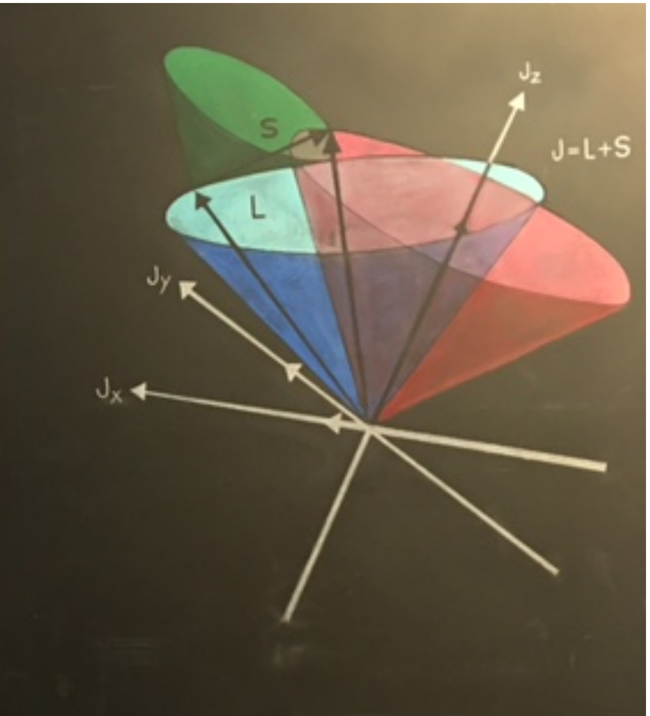
ACCESSING PARTON OAM AND GPDS IN EXPERIMENT

JEFFERSON LAB E&M PROBES MEETING
NOVEMBER 2-3, 2017



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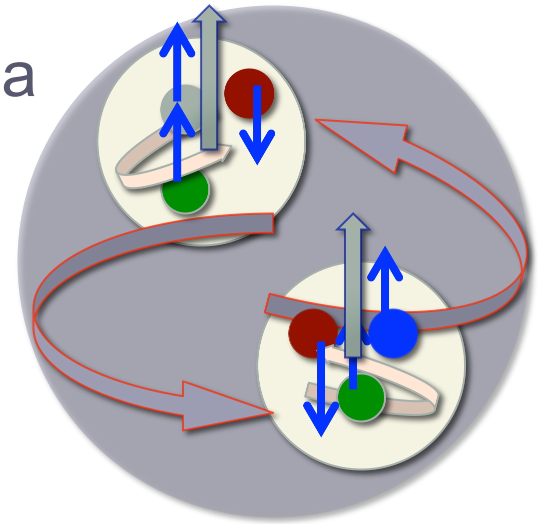

Based on

PHYSICAL REVIEW D **94**, 034041 (2016)

Parton transverse momentum and orbital angular momentum distributions

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The quark orbital angular momentum component of proton spin, L_q, can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of L_q and evaluations through lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization but can occur using longitudinally polarized targets.

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... and on

Lorentz Invariance and QCD Equation of Motion Relations for Generalized Parton Distributions and the Dynamical Origin of Proton Orbital Angular Momentum

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We derive new Lorentz Invariance and Equation of Motion Relations between twist-three Generalized Parton Distributions (GPDs) and moments in the parton transverse momentum, k_T, of twist-two Generalized Transverse Momentum-Dependent Distributions (GTMDs), as a function of the parton longitudinal momentum fraction x. Although GTMDs in principle define the observables for partonic orbital motion, experiments that can unambiguously detect them appear remote at present. The relations presented here provide a solution to this impasse in that, e.g., the orbital angular momentum density is connected to directly measurable twist-three GPDs. Out of 16 possible Equation of Motion relations that can be written in the T-even sector, we focus on three helicity configurations that can be detected analyzing specific spin asymmetries: two correspond to longitudinal proton polarization and are associated with quark orbital angular momentum and spin-orbit correlations; the third, obtained for transverse proton polarization, is a generalization of the relation obeyed by the g_2 structure function. We also exhibit an additional relation connecting the off-forward extension of the Sivers function to an off-forward Qiu-Sterman term.

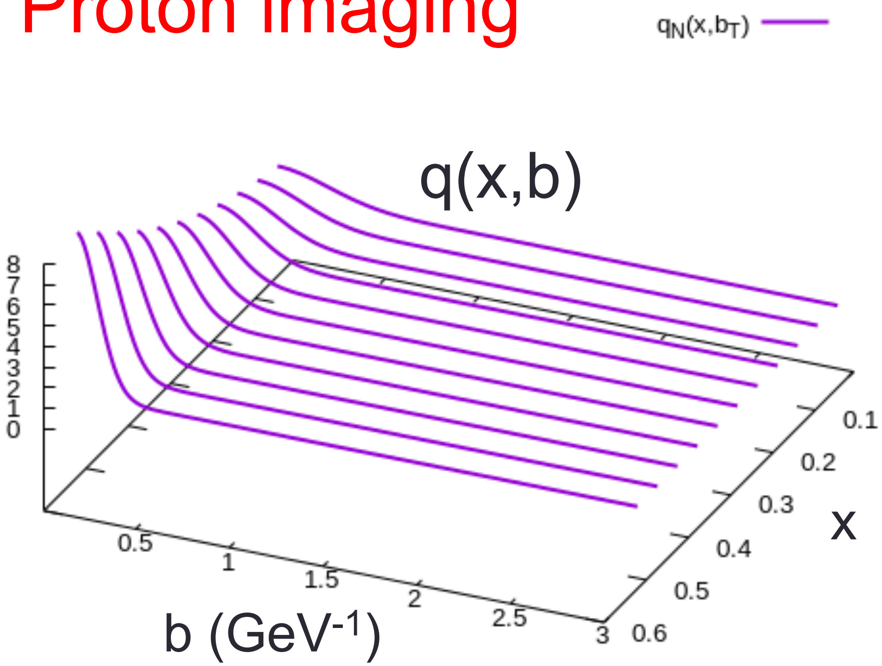
arXiv:1709.xxxx [hep-ph]

Outline

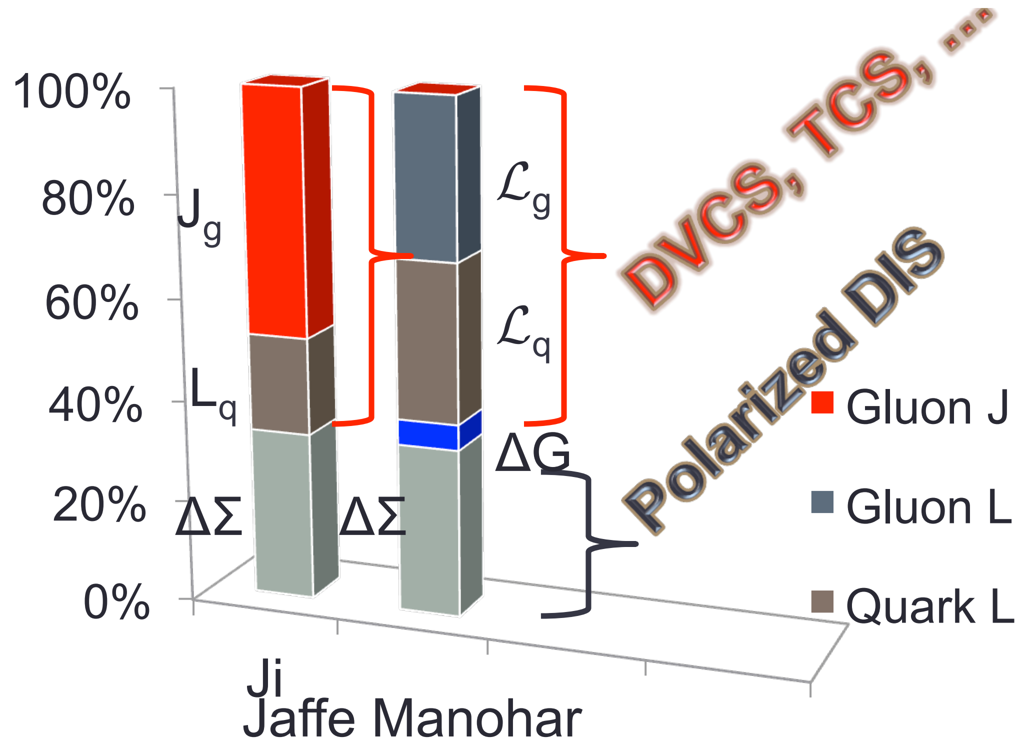
1. Introduction
2. Towards Proton Imaging
3. Orbital Angular Momentum in QCD
4. OAM and twist three GPD $\tilde{E}_{2T}$
5. Multivariate Analysis
6. Nuclei

Why are we measuring GPDs?

Proton imaging



A. Rajan, S.L.



Angular Momentum Budget

QCD Energy Momentum Tensor

$T^{\mu\nu}$ → $M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu}$ **Angular Momentum density**

Energy density

$\frac{E^2+B^2}{2}$	S_x	S_y	S_z
S_x	σ_{xx}	σ_{xy}	σ_{xz}
S_y	σ_{yx}	σ_{yy}	σ_{yz}
S_z	σ_{zx}	σ_{zy}	σ_{zz}

Momentum density

Shear stress

Pressure

$$\vec{S} = \vec{E} \times \vec{B}$$

In QCD

operator

$$T^{\mu\nu} = \frac{1}{4} i q \bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{2}g^{\mu\nu}F^{2}\right\}$$

quark field

gluon field

observable

$$\langle P \mid T^{\mu\nu} \mid P\rangle$$

A closer look at mass

$$H_{QCD} = \langle T^{00} \rangle = H_q + H_g + H_m + H_a$$

$$H_q = \langle \bar{\psi}(-i\,\mathbf{D}\cdot\gamma)\psi \rangle$$ quark energy

$$H_g = \left\langle \frac{1}{2}(E^2 + B^2) \right\rangle$$ gluon field energy

$$H_m = \left\langle m\,\bar{\psi}\left(1 + \frac{1}{4}\gamma_m\right)\psi \right\rangle$$ quark mass anomaly

$$H_a = \left\langle \frac{1}{4}\beta(g)(E^2 - B^2) \right\rangle$$ gluon anomaly

