Imaging Hadrons using Lattice QCD

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Exploring Hadrons with Electromagnetic Probes: Structure, Excitations, Interactions

Introduction

- Measures of Hadron Structure and Lattice QCD
- 1-D hadron Structure - Parton Distribution Functions and Form Factors
- 3-D Measures: (Moments of)
  - Generalized Parton Distributions
  - TMDs
- New Developments in LQCD: LaMET, Quasi-distributions, Pseudo-Distributions
- Summary

Measures of Hadron Structure

5D

Wigner distributions

Transverse Momentum Dependent Distributions (TMDs)

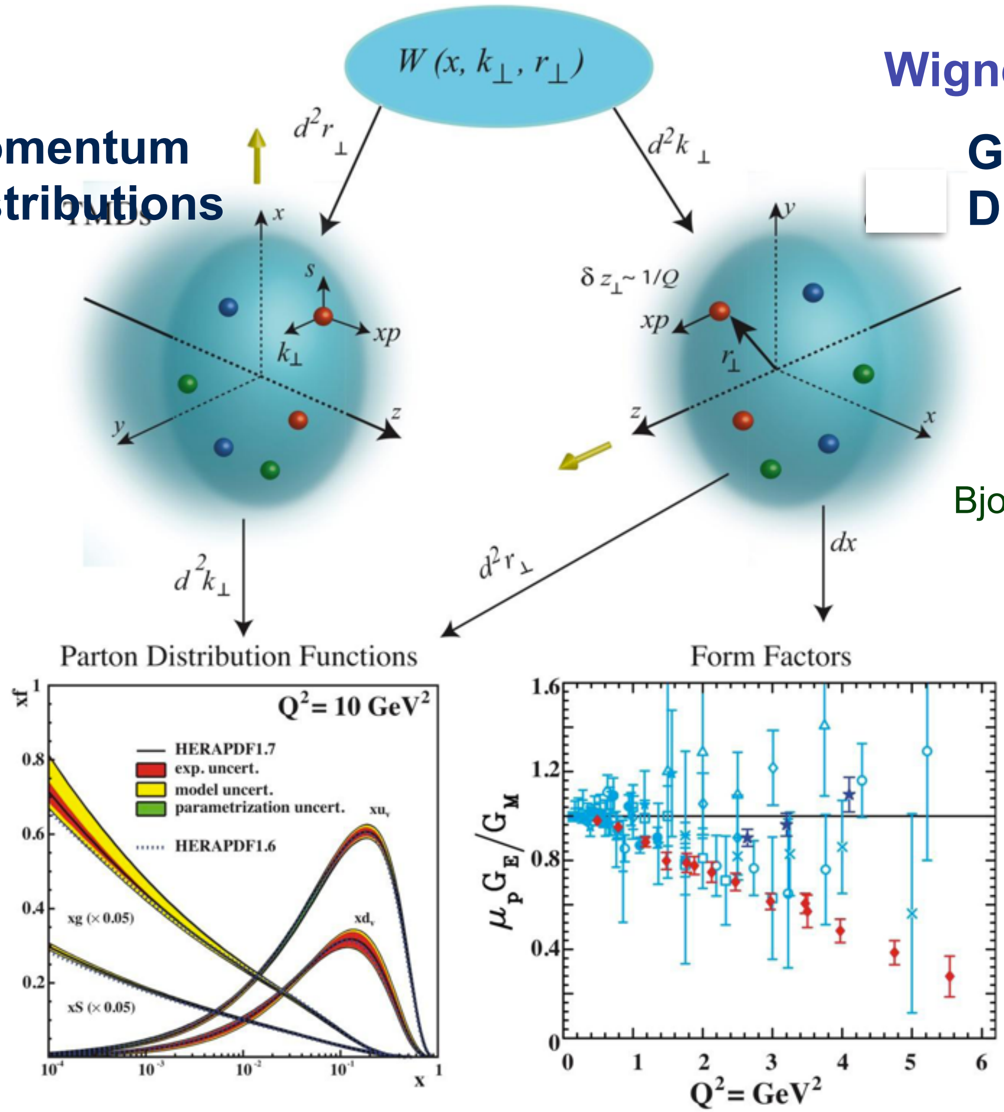
Generalized Parton Distributions (GPDs)

3D

Bjorken-x and transverse momentum

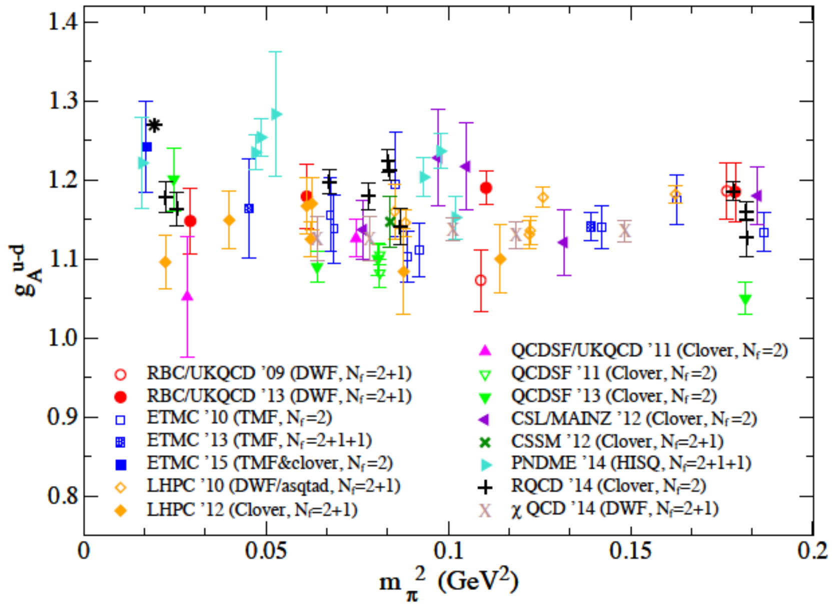
Bjorken-x and impact parameter

1D



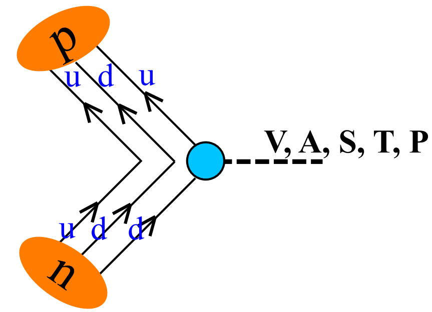
1D Structure - Charges and Precision

M Constantinou, arXiv:1511.00214



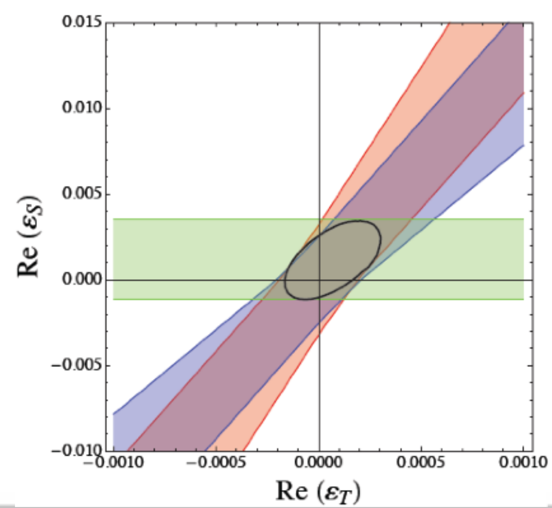
- Governs beta-decay rate
- Important for proton-proton fusion rate in solar models
- Benchmark for lattice QCD calculations of hadron structure

e.g. novel interactions probed in ultra-cold neutron decay



$$H_{eff} \supset G_F\left[\varepsilon_S\, \bar{u}d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T\, \bar{u}\sigma_{\mu\nu}d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e\right]$$

$$g_S = Z_S \langle p|\bar{u}d|n\rangle \quad g_T = Z_T\, \langle p|\bar{u}\,\sigma_{\mu\nu}\, d|n\rangle$$



