

# Transverse densities of nucleons and resonances

C. Weiss (JLab), Exploring hadrons with EM probes, JLab, 02-Nov-17  
based on work with J.M. Alarcon, A. Hiller Blin, C. Granados



**Objective:** Explain/predict structure of nucleon and baryon resonances using model-independent methods of strong interaction physics

Chiral effective field theory

large-distance dynamics, controlled accuracy, predictive

Dispersion theory

analyticity, global properties, spectrum  $\leftrightarrow$  structure

Large- $N_c$  QCD

parametric expansion,  $N \leftrightarrow \Delta$ , connection with QCD

## Structures

EM form factors and transverse densities  $N \rightarrow N$ ,  $N \rightarrow N^*$ ,  $N^* \rightarrow N^*$   $\leftarrow$

Form factors of scalar, axial, twist-2 operators (GPDs)

$\pi NN^*$  vertices,  $NN \rightarrow NN^*$  amplitudes

Granados, Weiss, JHEP 1401, 092 (2014); PRC 92, 025206 (2015); JHEP 1507, 170 (2015); JHEP 1606, 075 (2016).

Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 964, 18 (2017)

Alarcon, Weiss, arXiv:1707.07682; arXiv:1710.06430.

- Form factors and transverse densities
- Peripheral transverse densities  $b = \mathcal{O}(M_\pi^{-1})$

Dispersive representation

$\chi$ EFT calculation

Dispersively Improved  $\chi$ EFT (DI $\chi$ EFT)

Nucleon EM densities

$SU(3)$  flavor and octet baryons

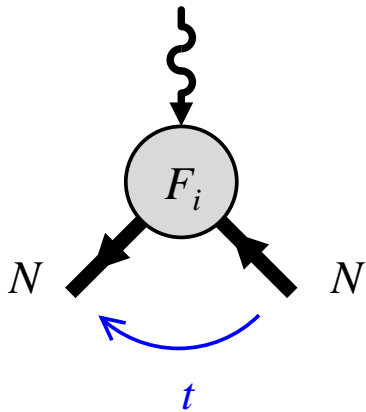
Large- $N_c$  relations

- Resonance form factors and densities

Unstable particles,  $N \rightarrow N^*$ ,  $N^* \rightarrow N^*$

- Applications and extensions

Scalar, axial form factor, EM tensor



- Current matrix element

$$\langle N' | J_\mu | N \rangle \rightarrow F_1(t), F_2(t) \quad \text{invariant FFs}$$

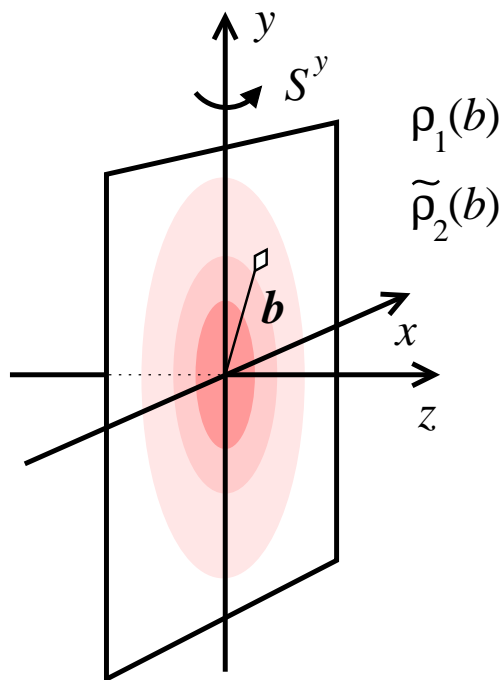
- Transverse densities

Soper 76, Burkardt 00, Miller 07

$$F_{1,2}(t = -\Delta_T^2) = \int d^2b e^{i\Delta_T b} \rho_{1,2}(b) \quad \text{Fourier}$$