H_m and H_a: trace anomaly

X. Ji (1995)

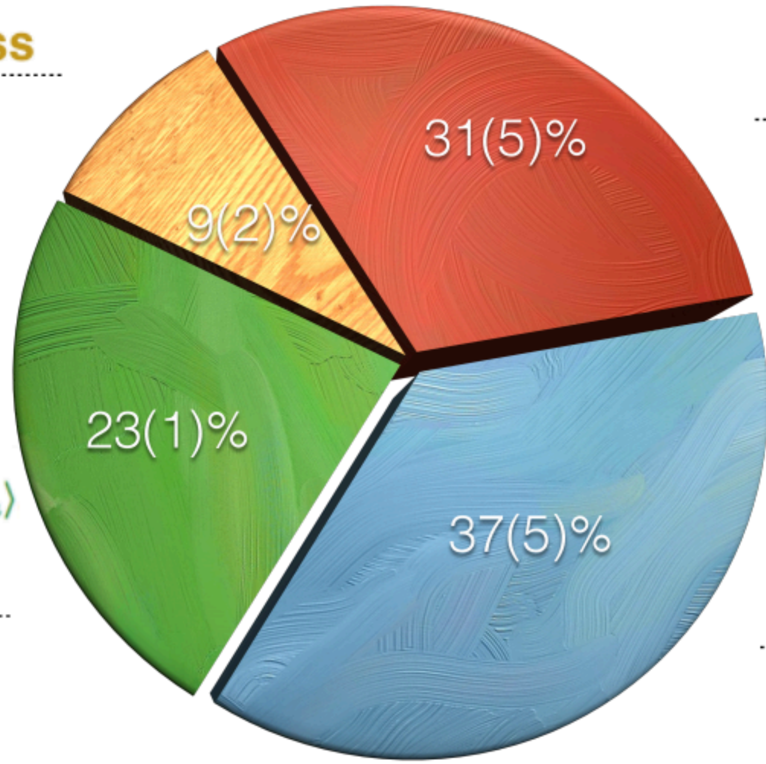
The nucleon mass in Lattice QCD

$\langle H_m \rangle / M_N = 9(2)\%$

quark mass

$\langle H_E \rangle = \frac{3}{4}\langle x \rangle_q M - \frac{3}{4}\langle H_m \rangle$

quark energy



$\langle x \rangle_q = 50(7)\%$ and
$\langle x \rangle_g = 50(7)\%$

$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \langle H_a \rangle$

QCD anomaly

$\langle H_g \rangle = \frac{3}{4}\langle x \rangle_g M$

glue energy

Y. Yang et al., χQCD Coll

… a closer look at spin

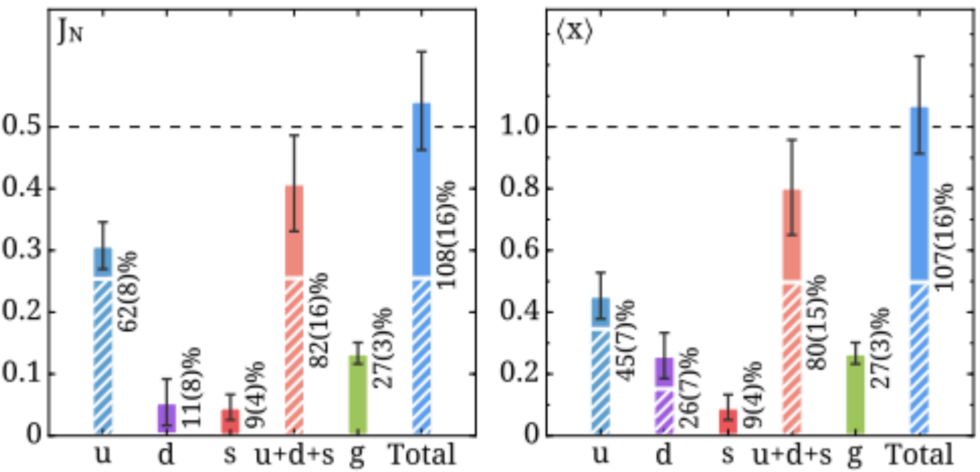
$$M^{+12} = \psi^{\dagger}\, \sigma^{12}\, \psi \; + \; \psi^{\dagger}\left[\vec{x}\times\left(-i\vec{D}\right)\right]^{3}\psi \; + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^{3}$$

quark spin

quark orbital angular momentum

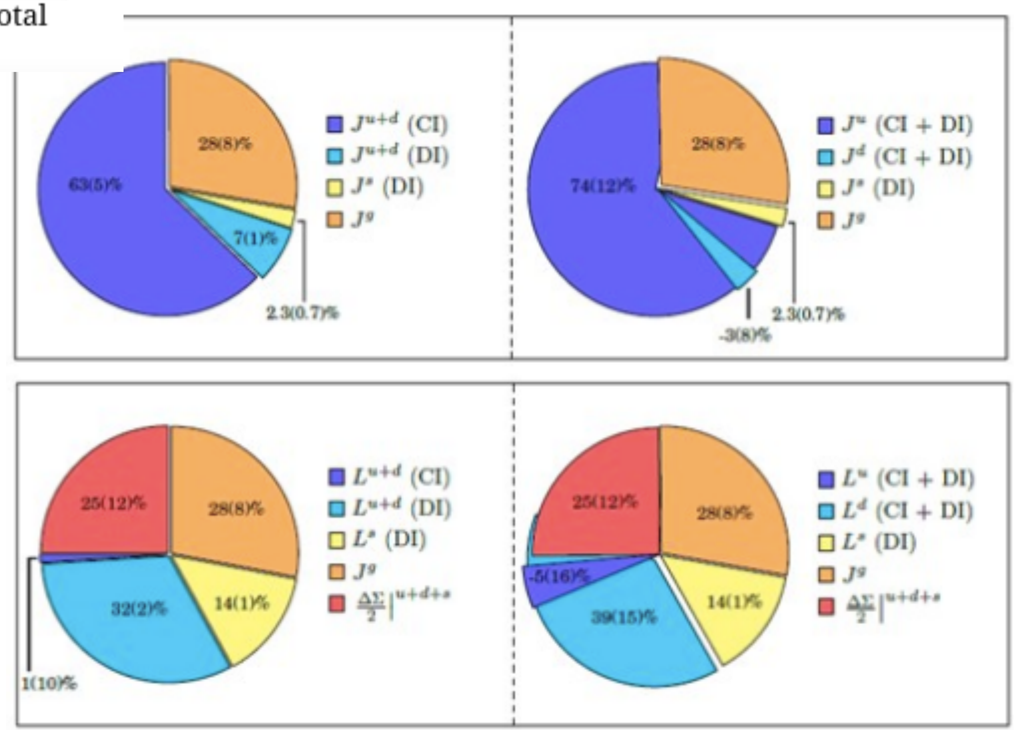
gluon angular momentum

The nucleon spin in Lattice QCD

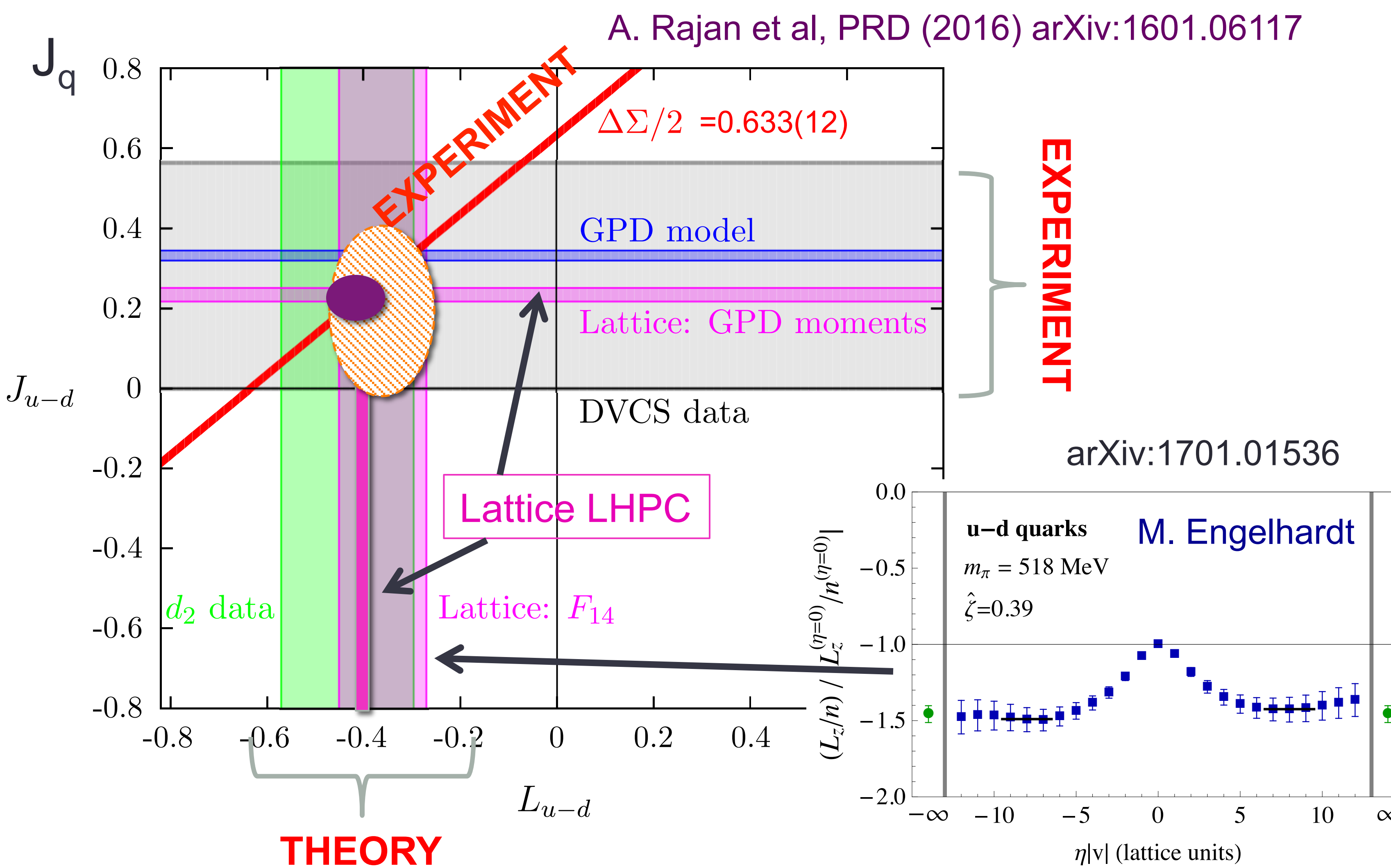


C. Alexandrou et al., PRL (2017)

χQCD Coll., Deka et al. PRD (2015)

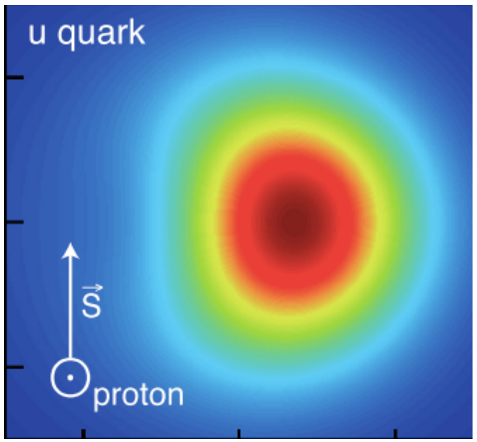


Quark sector : $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$ q=u-d



To understand both **mass** and **spin** (orbital motion) we need to be able to describe and measure both **space** and **momentum** distributions of **quarks** and **gluons** inside the proton

spatial distribution of transversely polarized proton



Color charge Flux tube

“light (quark) pair creation seems to occur non-localized and instantaneously.”

