

Insights into Strong Forces through GPDs

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Outline

- **Introduction**
hard-exclusive reactions \rightarrow GPDs
tomography, J_i sum rule, and more
- **Energy-momentum tensor**
EMT form factors & D -term
last unknown global property(!)
- **D -term**
What do we know?
theory & experiment
- **Physical interpretation**
3D densities: limitations & uses
stress tensor and stability
- **Applications**
insights in mechanical stability
from hard-exclusive reactions at JLab ...
to $c\bar{c}$ pentaquark spectroscopy at LHCb
- **Outlook**

Introduction

- tons of information \in GPDs

- generalize
form factors, PDFs

$$\int dx H^q(x, \xi, t) = F_1^q(t)$$

$$\lim_{\Delta \rightarrow 0} H^q(x, \xi, t) = f_1^q(x)$$

- explore impact parameter space
allow tomography (M. Burkardt, ...)

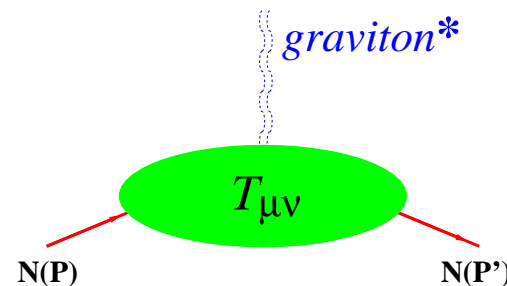
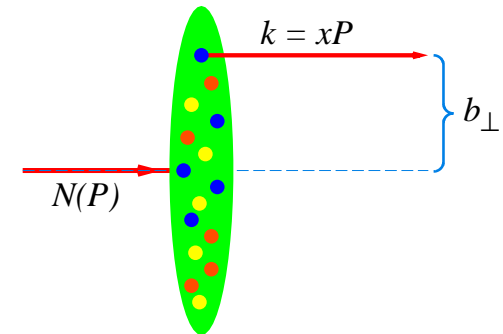
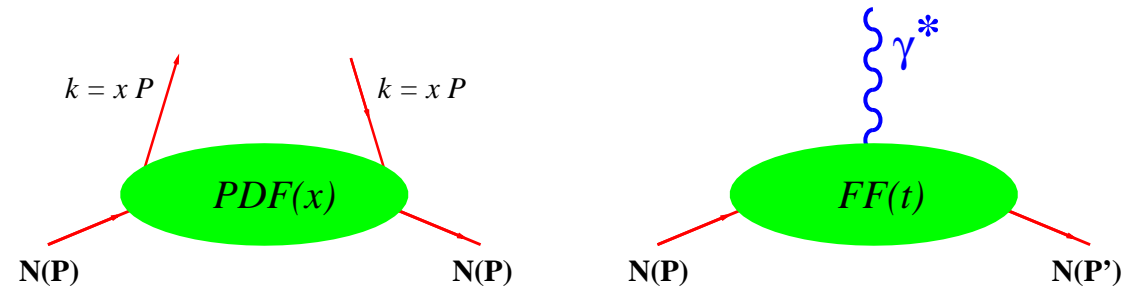
$$H^q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \left[\lim_{\xi \rightarrow 0} H^q(x, \xi, t) \right] e^{i\Delta_\perp b_\perp}$$

- allow to access (polynomiality)
gravitational form factors

$$\int dx x H^q(x, \xi, t) = A^q(t) + \xi^2 D^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - \xi^2 D^q(t)$$

- **gravity** couples to
energy momentum tensor
EMT = most fundamental quantity



nucleon EMT form factors

$$\langle p' | \hat{T}_{\mu\nu}^{q,g} | p \rangle = \bar{u}(p') \left[\begin{aligned} & \mathbf{A}^{q,g}(t) \frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} \\ & + \mathbf{B}^{q,g}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{4M_N} \\ & + \mathbf{D}^{q,g}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} \pm \bar{c}(t) g_{\mu\nu} \end{aligned} \right] u(p)$$

- $\hat{T}_{\mu\nu}^q, \hat{T}_{\mu\nu}^g$ both gauge-invariant (not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}^q + \hat{T}_{\mu\nu}^g$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow A^q(0) + A^g(0) = 1$ (100% of nucleon momentum carried by quarks + gluons)
spin $\Leftrightarrow B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)*
- property: **D-term** $\Leftrightarrow D^q(0) + D^g(0) \equiv \mathbf{D} \rightarrow$ unconstrained! **Unknown! Last global unknown!**

$$\begin{aligned} 2P &= (p' + p) \\ \Delta &= (p' - p) \\ t &= \Delta^2 \end{aligned}$$

notation: $A^q(t) + B^q(t) = 2J^q(t)$
 $D^q(t) = \frac{4}{5} d_1^q(t) = \frac{1}{4} C^q(t)$ or $C^q(t)$
 $A^q(t) = M_2^q(t)$

* also expressed as: vanishing of total gravitomagnetic moment

last global unknown: How do we learn about hadrons?

$|N\rangle =$ **strong** interaction particle. Use other forces to probe it!

em: $\partial_\mu J_{\text{em}}^\mu = 0 \quad \langle N' | J_{\text{em}}^\mu | N \rangle \longrightarrow Q, \mu, \dots$

weak: PCAC $\quad \langle N' | J_{\text{weak}}^\mu | N \rangle \longrightarrow g_A, g_p, \dots$

gravity: $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0 \quad \langle N' | T_{\text{grav}}^{\mu\nu} | N \rangle \longrightarrow M, J, D, \dots$

global properties:	Q_{prot}	=	$1.602176487(40) \times 10^{-19} \text{C}$
	μ_{prot}	=	$2.792847356(23) \mu_N$
	g_A	=	$1.2694(28)$
	g_p	=	$8.06(0.55)$
	M	=	$938.272013(23) \text{ MeV}$
	J	=	$\frac{1}{2}$
	D	=	?

and more:
 t -dependence
 parton structure, etc

$\hookrightarrow D =$ "last" global unknown
 which value does it have?
 what does it mean?