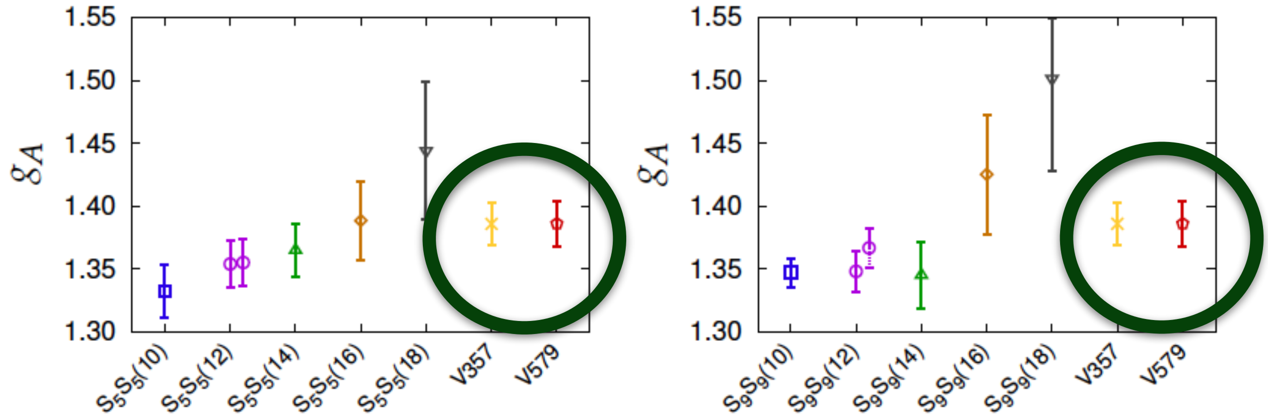
R Gupta, 2014

Systematic Uncertainties

Yoon et al., Phys. Rev. D 93, 114506 (2016)

Failure to isolating **ground state** leads to important systematic uncertainty.

Variational Method



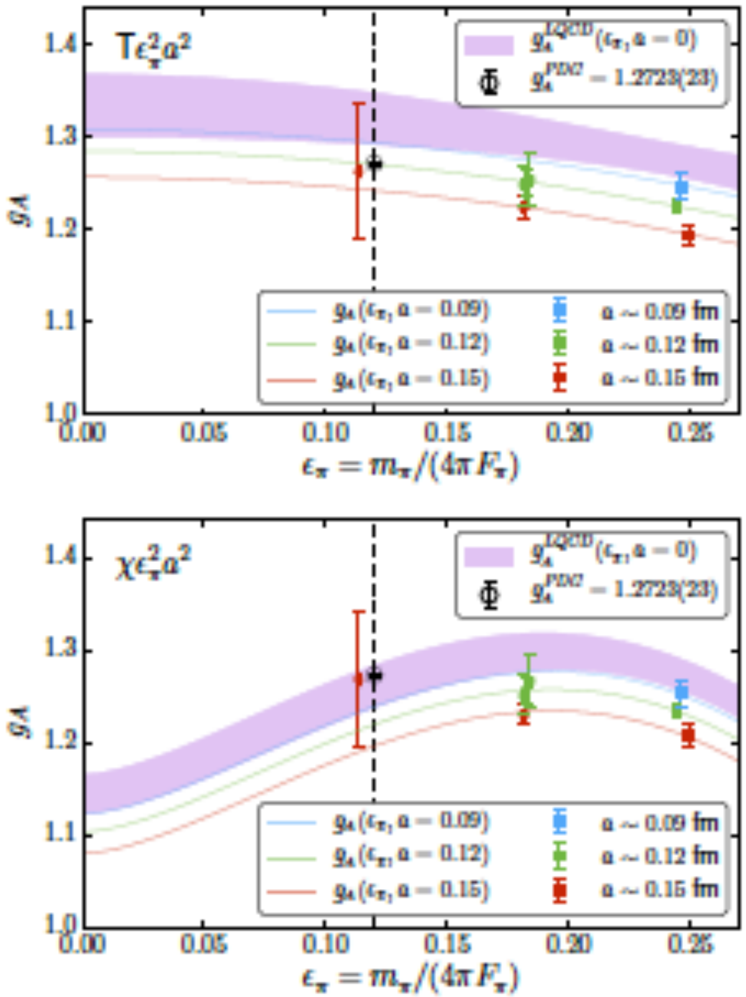
Yoon et al., Phys. Rev. D 95, 074508 (2017)

ID	Lattice Theory	a fm	M_π(MeV)	g_A^{u-d}	g_S^{u-d}	g_T^{u-d}	g_V^{u-d}
$a127m285$	2+1 clover-on-clover	0.127(2)	285(6)	1.249(28)	0.89(5)	1.023(21)	1.014(28
$a12m310$	2+1+1 clover-on-HISQ	0.121(1)	310(3)	1.229(14)	0.84(4)	1.055(36)	0.969(22
$a094m280$	2+1 clover-on-clover	0.094(1)	278(3)	1.208(33)	0.99(9)	0.973(36)	0.998(26
$a09m310$	2+1+1 clover-on-HISQ	0.089(1)	313(3)	1.231(33)	0.84(10)	1.024(42)	0.975(33
$a091m170$	2+1 clover-on-clover	0.091(1)	166(2)	1.210(19)	0.86(9)	0.996(23)	1.012(21
$a09m220$	2+1+1 clover-on-HISQ	0.087(1)	226(2)	1.249(35)	0.80(12)	1.039(36)	0.969(32
$a09m130$	2+1+1 clover-on-HISQ	0.087(1)	138(1)	1.230(29)	0.90(11)	0.975(38)	0.971(32

Consistency between different actions

Matrix Elements of 1st excited state?

Feynman-Hellman Method



Berkowitz et al, arXiv:1704.01114

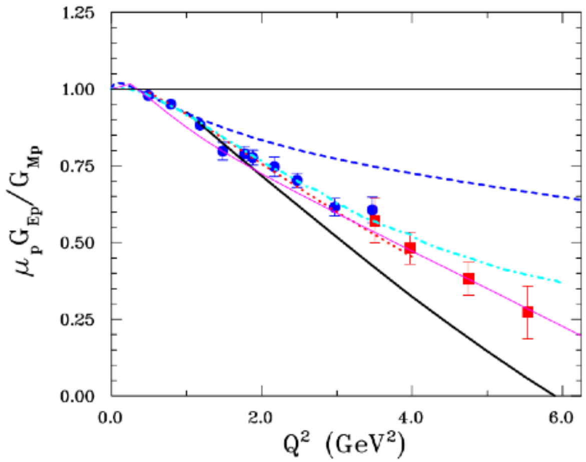
Calculation using Feynman-Hellman Theory

$$H = H_0 + \lambda H_\lambda$$

$$\frac{\partial E_n}{\partial \lambda} = \langle n \mid H_\lambda \mid n \rangle$$