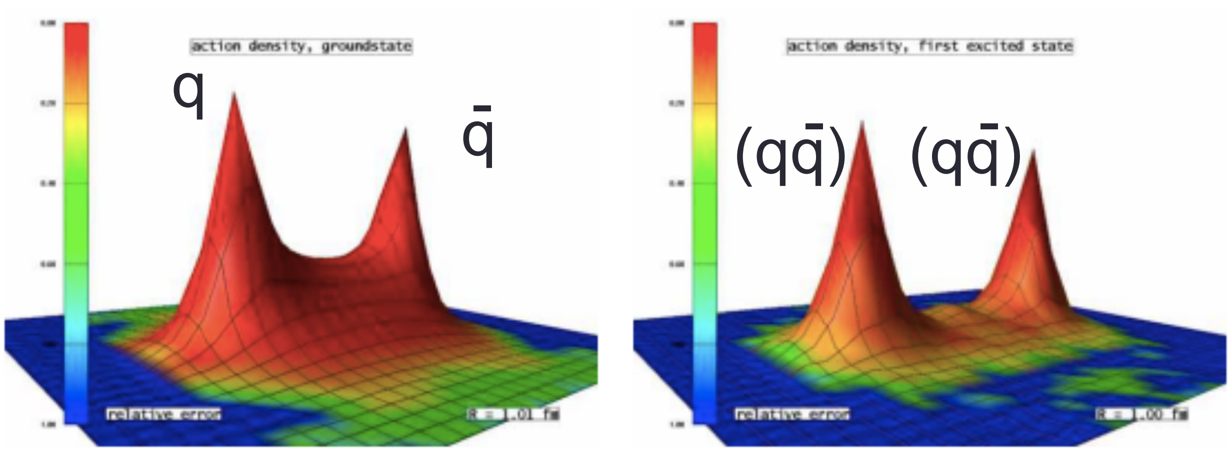


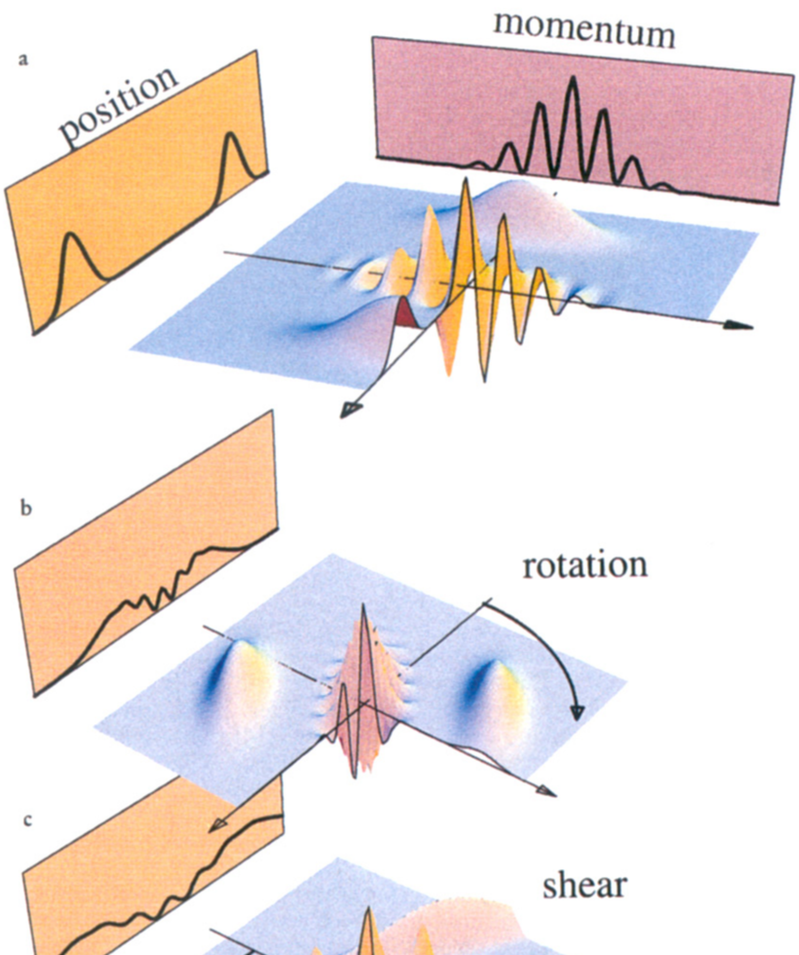
Figure 3: Action density distribution for the ground state and the first excitation.

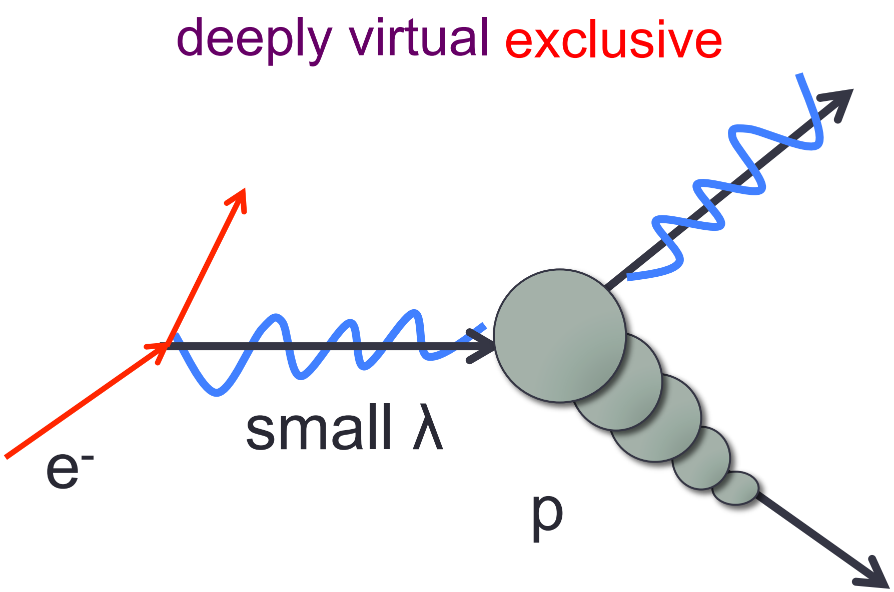
G. Bali et al., PoS LAT2005 (2006) 308

DVCS type experiments allow us to test the dynamical picture behind the mass and spin structure of matter and to “see” inside the proton

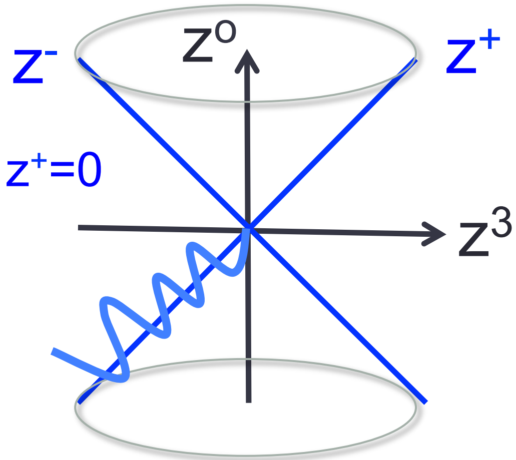
How can this work for a relativistic system where all components are moving nearly at the speed of light?

Wigner/phase space distributions at the femtoscale





A quick ride on the Light Cone



$$p^{\pm} = \frac{p^o \pm p^3}{\sqrt{2}}$$

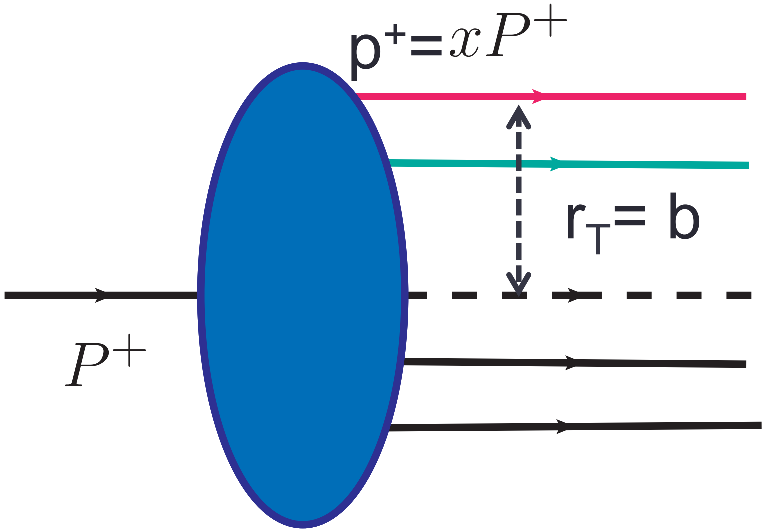
For a particle with large momentum in the *longitudinal* direction and limited *transverse* momentum ➔ p^+ is large and p^- is small

Fourier transform

$$(pz) = \underline{p^+ z^-} + \underline{p^- z^+} - p_T \cdot z_T$$

p^+ is conjugate to **z^-** and **p^-** is conjugate to **z^+**

The Proton Relativistic Wave Function: Poincaré Invariance



Center of P^+

$$\vec{R}_T = \frac{1}{P^+} \sum_i (x_i P^+) \vec{r}_T^{\,i}$$

- P^+ plays the role of mass
- "The subgroup of the Poincaré group that leaves the surface z^+=const invariant, is isomorphic to the Galilean group in 2D"
- We can disentangle the transverse components from the time components in boosts ➔ boosts in transverse plane are kinematical