D-term in theory

pions

- free Klein-Gordon field $D = -1$
(Pagels 1965; Hudson, PS 50 years later)
- Goldstone bosons of chiral symmetry breaking $D = -1$ in soft pion limit
Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- chiral perturbation theory for Goldstone bosons
Donoghue, Leutwyler (1991)

$$D_\pi = -1 + 16a \frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2} I_\pi - \frac{m_\pi^2}{3F^2} I_\eta + \mathcal{O}(E^4)$$

$$D_K = -1 + 16a \frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2} I_\eta + \mathcal{O}(E^4)$$

$$D_\eta = -1 + 16a \frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2} I_\pi + \frac{8m_K^2}{3F^2} I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2} I_\eta + \mathcal{O}(E^4)$$

where

$$a = L_{11}(\mu) - L_{13}(\mu)$$

$$I_i = \frac{1}{48\pi^2} (\log \frac{\mu^2}{m_i^2} - 1)$$

$$i = \pi, K, \eta.$$

$$\Rightarrow D_\pi = -0.97 \pm 0.01$$

$$D_K = -0.77 \pm 0.15$$

$$D_\eta = -0.69 \pm 0.19$$

nuclei

- nuclei in liquid drop model $D = -0.2 \times A^{7/3}$ → potential for DVCS wit nuclei!
Maxim Polyakov (2002) (see below)
- nuclei in Walecka model
Guzey, Siddikov (2006)

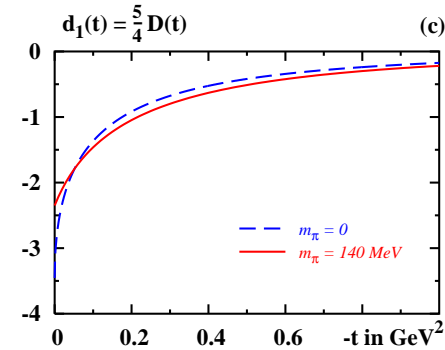
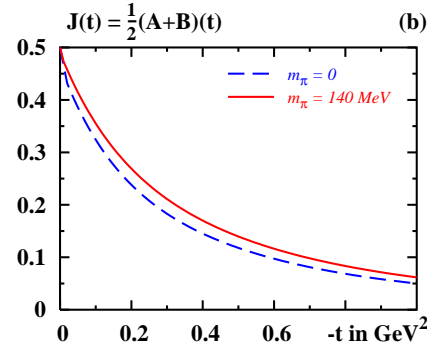
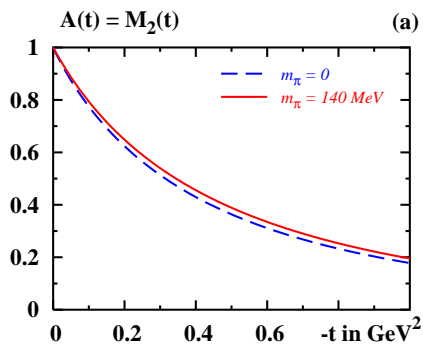
$$\begin{aligned} {}^{12}\text{C} : D &= -3 \\ {}^{16}\text{O} : D &= -58 \\ {}^{40}\text{Ca} : D &= -610 \\ {}^{90}\text{Zr} : D &= -3300 \\ {}^{208}\text{Pb} : D &= -19700 \end{aligned}$$

Q-balls

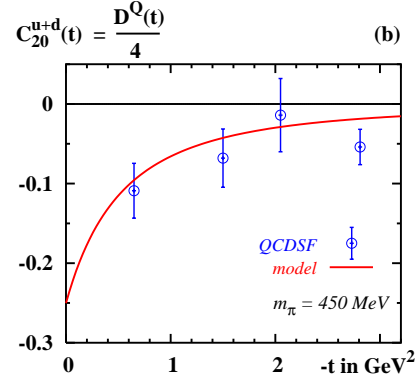
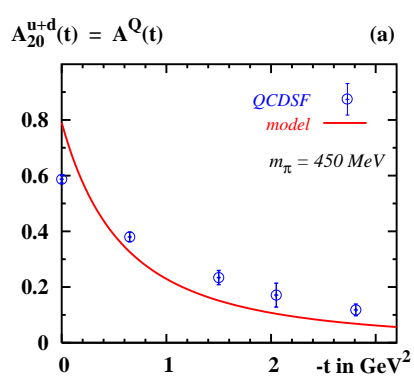
- Q-balls, non-topological solitons in strongly interacting theory: $90 \leq (-D) \leq \infty$
Mai, PS (2012)
- N^{th} excited Q-ball state (decay into ground states): $D = -\text{const } N^8$
Mai, PS (2012)
- Q-cloud limit, most extreme instability we could find: $D = -\text{const}/\varepsilon^2$ in the limit $\varepsilon \rightarrow 0$
Cantara, Mai, PS (2016)
- Q-cloud excitations, even more extreme instability: $D < 0$ divergent and even more negative
Bergabo, Cantara, PS (2017)

nucleon

- bag model (always good starting point!) $D < 0$ due to bag pressure!
Ji, Melnitchouk, Song (1997); Neubelt, Sampino, PS (2017+)
- chiral quark soliton model
Petrov et al 1998, Goeke et al, PRD75 (2007) 094021



- lattice D^Q : QCDSF Collaboration, Gockeler et al, PRL92 (2004) 042002 & hep-ph/0312104



- χ PT cannot predict D -term, but $d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5k g_A^2 M_N}{64 \pi f_\pi^2} m_\pi + \dots$, $\overset{\circ}{d}'_1(0) = -\frac{k g_A^2 M_N}{32 \pi f_\pi^2 m_\pi} + \dots$
 $k = 1$ for finite N_c , and $k = 3$ for $N_c \rightarrow \infty$ Belitsky, Ji (2002), Diehl et al (2006)
- dispersion relations Barbara Pasquini, Maxim Polyakov, Marc Vanderhaeghen (2014)

D-term in experiment

- HERMES Ellinghaus [HERMES Collaboration], NPA711, 171 (2002); PRD 75, 011103 (2007)
- JLab talk by V. Burkert at SPIN 2016 in Urbana-Champaign, September 25-30, 2016; PRL115 (2015); arXiv:1707.03361; PR12-16-010; waiting!
- COMPASS PoS DIS 2016, 235 (2016).

D-term in GPD parameterizations

- non-zero value favored, tendency negative, still more work
Kumericki & Müller, Kroll & Goloskokov, ...

interpretation in terms of 3D-densities

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$ (!)

- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r} \rho_E(\vec{r}) e^{i\vec{\Delta}\vec{r}} \rightarrow$ charge distribution
Sachs, PR126 (1962) 2256

$$\hookrightarrow Q = \int d^3\vec{r} \rho_E(\vec{r})$$

- static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle \rightarrow$ mechanical properties of nucleon
M.V.Polyakov, PLB 555 (2003) 57

$$\hookrightarrow M_N = \int d^3\vec{r} T_{00}(\vec{r}), \text{ etc}$$

limitations (\exists in contrast to 2D Fourier transforms \leftrightarrow tomography)

well known since earliest days (Sachs, 1962)

comprehensive studies, e.g. by

- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2;
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect),
- G. Miller, PRC80 (2009) 045210 (toy model, dramatic effect)

No doubt: mathematical operation well-defined

The question: is the concept justified?

Answer: **yes**, modulo **corrections!**

How large are the corrections?

illustration in simplest framework

$\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2$ free neutral elementary point-like scalar particle
 (“Higgs” modulo standard model corrections...)

evaluate EMT:

$$\langle \vec{p}' | \hat{T}^{\mu\nu}(x) | \vec{p} \rangle = e^{i(p'-p)x} \frac{1}{2} \left\{ P^\mu P^\nu A(t) + \left(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) D(t) \right\}, \quad A(t) = -D(t) = 1$$

compute **energy density**

$$\mathbf{T}_{00}(\vec{r}) \stackrel{\text{gen.}}{=} m^2 \int \frac{d^3 \Delta}{E(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \left[A(t) - \frac{t}{4m^2} (A(t) + D(t)) \right] \quad \text{in Breit frame } E = E' = \sqrt{m^2 + (\vec{\Delta}/2)^2}$$

$$\stackrel{\text{here}}{=} \frac{m}{\sqrt{1 - \vec{\nabla}^2/(4m^2)}} \delta^{(3)}(\vec{r})$$

reproduces correctly: $\int d^3 r \mathbf{T}_{00}(\vec{r}) \stackrel{!}{=} m$

but yields also $\langle r_E^2 \rangle = \frac{1}{m} \int d^3 r r^2 \mathbf{T}_{00}(\vec{r}) \stackrel{!??}{=} \frac{3}{4m^2}$

corrections generate **mean square radius $\neq 0$** for point-like particle???

- take **heavy mass limit** to recover “correct” intuitive 3D description

$$T_{00}(\vec{r}) \longrightarrow m \delta^{(3)}(\vec{r}) \quad \text{for } m \rightarrow \text{large} \dots \quad \text{large with respect to what?}$$

- let’s give particle a **finite size R** (i.e. “**smear out**” δ -function)

$$T_{00}(\vec{r})_{\text{true}} \stackrel{\text{e.g.}}{=} m \frac{e^{-r^2/R^2}}{\pi^{3/2} R^3} \quad \text{“true energy density”}$$

$$\rightarrow \langle r_E^2 \rangle = \langle r_E^2 \rangle_{\text{true}} \left(1 + \delta_{\text{rel}} \right) \simeq \langle r_E^2 \rangle_{\text{true}} \quad \text{if } \delta_{\text{rel}} = \frac{1}{2m^2 R^2} \ll 1 \quad \left(\text{it is } \langle r_E^2 \rangle_{\text{true}} = \frac{3}{2} R^2 \text{ if Gaussian} \right)$$

- **for nuclei** ($M_A \sim M_N A$, $R_A \sim R_0 A^{1/3}$) $\rightarrow \delta_{\text{rel}} \sim 0.16 A^{-8/3} \lesssim 10^{-4}$ (for ${}^4\text{He}$, less for heavier)

- **nucleon** ($M_N \sim 940 \text{ MeV}$, $R_N \sim 1 \text{ fm}$) $\rightarrow \delta_{\text{rel}} \sim 3\% \stackrel{!}{\ll} 1$ small enough!?

- **large- N_c nucleon** ($M_N \sim N_c$, $R_N \sim N_c^0$) $\rightarrow \delta_{\text{rel}} \sim \frac{1}{N_c^2} \stackrel{!!}{\ll} 1$ conceptually small enough!

remark:

$\frac{1}{N_c}$ *only small parameter of QCD at all energies* (S. Coleman in “Aspects of Symmetry”)

(... besides large- N : powerful theoretical method, much more general than QCD)

- important distinction:

2D densities = partonic probability densities (unitarity)

must be exact! → M. Burkardt (2000)

are exact ✓

work for any particle, including pion

vs

3D densities = mechanical response functions

correlation functions subject to corrections → M. Polyakov (2002)

okay, if corrections acceptably small ✓

work for nucleon or heavier

- besides: no 2D interpretations exist for stress tensor and pressure
inherently 3D concepts, have to pay the prize for (and deal with corrections)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle$

all formulae correct, interpretation in terms of 3D-densities has limitations (see above)

$$\int d^3r T_{00}(\vec{r}) = M_N \quad \text{known}$$

$$\int d^3r \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}) = \frac{1}{2} \quad \text{known}$$

$$-\frac{2}{5} M_N \int d^3r \left(r^i r^j - \frac{r^2}{3} \delta^{ij} \right) T_{ij}(\vec{r}) \equiv D \quad \text{new!}$$

with: $T_{ij}(\vec{r}) = \mathbf{s}(\mathbf{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + \mathbf{p}(\mathbf{r}) \delta_{ij} \quad \text{stress tensor}$

