Hadron form factors and transverse charge and spin densities

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Workshop on Exploring Hadrons with Electromagnetic Probes: Structure, Excitations, Interactions
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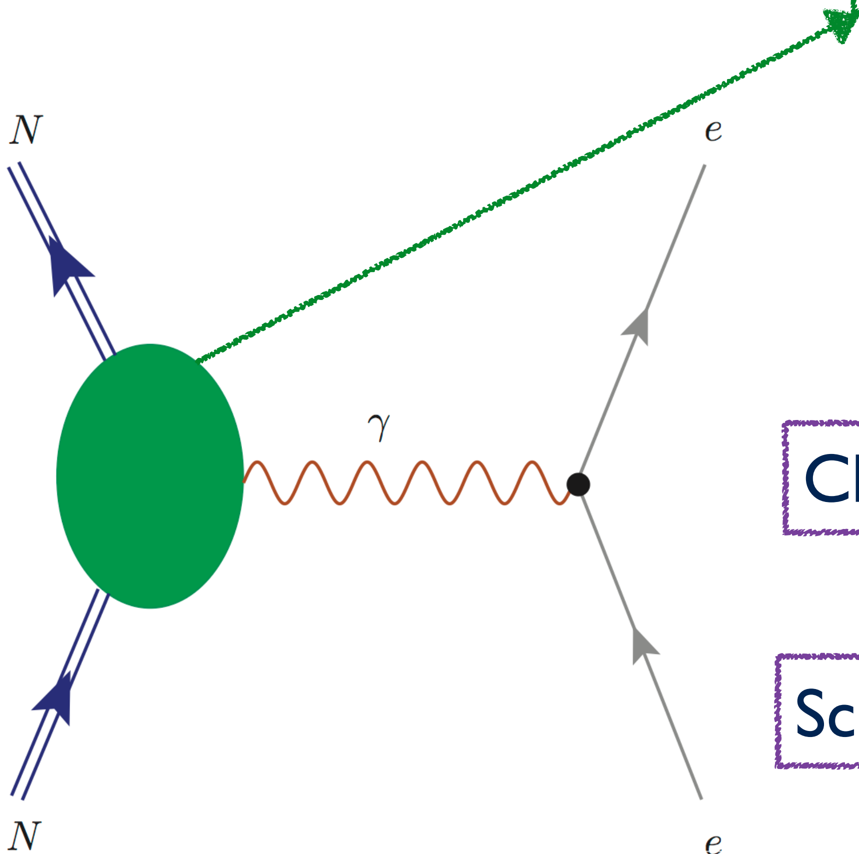
Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



Electromagnetic form factors

↑

3-D Fourier Transform

↑

Charge & Magnetisation Densities

↑

Schroedinger Eq. & Wave functions

Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \rho(\mathbf{r}) \quad \rightarrow \quad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

However, the initial and final momenta are different in a relativistic case. Thus, the initial and final wave functions are different.

→ **Probability interpretation is wrong in a relativistic case!**

↓

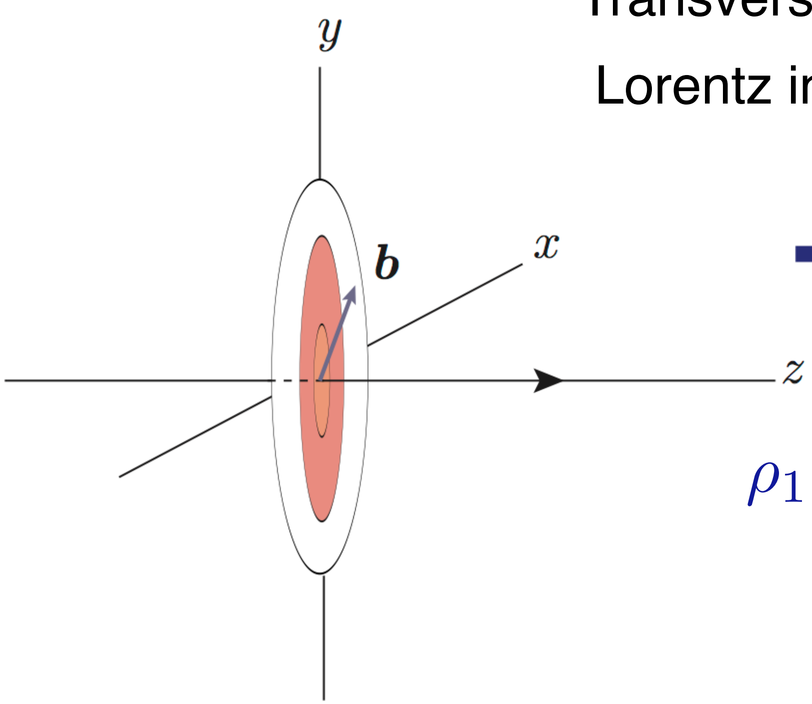
We need a correct interpretation of the form factors

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

G.A. Miller, PRL **99,** 112001 (2007)

Interpretation of the EMFFs

Modern understanding of the form factors



Transverse Charge densities $\rho(\mathbf{b})$

Lorentz invariant: independent of any observer.

$\boldsymbol{p}$ → Infinite momentum framework

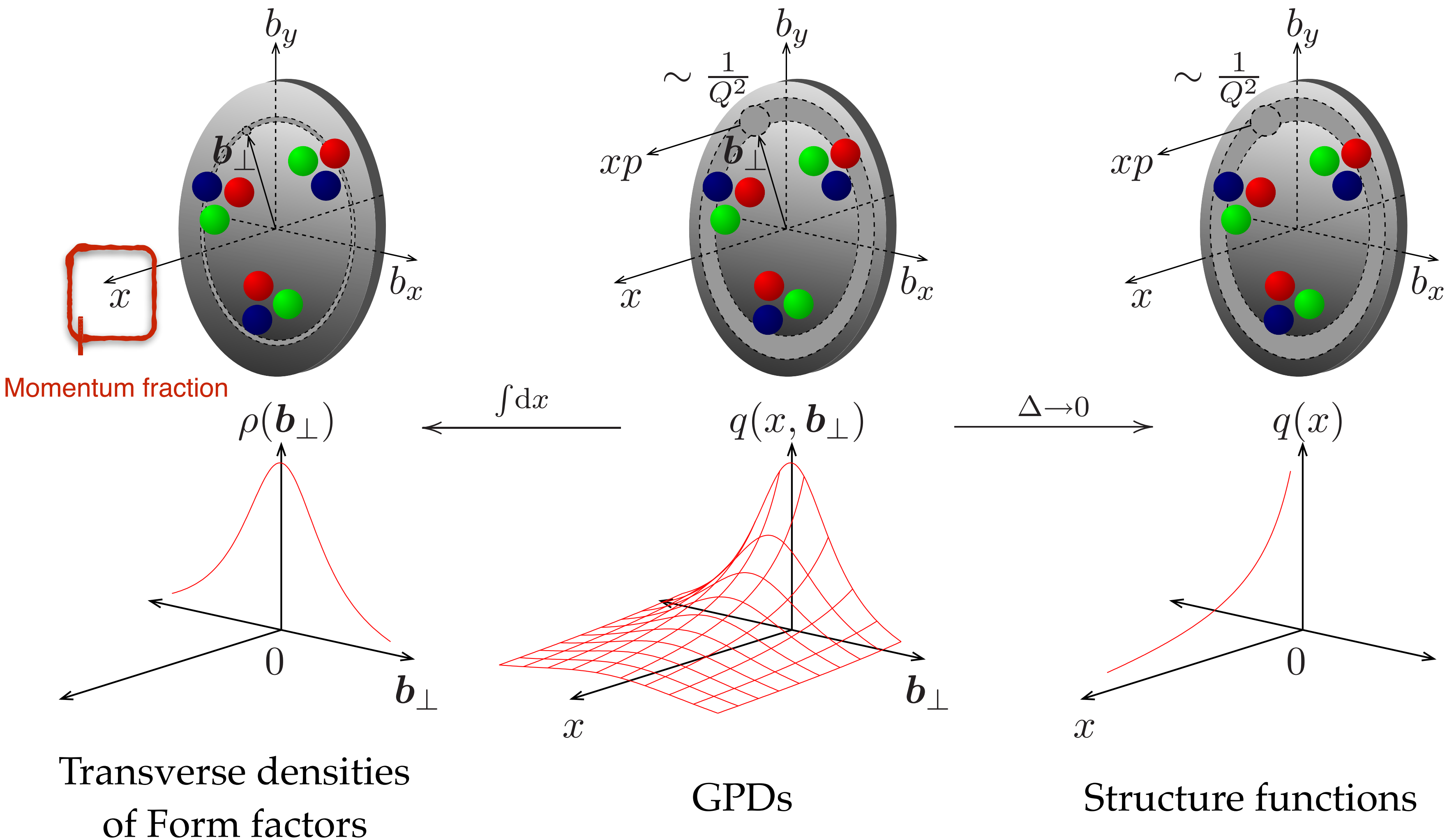
$$\rho_1(b) = \sum_q e_q^2 \int dx\, f_{q-\bar{q}}(x, \mathbf{b})$$