Charge/magnetization density, or spin-indep/dependent

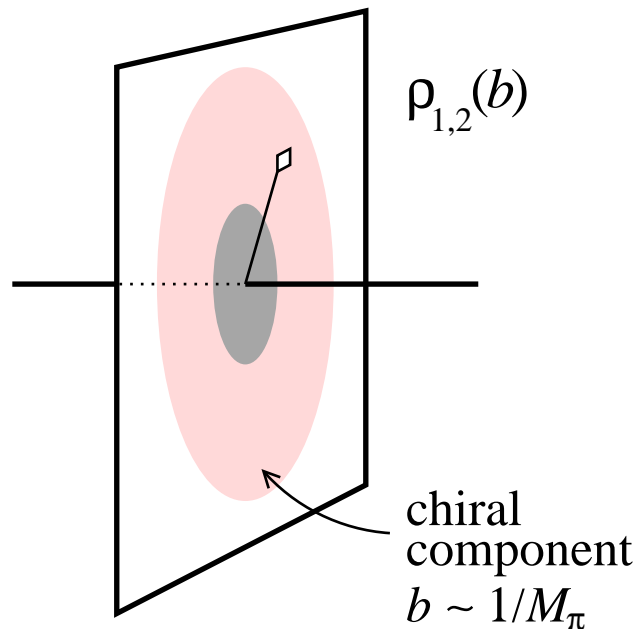
Density at fixed light-front time  $x^+ = x^0 + x^3$ ,  
boost-invariant, appropriate for relativistic systems



- Connection with GPDs/QCD

$$\rho_1(b) = \sum_q e_q \int_0^1 dx [q(x, b) - \bar{q}(x, b)]$$

Dual role of transverse densities:  
Accessible through low-energy elastic FFs,  
interpretable in terms of QCD DoF



- Peripheral densities  $b = \mathcal{O}(M_\pi^{-1})$

Governed by chiral dynamics:  
universal, model-independent

Calculable using  $\chi$ EFT + dispersion theory  
[Strikman, CW 10](#); [Granados CW 13](#)

- Theoretical interest

Distance as parameter

Proper definition of mesonic component

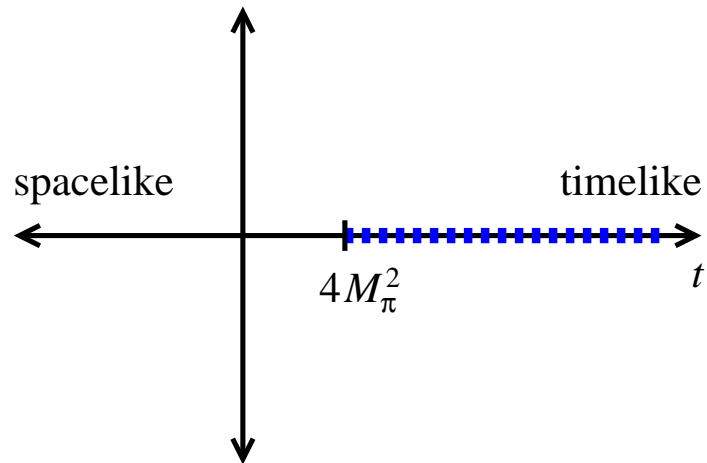
Space-time picture of chiral dynamics

- Practical interest

Low- $|t|$  form factors, proton size

Connection w. peripheral quark/gluon structure

# Dispersive representation

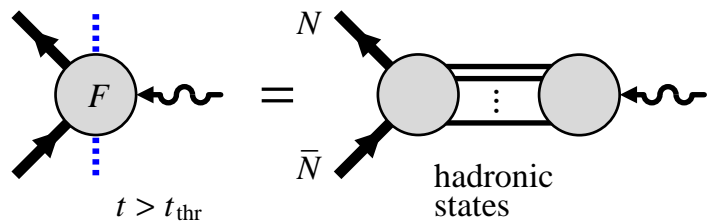


- Dispersive representation of form factor

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t' - t - i0} \frac{\text{Im } F(t')}{\pi}$$

Process: Current  $\rightarrow$  hadronic states  $\rightarrow N \bar{N}$

Unphysical region:  $\text{Im } F(t')$  from theory  
 Frazer, Fulco 60; Höhler et al 74; Hoferichter et al 14