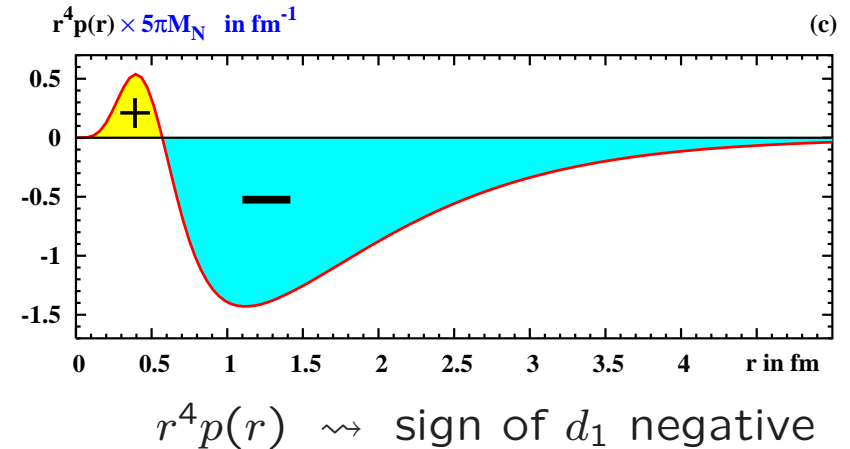
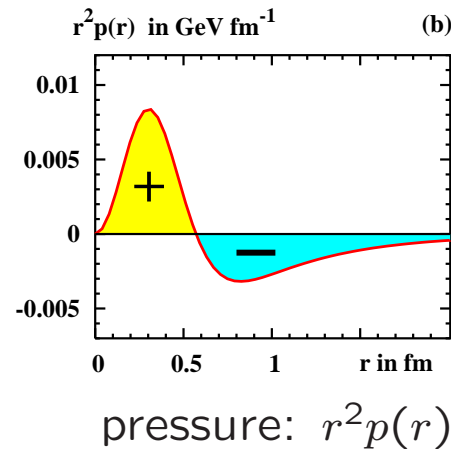
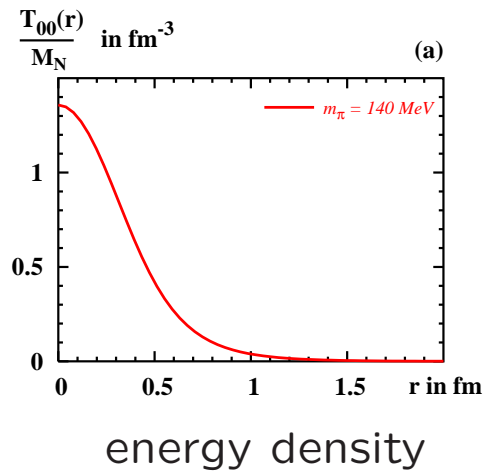
$\left. \begin{array}{l} \mathbf{s}(\mathbf{r}) \text{ related to distribution of } \textit{shear forces} \\ \mathbf{p}(\mathbf{r}) \text{ distribution of } \textit{pressure} \text{ inside hadron} \end{array} \right\} \longrightarrow \text{“mechanical properties”}$

- **relation to stability:** EMT conservation $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

\hookrightarrow necessary condition for stability $\int_0^\infty dr r^2 p(r) = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15} M_N \int_0^\infty dr r^4 s(r) = 4\pi M_N \int_0^\infty dr r^4 p(r) \quad \hookrightarrow \text{shows how internal forces balance}$$

- lessons from model



$$T_{00}(0) = 1.70 \text{ GeV}/\text{fm}^3 \approx 3 \times 10^{15} \rho(\text{H}_2\text{O}) \approx 13 \times (\text{nuclear density})$$

$$p(0) = 0.23 \text{ GeV}/\text{fm}^3 \approx 4 \times 10^{34} \text{ N}/\text{m}^2 \gtrsim 100 \times (\text{pressure in center of neutron star})$$

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

... how does it look like in QCD? We do not know.

Application I: investigating forces

question: how do strong forces balance inside the nucleon?

- answer in **model**: strong cancellation of **repulsive forces** due to quark core, and **attractive forces** from pion cloud
cf. $V_{\text{conf}}(r) \approx kr$ with $k \approx 1 \text{ GeV/fm}$
- answer in **QCD**: we do not know nice pictures, attractive insights

be aware:

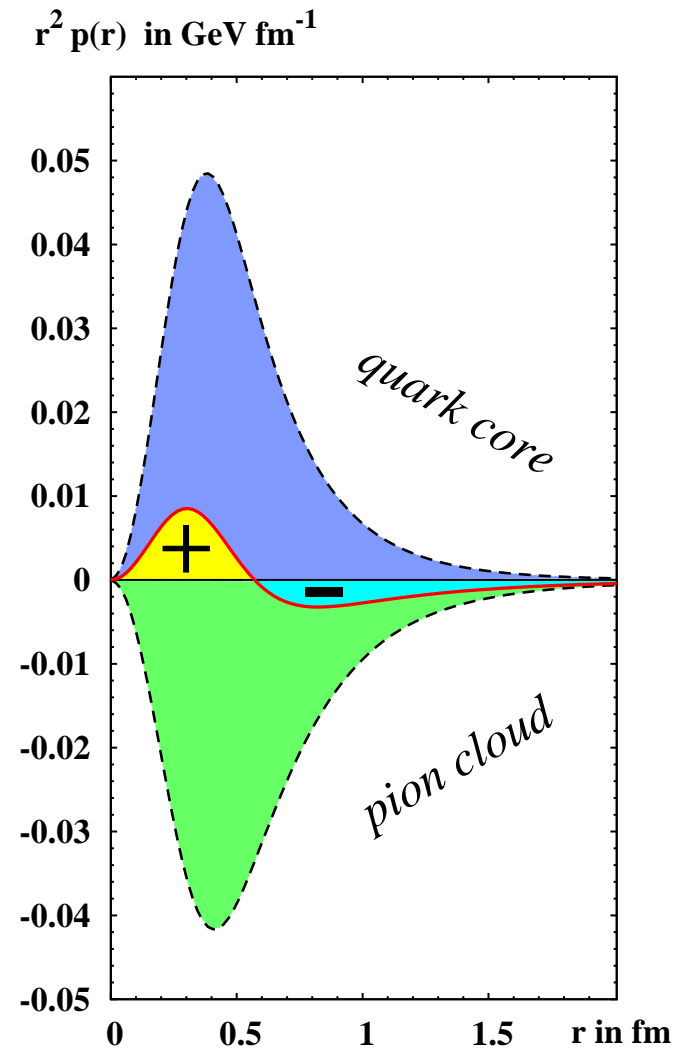
proton $\tau_{\text{prot}} > 2.1 \times 10^{29} \text{ years}$

neutron $\tau_{\text{neut}} = 14 \text{ min } 40 \text{ sec}$

Δ -resonance $\tau_{\Delta} \sim 10^{-23} \text{ sec}$

→ necessary condition!