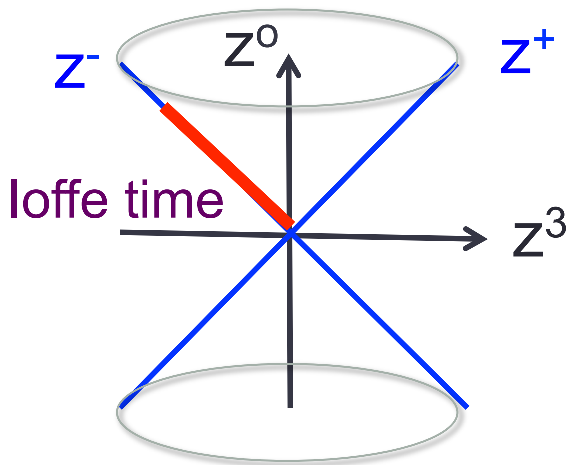
Implication

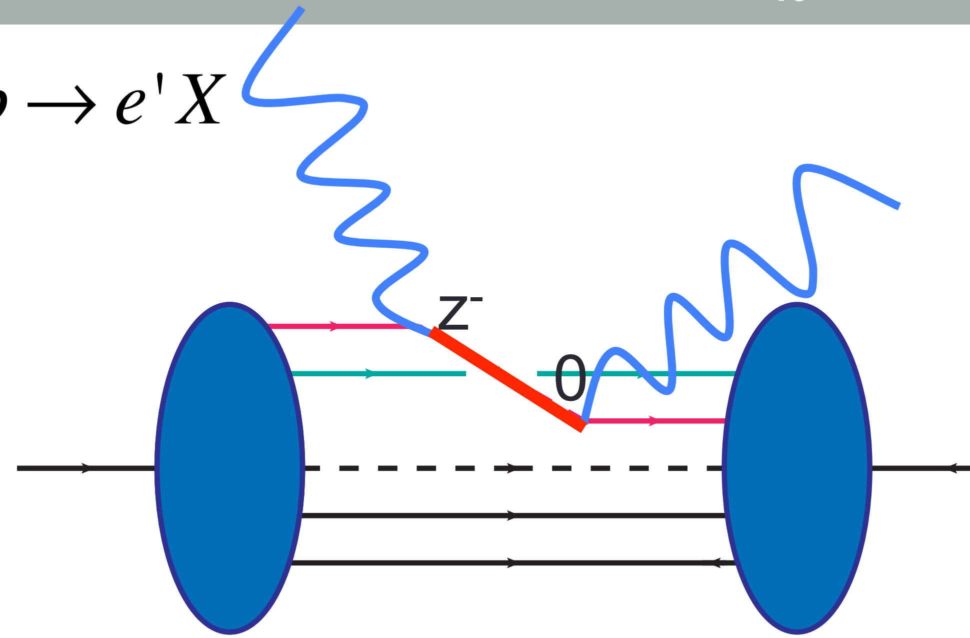
We can map out faithfully the spatial quark distributions in the transverse plane (no modeling/approximation)

$$q(x,\vec{b}) = \frac{dn}{dx\,d^2\vec{b}}$$

Soper (1977), Burkardt (2001)

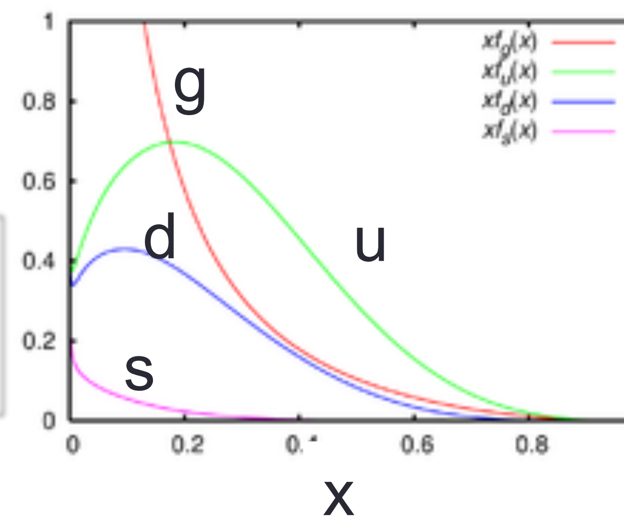
Deep Inelastic Scattering $e\,p \to e'\,X$



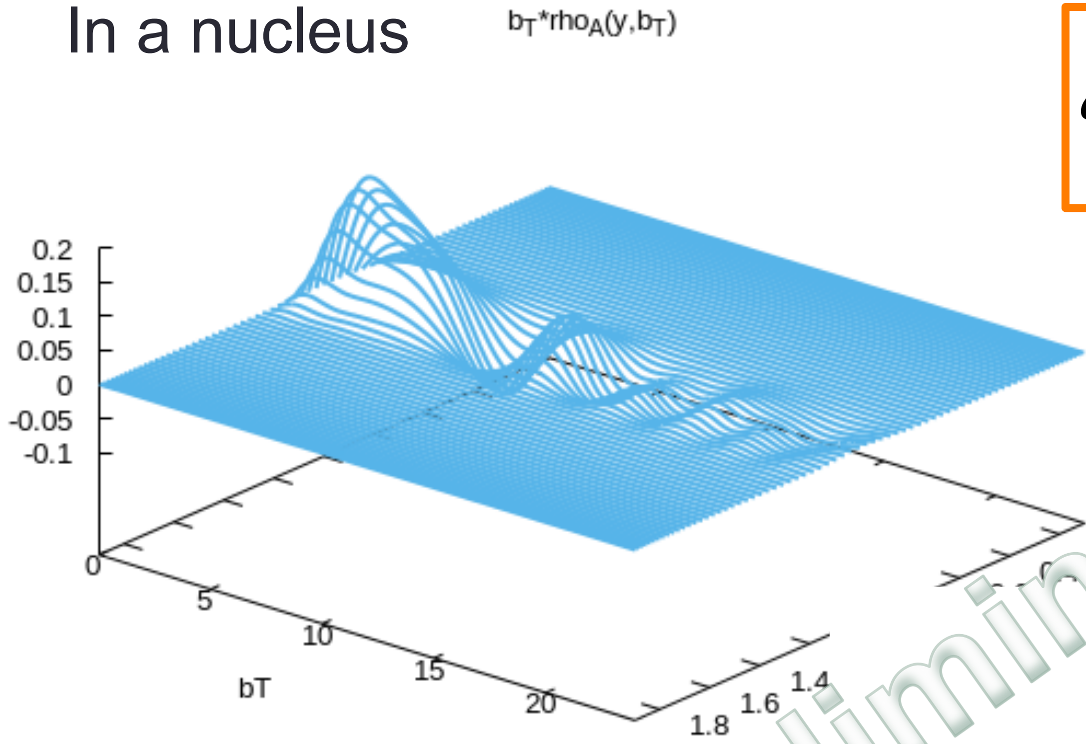


z^- is conjugate to $p^+ = xP^+$

$$\int d^2b\, q(x,b) \equiv q(x)$$

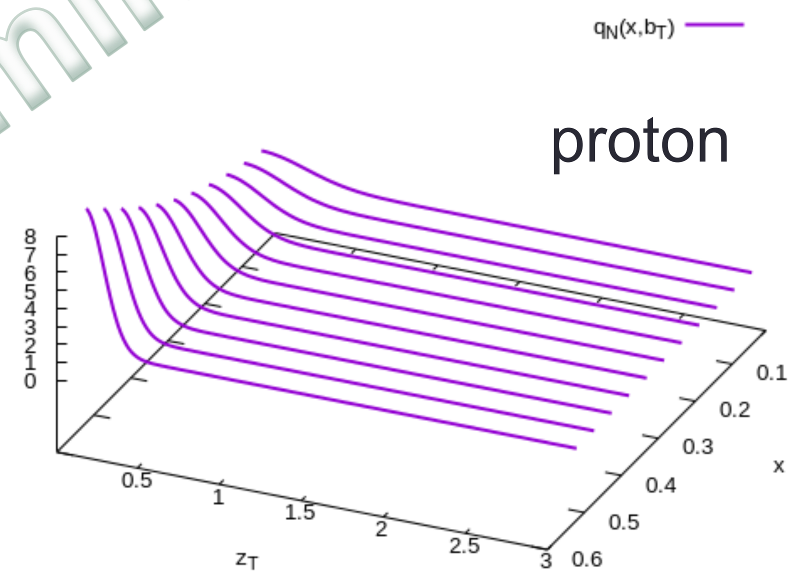


In a nucleus



$$q(x,\vec{b}) = \frac{dn}{dx\, d^2\vec{b}}$$

proton



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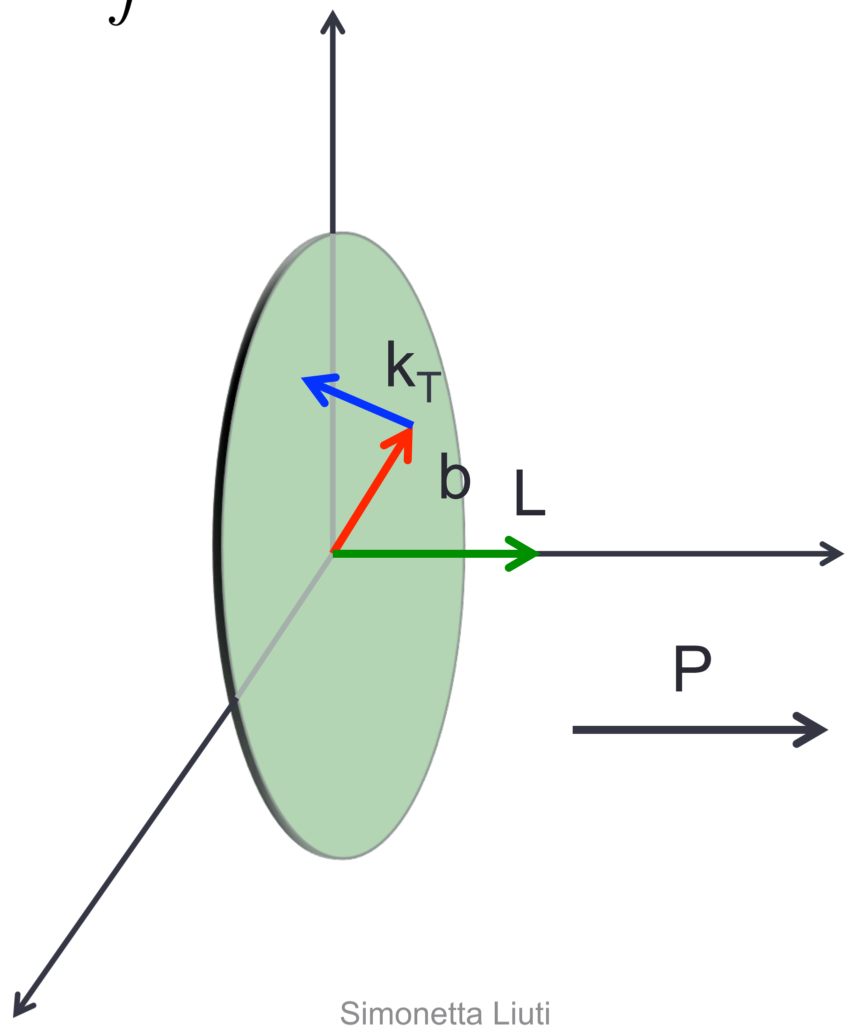
Partonic OAM: Wigner Distributions