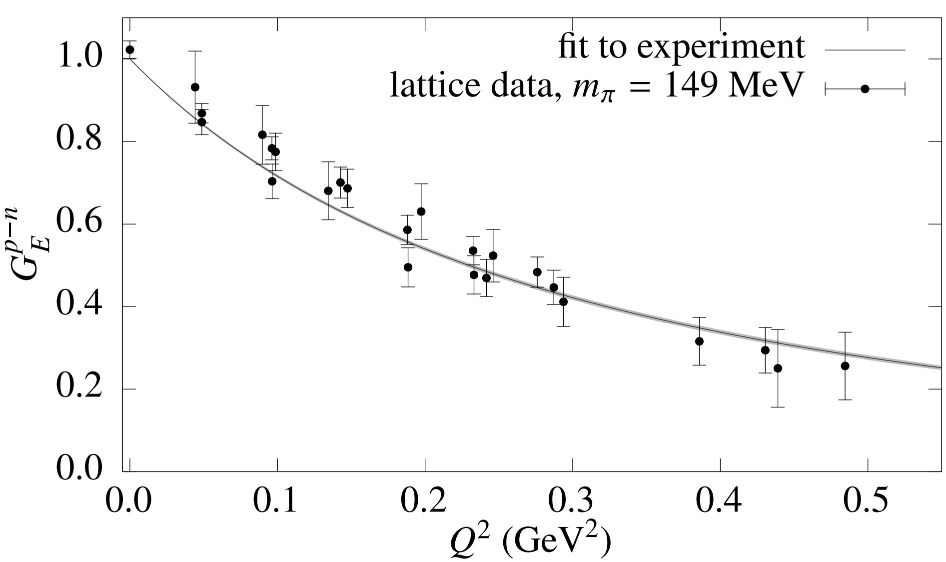
Reduces to calculation of energy-shift of two-point functions ***but*** repeat the calculation for each operator

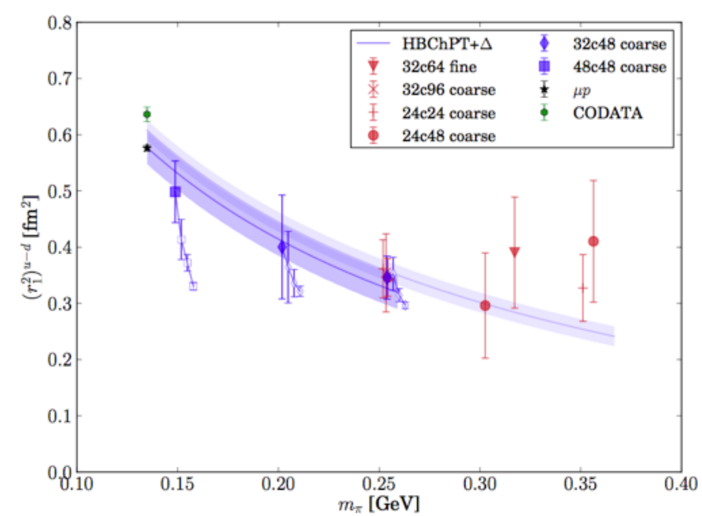
1D Structure: EM Form Factors



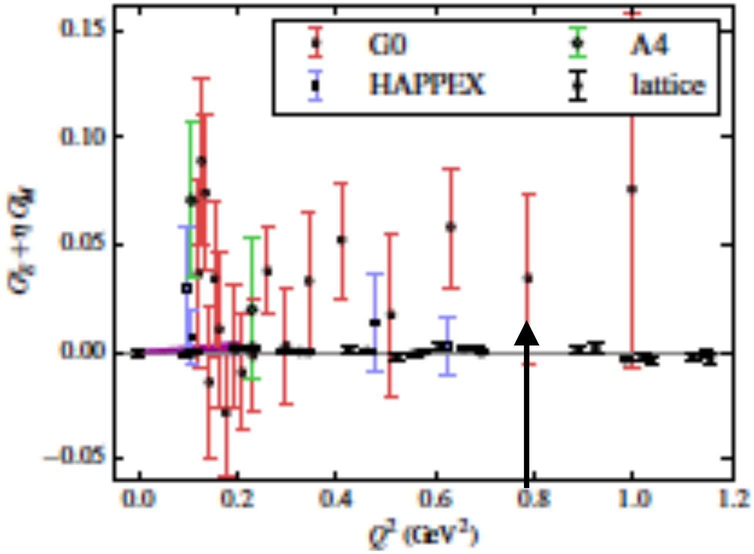
Large Q² behavior: Hall C at JLab to 15 GeV²

Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)





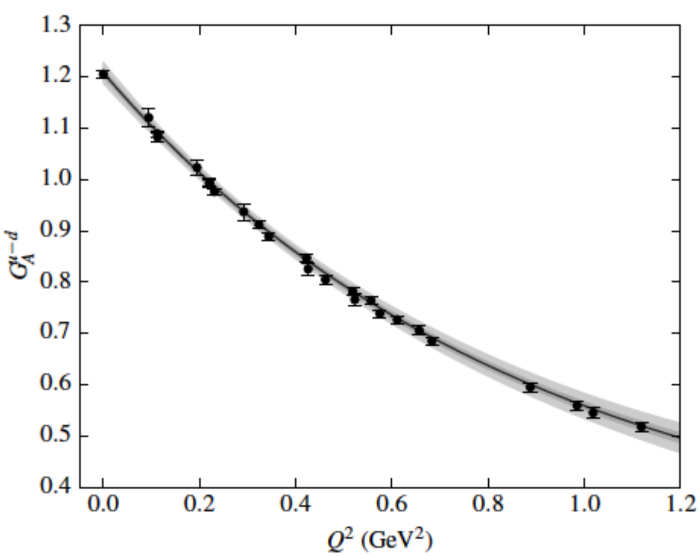
Sea Quark Contributions

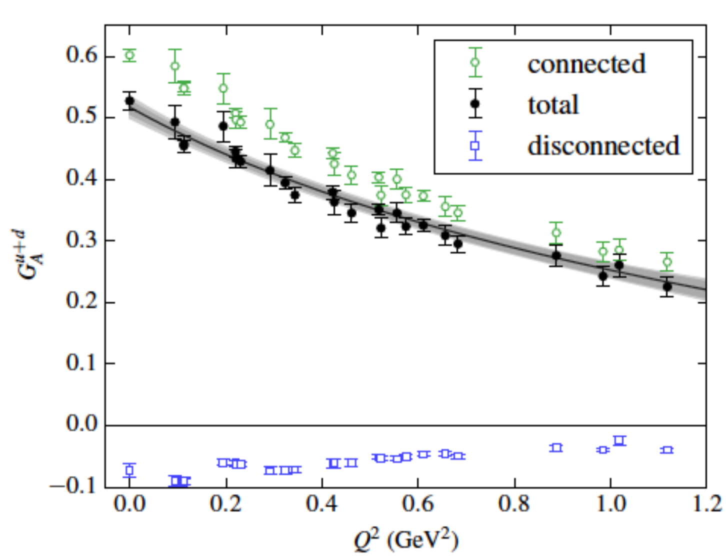


J. Green, K. Orginos *et al.*, Phys. Rev. D 92, 031501 (2015); Phys. Rev. D 95, 114502 (2017

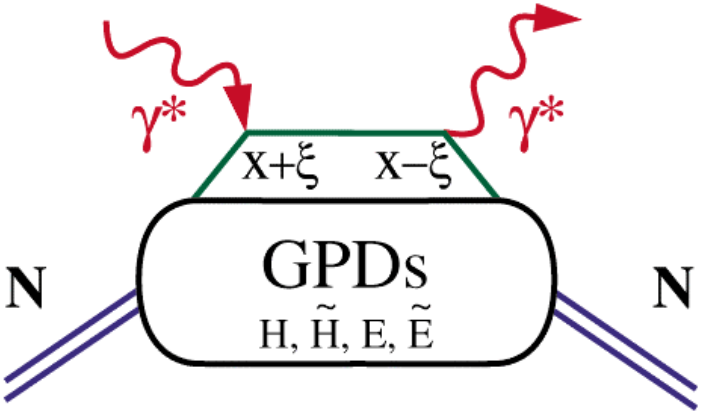
Using *Hierarchical Probing* - A. Stathopoulos, J. Laeuchli, K. Orginos (2013)

