Pion Distribution Amplitude from Euclidean Correlation functions

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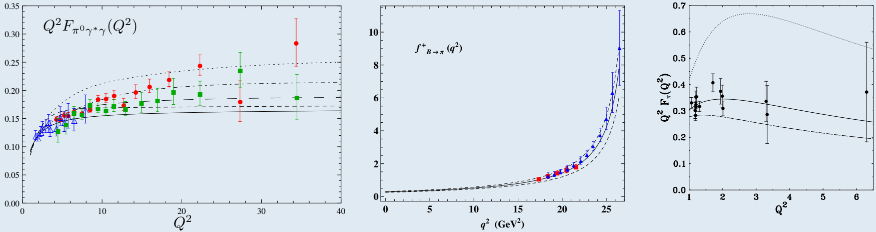
RQCD Collaboration: G. Bali, V.M. Braun, M. Göckeler, M. Gruber, F. Hutzler, P. Korcyl, B. Lang, A. Schäfer, P. Wein, J.-H. Zhang

Hard exclusive pion production
is sensitive to pion valence quark distribution at small transverse separations: pion DA



A vast field: $\gamma^* \to \gamma\pi(\eta,\eta')$, pion electroproduction, $B \to \pi\ell\nu$, $B \to \pi\pi$ etc.

V.B. *et al.* (RQCD Collaboration), PRD **92** (2015) 014504

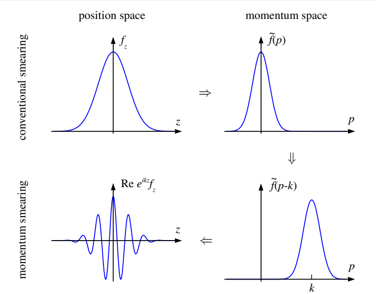
$$\langle \xi^2 \rangle^{\overline{\rm MS}} = \int_0^1 du\,(2u-1)^2 \phi_\pi(u,\mu) \;=\; 0.2361(41)(39)(?)$$

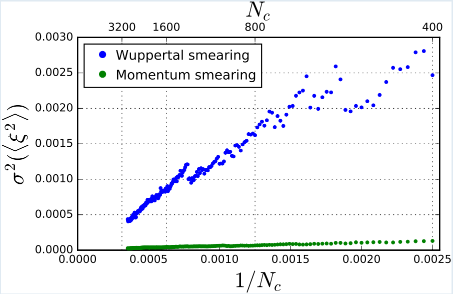
$$a_2^{\overline{\rm MS}} = \frac{7}{18}\int_0^1 du\, C_2^{3/2}(2u-1)\,\phi_\pi(u,\mu) \;=\; 0.1364(154)(145)(?)$$

$\mu = 2$ GeV

Momentum smearing

G. Bali, B. Lang, B. Musch, A. Schäfer,1602.05525

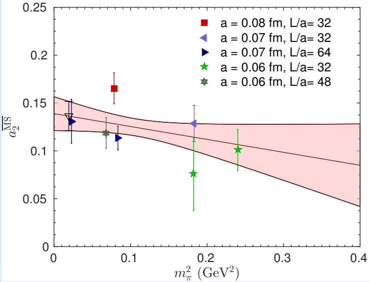




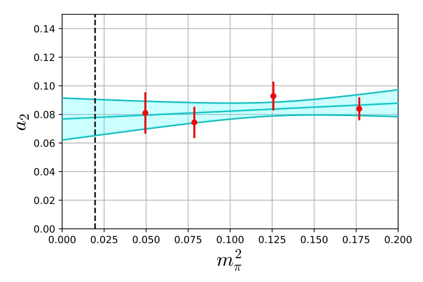
Right: Squared error as a function of the statistics;
Ensemble "H105" for $\vec{n}_{\vec{p}} = (110), (101), (011)$

- Momentum smearing: 2 inversions for each momentum (6 inversions)
- Wuppertal smearing: 1 inversion + additional Fourier sums

