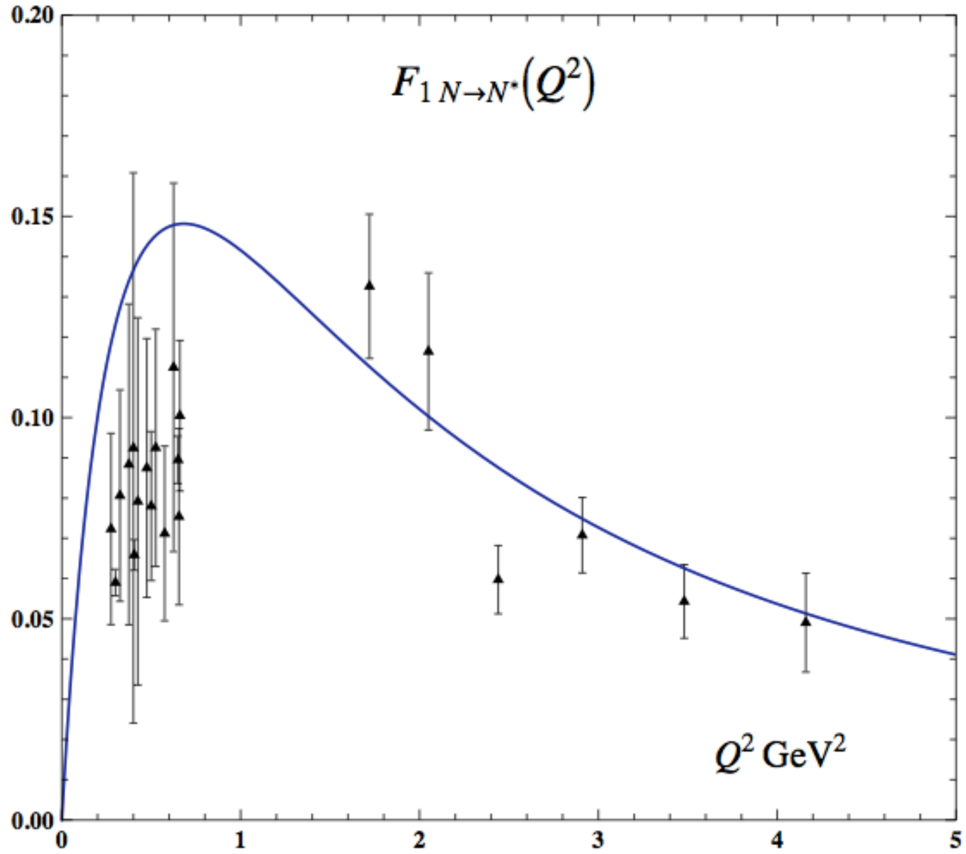
- Compute spin non-flip EM transition $N(940) \to N^*(1440)$
$\left(n = 0, L = 0, s_z = \frac{1}{2} \to n = 1, L = 0, s_z = \frac{1}{2}\right)$

$$\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$$

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Find

$$F^p_{1\,N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$



Data from I. Aznauryan, *et al.* CLAS (2009)