GPDs

Form factors

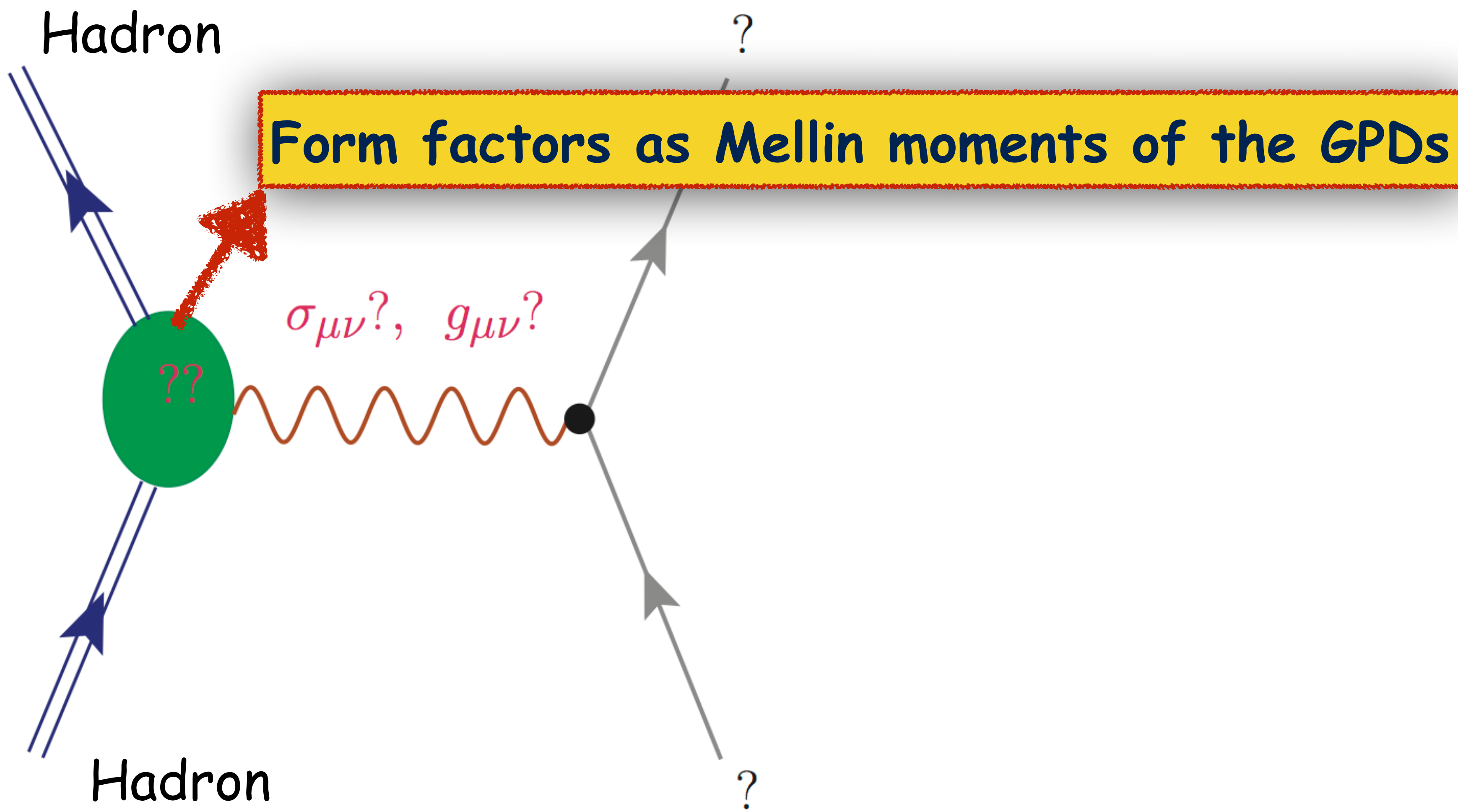
$$F(q^2) = \int d^2 b e^{i\mathbf{q}\cdot\mathbf{b}} \rho(\mathbf{b})$$

Hadron Tomography



Generalised Parton Distributions

Probes are unknown for **Tensor form factors**
and the **Energy-Momentum Tensor form factors!**



Structure of hadrons

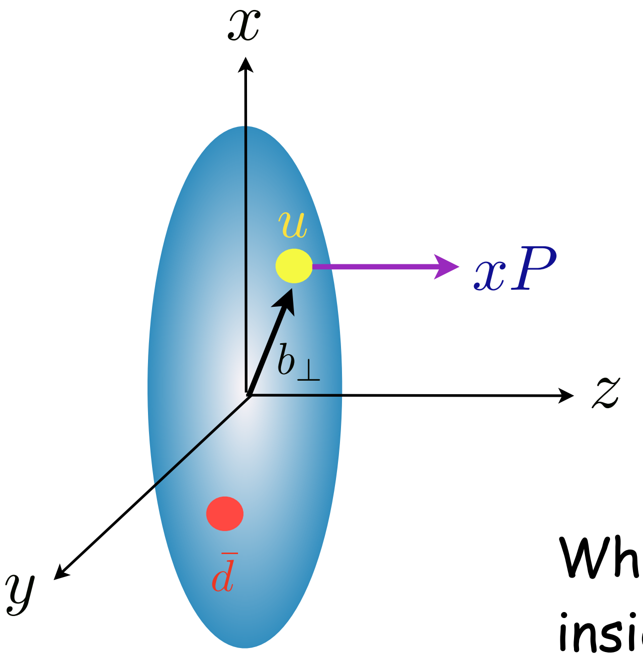
Modern approach: Generalized parton distributions make it possible to get access to these EMT & tensor form factors.

Generalized form factors as Mellin moments of the GPDs

In the present talk, I would like to review various form factors of the pion with its transverse charge & spin structures.

The spin structure of the Pion

Vector & Tensor Form factors of the pion



Pion: Spin S=0

Longitudinal spin structure is trivial.

$$\langle \pi(p')|\bar{\psi}\gamma_3\gamma^5\psi|\pi(p)\rangle \;=\; 0$$

What about the transversely polarized quarks inside a pion?

→ Internal spin structure of the pion

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Spin probability densities in the transverse plane

A_{n0}: Vector densities of the pion, B_{n0}: Tensor densities of the pion

$$\int_{-1}^{1} dx\, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2)$$

Vector and Tensor form factors of the pion

$$\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)$$

$$\langle \pi^+(p_f) | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(p_i) \rangle = \mathcal{AS}\left[\frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\mathrm{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} B_{ni}(Q^2)\right]$$

Nonlocal chiral quark model

Gauged Effective Nonlocal Chiral Action

$$S_{\rm eff}[m,\pi] = -\mathrm{Spln}\left[i\not{D} + im + i\sqrt{M(iD,\,m)}U^{\gamma_5}(\pi)\sqrt{M(iD,\,m)}\right]$$

$$D_\mu = \partial_\mu - i\gamma_\mu V_\mu$$

The nonlocal chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- "Derived" from QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter

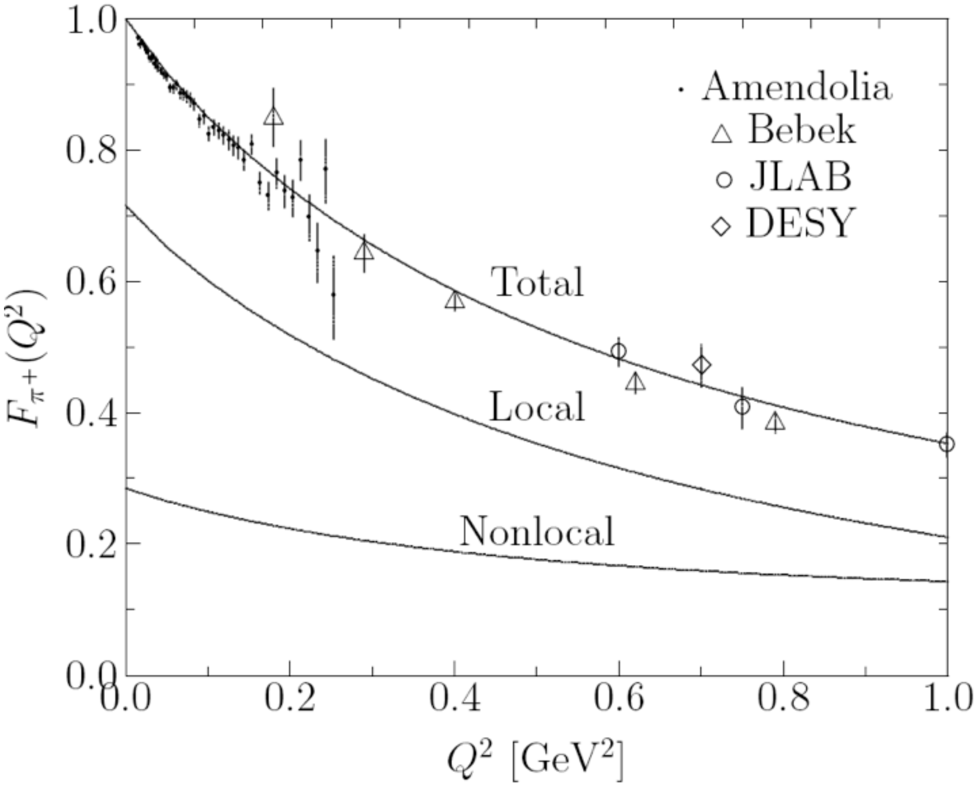
$\rho \approx 0.3\,\mathrm{fm},\ \ R \approx 1\,\mathrm{fm}$ $\qquad\qquad$ $\mu \approx 600\,\mathrm{MeV}$

D. Diakonov & V. Petrov Nucl.Phys. B272 (1986) 457
H.-Ch.K, M. Musakhanov, M. Siddikov Phys. Lett. B **608**, 95 (2005).
Musakhanov & H.-Ch. K, Phys. Lett. B **572**, 181-188 (2003)

EM Form factor of the pion

EM form factor (A_{10})

$$\langle \pi(p_f)|\psi^\dagger\gamma_\mu\hat{Q}\psi|\pi(p_i)\rangle = (p_i+p_f)A_{10}(q^2)$$



$$\sqrt{\langle r^2\rangle} = 0.675\,\mathrm{fm}$$

$$\sqrt{\langle r^2\rangle} = 0.672 \pm 0.008\,\mathrm{fm}\ (\mathrm{Exp})$$

$$F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1+Q^2/M^2}$$

M(Phen.): 0.714 GeV

M(Lattice): 0.727 GeV

M(XQM): 0.738 GeV

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

Tensor Form factor of the pion

- Generalized tensor form factors

$$\langle \pi^+(p_f)|q^\dagger(0)\sigma_{ab}q(0)|\pi^+(p_i)\rangle = [(p_i\cdot a)(p_f\cdot b)-(p_i\cdot b)(p_f\cdot a)]\frac{B_{10}(Q^2)}{m_\pi}, \qquad \sigma_{ab}=\sigma_{\mu\nu}a^\mu b^\nu$$

$$\langle \pi^+(p_f)|q^\dagger(0)\sigma_{ab}(i\overleftrightarrow{D}D\cdot a)q(0)|\pi^+(p_i)\rangle = \{(p\cdot a)[(p_i\cdot a)(p_f\cdot b)-(p_i\cdot b)(p_f\cdot a)]\}\frac{B_{20}(Q^2)}{m_\pi}$$

- Transverse spin density

$$\rho_1(b_\perp, s_x=\pm1)=\frac{1}{2}\left[A_{10}(b^2)\mp\frac{b\sin\theta_\perp}{m_\pi}B_{10}(b^2)'\right]$$