- Transverse densities

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im } F(t)}{\pi}$$

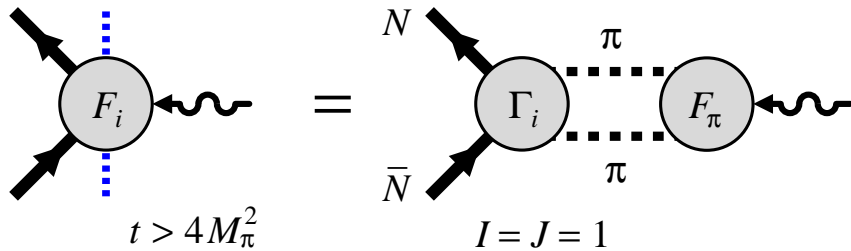
$K_0 \sim e^{-b\sqrt{t}}$  exponential suppression of large  $t$

Distance  $b$  selects masses  $\sqrt{t} \sim 1/b$ : Filter  
 Strikman, CW 10; Miller, Strikman, CW 11

Isovector:  $\pi\pi$  (incl.  $\rho$ ),  $4\pi, \dots$   
 Isoscalar:  $3\pi$  (incl.  $\omega$ ),  $K\bar{K}$  (incl.  $\phi$ ),  $\dots$

Peripheral densities  $\longleftrightarrow$  low-mass states

# Spectral functions



$$\begin{aligned} \text{Im}F_i(t) &= \frac{k_{\text{cm}}^3}{\sqrt{t}} \Gamma_i(t) F_\pi^*(t) \\ &= \underbrace{\frac{k_{\text{cm}}^3}{\sqrt{t}} \frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \underbrace{|F_\pi(t)|^2}_{\text{Data}} \end{aligned}$$

- Elastic unitarity relation

Timelike pion FF  $F_\pi(t)$ ,  $\pi\pi \rightarrow N\bar{N}$  partial-wave amplitude  $\Gamma_i$

Functions have same phase — Watson's theorem

Relation valid up to  $t = 16 M_\pi^2$ , in practice up to  $t \sim 1 \text{ GeV}^2$

Includes  $\rho$  as  $\pi\pi$  resonance

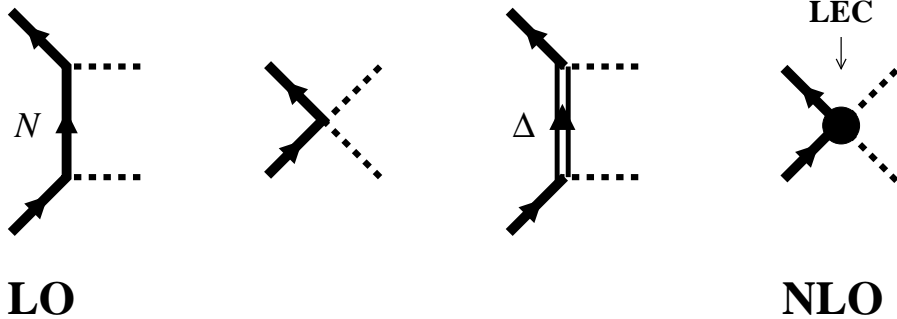
- New  $\chi$ EFT-based approach

Calculate  $\Gamma_i/F_\pi$  in  $\chi$ EFT — free of  $\pi\pi$  rescattering, well convergent

Multiply with  $|F_\pi|^2$  from  $e^+e^-$  data — includes  $\pi\pi$  rescattering,  $\rho$  resonance

*N/D* method. Many theoretical advantages. Predictive!