Combination *measured* in expt





Generalized Parton Distributions



D. Muller *et al* (1994), X. Ji, Radyushkin (1996)

$$\bar{u}(P')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+\kappa}\Delta_k}{2m}E(x,\xi,t)\right)u(P) =$$

$$\int_{-\infty}^{\infty}\frac{d\omega^-}{4\pi}e^{-i\xi P^+\omega^-}\langle P' \mid T\bar{\psi}(0,\omega^-,O_{\mathrm{T}})W(\omega^-,0)\gamma^+\frac{\lambda^a}{2}\psi(0) \mid P\rangle$$

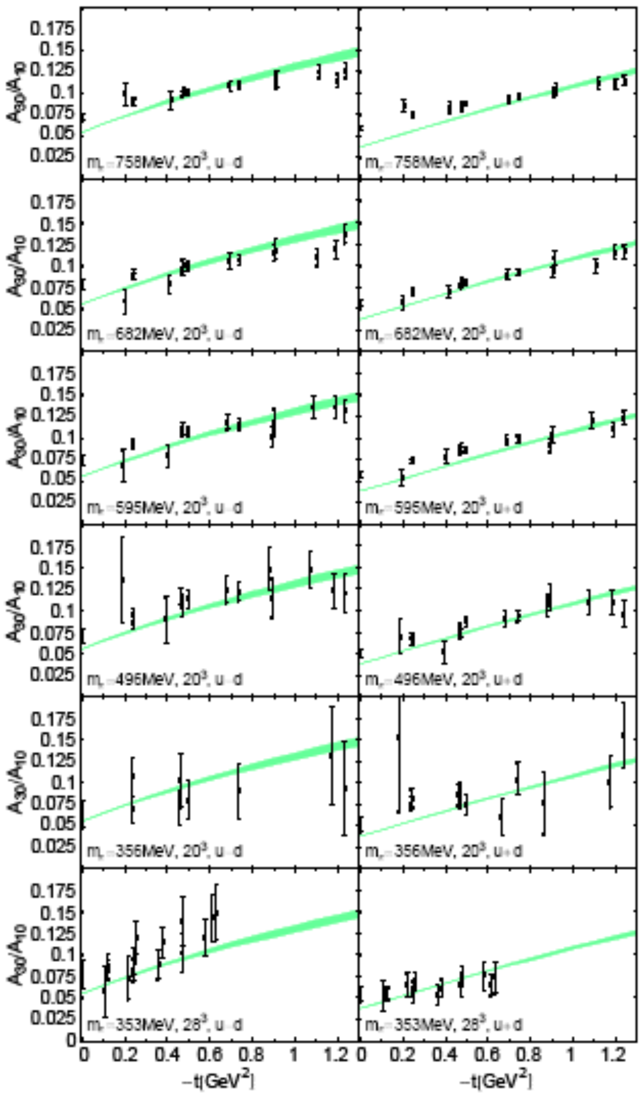
- Light-cone distributions not accessible in Euclidean-space QCD

$$\int_{-1}^{1} dx\, x^{n-1}\left[\begin{array}{c} H(x,\xi,t) \\ E(x,\xi,t) \end{array}\right] = \sum_{k=0}^{(n-1)/2}(2\xi)^{2k}\left[\begin{array}{c} A_{n,2k}(t) \\ B_{n,2k}(t) \end{array}\right] \pm \delta_{n,\mathrm{even}}(2\xi)^n C_n(t)$$

$$\downarrow$$

$$\mathcal{O}^{\mu_1\ldots\mu_n} = i^{n-1}\bar{\psi}\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_n\}}\frac{\lambda^a}{2}\psi$$

Parametrizations of GPDs



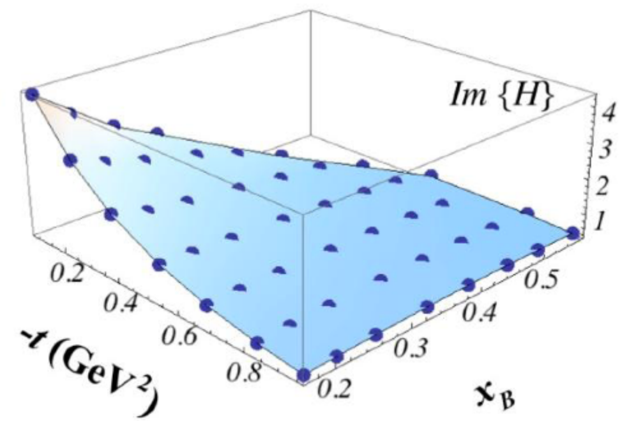
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); Phys.Rev.D82:094502,2010