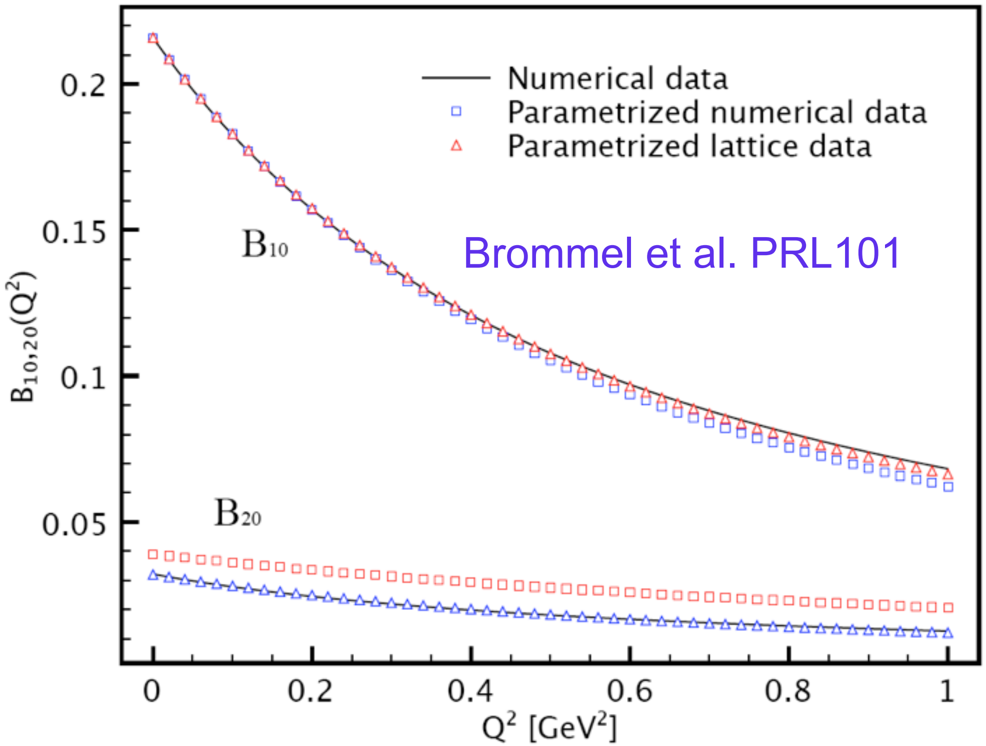
$$F(b_\perp^2)=\frac{1}{(2\pi)^2}\int d^2q_\perp e^{-ib_\perp\cdot q_\perp}F(q_\perp^2)=\frac{1}{2\pi}\int_0^\infty Q\,dQ\,J_0(bQ)F(Q^2)$$

$$\frac{\partial F(b_\perp^2)}{\partial b_\perp^2}\equiv F'(b_\perp^2)=-\frac{1}{4\pi b}\int_0^\infty Q^2\,dQ\,J_1(bQ)F(Q^2)$$

S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

For the kaon, S.i. Nam & HChK, Phys. Lett. **B**707, 546 (2012)

Tensor Form factor of the pion



RG equation for the tensor form factor

$$B_{10}(Q^2,\,\mu) = B_{10}(Q^2,\,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma/2\beta_0}$$

$$\gamma_1 = 8/3,\ \gamma_2 = 8,\ \beta_0 = 11N_c/3 - 2N_f/3$$

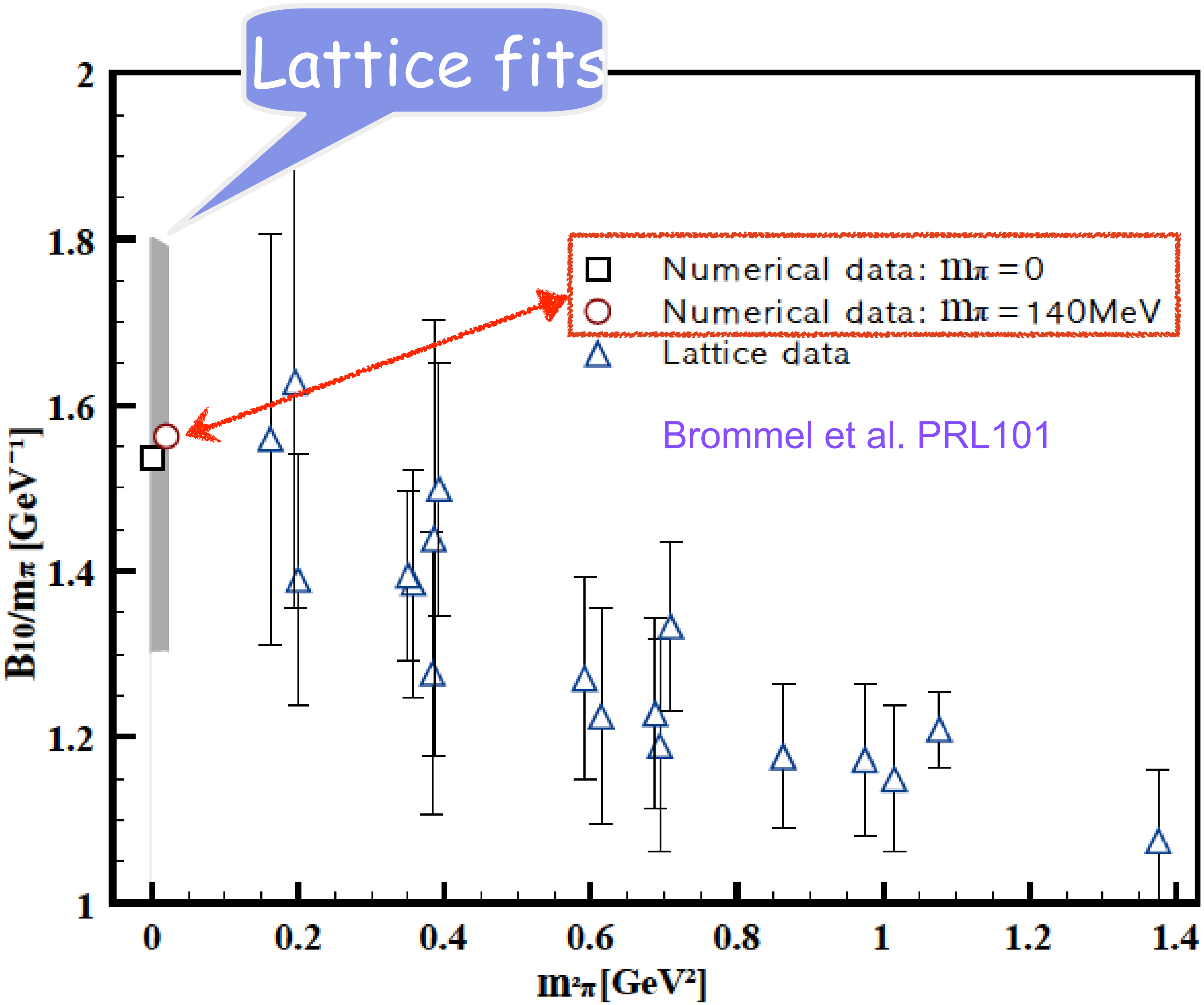
p-pole parametrization for the form factor

$$B_{10}(Q^2) \ = \ B_{10}(0)\left[1+\frac{Q^2}{pm_p^2}\right]^{-p}$$

S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

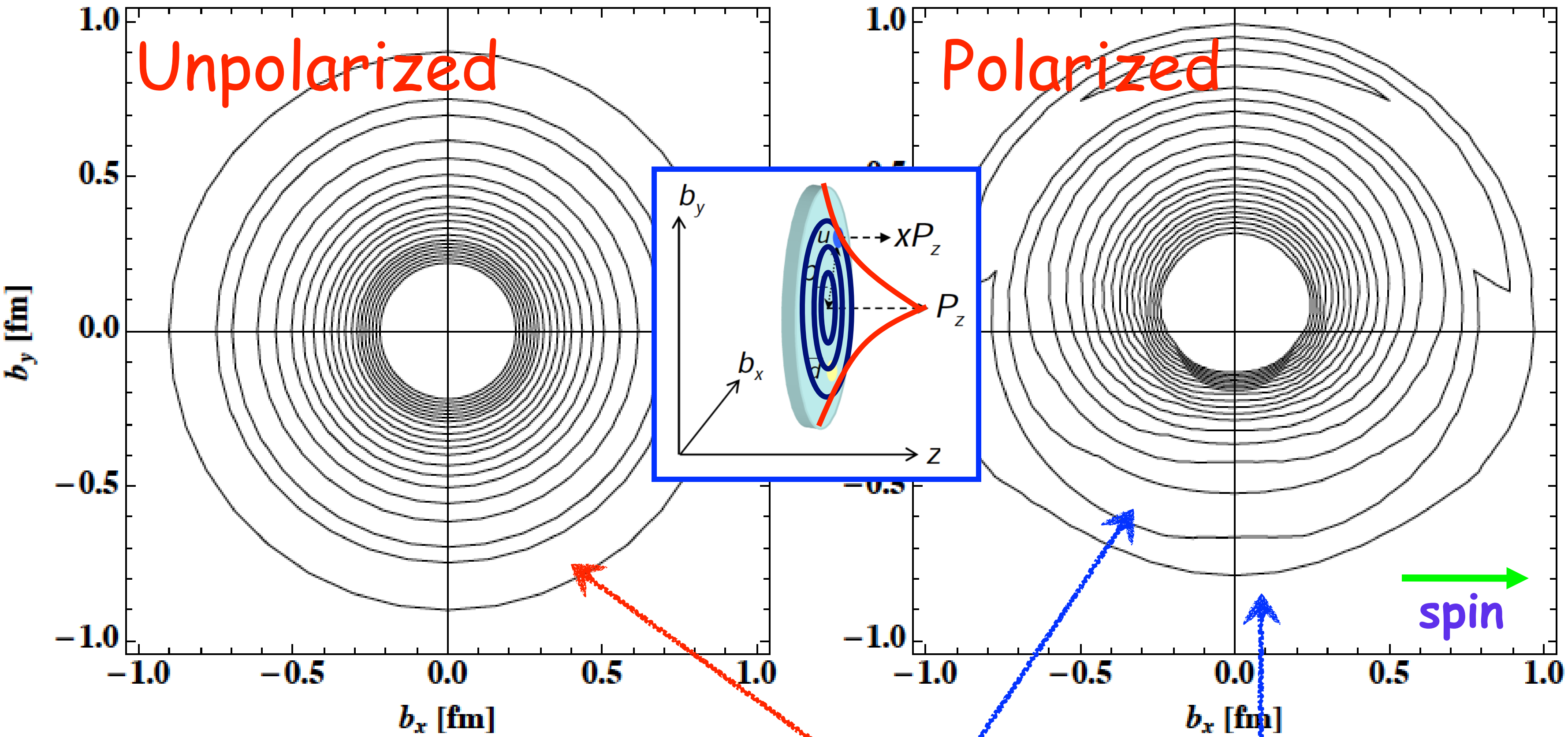
For the kaon, S.i. Nam & HChK, Phys. Lett. **B**707, 546 (2012)