# Spectral functions II



$\pi\pi \rightarrow N\bar{N}$  amplitude in  $\chi$ EFT

- Relativistic  $\chi$ EFT

Expansion in  $(M_\pi, k_\pi)/\Lambda_\chi$

Controlled accuracy,  
systematic improvement

$\pi, N, \Delta$  as effective DoF

- Spectral function results

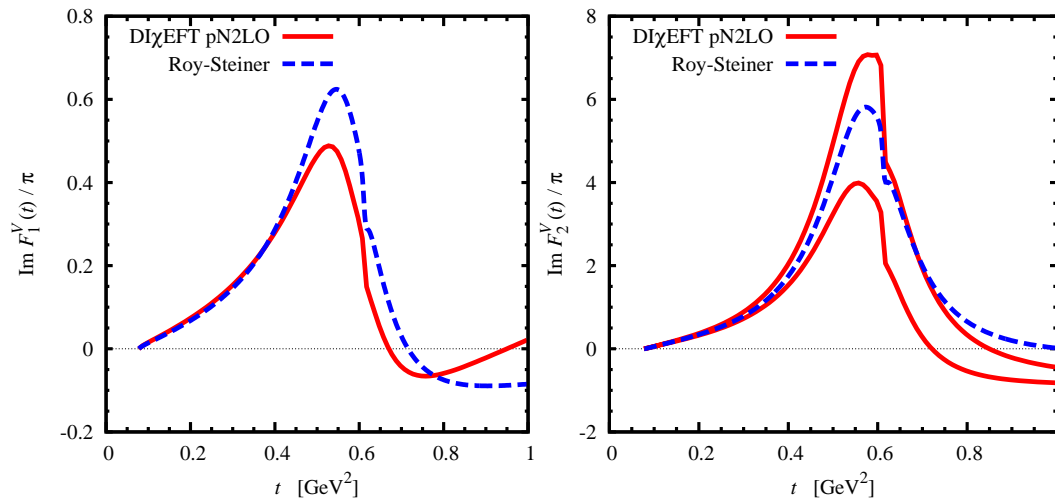
New method includes  $\pi\pi$  rescattering,  $\rho$  resonance

Dramatic improvement over conventional  $\chi$ EFT calculations

Good convergence in higher orders

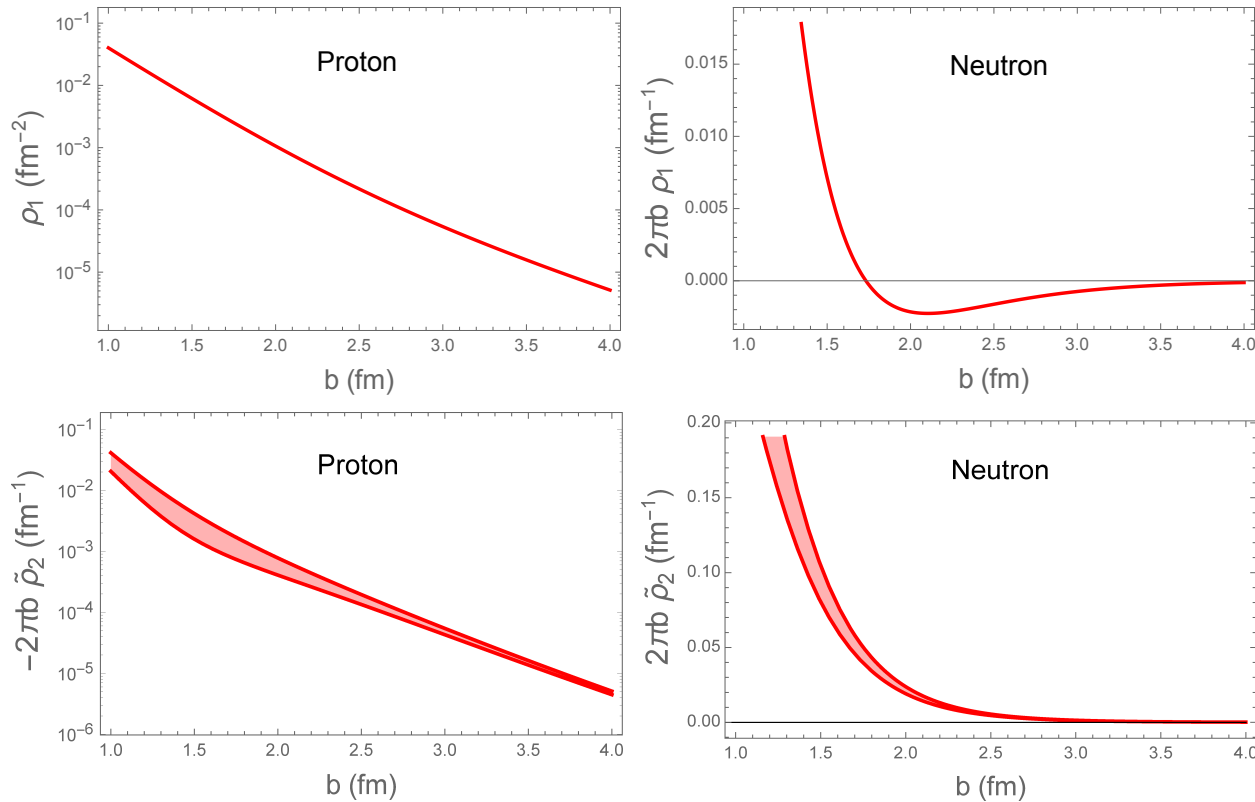
Possible to compute spectral functions up to  $\sim 1 \text{ GeV}^2$

