Quarkonium in Thermal Environment

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JLab related

• Frascati-Argonne-Swansea-Trinity-Sejong-Utah-Maynooth (FASTSUM):

☐ M.-P. Lombardo, D.K. Sinclair, G. Aarts, C. Allton, S. Ryan, SK, B. Oktay, J.-I. Skullerud + many others

 \Box anisotropic lattices, T = 0 HadSpec parameters

□ PRL106(2011)061602, JHEP111(2011)103, JHEP1303(2013)084, JHEP1312(2013)064, JHEP1407(2014)097

• SK (Sejong), P. Petreczky (BNL), A. Rothkopf (Heidelberg)

□ JLab clusters via USQCD

PRD91(2015)054511

Outline







modification in the spectral behavior of

quarkonium at $T \neq 0$



• sequential suppression of $\Upsilon(1S, 2S, 3S)$: CMS, PRL109 (2012) 222301

- Investigation of QGP properties requires comparison between the baseline (p, p) and relativistic heavy ion collisions
- heavy quark system is one of better understood hadronic systems
- heavy quark mass scale(*M*) is large and the strong coupling at the mass scale is "small"

 \rightarrow separation of bound state dynamics from short distance perturbative dynamics

•effective field theory descriptions : NRQCD (pNRQCD), HQET (cf. G.T.Bodwin, E. Braaten, G.P. Legage, PRD51 (1995) 1125, N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys 77 (2005) 1423, N. Isgur and M. Wise, PLB 237 (1990) 527)

• NRQCD is an effective field theory: separation of perturbative UV physics (> M_b) and non-perturbative IR physics

• inclusive decay rates = partonic decay rate × the probability for heavy quark to meet anti-heavy quark (cf. E.Braaten, G.T.Bodwin, G.P.Lepage, PRD51 (1995) 1125)

 \bullet allows accurrate measurements (e.g., lattice calculation of $\alpha_{\mathcal{S}}$ using quarkonium spectrum)

• long distance matrix elements can be calculated by lattice method (e.g, G.T.Bodwin, D.K.Sinclair, SK, PRL77 (1996) 2376)

• T. Matsui and H. Satz, PLB178 (1986) 416.



• T. Matsui and H. Satz, PLB178 (1986) 416.



F. Karsch, M.T. Mehr, and H. Satz, Z.Phys.C37 (1988) 617





cf. A. Mocsy, 0811.0337

• qualitatively, melting of quarkonium at $T \neq 0$ can be understood in terms of screening potential and imaginary part

$$V(r) = -\alpha_s C_F \left[m_D + \frac{\exp(-m_D r)}{r} \right] - i\alpha_s C_F T \phi(m_D r)$$
(1)

(cf. M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP0703(2007)054)

however quantitative study requires first principle calculation
 → lattice gauge theory method

- use lattice version of NRQCD for quarkonium at $T \neq 0$
- to keep NRQCD remain valid as an effective field theory, T << Mb

• for the study of in-medium bottomonium, study bottomonium correlator using lattice NRQCD

$$G(\tau) = \sum_{\vec{x}} \langle \phi^{\dagger}(\vec{x},\tau;\vec{0},0)\phi(\vec{x},\tau;\vec{0},0)\rangle$$
(1)

$$G(\tau) = \sum_{n} e^{-E_{n}\tau} |\langle 0|\phi(0)|n\rangle|^{2}$$
(1)

• if the states are well defined stationary states (T = 0),

$$ightarrow G(\tau) \sim a_0 e^{-E_0 \tau} + a_1 e^{-E_1 \tau} + a_2 e^{-E_2 \tau} + \cdots$$
 (2)

usual χ^2 fitting is sufficient

• for in-medium bottomonium, the states are no longer narrow

 \rightarrow spectral information is needed unless the functional form at $\mathcal{T}\neq 0$ is known

spectral representation

$$G_{\Lambda}(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau, \vec{x}) \Lambda \psi(\tau, \vec{x}) \overline{\psi}(0, \vec{0}) \Lambda \psi(0, \vec{0}) \rangle$$
(1)

$$= \int \frac{d^3 \rho}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \kappa(\tau, \omega) \rho_{\Lambda}(\omega, \vec{\rho})$$
(2)

and

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.$$
(3)

• the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

- numerically ill-posed problem
- Maximum Entropy Method is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)

$$G_{\Lambda}(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau, \vec{x}) \Lambda \psi(\tau, \vec{x}) \overline{\psi}(0, \vec{0}) \Lambda \psi(0, \vec{0}) \rangle$$
(1)
$$= \int \frac{d^3 p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\Lambda}(\omega, \vec{p})$$
(2)

and

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.$$
(3)

- known to have problems (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)
- both the kernel($K(\tau, \omega)$) and the spectral density($\rho_{\Gamma}(\omega, \vec{p})$) depend on temperature

• In NRQCD, with $\omega=2\textit{M}+\omega'$ and $\textit{T}/\textit{M}<<1,\,\textit{K}(\tau,\omega)\rightarrow\textit{e}^{-\omega\tau}$

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')$$
(1)

- inverse Laplace transform problem
- Maximum Entropy Method (G. Aarts et al, JHEP1111 (2011) 103)
- new improved Bayesian method (Burnier-Rothkopf, PRL111 (2013) 182003)

Bayesian methods

• given $\textit{G}(\tau)$ which is calculated on lattice, what is the spectral function, $\rho(\omega)$?

· Bayes theorm

$$P[X|Y] = P[Y|X]P[X]/P[Y]$$

• in other words

 $\textit{P}[\rho|\textit{D},\textit{H}] \propto \textit{P}[\textit{D}|\rho,\textit{H}]\textit{P}[\rho|\textit{H}]$

• systematic inclusion of prior knowledge (H)

$$P[D|\rho,H] = e^{-L}, \ L = \frac{1}{2}\sum_{i}(D_i - D_i^{\rho})^2/\sigma_i^2$$

and

$$P[\rho|H] = e^{-S}, S = S[\rho(\omega), m(\omega)]$$

where *S* is the prior and $m(\omega)$ is default model

Bayesian methods

• Shannon-Jaynes entropy for *S* (cf. Asakwa, Hatsuda, Nakahara, Prog. Part.Nucl.Phys. 45 (2001) 459)

$$S_{SJ} = \alpha \int d\omega \left(
ho - m -
ho \log(rac{
ho}{m})
ight)$$

• new prior (cf. Y.Burnier, A. Rothkopf, PRL111 (2013) 182003)

$$S_{BR} = \alpha \int d\omega \, \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right)$$

Using MEM

• FASTSUM, JHEP1407 (2014) 097 : S-wave



Using MEM

• FASTSUM, JHEP1407 (2014) 097 : P-wave



Different Bayesian Method

• S.K. A. Rothkopf, P. Petreczky, PRD 91 (2015) 054511



Different Bayesian Method

• S.K. A. Rothkopf, P. Petreczky, PRD 91 (2015) 054511



Current efforts

• FASTSUM runs simulations which increase N_{τ} while fixing all the other parameters



ω [GeV]

Current efforts

• KPR experiments with modifying Bayesian regulator