Tensor Form factor of the pion



S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

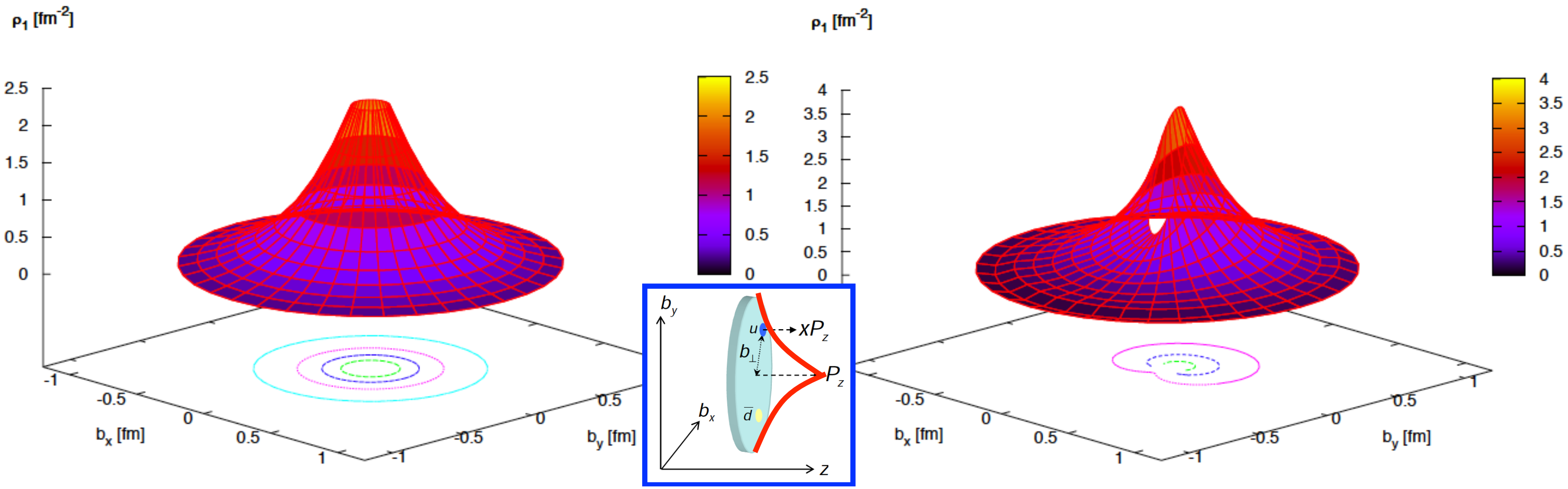
Spin density of the quark



$$\rho_1\left(b_\perp, s_x = \pm\frac{1}{2}\right) = \frac{1}{2}\left[A_{10}(b^2) \mp \frac{b\sin\theta}{m_\pi} B'_{10}(b^2)\right]$$

Polarization

Spin density of the quark



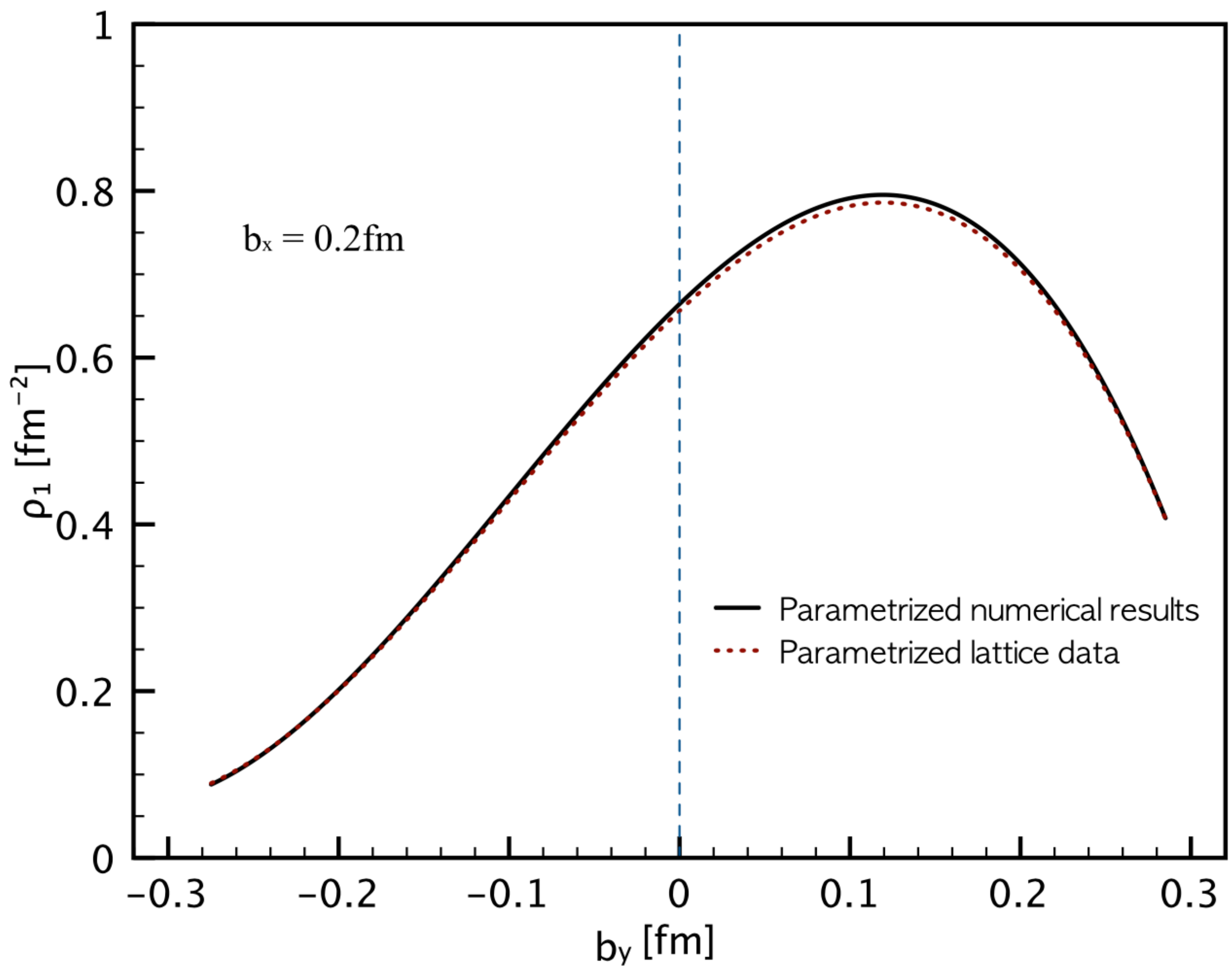
Significant distortion appears for the polarized quark!

$m_\pi = 140$ MeV	$B_{10}(0)$	m_{p_1} [GeV]	$\langle b_y \rangle$ [fm]	$B_{20}(0)$	m_{p_2} [GeV]
Present work	0.216	0.762	0.152	0.032	0.864
Lattice QCD [7]	0.216 ± 0.034	0.756 ± 0.095	0.151	0.039 ± 0.099	1.130 ± 0.265

Results are in a good agreement with the lattice calculation!

S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

Spin density of the quark



S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

EMT form factor of the pion

- Energy-momentum tensor (Gravitational) form factor: A_{20}, A_{22}

M. V. Polyakov and C. Weiss, Phys. Rev. D 60, 114017 (1999)

Isoscalar vector GPD

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\left\langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\right\rangle$$

Its second moments

$$\int dx\, x H_\pi^{I=0}(x,\,\xi,t) \;=\; A_{2,0}(t) + 4\xi^2 A_{2,2}(t)$$

Energy-momentum tensor form factor

$$\left\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\right\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 4P_\mu P_\nu \Theta_2(t)\right],$$

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

EMT form factor of the pion

- Energy-momentum tensor operator

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{\mu} i \overleftrightarrow{\partial}_{\nu\}}\psi(x).$$

- The pion mass

$$\langle \pi^a(p)|T_{00}(0)|\pi^b(p)\rangle\big|_{t=0} \;=\; 2m_\pi^2\Theta_2(0)\delta^{ab}$$

- Spatial component → Pressure of the pion

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\big|_{t=0} \;=\; \delta^{ab}\frac{3}{2}t\,\Theta_1(t)\bigg|_{t=0}$$

The pressure of any particle must be equal to zero:
Stability condition (von Laue condition)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

EMT form factor of the pion

- Pressure of the pion (In the chiral limit, the pressure vanishes trivially.)

$$\mathcal{P} = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle$$

$$= \frac{12N_c m M}{f_\pi^2} \int d\tilde{l} \frac{-l^2}{[l^2 + \overline{M}^2]^2} + \frac{12N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + \overline{M}^2]^3}$$

$$p^2 = -m_\pi^2$$

Quark condensate

$$i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{l} \frac{\overline{M}}{[l^2 + \overline{M}^2]}$$

Pion decay constant

$$f_\pi^2 = 4N_c \int_0^1 dx \int d\tilde{l} \frac{M\overline{M}}{[l^2 + \overline{M}^2 + x(1-x)p^2]^2}$$

The pressure of any particle must be equal to zero:
Stability condition (von Laue condition)

$$\mathcal{P} = \frac{3M}{f_\pi^2 \overline{M}} \left(m \langle \bar{\psi}\psi \rangle + m_\pi^2 f_\pi^2 \right) = 0$$