Many applications!



Alarcon, Blin, Vicente Vacas, Weiss, NPA 964 18 (2017)  
NLO: Alarcon, Weiss, arXiv:1710.06430; in progress.

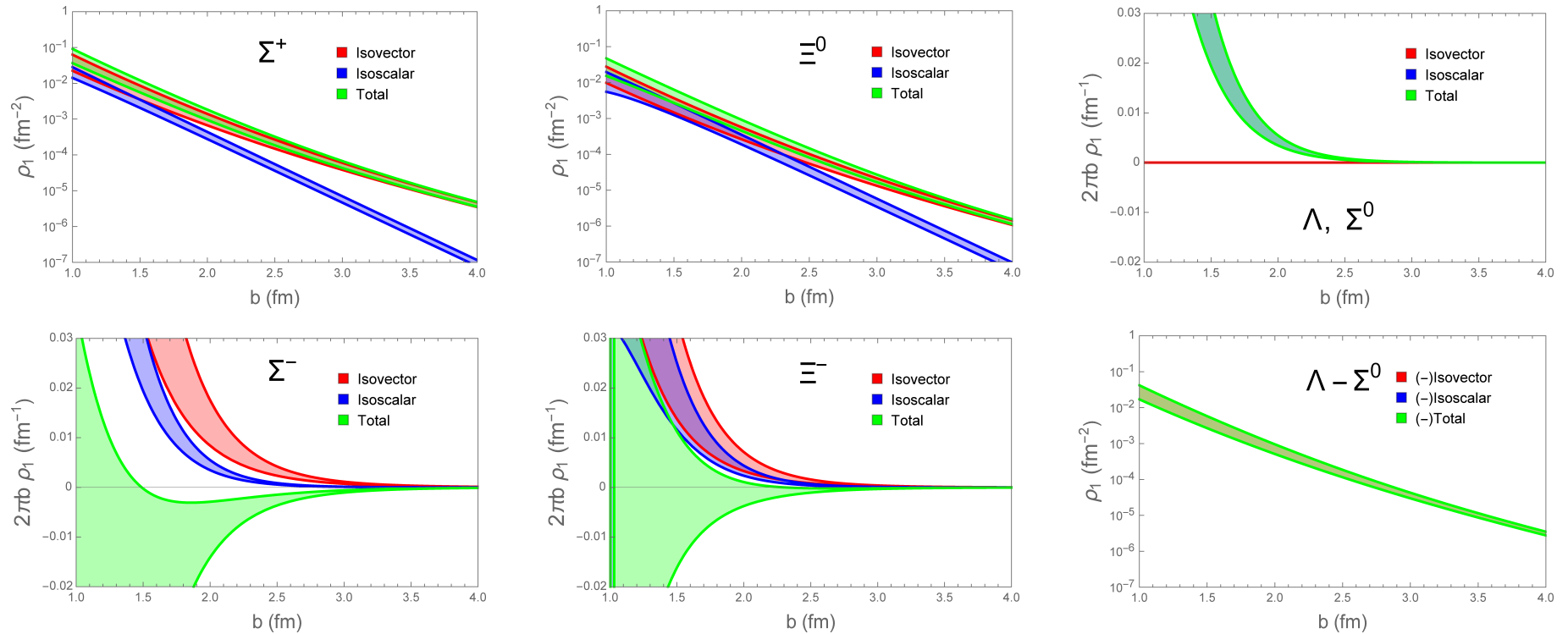
# Peripheral densities in nucleon



- Use  $\text{DI}\chi\text{EFT}$  spectral functions to calculate peripheral transverse densities
- Peripheral isovector densities predicted down to  $b \sim 1$  fm with controlled accuracy  
Isoscalar densities from empirical parametrization with  $\omega, \phi$
- Peripheral nucleon structure can be computed from first principles!  
[Alarcon, Blin, Vicente Vacas, Weiss, NPA 964 18 \(2017\)](#). NLO: [Alarcon, Weiss, arXiv:1710.06430](#); in progress.

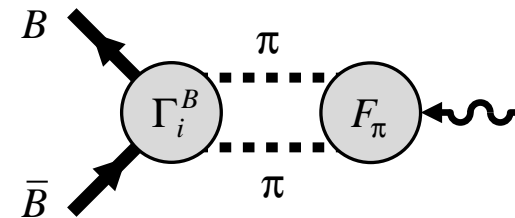


# SU3 flavor and octet baryons



- DI $\chi$ EFT extended to  $SU(3)$  flavor
- $\pi\pi$  spectral functions of octet baryon FFs  $K\bar{K}$  negligible at peripheral distances