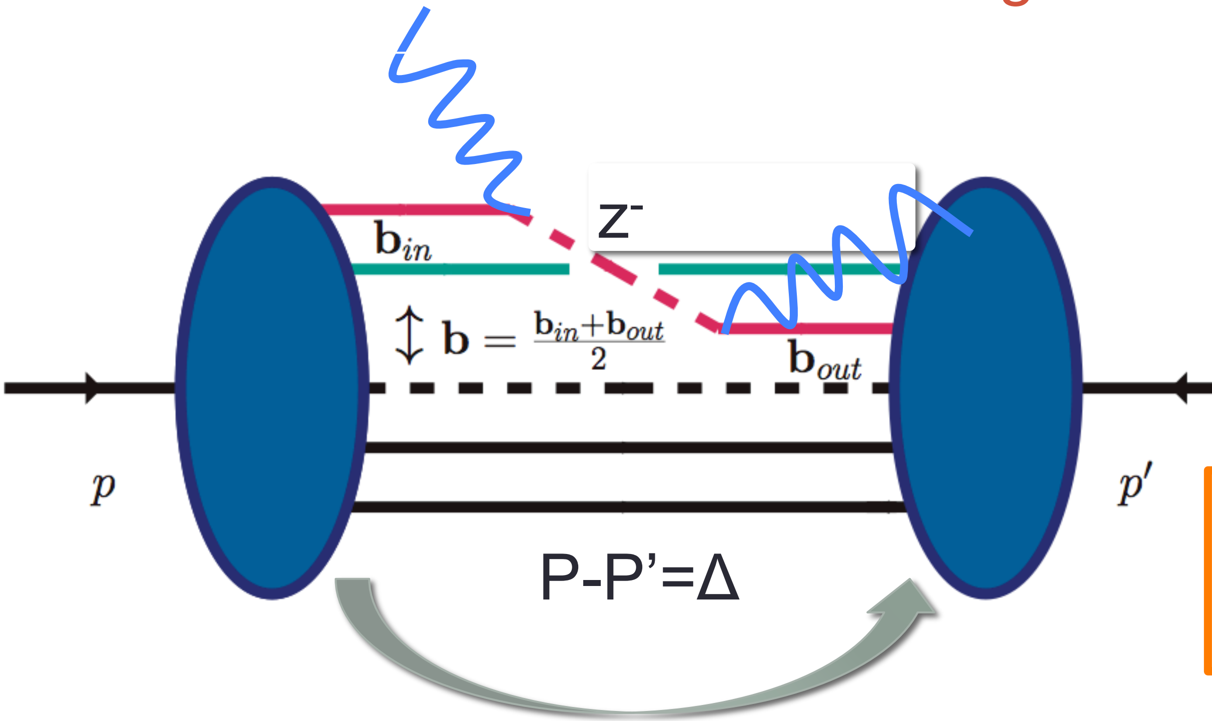
$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\,(\mathbf{b}\times\mathbf{k}_T)_z\,\mathcal{W}^{\mathcal{U}}(x,\mathbf{k}_T,\mathbf{b})$$

Hatta
Lorce, Pasquini,
Xiong, Yuan
Mukherjee



Wigner Distribution/GTMDs

A new generation of **deeply virtual** **exclusive** experiments probing **two** distinct distance scales will allow us to image **fermi size** structure



$$q(x,\vec{b}) = \frac{dn}{dx\,d^2\vec{b}}$$

- Δ_T Fourier conjugate of **b** = transverse position of the quark inside the proton
- xP^+ Fourier conjugate of **z-** = LC distance traveled by the struck quark between the initial and final photon scattering

Possible Observable for L_q

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = \langle b_T \times k_T\rangle_3(x)$$

$L_q(x)$

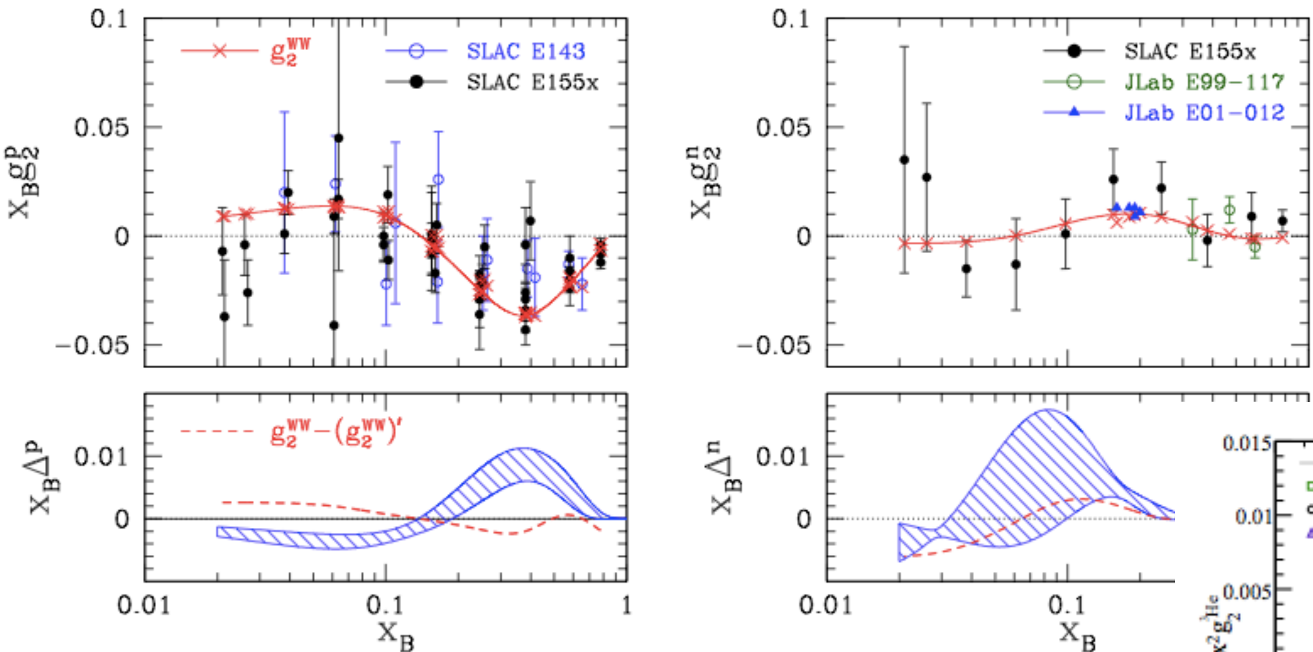
k_T moment of a GTMD
(Lorce and Pasquini)

$\xi=0$
$k_T \bullet \Delta_T=0$
$\Delta_T^2=0$

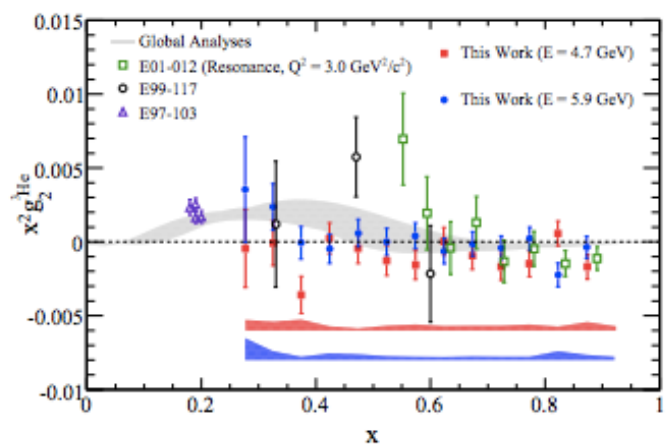
Can it be measured?

Analogously to studies of g_2 where…

$$\int d^2k_T \, \frac{k_T^2}{M^2} \, g_{1T}(x, k_T^2) = -\int_x^1 g_2(y)dy + \widehat{g}_T$$



Accardi, Bacchetta, Melnitchouk, Schlegel JHEP (2009)



D. Flay et al, PRC 2016

The effect of the two twist-three terms combined might be small but each individual contribution can be large

…F_{14} is connected to twist three GPDs through a generalized Lorentz Invariance Relation

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = -\int_x^1 dy\, \left[\widetilde{E}_{2T} + H + E\right]$$

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016), arXiv:1601.06117

OAM distribution emerges from LIR

$L_q(x)$ ➔ density $\qquad$ L_q ➔ integrated

$$F_{14}^{(1)} = -\int_x^1 dy\,(\tilde{E}_{2T} + H + E) \;\Rightarrow\; -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx x G_2$$

- $F^{(1)}{}_{14}$ and $\tilde{E}_{2T}$ give us the same information on the distribution in x of OAM!
- "In addition": we confirm and corroborate the global/integrated OAM result deducible from Ji et al

Different notation!

$$G_2 \rightarrow \tilde{E}_{2T} + H + E$$

Polyakov et al. $\qquad$ Meissner, Metz and Schlegel, JHEP(2009)

This is consistent with integrated OAM obtained by subtraction from Ji Sum Rule

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

L = J - S + 0

Measuring twist three GPDs gives us the same information on OAM as measuring k_T integrals GTMDs, but….

….we have referred so far only to Ji's OAM

Genuine "intrinsic" twist three terms

$$\mathcal{M}^{i}_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T}$$

$$\langle p', \Lambda' \mid \overline{\psi}(-z/2) \left[(\overrightarrow{\partial\!\!\!/} - ig A\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + ig A\!\!\!/)\Big|_{z/2} \right] \psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{\rm staple} + \mathcal{M}_{F_{14}}^{\rm staple}$$

Jaffe Manohar

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{\rm straight} + \mathcal{M}_{F_{14}}^{\rm straight}$$

Ji

By subtracting the two expressions

$$F_{14}^{(1)}\Big|_{\text{staple}} - F_{14}^{(1)}\Big|_{\text{staple}} = \mathcal{M}_{F_{14}}|_{\text{staple}} - \mathcal{M}_{F_{14}}|_{\text{straight}}$$

integrating

$$-\int dx\ F_{14}^{(1)}\Big|_{\text{diff}}\Big|_{\Delta_T=0} = -\frac{\partial}{\partial \Delta_i} i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', +|\bar{\psi}(0)\gamma^+ U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,+\rangle\Big|_{\Delta_T=0}$$

Difference between Jaffe-Manohar and Ji
(Hatta, Burkardt, 2013)

$$\mathcal{A} = \frac{d}{dx}\left(\mathcal{M}^{\text{staple}} - \mathcal{M}^{\text{straight}}\right)$$

LIR violating term is the difference between JM and Ji

x-Moments

M_0 $$\int dx \tilde{E}_{2T} = -\int dx (H+E) \quad \Rightarrow \int dx \left(\tilde{E}_{2T} + H + E\right) = 0$$

M_1 OAM Sum Rule $$\int dx x \tilde{E}_{2T} = -\frac{1}{2}\int dx x (H+E) - \frac{1}{2}\int dx \tilde{H}$$

M_2 $$\int dx x^2 \tilde{E}_{2T} = -\frac{1}{3}\int dx x^2 (H+E) - \frac{2}{3}\int dx x \tilde{H} - \frac{2}{3}\int dx x \, \mathcal{M}_{F_{14}}$$

Interpretation

M_1

Force acting on quark

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \quad i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p',\Lambda'|\bar\psi(0)\gamma^+ U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,\Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \quad -g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p',\Lambda'|\bar\psi(0)\gamma^+\gamma^5 U(0,sv)F^{+i}(sv)U(sv,0)\psi(0)|p,\Lambda\rangle$$

Non zero only for staple link

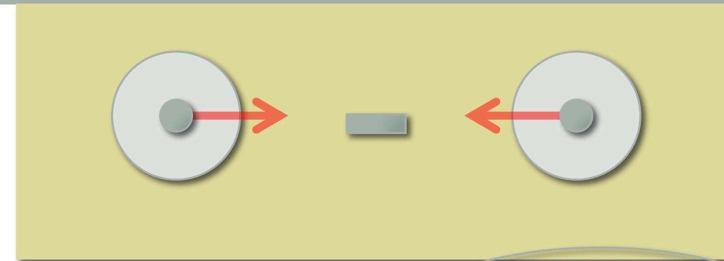
M_2

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2}\langle p',\Lambda'|\bar\psi(0)\gamma^+\gamma^5 F^{+i}(0)\psi(0)|p,\Lambda\rangle$$

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p',\Lambda'|\bar\psi(0)\gamma^+ F^{+j}(0)\psi(0)|p,\Lambda\rangle$$

“rest frame” interaction ≠ d_2

Other integrated relations: SPIN ORBIT!



$$\int dx x \left(E'_{2T} + 2\tilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \tilde{H} - \frac{1}{2}\int dx H + \frac{m}{2M}\int dx (E_T + 2\tilde{H}_T)$$

$$(L_z S_z)_q = \int dx x \left(E'_{2T} + 2\tilde{H}'_{2T} + \tilde{H}\right), \quad \kappa_T = \int dx \left(E_T + 2\tilde{H}_T\right), \quad e_q = \int dx\, H$$

$$\boxed{\frac{1}{2}\int dx x \tilde{H} = (L_z S_z)_q + \frac{1}{2} e_q - \frac{m_q}{2M}\kappa_T^q}$$

- ➢ Integral relation without connecting to spin-orbit Polyakov et al. (2000)
- ➢ Integral relation with "educated guess" for spin-orbit Lorce (2015)

Chiral symmetry breaking test!