Provide phenomenological guidance for GPD's

– ***CTEQ, Nucleon Form Factors, Regge***

Comparison with *Diehl et al, hep-ph/0408173*

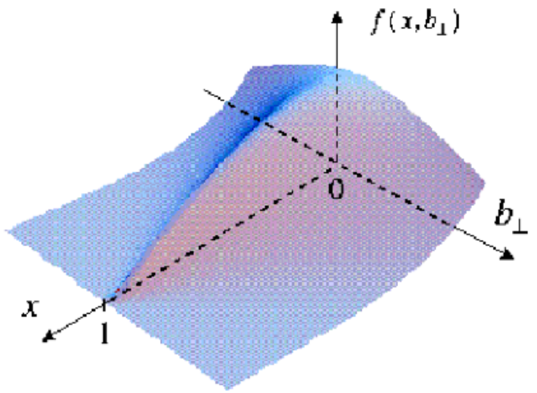
Important Role for LQCD

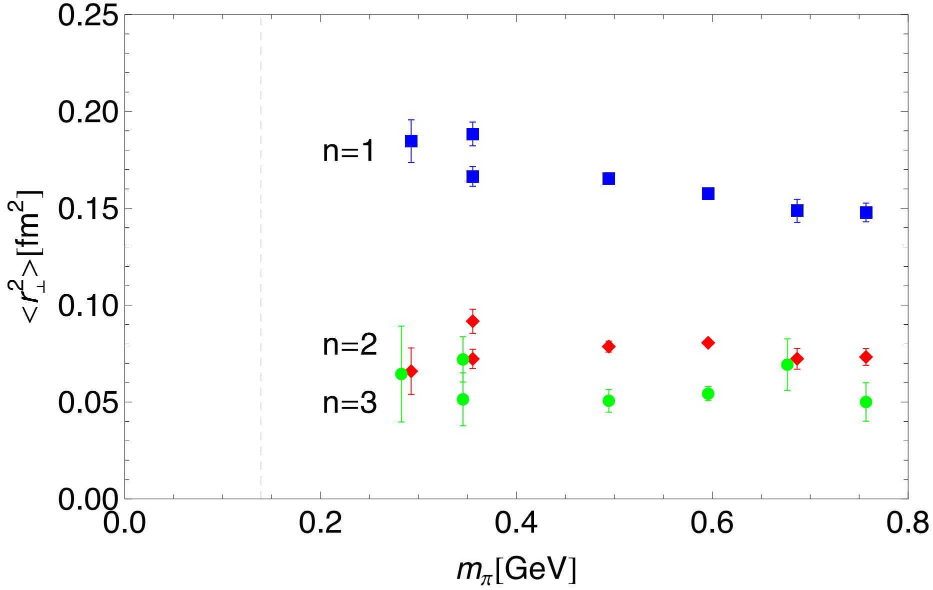


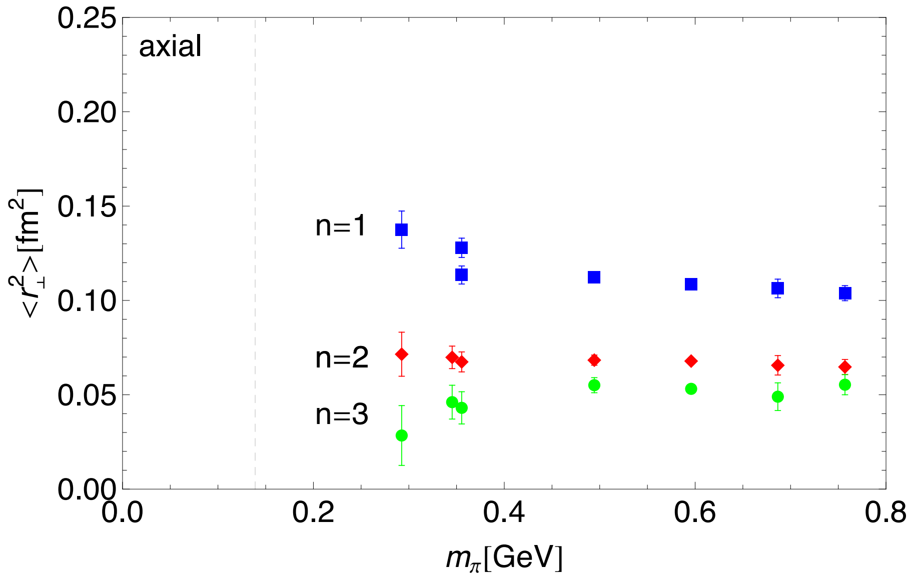
Charge Radius of GFFs

Lattice results consistent with narrowing of transverse size with increasing x

Flattening of GFFs with increasing n







Orbital Angular Momentum

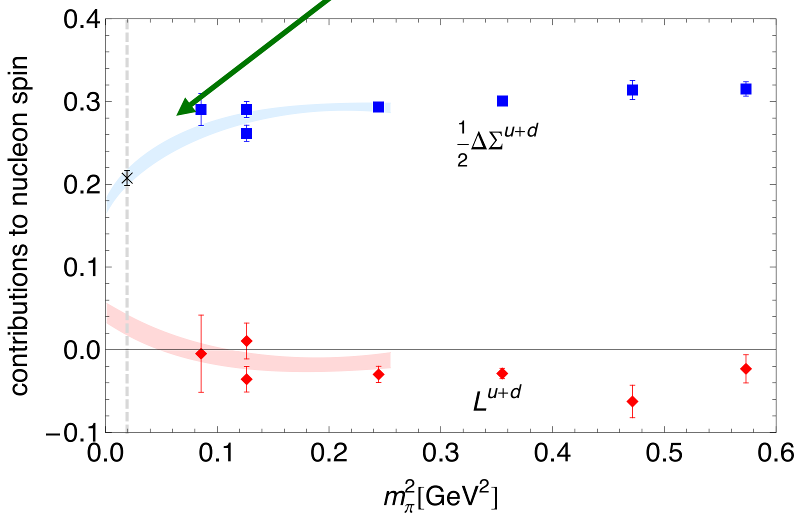
- **Total orbital angular momentum carried by quarks small**
- **Orbital angular momentum carried by individual quark flavours substantial.**

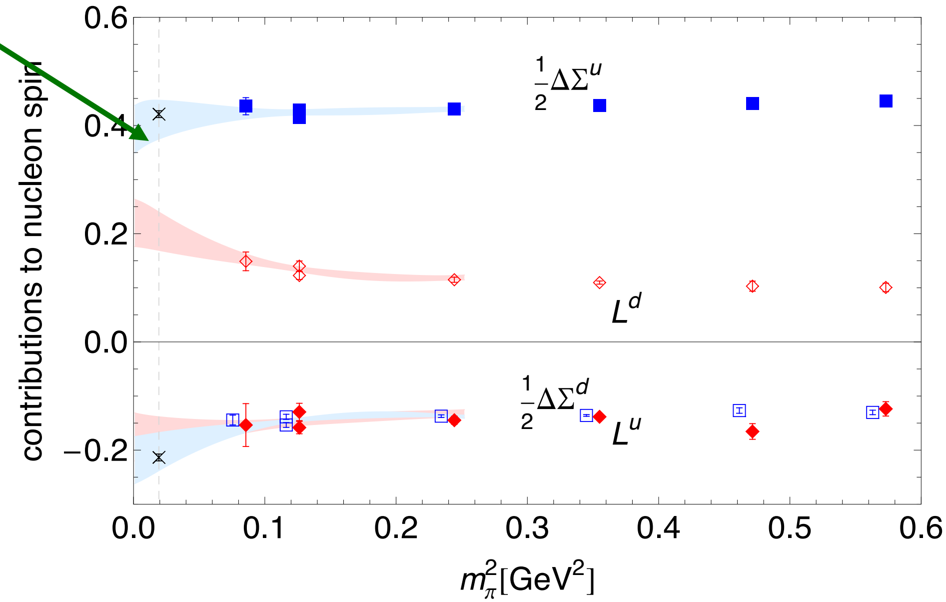
$$J^q = 1/2\left(A_{20}^q(t=0) + B_{20}^q(t-0)\right)$$
$$\Delta\Sigma^q/2 = \tilde{A}_{10}^q(t=0)/2$$
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

Mathur et al., *Phys.Rev. D62 (2000) 114504*

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

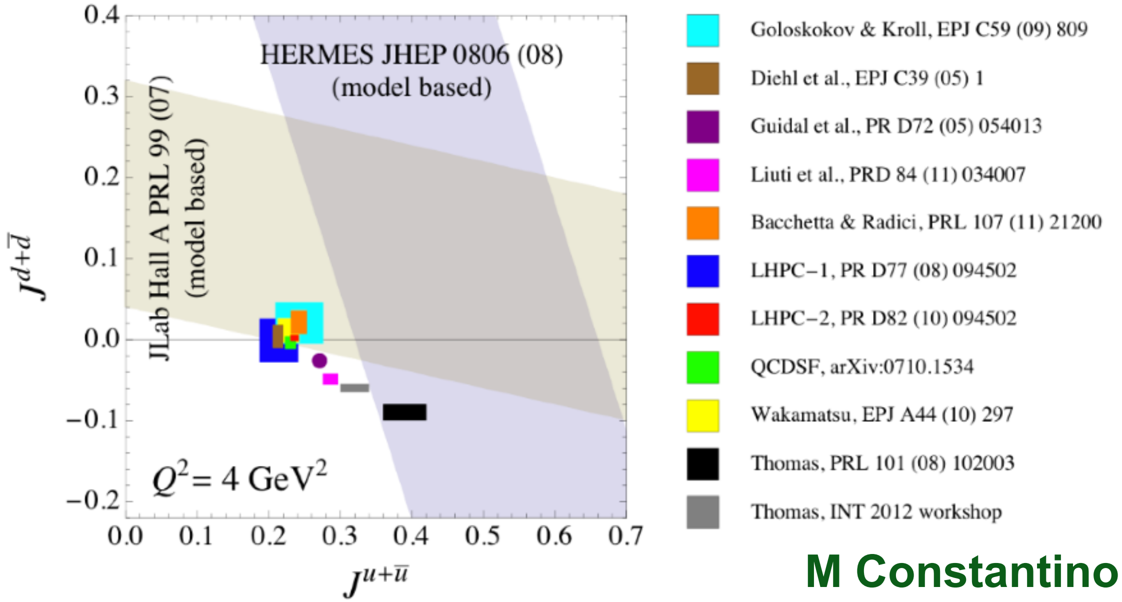
HERMES, PRD75 (2007)