(By the Gell-Mann-Oakes-Renner relation)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

EMT form factor of the pion

- LECs in the curved space: L_{11}, L_{12}, L_{13}

$$\Theta_1(t) = 1 + \frac{2}{f_\pi^2}\left[t(4L_{11} + L_{12}) - 8m_\pi^2(L_{11} - L_{13})\right]$$

$$\Theta_2(t) = 1 - \frac{2t}{f_\pi^2}L_{12}.$$

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}, L_{12} = -2L_{11} = -3.2 \times 10^{-3},$$

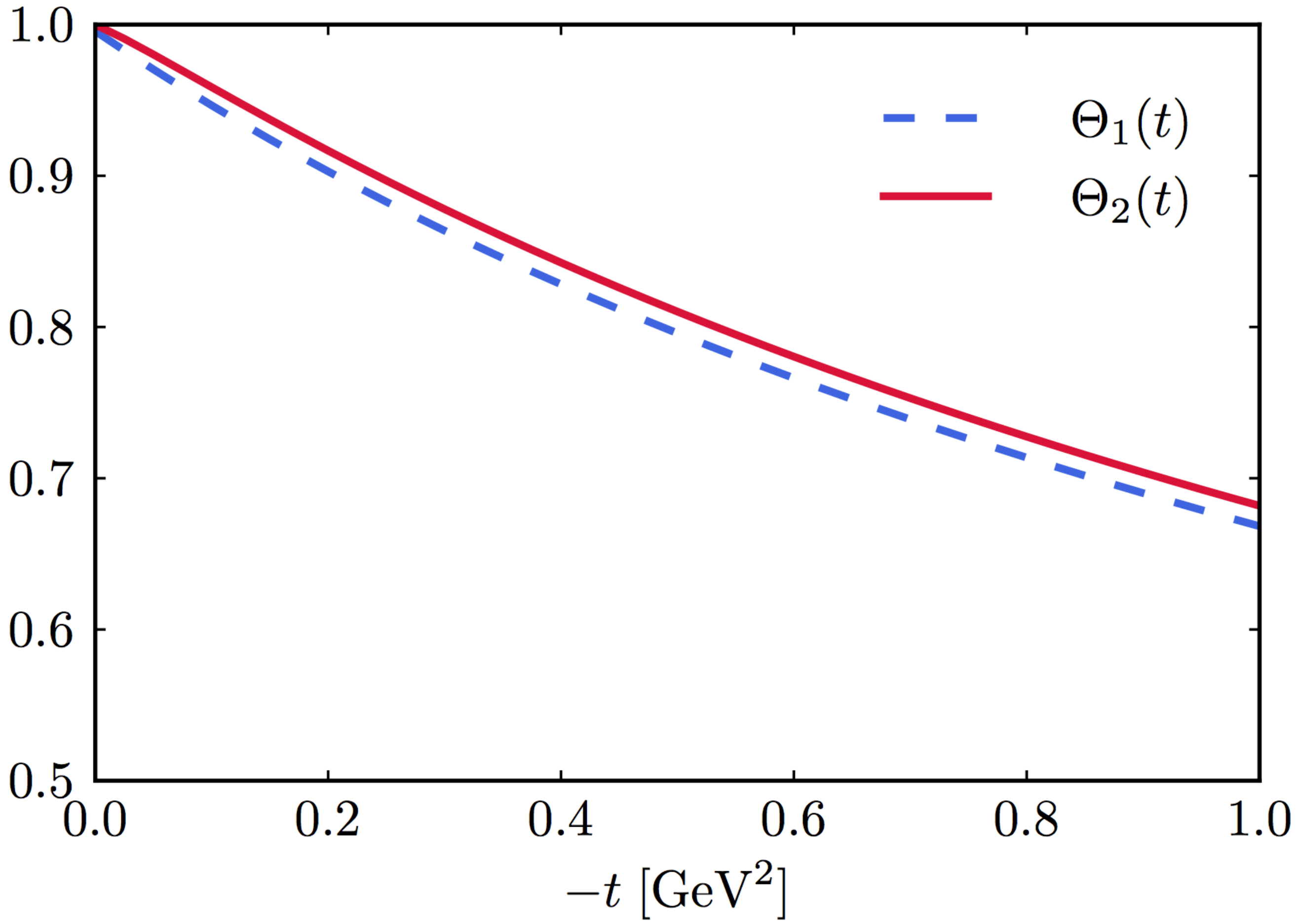
$$L_{13} = -\frac{N_c}{96\pi^2}\frac{M}{B_0}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) = 0.84 \times 10^{-3}$$

In XPT, $(L_{11} = 1.4, \quad L_{12} = -2.7, \quad L_{13} = 0.9 \text{ in unit of } 10^{-3})$

