Baryon Resonances from Lattice QCD

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$N^* @ \text{high } Q^2, \ 2011$

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Auspices of the Hadron Spectrum Collaboration
Lattice QCD

**Goal:** resolve highly excited states

\[ N_f = 2 + 1 \ (u,d + s) \]

**Anisotropic lattices:**

\[ (a_s)^{-1} \sim 1.6 \text{ GeV}, \ (a_t)^{-1} \sim 5.6 \text{ GeV} \]
Spectrum from variational method

Two-point correlator

\[
C'(t) = \langle 0 | \Phi'(t) \Phi(0) | 0 \rangle
\]

\[
C'(t) = \sum_n e^{-E_n t} \langle 0 | \Phi'(0) | n \rangle \langle n | \Phi(0) | 0 \rangle
\]

Matrix of correlators

\[
C(t) = \begin{bmatrix}
\langle 0 | \Phi_1(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2(0) | 0 \rangle & \cdots \\
\langle 0 | \Phi_2(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2(0) | 0 \rangle & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}
\]

Diagonalize:

eigenvalues → spectrum
eigenvectors → wave function overlaps

Each state optimal combination of \( \Phi_i \)

\[
\Omega_n = v^n_1 \Phi_1 + v^n_2 \Phi_2 + \ldots
\]

Benefit: orthogonality for near degenerate states
Operator construction

Baryons: permutations of 3 objects

Permutation group $S_3$: 3 representations

- **Symmetric**: 1-dimensional
  - e.g., $uud + udu + duu$
- **Antisymmetric**: 1-dimensional
  - e.g., $uud - udu + duu - ...$
- **Mixed**: 2-dimensional
  - e.g., $udu - duu$ & $2duu - udu - uud$

Color antisymmetric $\rightarrow$ Require Space [Flavor Spin] symmetric

Classify operators by these permutation symmetries:
- Leads to rich structure
Orbital angular momentum via derivatives

Couple derivatives onto single-site spinors: Enough D’s – build any J,M

\[ \mathcal{O}^{JM} \leftarrow (CGC')_{i,j,k} \left[ \vec{D} \right]_i \left[ \vec{D} \right]_j [\Psi]_k \]

Only using symmetries of continuum QCD

\[ \text{Ops} \leftarrow \text{Derivatives} \quad \begin{bmatrix} \text{Flavor} & \text{Dirac} \end{bmatrix} \]

Use all possible operators up to 2 derivatives (transforms like 2 units orbital angular momentum)
3-quark operators with up to two covariant derivatives – projected into definite isospin and continuum $J^P$

Spatial symmetry classification:

Nucleons: $N^{2S+1}L_{\pi}J^P$

By far the largest operator basis ever used for such calculations

Symmetry crucial for spectroscopy
Spin identified Nucleon & Delta spectrum

$m_\pi \sim 520$MeV

Statistical errors < 2%

arXiv:1104.5152
Spin identified Nucleon & Delta spectrum

SU(6)xO(3) counting
No parity doubling

$m_\pi \sim 520\text{MeV}$

arXiv:1104.5152
Spin identified Nucleon & Delta spectrum

Discern structure: wave-function overlaps

\[
m_\pi \sim 520\text{MeV}
\]

[Image of graph showing the spectrum of nucleon and delta particles with identified states and wave functions.]
Significant mixing in $J^+$

Discern structure: wave-function overlaps

$N^*$

13 levels/ops

$\Delta^*$

8 levels/ops

$^2S_s^2S_M^4S_M$

$^2D_S^2D_M^4D_M$ $^2P_A$
Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) O(3) but heavily mixed

Discrepancies?

Operator basis – spatial structure

What else?
Multi-particle operators
Spectrum of finite volume field theory

**Missing states:** “continuum” of multi-particle scattering states

**Infinite volume:** continuous spectrum

\[ E(p) = 2 \sqrt{m^2_\pi + p^2} \]

**Finite volume:** discrete spectrum

Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift

\[ \Delta E(L) \leftrightarrow \delta(E) : \text{Lüscher method} \]
Finite volume scattering

**Lüscher method**
- scattering in a periodic cubic box (length $L$)
- finite volume energy levels $E(L) \rightarrow \delta(E)$

E.g. just a single elastic resonance

At some $L$, have discrete excited energies

\[ \pi \pi \rightarrow \rho \rightarrow \pi \pi \]
\[ \pi N \rightarrow \Delta \rightarrow \pi N \]
I=1 $\pi\pi$ : the "$\rho$"

Extract $\delta_1(E)$ at discrete E

Extracted coupling: stable in pion mass

$g_{\rho\pi\pi}$

$\sin^2(\delta)$

$aE_{CM}$

$m_{\pi}^2$ (GeV$^2$)

Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288
What is a form-factor off of a resonance?