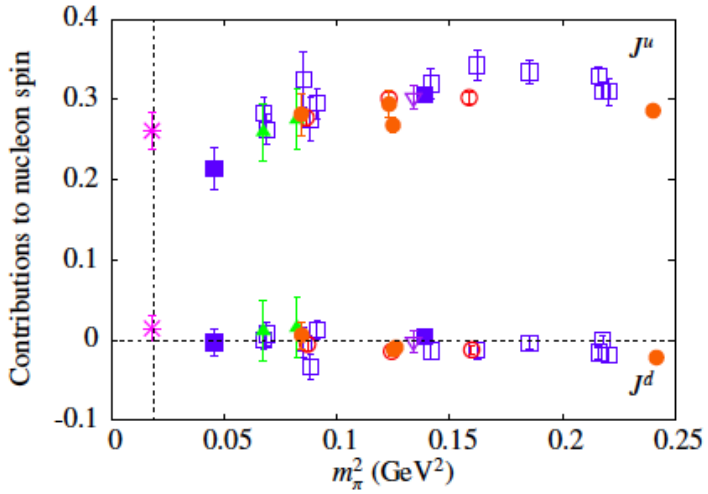


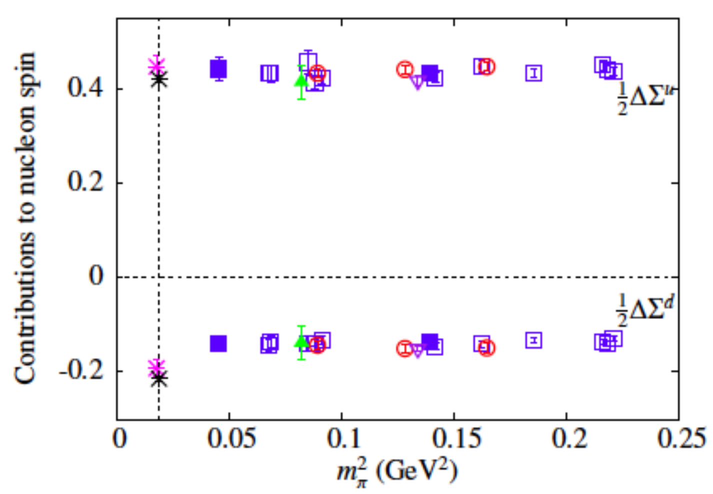
Disconnected contributions neglected.

Origin of Nucleon Spin - II



M Constantinou, arXiv:1511.00214

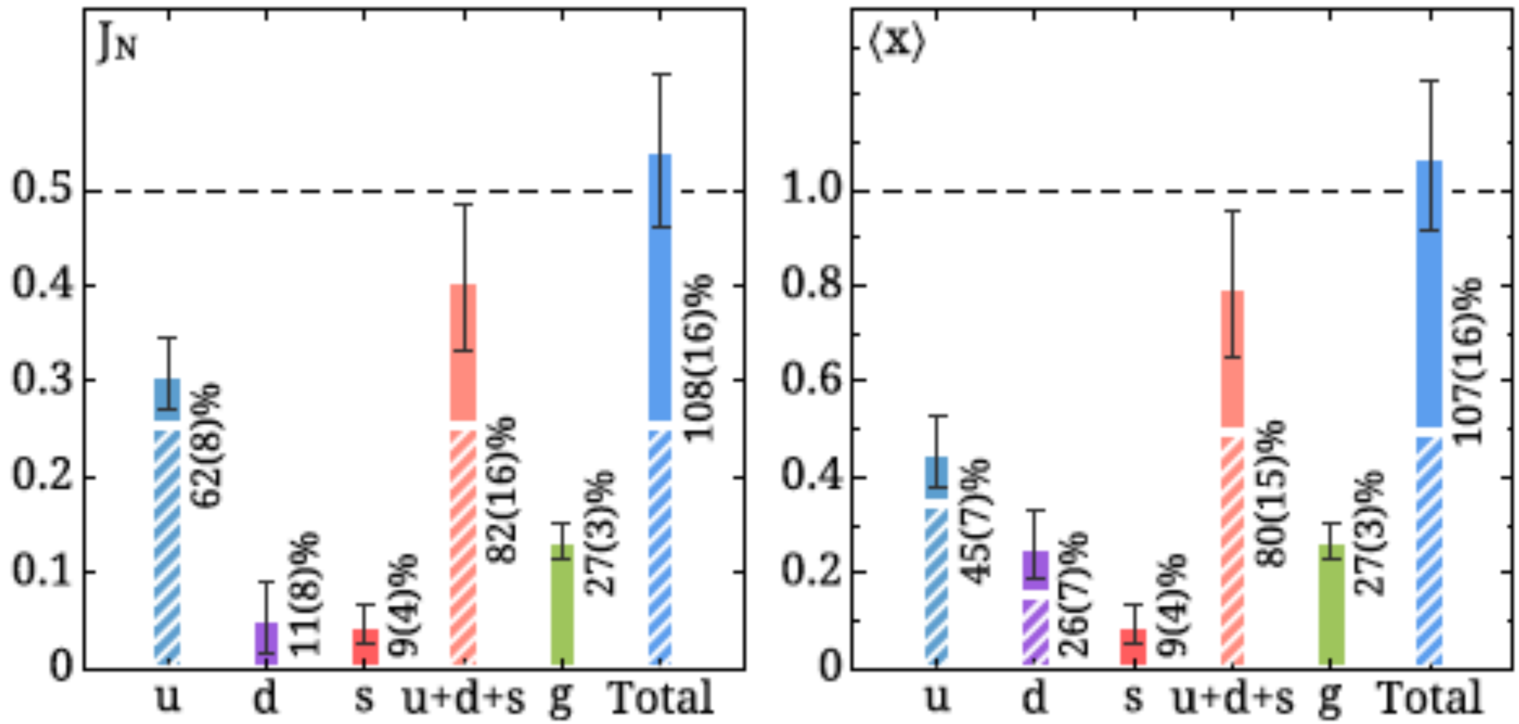




Spin and Momentum Decomposition

Gluonic observables "statistically challenging"

Twisted-Mass Fermions: C.Alexandrou et al, arXiv:1706.02973

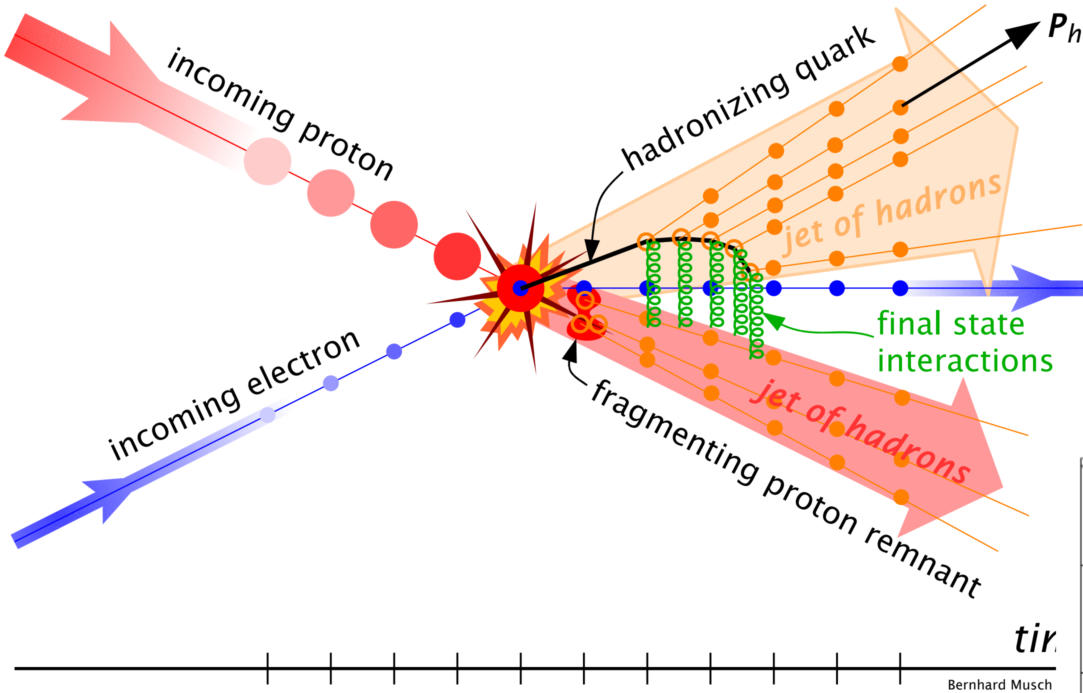


→ Momentum and Spin Sum Rules Satisfied

Transverse momentum distributions (TMDs)