- Peripheral densities of octet baryons, quark flavor separation  $u/d/s$

Alarcon, Blin, Vicente Vacas, CW, 2017



- Study scaling behavior of non-perturbative QCD quantities with  $N_c$ :  
 Meson and baryon masses, current matrix elements, hadronic couplings, ...  
'tHooft 73, Witten 79

$N_c$  scaling can be established on general grounds

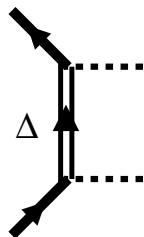
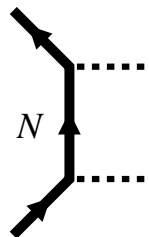
Parametric classification, hierarchy of structures, qualitative insight

- $N_c$  scaling of nucleon transverse densities

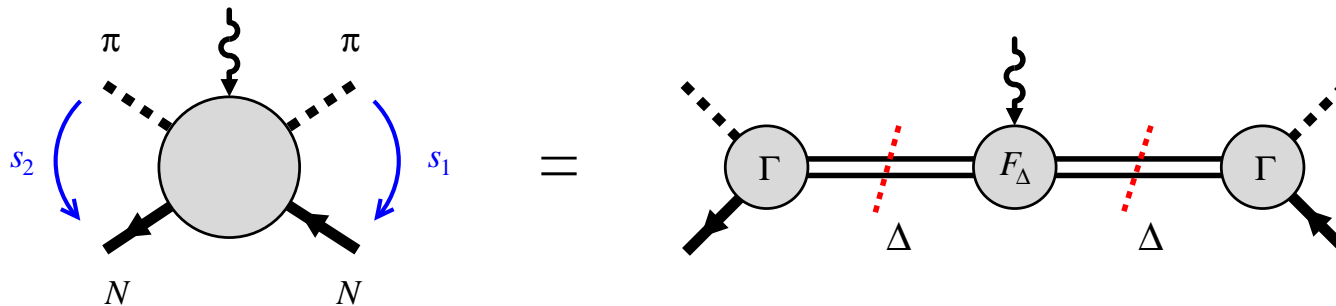
$$\rho_1(b) = \mathcal{O}(N_c^0), \quad \tilde{\rho}_2(b) = \mathcal{O}(N_c), \quad \text{for } b = \mathcal{O}(N_c^0)$$

- $\chi$ EFT results have correct  $N_c$  scaling if  $\Delta$  isobar included as dynamical DoF

Cohen, Broniowski 92. Transverse densities and GPDs: Strikman CW 04, 09, 11; Granados, CW 13