Transverse proton spin (unpolarized quark)

$$-x\left(F_{23}+\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{24}\right)+\quad\frac{1}{2M^2}\left(\Delta_T^2G_{13}+k_T\cdot\Delta_TG_{12}\right)+\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{12}$$

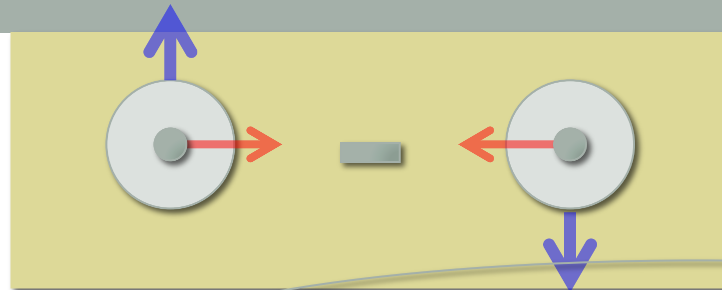
$$+\quad\frac{\Delta_T^i}{2M\Delta_T^2}\left((\Delta_1-i\Delta_2)\mathcal{M}_{-+}^{i,S}+(\Delta_1+i\Delta_2)\mathcal{M}_{+-}^{i,S}\right)=0.$$

⇓

$$f_{1T}^{\perp(1)}=-F_{12}^{o(1)}=\mathcal{M}_{F_{12}}|_{\Delta_T=0}$$

Sivers function

Qiu-Sterman term



$$H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} = \frac{P^2}{M^2}\int_x^1 \frac{dy}{y}\tilde{H} + \frac{m}{M}\left[\frac{1}{x}\left(H_T - \frac{\Delta_T^2}{4M^2}E_T\right) - \int_x^1 \frac{dy}{y^2}\left(H_T - \frac{\Delta_T^2}{4M^2}E_T\right)\right.$$

$$+\frac{\Delta_T^2}{4M^2}\left[\frac{1}{x}(H+E) - \int_x^1 \frac{dy}{y^2}(H+E)\right] + \left[\frac{\mathcal{M}_{G_{12}}}{x} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{G_{12}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{G_1}$$

$$g_2 = -\left(g_1 - \int_x^1 \frac{dy}{y}g_1\right) + \frac{m}{M}\left(\frac{1}{x}h_1 - \int_x^1 \frac{dy}{y^2}h_1\right) + \left(\tilde{g}_T - \int_x^1 \frac{dy}{y}\tilde{g}_T\right) + \int_x^1 \frac{dy}{y}\hat{g}_T$$

Original Wandzura Wilczek relation in forward limit

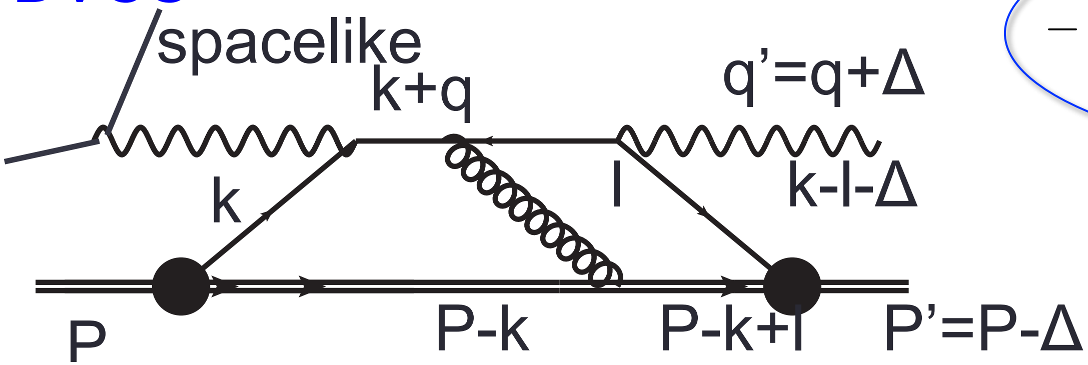
Chiral symmetry breaking test!

A probe of QCD at the amplitude level: color forces!

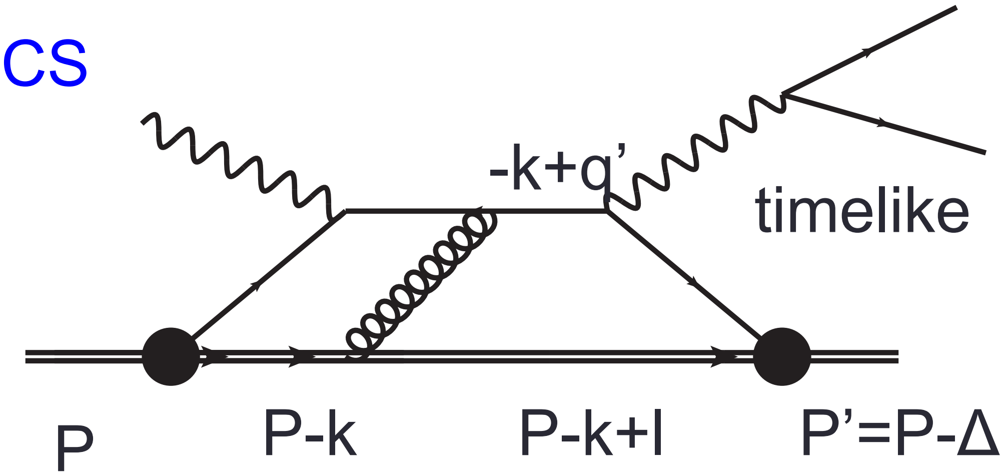
$$\tilde{E}_{2T} = \tilde{E}_{2T}^{WW} + \tilde{E}_{2T}^{(3)} + \tilde{E}_{2T}^{LIR}$$

$$-\left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

DVCS



TCS

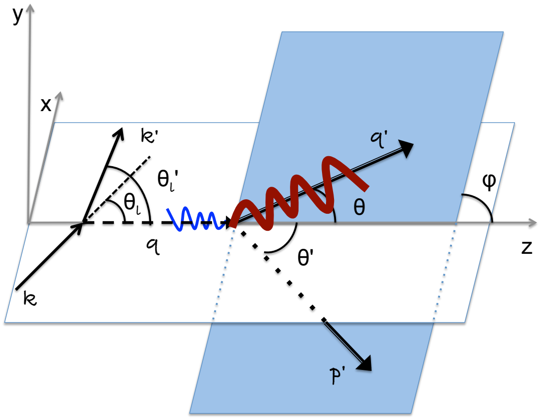


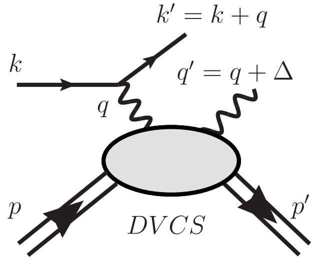
Test Universality!