from experiment, e.g., **SIDIS** (semi-inclusive deep inelastic scattering) + DY

HERMES, COMPASS, JLab 12 GeV , RHIC-spin, EIC, DY



Slide: B. Musch

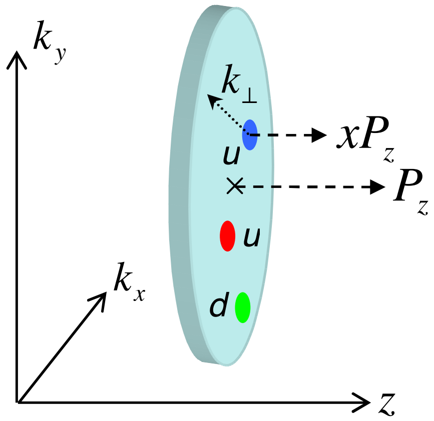
N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

Boer-Mulders

Sivers ← time-reversal odd

final state interactions!
explain large asymmetries otherwise forbidden!
signature of QCD!

TMDs in Lattice QCD



B. Musch, PhD Thesis; Haegler, Musch, Negele, Schafer arXiv:0908.1283

Introduce Momentum-space correlators

$$\Phi_\Gamma = \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\tilde{\Phi}_\Gamma(l;P,S)$$

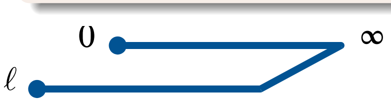
$$= \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\langle P,S\mid \bar{q}(l)\Gamma\mathcal{U}q(0)\mid P,S\rangle$$

continuum

$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$

along path from 0 to ℓ

Choice of path - retain gauge invariance

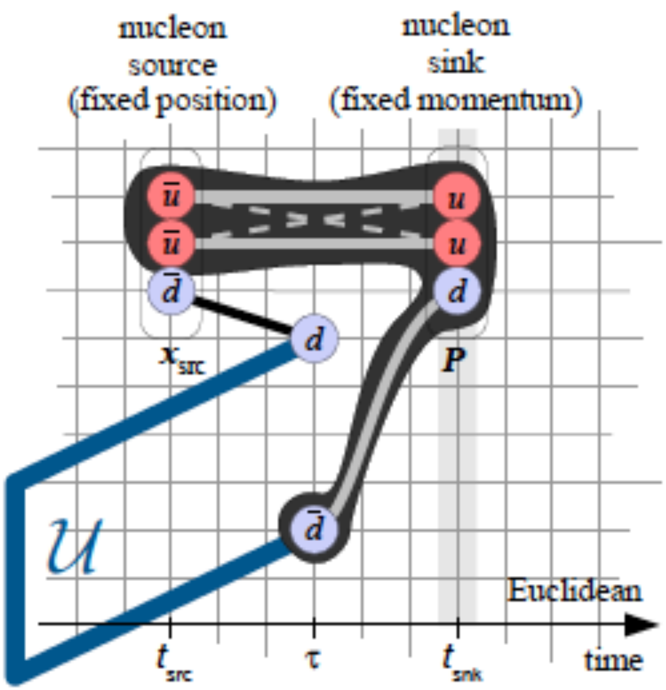


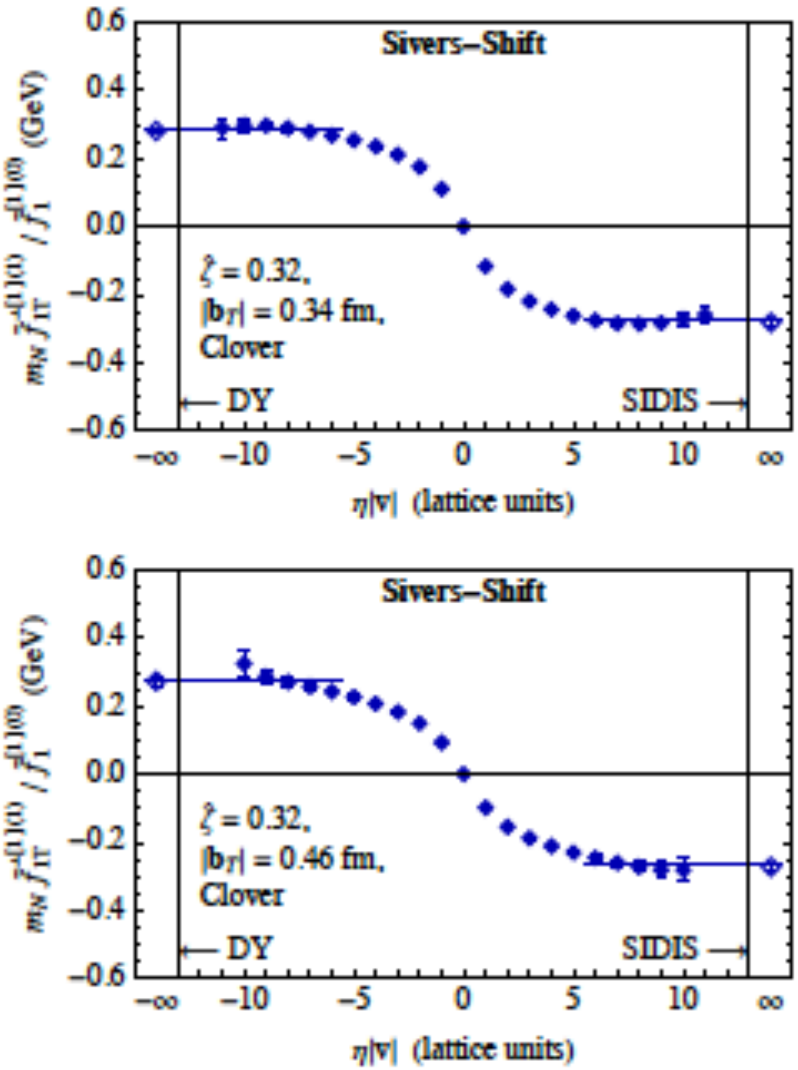
SIDIS: path runs to infinity

Lattice: equal time slice

Transverse momentum distributions (TMDs)

Lattice QCD





B. Musch et al., Phys.Rev. D85 (2012) 094510;
M. Engelhardt, Lattice 2014
Yoon et al, arXiv:1706.03606

Two Challenges….

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments* with respect to *Bjorken x*.