J. F. Donoghue and H. Leutwyler, Z. Phys. C 52, 343 (1991)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

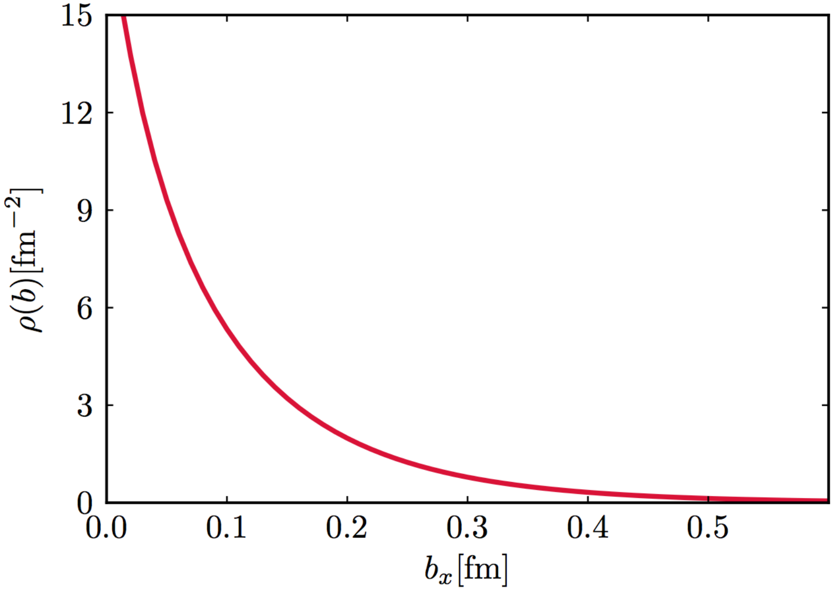
EMT form factor of the pion

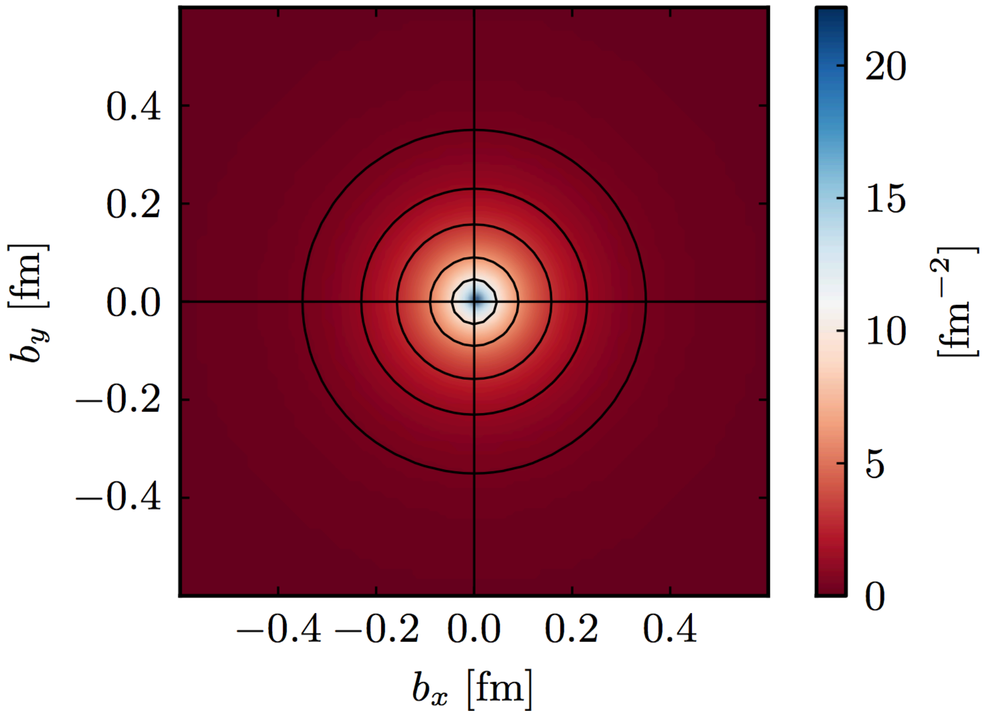


H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

Transvere charge density of the pion

$$\rho_{20}(b) = \int_0^\infty \frac{QdQ}{2\pi} J_0(bQ)\Theta_2(t)$$

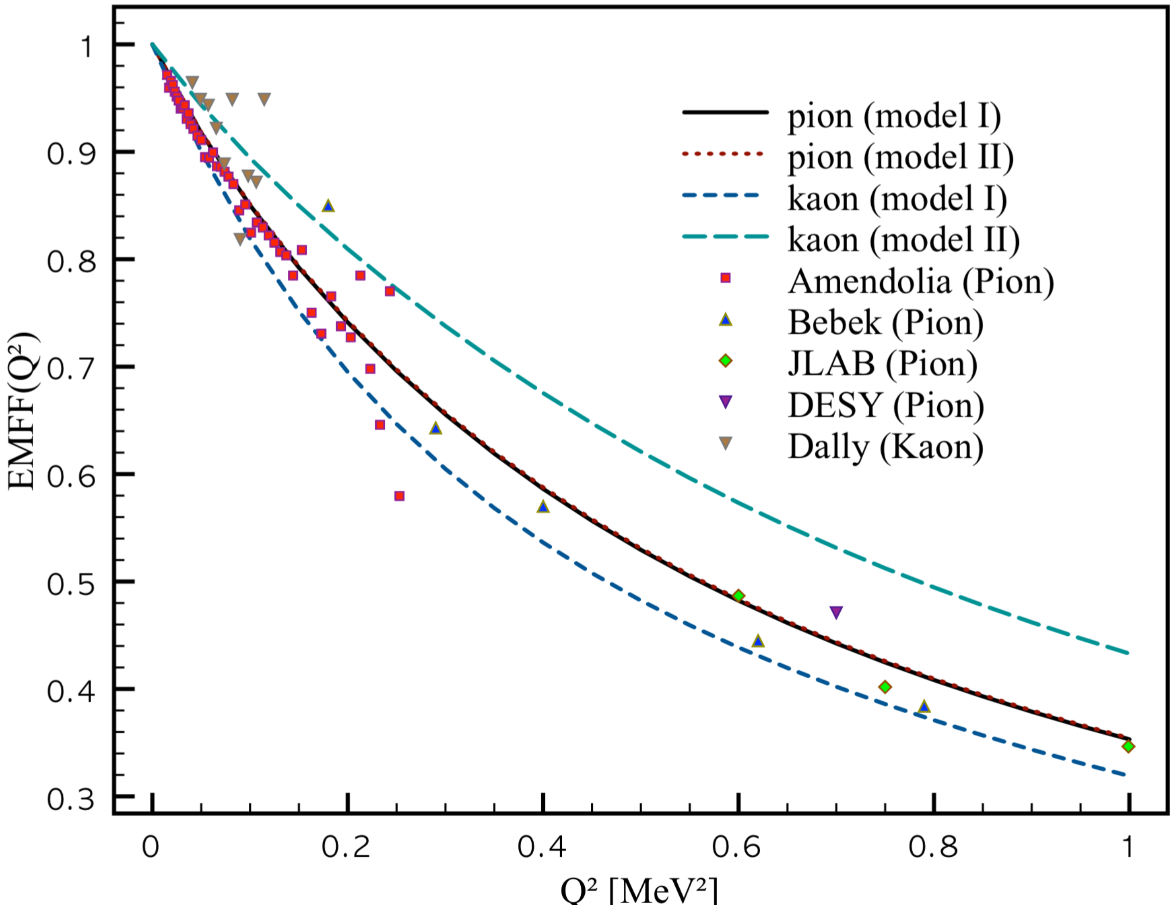




EM Form factor of the kaon

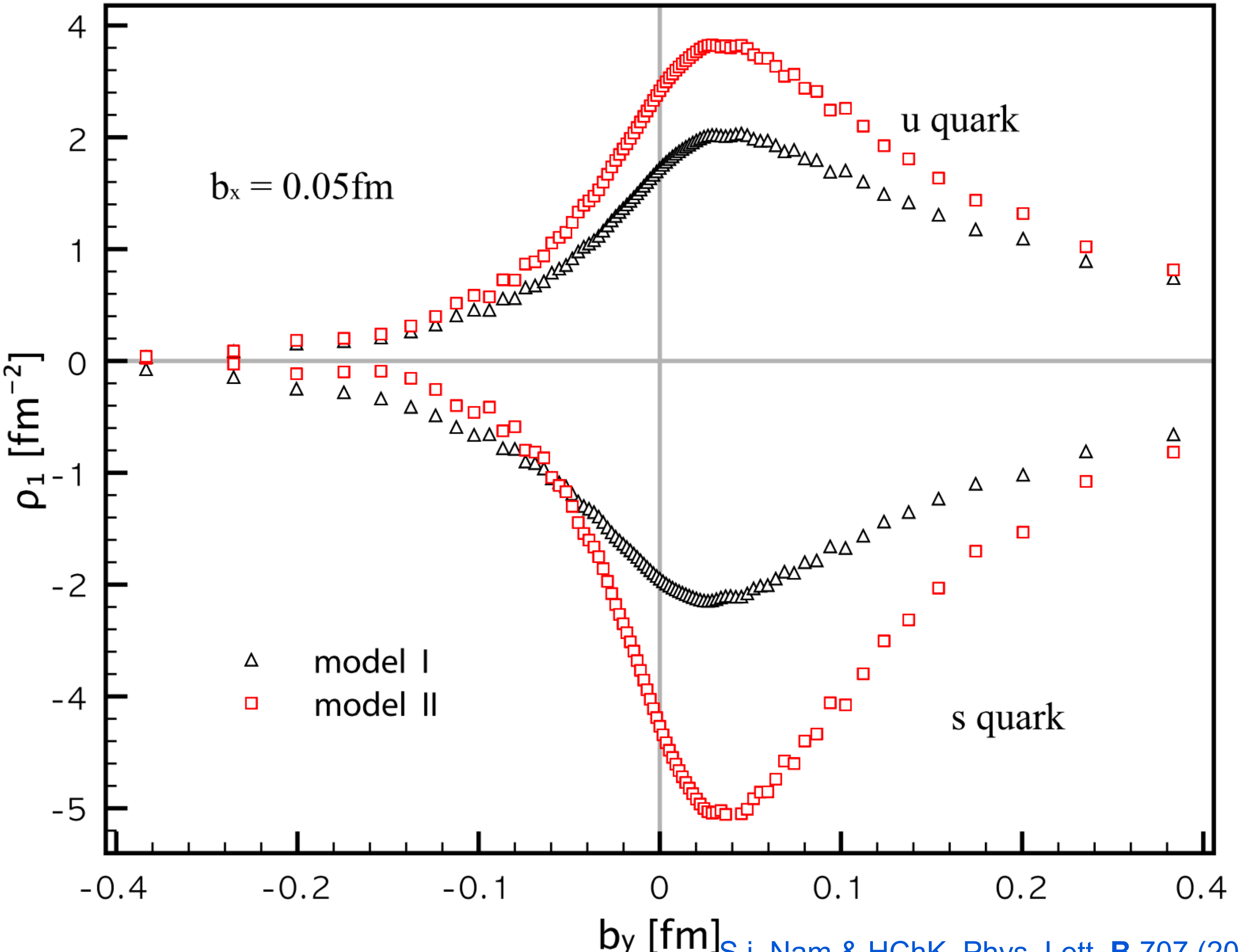
EM form factor (A_{10})

$$\langle K(p_f)|\psi^\dagger \hat{Q}\psi|\pi(p_i)\rangle = (p_i + p_f)A^K_{10}(q2)$$



S.i. Nam & HChK, Phys. Lett. **B** 707 (2012) 546

Kaon spin structure



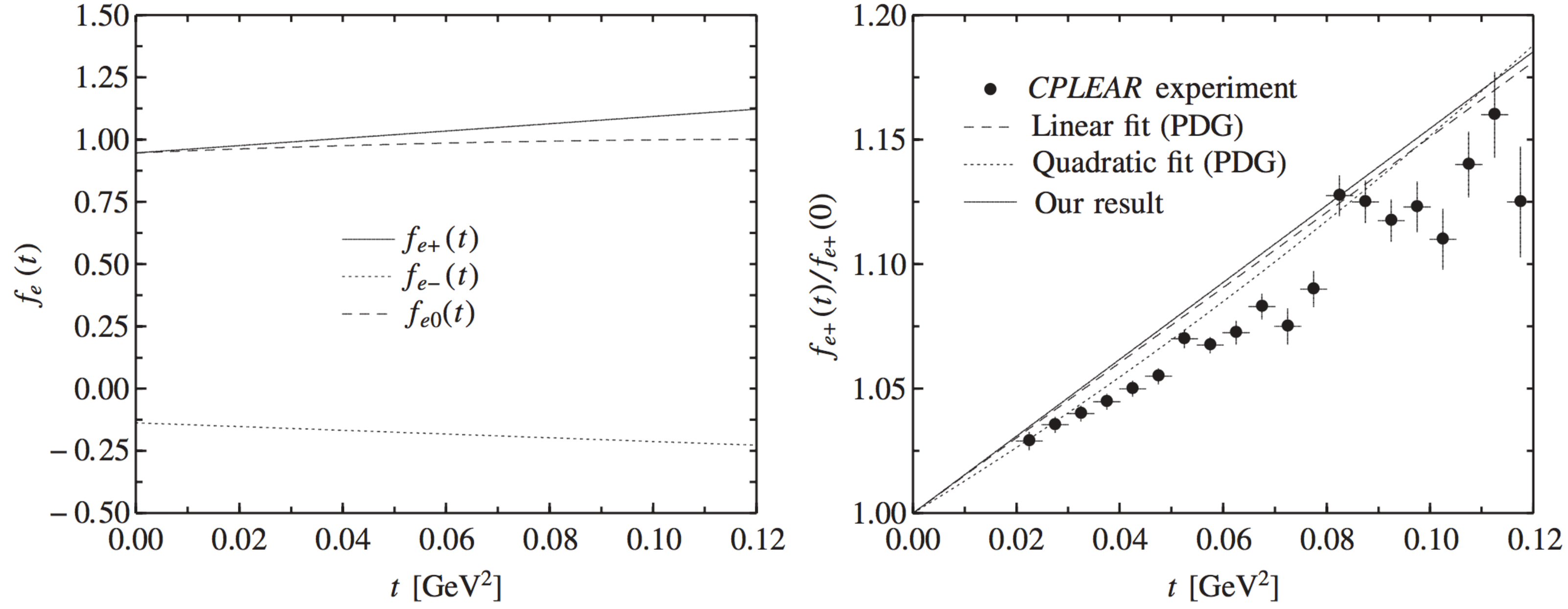
S.i. Nam & HChK, Phys. Lett. **B** 707 (2012) 546

Kaon to pion tensor form factors

- Kaon semileptonic decay (K_{l3})

  - Vector transitions

$$F_\mu^{K^0}(p_l,p_\nu) = \langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = (p_K+p_\pi)_\mu f_{l+}(t) + (p_K-p_\pi)_\mu f_{l-}(t)$$