- support for GPD program
- practical application!



in chiral quark soliton model

chiral symmetry breaking ✓

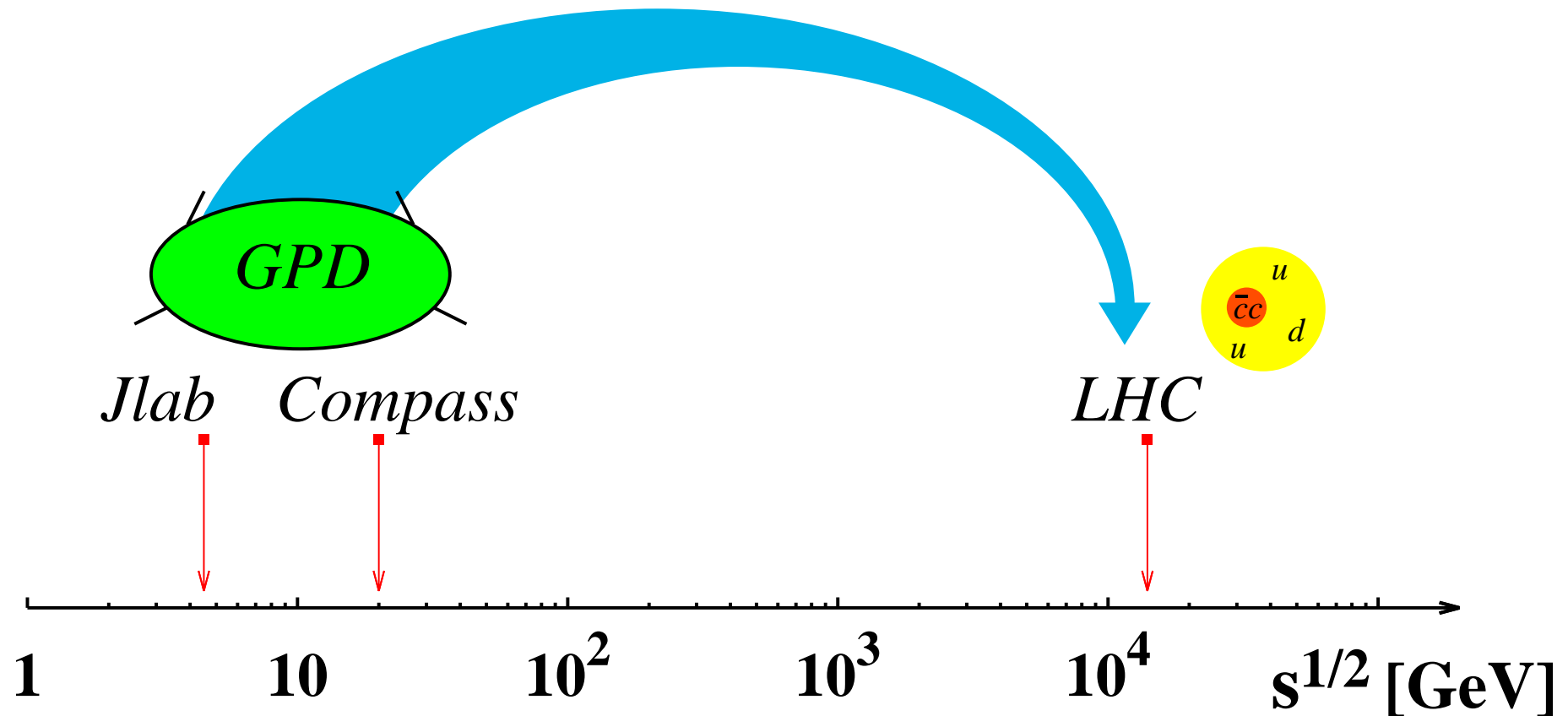
realization of QCD in large- N_c ✓

built on instanton vacuum calculus ✓

not bad, but after all a model ...

Goeke et al, PRD75 (2007)

Application II: from hard-exclusive reactions at JLab, COMPASS ...
... to spectroscopy of $\bar{c}c$ -pentaquarks at LHCb



not usual hadrons, not just any exotic hadron, $\bar{c}c$ -baryon bound states \rightarrow rich potential!

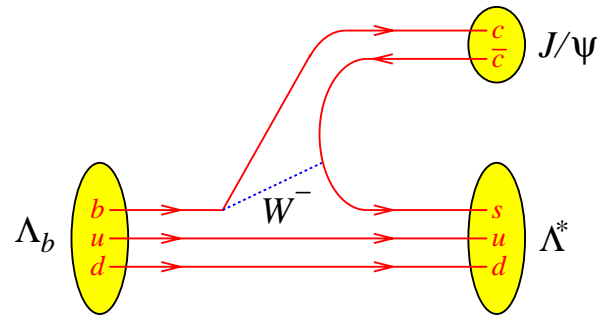
- hidden-charm pentaquarks in Λ_b^0 decays at LHCb

Aaij et al. PRL 115, 072001 (2015)

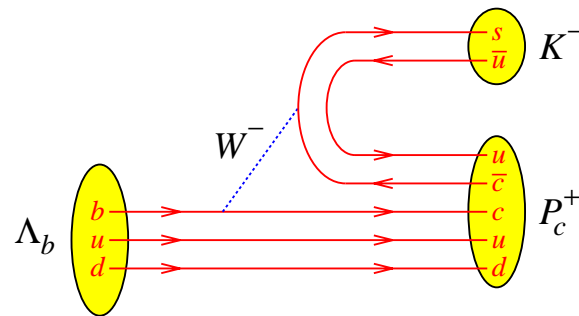
$\Lambda_b^0 \longrightarrow J/\Psi p K^-$ seen

Λ_b^0 $m = 5.6 \text{ GeV}, \tau = 1.5 \text{ ps}$
 J/Ψ $m = 3.1 \text{ GeV}, \Gamma = 93 \text{ keV}, \Gamma_{\mu^+\mu^-} = 6 \%$
 Λ^* $m = 1.4 \text{ GeV or more}, \Lambda^* \rightarrow K^- p \text{ in } 10^{-23} \text{ s}$

$\longrightarrow J/\Psi \Lambda^*$



$\longrightarrow J/\Psi P_c^+$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1) \%$	$J/\psi p$	$\frac{3}{2}^-$ or $\frac{5}{2}^+$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2) \%$	$J/\psi p$	$\frac{3}{2}^{\pm}$ or $\frac{3}{2}^-$

appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- theoretical approach**

$R_{J/\psi} \ll R_N \Rightarrow$ non-relativistic multipole expansion [Gottfried, PRL 40 \(1978\) 598](#)
 baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$$V_{\text{eff}} = -\frac{1}{2} \alpha \vec{E}^2 \quad \text{Voloshin, Yad. Fiz. 36, 247 (1982)}$$

- chromoelectric polarizability**

$$\begin{aligned} \alpha(1S) &\approx 0.2 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S) &\approx 12 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S \rightarrow 1S) &\approx \begin{cases} -0.6 \text{ GeV}^{-3} \text{ (pert),} \\ \pm 2 \text{ GeV}^{-3} \text{ (pheno),} \end{cases} \end{aligned}$$

in heavy quark mass limit & large- N_c limit
 \rightsquigarrow “perturbative result” [Peskin, NPB 156 \(1979\) 365](#)

value for $2S \rightarrow 1S$ transition from
 phenomenological analysis of $\psi' \rightarrow J/\psi \pi \pi$ data
[Voloshin, Prog. Part. Nucl. Phys. 61 \(2008\) 455](#)