$N, \Delta$  cancel in  $\rho_1(b)$   
 amplify  $\tilde{\rho}_2(b)$



- Structure of unstable particle

S-matrix theory: Stable-particle amplitude  $\pi N \rightarrow \pi N$ ,  $\Delta$  as pole in 2-particle channels  $s_{1,2} = M_\Delta^2$  complex, residue factorizes

Resonance structure defined at complex pole

Can be implemented in  $\chi$ EFT

Ledwig et al 10. Alarcon, Blin, CW, in progress

- Form factors and densities of  $\Delta$  isobar

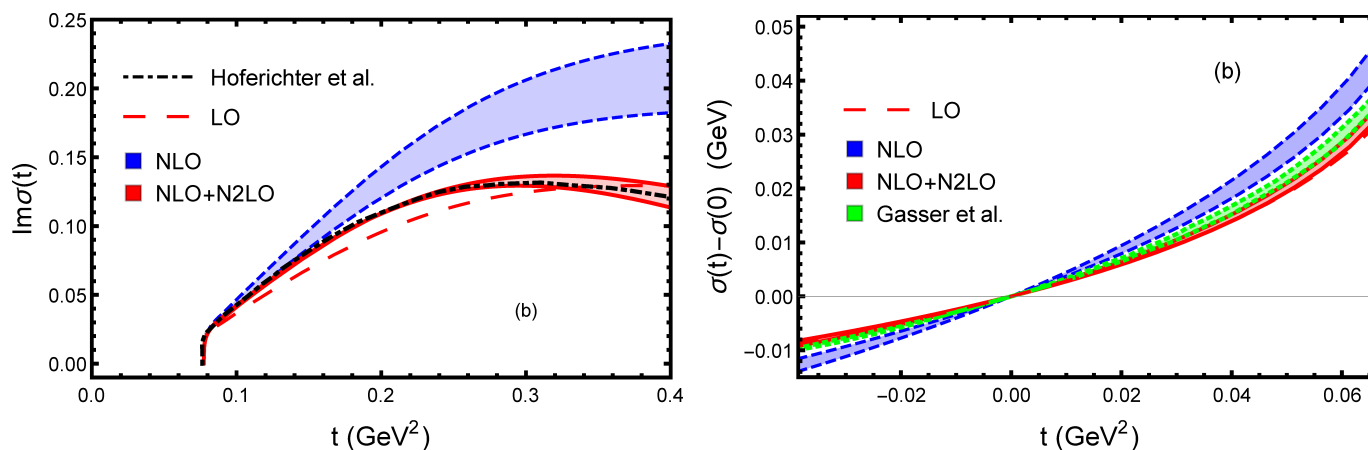
New spin structures because of  $S = \frac{3}{2}$

Lorce 09

LQCD results

Alexandrou et al 08; Aubin, Orginos, Pascalutsa, Vanderhaeghe 08

- Electromagnetic form factors at low  $t < 0$  and derivatives [Alarcon, CW, arXiv:1710.06430](#)
- Scalar form factor and mass distribution in nucleon [Alarcon, CW, arXiv:1707.07682](#)
- Energy-momentum tensor form factors and angular momentum [Granados, CW 2017](#)
- Peripheral GPDs and high-energy processes [Strikman, CW 04](#)
- Isoscalar-vector form factor from 3-body unitarity [Alarcon, Passemar, Piloni, CW; in progress](#)



Scalar spectral function and form factor from DIχEFT. Alarcon, CW, arXiv:1707.07682

- Transverse densities are an essential tool for hadron structure studies

Provide objective spatial representation of relativistic system

Connect low-energy dynamics with partonic structure in QCD

- Peripheral transverse densities can be computed with controlled accuracy by combining  $\chi$ EFT and dispersion theory (DI $\chi$ EFT)

Includes  $\pi\pi$  rescattering and  $\rho$  resonance

- Numerous applications and extensions

- [● Not covered here: Light-front time-ordered formulation of chiral dynamics, chiral  $N \rightarrow \pi N$  wave functions, quantum-mechanical interpretation]

[Granados, Weiss, PRC 92, 025206 \(2015\); JHEP 1507, 170 \(2015\); JHEP 1606, 075 \(2016\).](#)