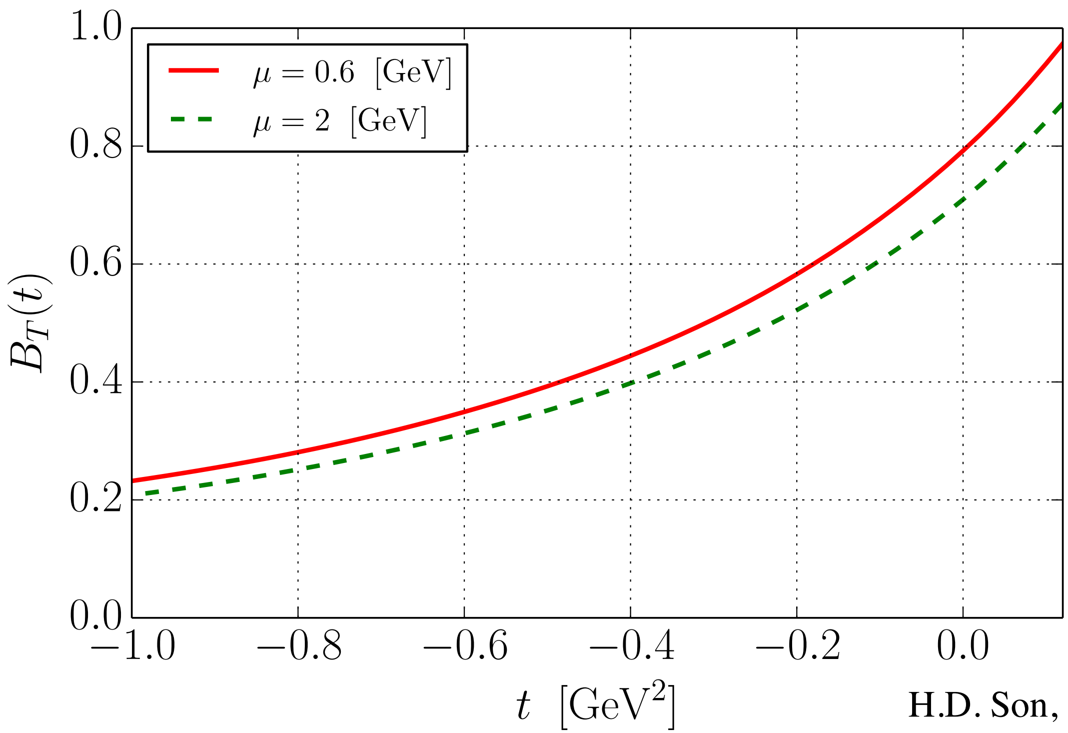
S.i.Nam & HChK, PRD **75,** 094011 (2007)

Kaon to pion tensor form factors

- Kaon semileptonic decay (K_{l3})

  - Tensor transitions

$$F^{K^0}_{\mu\nu}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\sigma_{\mu\nu}u|K^0(p_K)\rangle = \frac{p_{K\mu}p_{\pi\nu} - p_{K\nu}p_{\pi\mu}}{m_K} B_T^{K\pi}(t)$$



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Kaon to pion GPDs

Weak GPDs

$$2P^+H_\phi^{K\pi}(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \pi^-(p_f)|\bar{s}(-\frac{z^-}{2})\gamma^+ \left[-\frac{z^-}{2},\frac{z^-}{2}\right] u(\frac{z^-}{2})|K^0(p_i)\rangle \Big|_{z^+=z_\perp=0}$$

$$\frac{P^{[+}t^{j]}}{m_K} E_\phi^{K\pi}(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \pi^-(p_f)|\bar{s}(-\frac{z^-}{2}) i\sigma^{+j} \left[-\frac{z^-}{2},\frac{z^-}{2}\right] u(\frac{z^-}{2})|K^0(p_i)\rangle \Big|_{z^+=z_\perp=0}$$

→ We can define the generalized transition form factors & transverse charge densities.

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi=0,b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x,\xi=0,b_\perp^2) = B_{n0}(b_\perp^2)$$

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Kaon to pion tensor form factors

- Generalized form factors for kaon transitions: n =1

$$\langle \pi^-(p_f)|\bar{s}(0)\gamma_\mu u(0)|K^0\rangle = 2P_\mu A_{10}^{K\pi}(t) + q_\mu C_{10}^{K\pi}(t)$$

$$\langle \pi^-(p_f)|\bar{s}(0)\sigma_{\mu\nu} u(0)|K^0\rangle = \frac{p_{i\mu}p_{f\nu} - p_{i\nu}p_{f\mu}}{m_K} B_{10}^{K\pi}(t)$$

- Quarks with definite transverse polarization **s**

$$\frac{1}{2}\bar{\psi}\left[\gamma^+ - s^j i\sigma^{+j}\gamma_5\right]\psi$$

$$\sigma^{\mu\nu}\gamma_5 = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} i\sigma_{\alpha\beta}$$

- Transverse transition density

$$\rho^{K\pi}(b_\perp, s_\perp) = \int dX \rho(X, b_\perp, s_\perp) = \frac{1}{2}\left[A_{10}^{K\pi}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_K}\frac{\partial B_{10}^{K\pi}(b_\perp^2)}{\partial b_\perp^2}\right]$$

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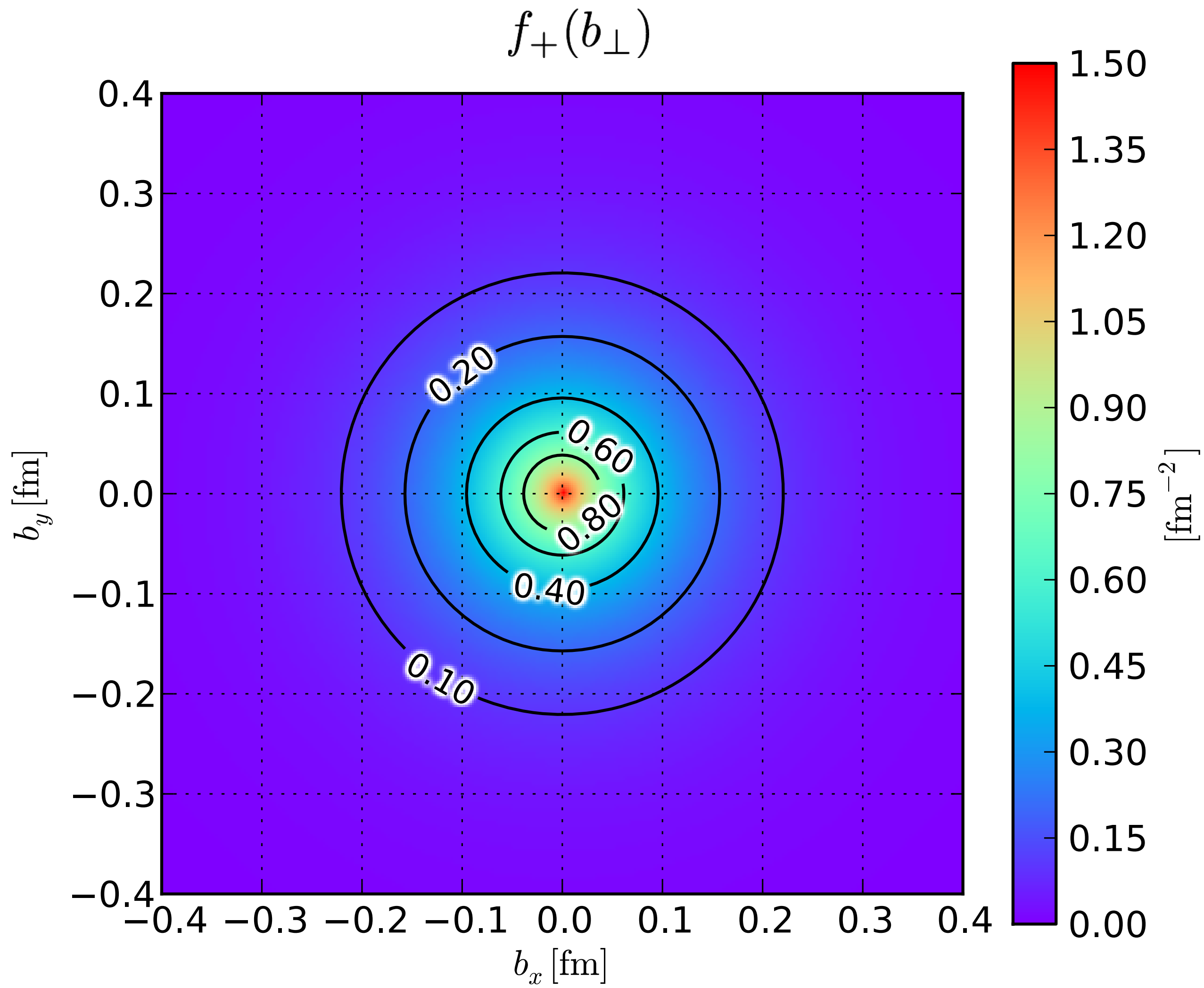
Kaon to pion transverse charge densities



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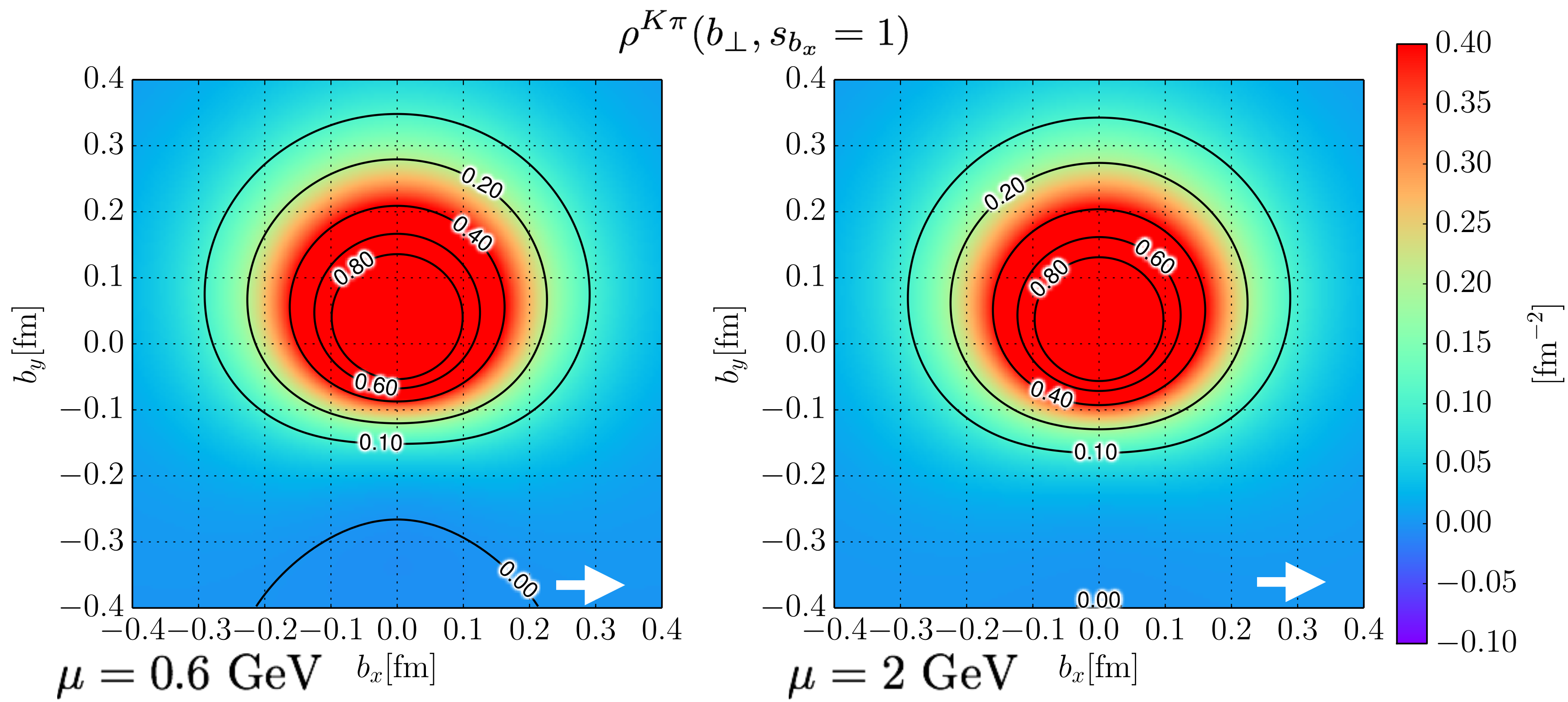
Kaon to pion transverse spin densities