- chromoelectric field strength:**

$$\vec{E}^2 = g^2 \left(\frac{8\pi^2}{bg^2} T^\mu{}_\mu + T_{00}^G \right)$$

$b = \frac{11}{3} N_c - \frac{2}{3} N_F$ leading coeff. of β -function
 g = strong coupling at low (nucleon) scale $\lesssim 1$ GeV
 g_s = strong coupling at scale of heavy quark ($g_s \neq g$)
 $T_{00}^G = \xi T_{00}$ with ξ = fractional contributions of gluon to M_N
 $T^\mu{}_\mu = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r} \frac{r^j}{r} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$

- universal effective potential**

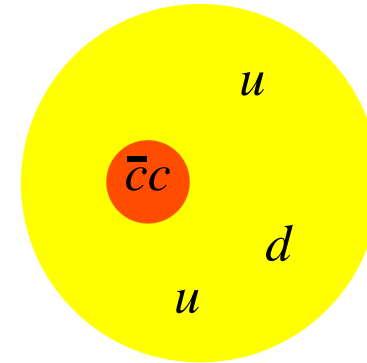
$$V_{\text{eff}} = -\frac{1}{2} \alpha \frac{8\pi^2}{b} \frac{g^2}{g_s^2} \left[\nu T_{00}(r) + 3p(r) \right], \quad \nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}$$

$\nu \approx 1.5$ estimate by [Eides et al, op. cit.](#)
[Novikov & Shifman, Z.Phys.C8, 43 \(1981\);](#)
[X. D. Ji, Phys. Rev. Lett. 74, 1071 \(1995\)](#)

- **in future GPDs can help:** GPDs \Rightarrow EMT form factors \Rightarrow EMT densities $\Rightarrow V_{\text{eff}}$
- **currently:** use chiral quark soliton model (Eides et al, 2015); Skyrme (Perevalova et al 2016)

- **solve quarkonium-nucleon bound state**

$$\text{solve } \left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r) \right) \psi = E_{\text{bind}} \psi$$



- **results:**

nucleon and J/ψ form no bound state

nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV in $L = 0$ channel, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ if $\alpha(2S) \approx 17 \text{ GeV}^{-3}$ (consistent with guideline from pert. calc.)

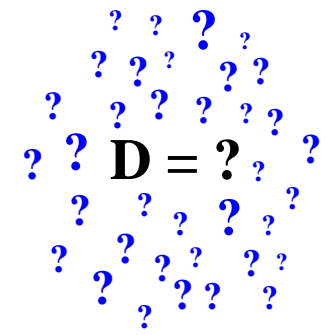
- **new predictions:**

also Δ and $\psi(2S)$ form a bound state! (Perevalova et al 2016)

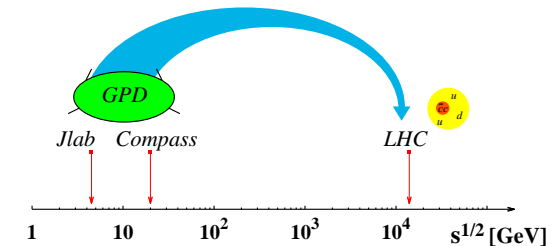
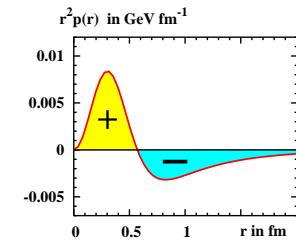
SU(3) octet baryons (N, Λ, Σ, Ξ) form bound states (Eides et al, arXiv:1709.09523)

- to be tested in experiment (LHCb; JLab, Meiziani et al arXiv:1609.00676)

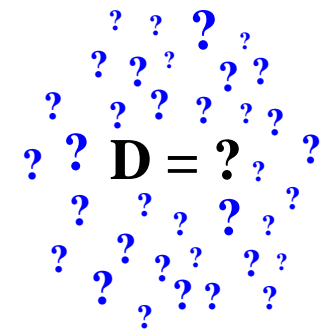
Summary & Outlook



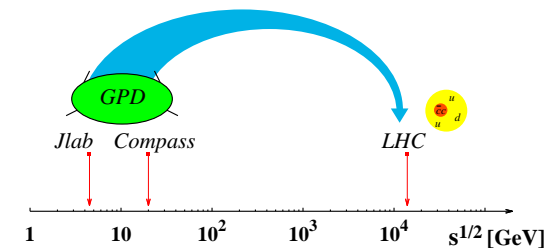
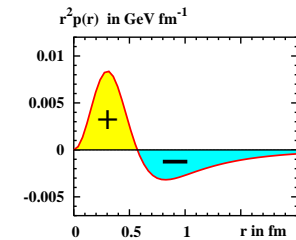
- **GPDs** → lots of new information!
- ⇔ form factors of **energy momentum tensor**
mass decomposition, spin decomposition, and *D*-term!
- **D-term**: last unknown global property
interesting for its own sake
- 3D interpretation (applicable for $N+$ heavier): **forces**
not how partons distributed, but why they stay together!
- development: **quarkonium-baryon** interaction V_{eff}
 $P_c^+(4450)$ at LHCb, new predictions, to be tested



Summary & Outlook



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Thank you!

Supporting slides

Energy-momentum tensor (EMT)

- (nearly) every introductory QFT text book* starts with discussing the EMT in first chapter
- why? If we can solve a theory (e.g. free theory):
 - construct $T_{\mu\nu}$ + generators of Poincaré group
 - learn what is the $\underbrace{\text{mass}}_{T_{00}}$, $\underbrace{\text{spin}}_{\varepsilon^{ijk}x_j T_{0k}}$, $\underbrace{D\text{-term}}_{T_{ij}}$ of particle
- even if theory cannot be solved, studies of EMT insightful
 - prominent example: **Ji sum rule** PRL 78 (1997) 610
$$\int dx x \left(H^q(x, \xi, t) + E^q(x, \xi, t) \right) = A^q(t) + B^q(t) \xrightarrow{t \rightarrow 0} 2J^q(0)$$
 - mass-decomp. (recent workshops) + more!

* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*

$\hat{T}_\mu^\mu \equiv \frac{\beta}{2g} F^{\mu\nu} F_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q$ Adler, Collins, Duncan, PRD15 (1977) 1712;

Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

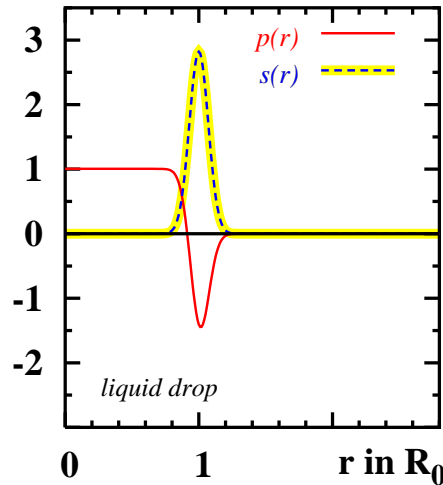
particle	J^π	mass [GeV]	size [fm]	δ_{rel}
pion	0^-	0.14	0.67	2.2
kaon	0^-	0.49	0.56	2.5×10^{-1}
η -meson	0^-	0.55	0.68	1.4×10^{-1}
η_c -meson	0^-	2.98	0.26	3.8×10^{-2}
proton	$\frac{1}{2}^+$	0.94	0.89	2.8×10^{-2}
deuteron	1^+	1.88	2.14	1.2×10^{-3}
^4He	0^+	3.73	1.68	5.0×10^{-4}
^{12}C	0^+	11.2	2.47	2.6×10^{-5}
^{20}Ne	0^+	18.6	3.01	6.2×10^{-6}
^{32}S	0^+	29.8	3.26	2.1×10^{-6}
^{56}Fe	0^+	52.1	3.74	5.1×10^{-7}
^{132}Xe	0^+	123	4.79	5.6×10^{-8}
^{208}Pb	0^+	194	5.50	1.7×10^{-8}
^{244}Pu	0^+	227	5.89	1.1×10^{-8}

from:
Hudson, PS
“ D -terms of
spin-0 systems”
new preprint (2017)

- intuition on shear forces and pressure

liquid drop

$p(r)$ & $s(r)$ in units of p_0



radius R_0

inside pressure p_0

surface tension $\gamma = \frac{1}{2}p_0R_0$

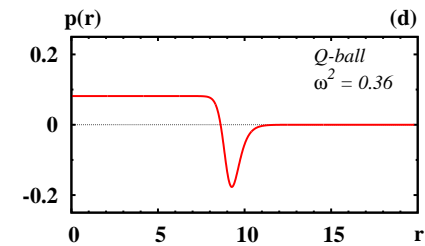
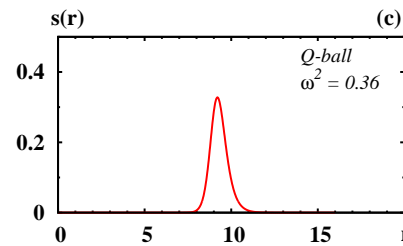
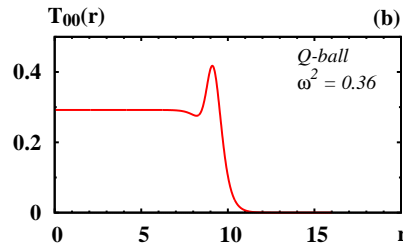
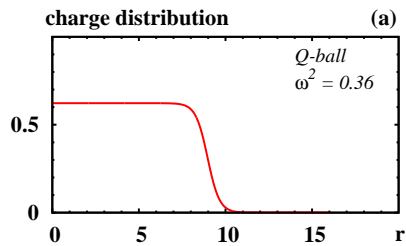
$$s(r) = \gamma \delta(r - R_0)$$

$$p(r) = p_0 \Theta(R_0 - r) - \frac{1}{3}p_0R_0 \delta(r - R_0)$$

application: liquid drop model of nucleus

(M.V.Polyakov, PLB 555 (2003) 57)

realized in field theoretical Q -ball system



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V \text{ with U(1) global symm.}, V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3, \Phi(t, \vec{r}) = e^{i\omega t} \phi(r)$$

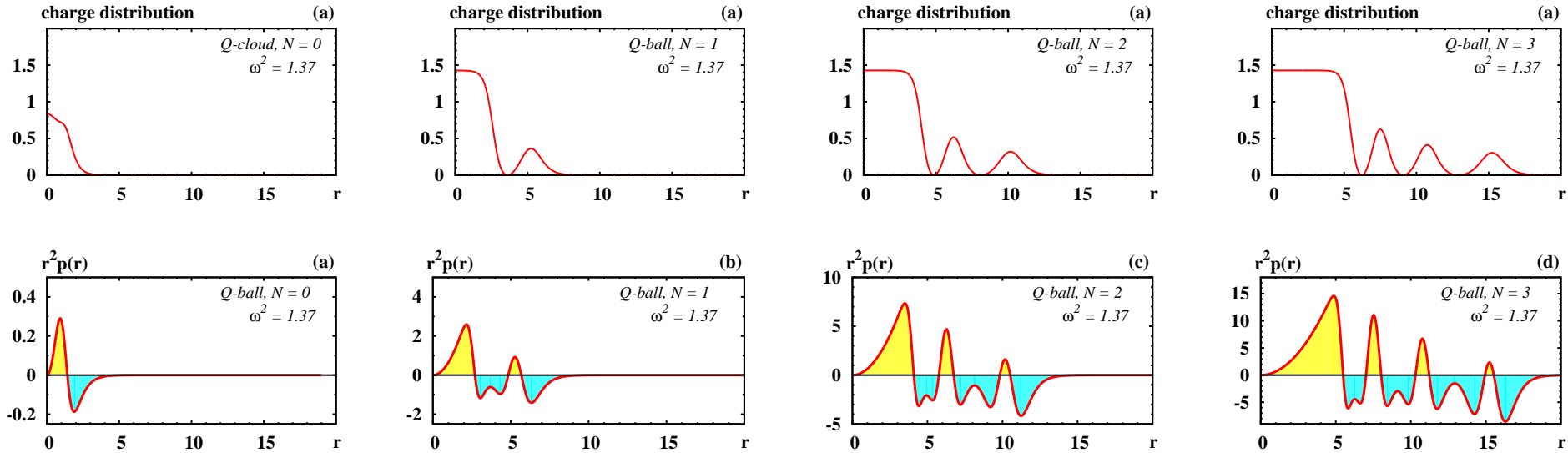
S. R. Coleman, NPB262 (1985) 263, 269 (1986) 744E; M. Mai, PS PRD86 (2012) 076001

to satisfy $\int_0^\infty dr r^2 p(r) = 0 \rightarrow p(r)$ must have a zero! Could it have more zeros?