B. Kriesten, in progress

Measuring GPDs in Deeply Virtual Exclusive Experiments

$$e\,p \rightarrow e'\gamma\,p'$$

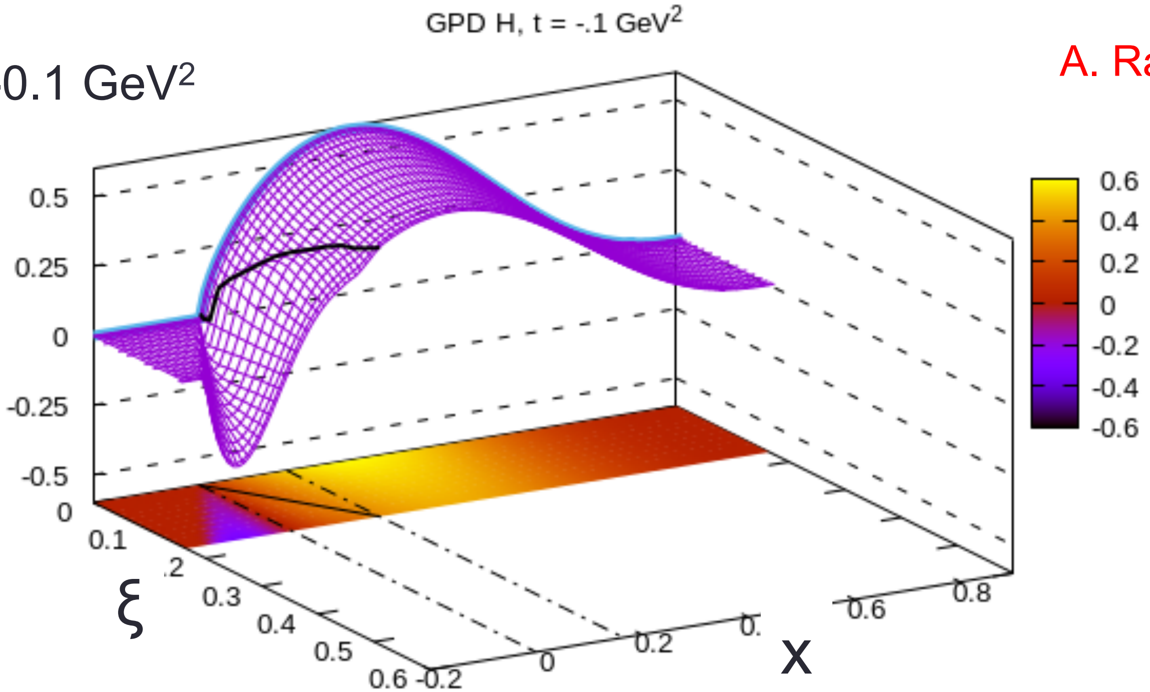




2/18/16

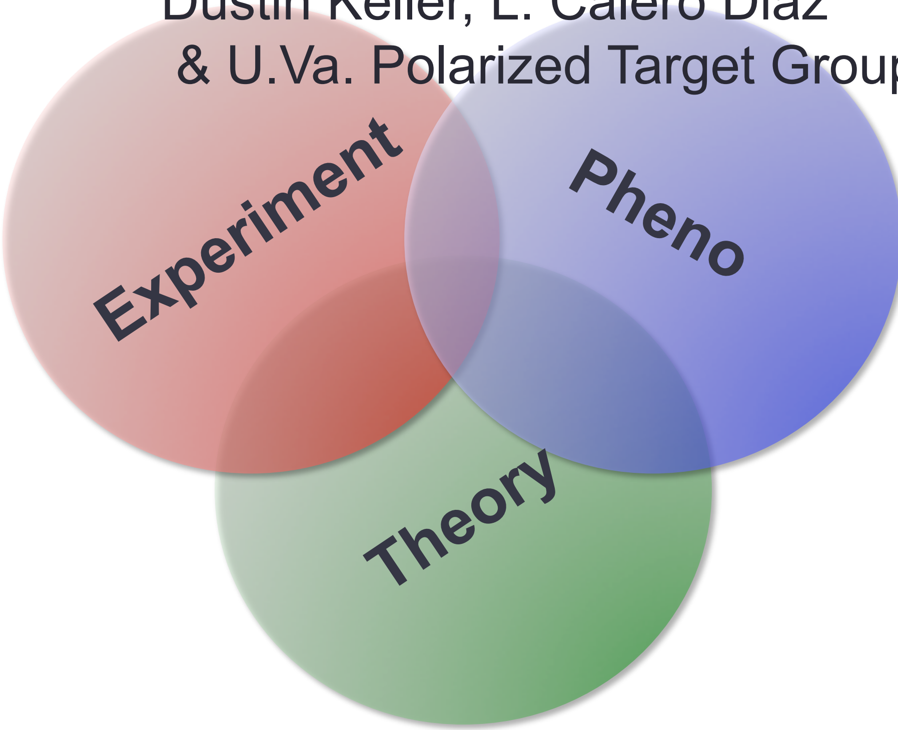
GPD H t=-0.1 GeV^2

A. Rajan, S.L.



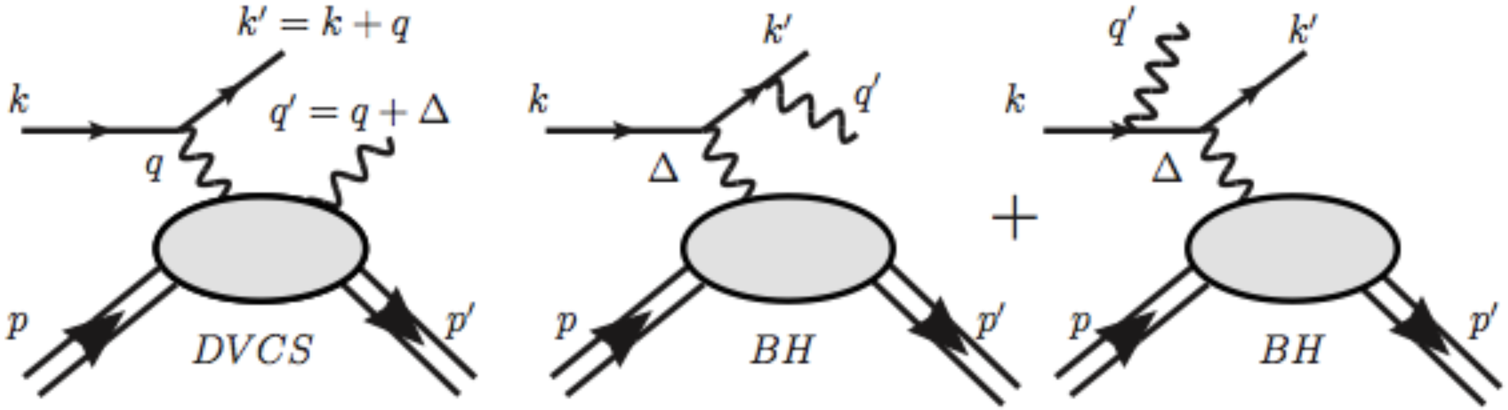
How do we detect all this?

Dustin Keller, L. Calero Diaz
& U.Va. Polarized Target Group



G. Goldstein, O. Gonzalez Hernandez, B. Kriesten, A. Meyer, A. Rajan,

Deeply Virtual Exclusive Photoproduction



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T\right|^2 ,$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

Demystification of “harmonics”

“Exact” Rosenbluth-like separation

BH unpolarized cross section

$$\sigma_{BH} = \Gamma \left[A(y,t,\gamma,Q^2,\phi)\frac{F_1+\tau F_2^2}{M^2} + B(y,t,\gamma,Q^2,\phi)\tau G_M^2(t) \right]$$

DVCS unpolarized cross section

$$\sigma^{UU} = \frac{\Gamma}{Q^2(1-\epsilon)}\left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}\right]$$

$$F_{UU,T} = 2(F_{++}^{11} + F_{+-}^{11} + F_{-+}^{11} + F_{--}^{11}),$$

Twist 2

$$F_{UU,L} = 2F_{++}^{00}$$

Twist 4

$$F_{UU}^{\cos\phi} = \mathrm{Re}\left[F_{++}^{01} + F_{--}^{01}\right]$$

Twist 3

$$F_{UU}^{\cos 2\phi} = \mathrm{Re}\left[F_{++}^{1-1} + F_{+-}^{1-1} + F_{-+}^{1-1} + F_{--}^{1-1}\right]$$

Photon helicity flip: transverse gluons

$$F^{11}_{++} = (1-\xi^2) \mid \mathcal{H} + \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* + \widetilde{\mathcal{H}})^* (\mathcal{E} + \widetilde{\mathcal{E}}) + (\mathcal{H} + \widetilde{\mathcal{H}})(\mathcal{E}^* + \widetilde{\mathcal{E}}^*) \right]$$

$$F^{11}_{--} = (1-\xi^2) \mid \mathcal{H} - \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* - \widetilde{\mathcal{H}})^* (\mathcal{E} - \widetilde{\mathcal{E}}) + (\mathcal{H} - \widetilde{\mathcal{H}})(\mathcal{E}^* - \widetilde{\mathcal{E}}^*) \right]$$

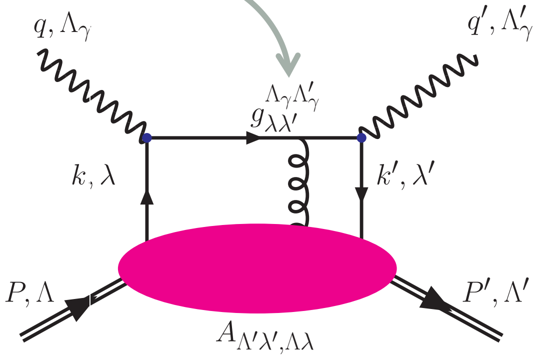
$$F^{11}_{+-} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} + \xi \widetilde{\mathcal{E}} \mid^2$$

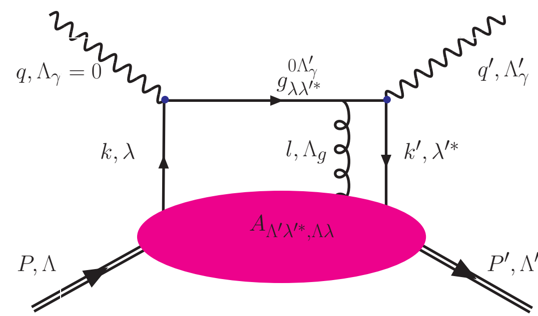
$$F^{11}_{-+} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} - \xi \widetilde{\mathcal{E}} \mid^2$$

Twist 3

$$f^{01}_{\Lambda\Lambda'} = g^{01}_{-*+} \otimes A_{\Lambda'+,\Lambda-^*} + g^{01}_{-+^*} \otimes A_{\Lambda'+^*,\Lambda-} + g^{01}_{+^*-} \otimes A_{\Lambda'-,\Lambda+^*} + g^{01}_{+-^*} \otimes A_{\Lambda'-^*,\Lambda+}$$

"Bad" component (exchanged gluon flips the quark chirality)





Connecting the DVCS formalism with the TMD/GPD/GTMD comprehensive parametrizations

Bacchetta et al JHEP02 (2007), Meissner Metz and Schlegel, JHEP08 (2009)

Example

$$A_{+-,++^*} = \frac{1}{2}\left(\tilde{E}_{2T} - \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

$$A_{+-^*,++} = \frac{1}{2}\left(-\tilde{E}_{2T} + \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

$$\vdots$$

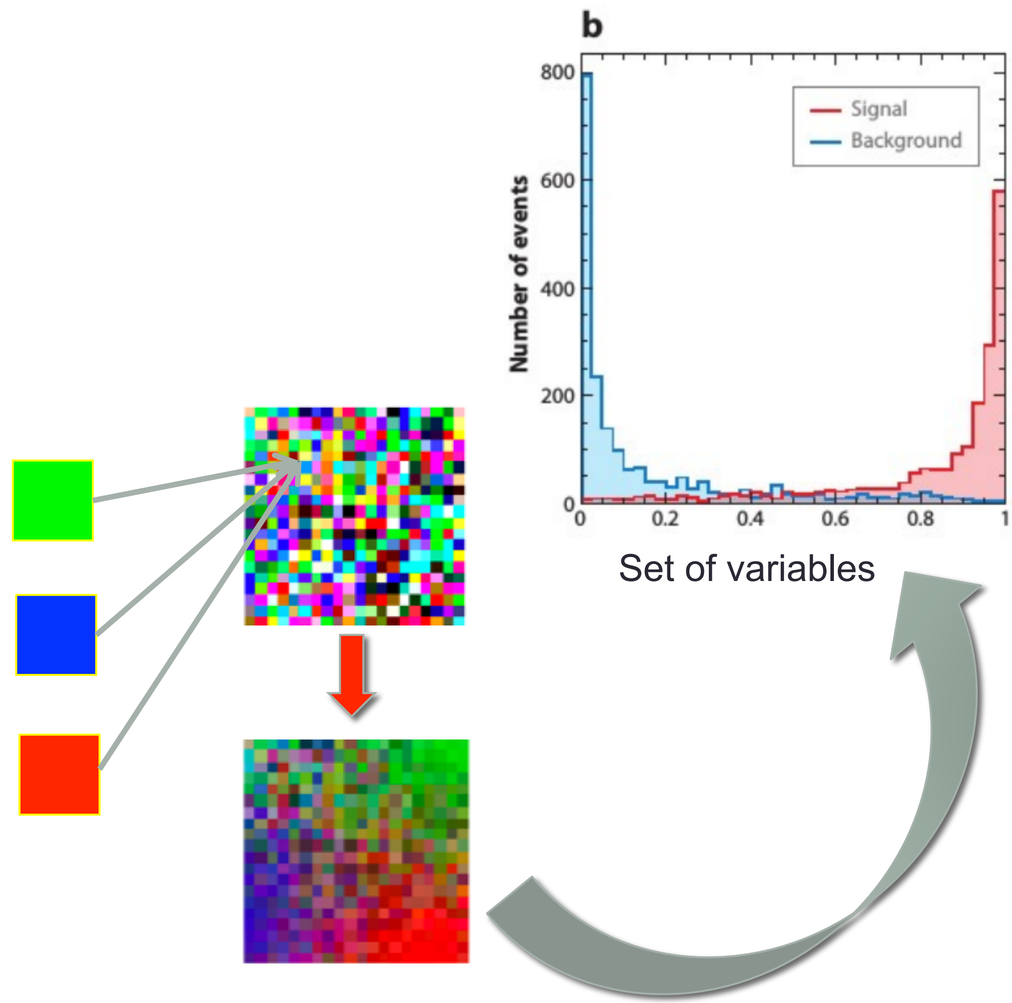
Spin Orbit interaction

Orbital angular momentum

IMPORTANT MESSAGE: Twist two and Twist three GPDs can be treated and should be treated simultaneously within "New generation" analysis with multivariate techniques