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

$$\downarrow$$

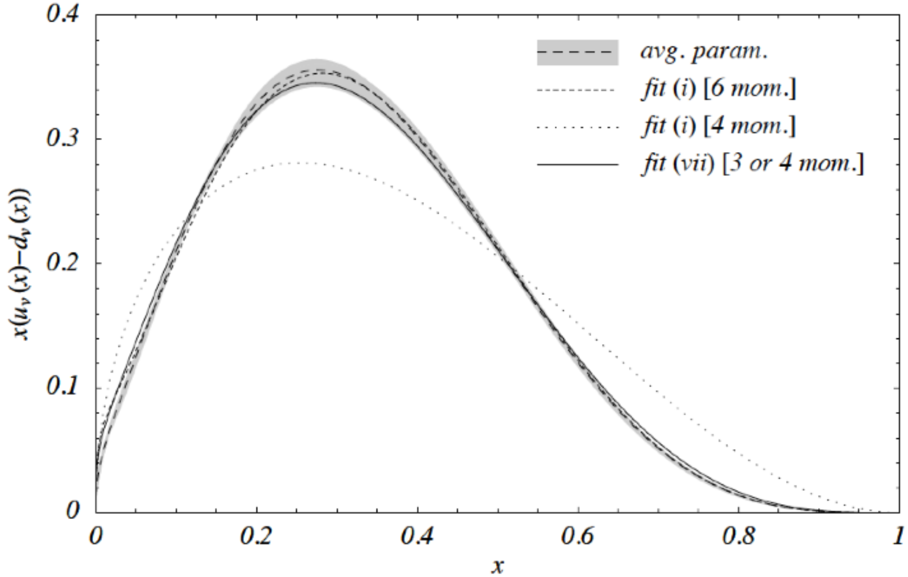
$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n} a^{(n)}$$

  - *Generalized Parton Distributions (off-forward): GPDs*
  - *Quark Distribution Amplitudes in exclusive processes: PDAs*
  - *(Transverse-Momentum-Dependent Distributions): TMDs*

- Discretisation, and hence reduced symmetry of the lattice, introduces power-divergent mixing for N >3 moment.

Higher Moments of Parton Distributions

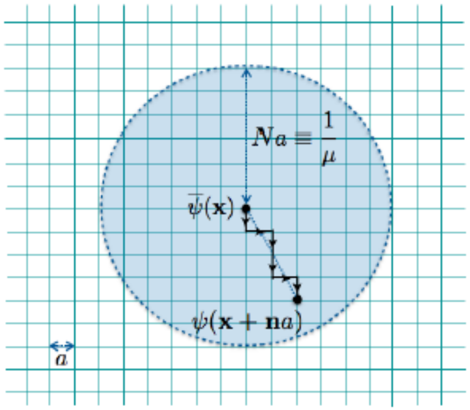
$$x(u_v(x) - d_v(x)) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$



IsoVector Distribution

Need to constrain parameters from phenomenology.

Detmold, Melnitchouk, Thomas
Eur.Phys.J.direct C3:1-15,2001



Use **improved, extended operators** to reduce power-divergent mixing. c.f. restoration of rotational symmetry for interpolating operators in spectroscopy

Davoudi and Savage, PRD86, 054505 (2012)

Quasi Distributions

- A solution, LaMET (Large Momentum Effective Theory) was proposed by X.Ji

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')}\psi(0) \mid P > \\ + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

- Quasi distributions approach light-cone distributions in limit of large P^z

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

Y-Q Ma and J-W Qiu, arXiv:1404.6860

- Matching and evolution of quasi- and light-cone distributions

Carlson, Freid, arXiv:1702.05775
Isikawa et al., arXiv:1609.02018
Monahan and Orginos, arXiv:1612.01584
Orginos, Radyushkin , et al arXiv:1706.05373 (Pseudo Distributions)
Briceno, Hansen, Monahan, arXiv:1703.06072 (Euclidean Signature)

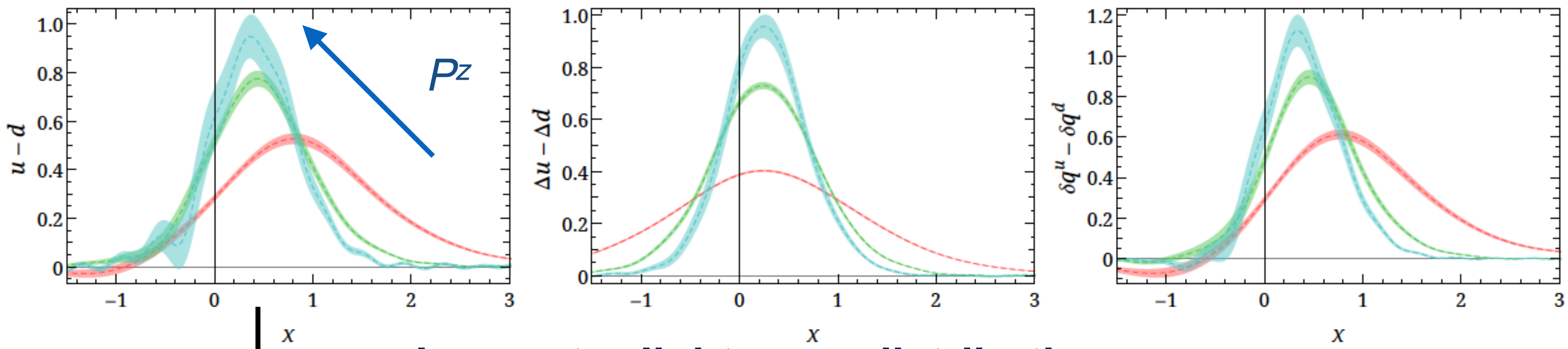
- Direct lattice calculation of hadronic tensor

K.F. Liu and S.J.Dong, PRL72, 1790 (1994); arXiv:1703.04690

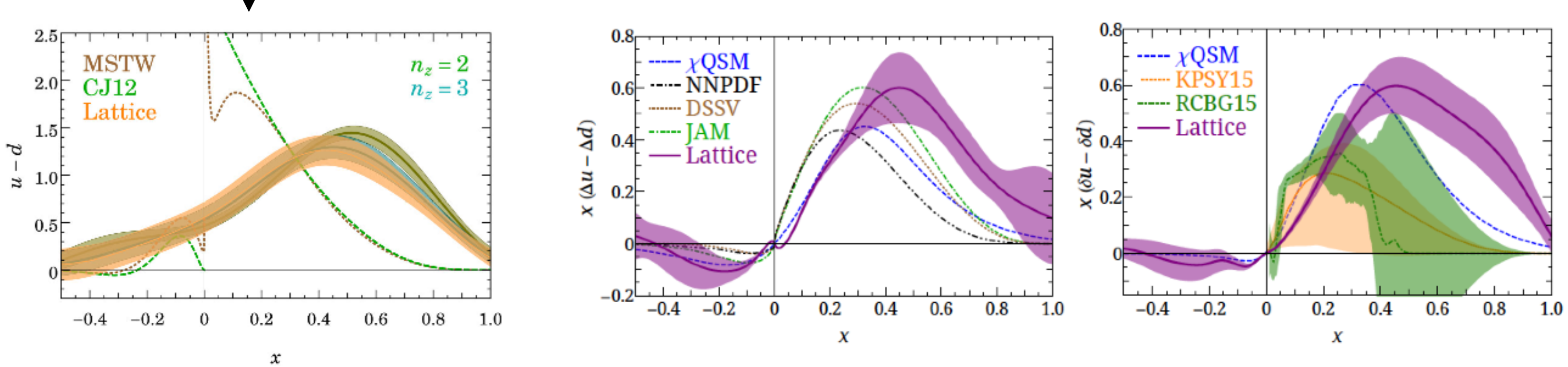
PDFs

H-W Lin, arXiv:1612.09366

Iso-vector quasi distributions



Iso-vector light-cone distributions



Alexandrou et al., arXiv:1610.03689

SUMMARY

- Lattice Calculations now have controlled uncertainties for certain key benchmark quantities, and can confront experiment.
  - Ji's sum rule
  - TMDs
  - Narrowing of hadron with increasing x
- Near Frontiers
  - sea quark and *gluonic* contributions to hadron structure.
  - Direct calculations of Bjorken-x dependence
- Capitalizing on Expt + LQCD + Phenomenology