N^{th} radial excitations of Q -balls

$N = 0$ ground state,
 $N = 1$ first excited state, etc

Mai, PS PRD86 (2012) 096002
 charge density exhibits N shells
 $p(r)$ exhibits $(2N + 1)$ zeros



$N > 0$ radial excitations all unstable

→ decay to ground state Q -balls of smaller total energy and same total charge

nevertheless $\int_0^\infty dr r^2 p(r) = 0$ always valid

→ necessary (not sufficient) condition \Leftrightarrow (local extremum of action, not global)

D -term always negative!

→ is it a theorem? → for Q -balls formulated Mai, PS PRD86 (2012) 076001

→ rigorous proof that $d_1 < 0$ for hadrons in QCD and other particles still awaiting

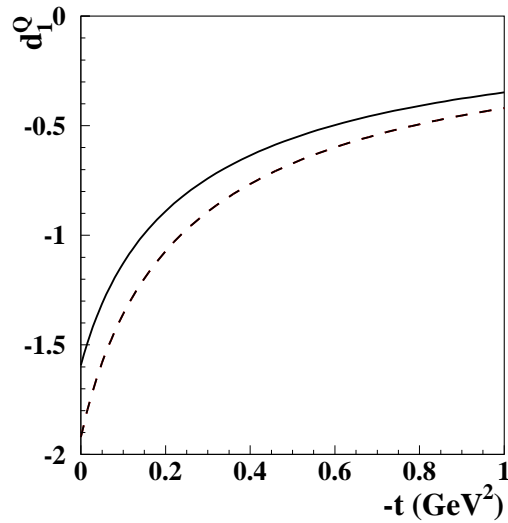
so far all D -terms negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q -balls, Q -clouds*

(Q -cloud: most extreme instability(!); parametric limit where Q -balls dissociate in free quanta; still $D < 0$)

D -term of nucleon from dispersion relations

- unsubtracted t -channel dispersion relations (sensitive to pion PDFs)

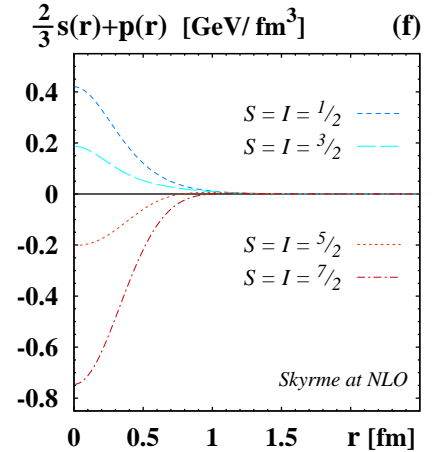
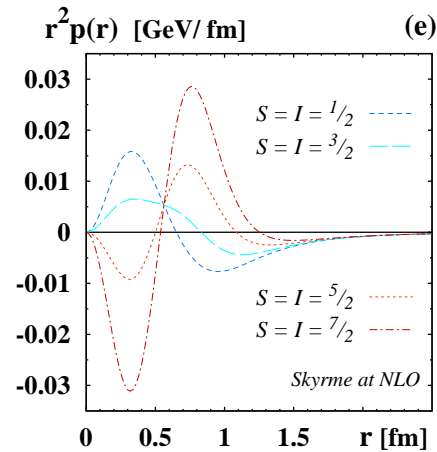
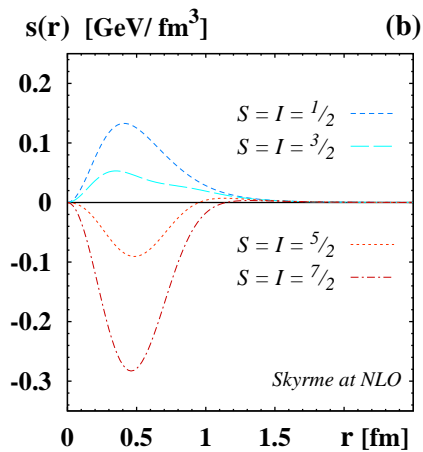
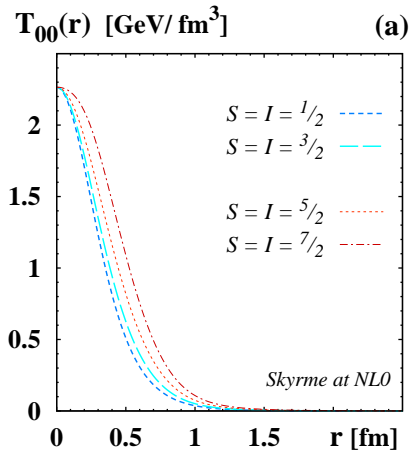
Barbara Pasquini, Maxim Polyakov, Marc Vanderhaeghen (2014)



Application III: nucleon, Δ , large- N_c artifacts

Witten 1979

in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \dots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
 Δ much more diffuse

$\int_0^\infty dr r^2 p(r) = 0$
 stability needs more:
 $p(r) > 0$ in center,
 negative outside
 okay for nucleon, Δ
 \implies implies $D < 0$

mechanical stability
 $T^{ij} da^i \geq 0$
 $\Leftrightarrow \frac{2}{3}s(r) + p(r) \geq 0$
 artifacts do not satisfy!
 \Rightarrow **have positive D-term!!**
That's why they do not exist!
 EMT: dynamical understanding
 Perevalova et al (2016)

\Rightarrow particles with positive D unphysical!!!

- **decay of $P_c(4450)$**

$M_{\psi(2S)} + M_N > 4450 \text{ MeV}$ so no decay to N and $\Psi(2S)$ possible

$\Gamma_{\psi(2S)} \sim 300 \text{ keV}$, “wait” for transition $(2S) \rightarrow (1S)$ governed by same V_{eff} but with small $\alpha(2S \rightarrow 1S)$ transition polarizability

transition “takes time,” after “completed,”

prompt decay to $J/\psi + \text{nucleon}$ (observed final states)

estimated width is tens of MeV \rightarrow compatible with data!

- $P_c^+(4380)?$

broader, no charmonium-nucleon bound-state, different mechanism