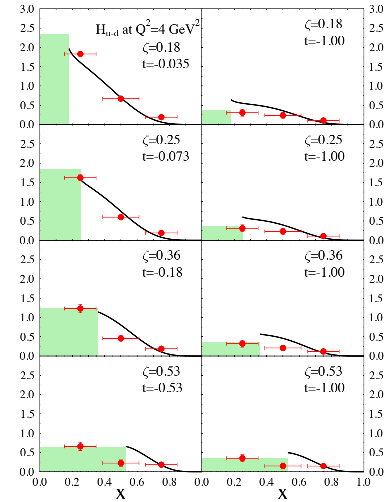
Dustin Keller, Andrew Meyer, Liliet Calero-Diaz

- Boosted Decision Trees
- Artificial Neural Networks
    - Self-Organizing Maps (E. Askanazi)



GPD Model: Flexible parametrization

Ahmad et al., using lattice moments

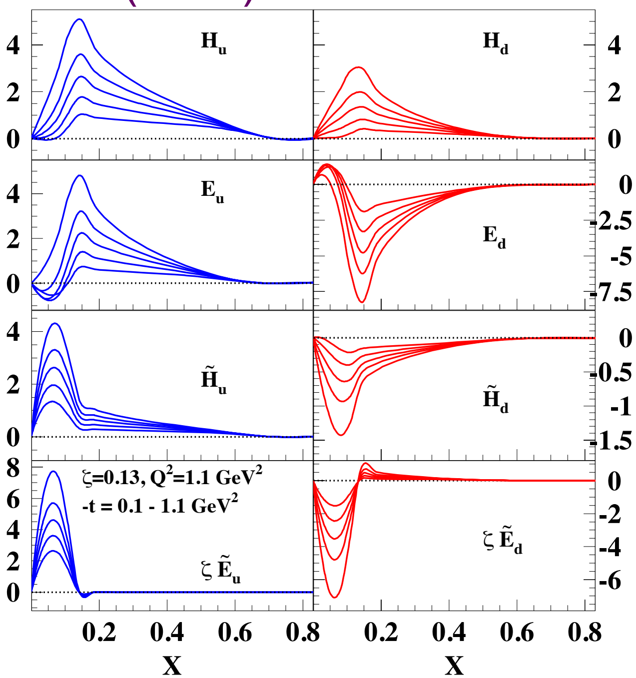


PRD75(2007) AHMAD HONKANEN S.L. TANEJA
EPJC63(2009) AHMAD HONKANEN, S.L. TANEJA

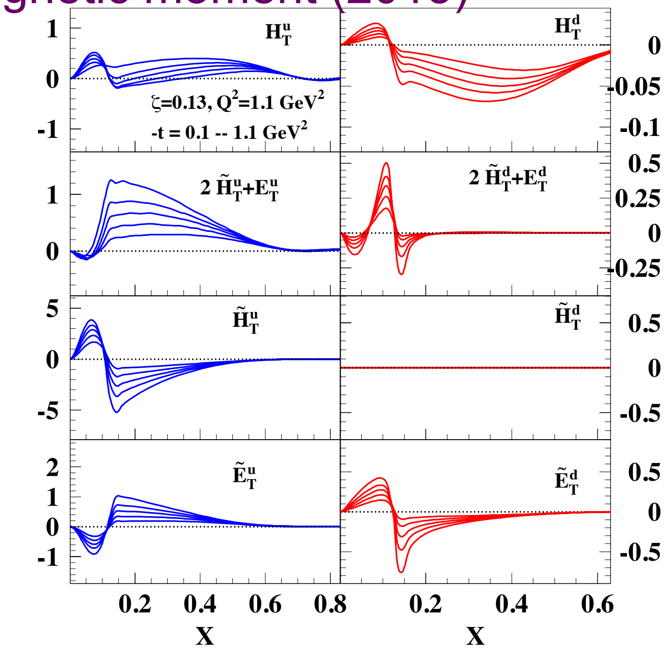
PRD84(2011)GOLDSTEIN GONZALEZ S.L.
PRC88(2013)GONZALEZ GOLDSTEIN S.L. KATHURIA

PRD91(2015) GOLDSTEIN GONZALEZ S,.L

Chiral Even (2011)



Chiral Odd --tensor charge and magnetic moment (2015)



Finally, nuclei

First Exclusive Measurement of Deeply Virtual Compton Scattering off ^{4}He: Toward the 3D Tomography of Nuclei

M. Hattawy,[1,2] N.A. Baltzell,[1,3] R. Dupré,[1,2,*] K. Hafidi,[1] S. Stepanyan,[3]
S. Bultmann,[4] R. De Vita,[5] A. El Alaoui,[1,6] L. El Fassi,[7] H. Egiyan,[3] F.X. Girod,[3]
M. Guidal,[2] D. Jenkins,[8] S. Liuti,[9] Y. Perrin,[10] B. Torayev,[4] and E. Voutier[10,2]
(The CLAS Collaboration)

Physics of the D-term

$$\int_{-A}^{A} dx H^A(x,\xi,t) = F^A(t)$$

$$\int_{-A}^{A} dx x H^A(x,\xi,t) = M_2^A(t) + \frac{4}{5} d_1^A(t)\xi^2,$$

d represents the spatial distribution of the shears forces (Polyakov Shuvaev)

$$d^Q(0) = -\frac{m_N}{2} \int d^3r \, T_{ij}^Q(\vec{r}) \left(r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right)$$

From S.L. and S.K. Taneja, PRC72(2005)

$$F^A(t) = F^{A,point}(t)F^N(t) \quad (54)$$

$$M_2^A(\xi,t) = M_2^{A,point}(t)M_2^N(t) + M_0^{A,point}(t)\frac{4}{5}d_1^N(t)\xi^2, \quad (55)$$

with $M_n^{A,point}(t) = \int dy y^{n-1} f_A(y,t)$, the nuclear moment obtained by considering "point-like" nucleons. At $\xi = 0$ one has:

$$M_2^A(t) = M_2^{A,point}(t)M_2^N(t), \quad (56)$$

related to the average value of the longitudinal momentum carried by the quarks in a nucleus:

$$\langle x(t)\rangle_A = \frac{M_2^A(t)}{F^A(t)} = \frac{M_2^{A,point}(t)}{F^{A,point}(t)}\frac{M_2^N(t)}{F^N(t)} = \langle y(t)\rangle_A\langle x(t)\rangle_N, \quad (57)$$

The D-term in a nucleus reads:

$$d_1^A(t) = M_0^{A,point}(t)d_1^N(t). \quad (58)$$

Is this factorization broken? First signature of non-nucleonic effects

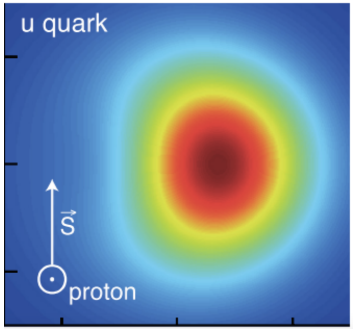
In liquid drop model

$$d_1^A(0) \propto \boxed{A^{7/3}}$$

$$d_1^A(0) \approx \frac{1}{1 - \frac{\langle E \rangle_A}{M} + \frac{2}{3}\frac{\langle p_\mu^2 \rangle_A}{M^2}} \propto \boxed{A \ln A}$$

Nuclear model taking into account virtuality

➢ Spin and 3D structure of Deuteron



$$\frac{1}{2}\int_{-1}^{1} dx x \left[H_q(x,0,0) + E_q(x,0,0)\right] = J_q$$

Nucleon (Ji, 1997)

$$\longrightarrow \frac{1}{2}\int_{-1}^{1} dx x \, H_2^q(x,0,0) = J_q$$

Deuteron (Taneja, Kathuria, SL, Goldstein, 2012)

Spin 1 nucleus GPD related to deuteron form factor, G_M:
measurable with transverse polarized target
(Crabb, Day, Keller)

Hopefully experimental studies of the hard exclusive processes will fill the gap in our understanding of the strong forces creating our world as we see it.

Maxim Polyakov (hep-ph/0210165)

Back up

This GPDs combination:

$$H + E = 2P^+ \int d^2k_T \int dk^-\, 2\left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5^F + A_6^F + \frac{P\cdot k - xP^2}{M^2}(A_8^F + xA_9^F)\right)$$

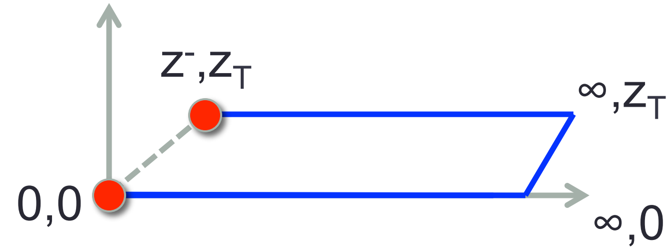
$$\widetilde{E}_{2T} = 2P^+ \int d^2k_T \int dk^-\,(-2)\left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5^F + A_6^F + \frac{(k_T\cdot\Delta_T)^2 - k_T^2\Delta_T^2}{M^2\Delta_T^2}A_9^F\right)$$

… and the derivative of F_{14}…

$$\frac{d}{dx}F_{14}^{(1)} = \frac{4P^+}{M^2}\int d^2k_T \int dk^-\left[(k\cdot P - xP^2)(A_8^F + xA_9^F) + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}A_9^F\right]$$

… are described by the same A_i amplitudes

Generalized LIR for a staple link



$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}^F + xA_{12}^F) + A_{14}^F + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8^F}{\partial(k\cdot v)} + x\frac{\partial A_9^F}{\partial(k\cdot v)}\right)\right]$$