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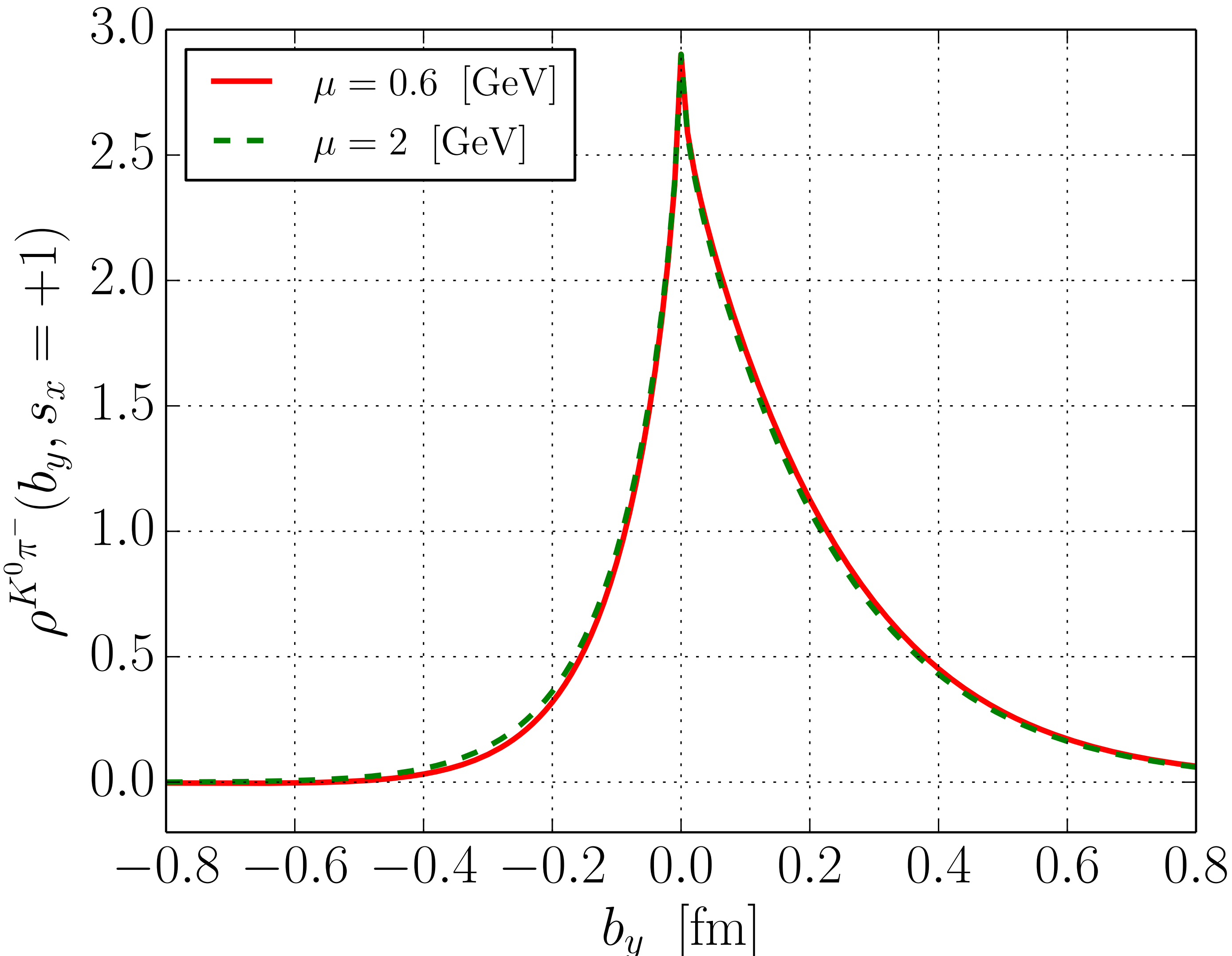
Kaon to pion transverse spin densities



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Radiative form factor of the pion

- Radiative pion (kaon) decays provide yet more information on their internal structures.

$$\pi^+ \to e^+ \nu_e \gamma \qquad \pi^+ \to e^+ \gamma \nu_e e^+ e^-$$

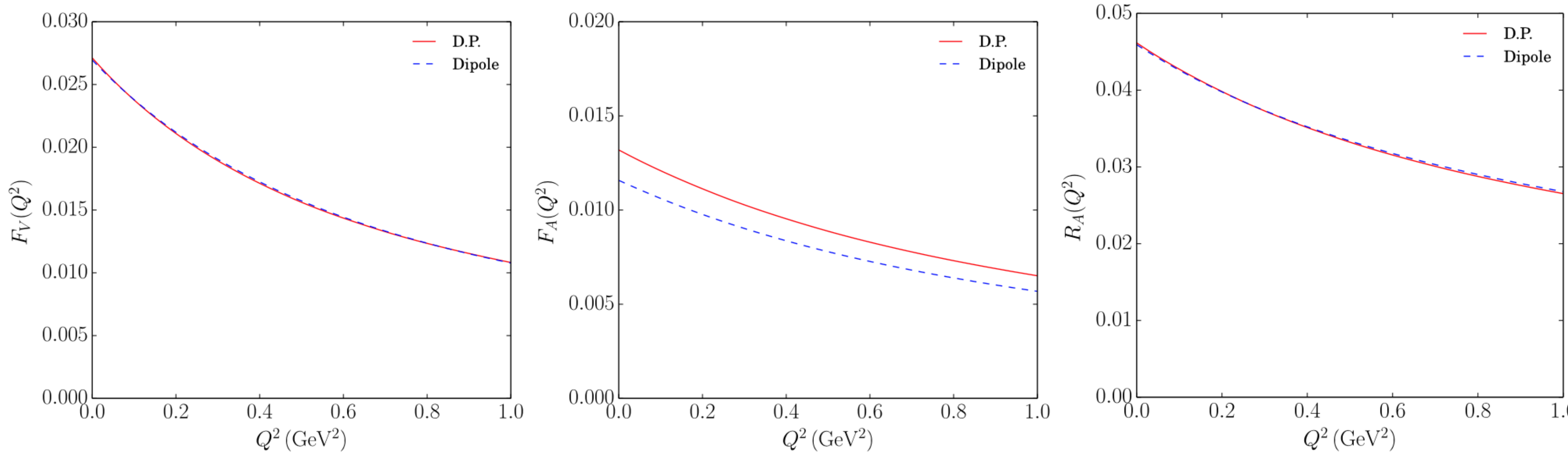
$$\langle \gamma(k) | V^\mu_{12}(0) | \pi^+(p) \rangle = -\frac{e}{m_\pi} \epsilon^*_\alpha F_V(q^2) \epsilon^{\mu\alpha\rho\sigma} p_\rho k_\sigma$$

$$\langle \gamma(k) | A^\mu_{12}(0) | \pi^+(p) \rangle = i e \epsilon^*_\alpha \sqrt{2} f_\pi \left[-g^{\mu\alpha} + q^\mu (q^\alpha + p^\alpha) \frac{F_\pi(k^2)}{q^2 - m_\pi^2} \right] + i \epsilon^*_\alpha \frac{e}{m_\pi} \left[F_A(q^2) \left(k^\mu q^\alpha - g^{\mu\alpha} q \cdot k \right) + R_A(q^2) \left(k^\mu k^\alpha - g^{\mu\alpha} k^2 \right) \right],$$

$$V^\mu_{12} = \bar{\psi} \gamma^\mu \frac{\tau_1 - i\tau_2}{2} \psi, \quad A^\mu_{12} = \bar{\psi} \gamma^\mu \gamma_5 \frac{\tau_1 - i\tau_2}{2} \psi$$

S. Shim, HChK, PLB 772 (2017) 687

Radiative form factor of the pion



	NJL [47]	NL NJL(A) [23]	χPT [20]	Experimental data	Present work	
					D.P.	Dipole
$F_V(0)$	0.0242	0.0270	0.0262(5)	0.0258(17) [15]	0.0271	0.0269
a_V		0.0191	0.0332(42)	0.10(6) [15]	0.0287	0.0280
$F_A(0)$	0.0239	0.0132	0.0106(36)	0.0117(17) [15]	0.0132	0.0116
a_A		0.012	0.0191(61)	–	0.0192	0.0193
$R_A(0)$				$0.059^{+0.009}_{-0.008}$ [12]	0.0462	0.0459

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Summary & Outlook

- We reviewed in this talk a series of recent works on the EM and spin structure of the pion, based on the nonlocal chiral quark model derived from the instanton vacuum.

Future programs, a personal view

- The transition tensor form factors of hadrons: Physics beyond the standard model
- Probing dark matter and dark photon, using the pion and kaon
- Physics of excited baryons: Transition form factors, transverse charge and spin densities.
- Internal view of heavy hadrons

→ **Internal shape of hadrons, literally hadron tomography.**

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!