What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

\[ \langle N | J_\mu | N^* \rangle_\infty \leftarrow \left[ \delta'(E) + \Phi'(E) \right] \langle N | J_\mu | N^* \rangle_{\text{volume}} \]

Requires excited level transition FF’s: some experience
- Charmonium E&M transition FF’s \((1004.4930)\)
- Nucleon 1st attempt: “Roper” -> N \((0803.3020)\)

Range: few \(\text{GeV}^2\)
Limitation: spatial lattice spacing
(Very) Large $Q^2$

Standard requirements: \[ \frac{1}{L} \ll m_\pi, m_N, Q \ll \frac{1}{a} \]

Cutoff effects: lattice spacing \((a_s)^{-1} \sim 1.6 \text{ GeV}\)

Appeal to renormalization group: *Finite-Size* scaling

Use short-distance quantity: compute perturbatively and/or parameterize

\[
R(Q^2) = \frac{F(s^2 Q^2)}{F(Q^2)}, \quad s = 2
\]

“Unfold” ratio only at low $Q^2 / s^{2N}$

\[
F(Q^2) = R(Q^2 / s^2) R(Q^2 / s^4) \cdots R(Q^2 / s^{2N}) \ F(Q^2 / s^{2N})
\]

For $Q^2 = 100 \ \text{GeV}^2$ and $N=3$, \(Q^2 / s^{2N} \sim 1.5 \ \text{GeV}^2\)

Initial applications: factorization in pion-FF

D. Renner
Hadronic Decays

Some candidates: determine phase shift
Somewhat elastic

\[ m_\pi \sim 400 \text{ MeV} \]
Prospects

• Strong effort in excited state spectroscopy
  - New operator & correlator constructions → high lying states

• Results for baryon excited state spectrum:
  - No “freezing” of degrees of freedom nor parity doubling
  - Broadly consistent with non-relativistic quark model
  - Add multi-particles → baryon spectrum becomes denser

• Short-term plans: resonance determination!
  - Lighter quark masses
  - Extract couplings in multi-channel systems

• Form-factors:
  - Use previous resonance parameters: initially, $Q^2 \sim$ few GeV$^2$
  - Decrease lattice spacing: $(a_s)^{-1} \sim 1.6$ GeV → 3.2 GeV, then $Q^2 \sim 10$ GeV$^2$
  - Finite-size scaling: $Q^2 \rightarrow 100$ GeV$^2$ ???
Backup slides

- The end
Baryon Spectrum

“Missing resonance problem”

- What are collective modes?
- What is the structure of the states?

Nucleon Mass Spectrum (Exp): 4*, 3*, 2*

PDG uncertainty on B-W mass

Nucleon spectrum
Phase Shifts demonstration: $I=2$ $\pi\pi$

- $\pi\pi$ isospin=2
- Extract $\delta_0(E)$ at discrete $E$

No discernible pion mass dependence

1011.6352 (PRD)
Phase Shifts: demonstration

\[ \pi\pi \ \text{isospin}=2 \]

\[ \delta_2(E) \]

Graph showing the phase shifts \( \delta_2 \) as a function of \( k^2 / \text{GeV}^2 \) for different energies (524 MeV, 444 MeV, and 396 MeV) and lattice sizes (16^3, 20^3, and 24^3). The graph includes data from Hoogland, Lesty, Cohen, and Durusoy.
Nucleon $J^-$

Overlaps

$$Z^n_i = \langle J^- | O^n_i | 0 \rangle$$

Little mixing in each $J^-$

Nearly “pure” $[S=1/2 \ & \ 3/2] \quad 1^-$
N & Δ spectrum: lower pion mass

Still bands of states with same counting
More mixing in nucleon N=2  J^+

m_π ~ 400 MeV
Operators are not states

Two-point correlator

\[ C(t) = \langle 0 | \Phi'(t) \Phi(0) | 0 \rangle \]

\[ C(t) = \sum_n e^{-E_n t} \langle 0 | \Phi'(0) | n \rangle \langle n | \Phi(0) | 0 \rangle \]

Full basis of operators: many operators can create same state

\[ \langle n; J^P | O_i^n | 0 \rangle = Z_i^n \]

States may have subset of allowed symmetries