2015



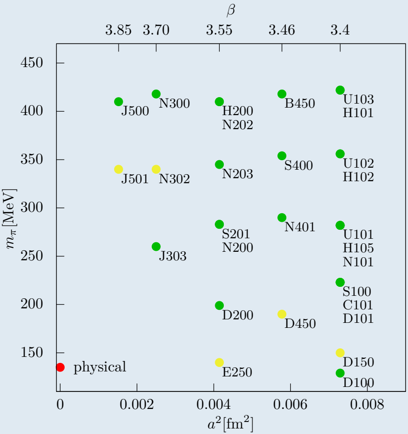
2017



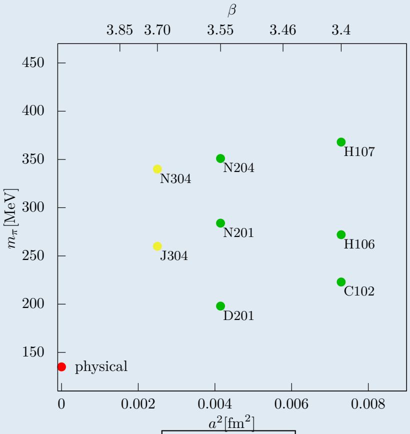
2015: $N_f = 2,\ a = 0.06 - 0.08$ fm

2017: $N_f = 2+1,\ a \sim 0.08$ fm $\to 0.04$ fm in 2018

CLS ensemble overview



$m_s + 2m_\ell =$ const



$\widehat{m}_s \approx$ const

E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

$\exists$ additional ensembles with $m_s = m_\ell$.

DAs/PDFs/GPDs from custom-made lattice (Euclidean) correlation functions

- Going beyond the second moment is not feasible:
  - — mixing with lower-dimensional operators
  - — adding more derivatives deteriorates signal-to-noise ratio

- General idea: perturbative factorization of Euclidean correlation functions

$$\langle H(p)|J_1(z)J_2(-z)|H(p)\rangle = C(z^2, p\cdot z; \mu_F) \otimes P(p\cdot z; \mu_F) \qquad z^2 < 0$$

  - —factorization in terms of PDFs is done in continuum (in $\overline{MS}$)
  - — small z necessary for factorization (and suppresses higher-twists)
  - — large $p\cdot z$ is necessary as a lever-arm on accessible momentum fractions
  - — additional *renormalization* factors may occur

- (Collinear) factorization in position space: light-ray operator product expansion

  Zavialov '76; Balitsky, Braun '89-'91

- Example: *Braun, Müller, EPJC,* **55**, *349 (2008)*

$$\langle 0|\, T\{\bar{q}(z)\gamma_\mu q(z)\bar{q}(-z)\gamma_\nu q(-z)\}\pi^0(p)\rangle \;=\; -\frac{5i}{9} f_\pi \epsilon_{\mu\nu\rho\sigma}\frac{z^\rho p^\sigma}{8\pi^2 z^4}\, T(p\cdot z, z^2)$$

$$T(p\cdot z, z^2) \;=\; \int_0^1 du\, e^{i(2u-1)p\cdot z} H(u,\mu_F^2 z^2, \alpha_s(\mu_F))\, \phi_\pi(u,\mu_F) \;+\; \mathcal{O}(z^2)$$

- Alternative

$$\langle 0|\, T\{\bar{q}(z)\not{z}\gamma_5[z,-z]q(-z)\}\pi^0(p)\rangle = if_\pi(pz)\int_0^1 du\, e^{i(2u-1)pz}\tilde{H}(u,\mu_F^2 z^2, \alpha_s(\mu_F))\phi_\pi(u,\mu_F) + \mathcal{O}(z^2)$$

- In both cases $H = 1 + c\alpha_s \ln z^2\mu^2$

What is better?

- Propagator: Physical observable, $\vec{z}$ direction arbitrary, extra handle from spinor structure
- Wilson line: cheaper, $\vec{z} = (0,0,z)$, no extra spinor structure, nonlocal RG factors

- or: heavy quark *W. Detmold, C.J.D. Lin, hep-lat/0507007*

$$\langle 0|\,T\{\bar q(z)\gamma_\mu c(z)\bar c(-z)\gamma_\nu q(-z)_\nu\}\pi^0(p)\rangle$$

- or: auxiliary scalar *U Aglietti et al, hep-ph/9806277; A Abada et al, hep-ph/0105221*

$$\langle 0|\,T\{\bar q(z)\gamma_\mu \phi(z)\bar\phi(-z)\gamma_\nu q(-z)_\nu\}\pi^0(p)\rangle$$

"Quasi-distribution"

- in analogy to "quasi-PDF", define *X Ji,1305.1539,; X Ji, 1506.00248*

$$\widetilde{\phi}_\pi(u,p_z) = \frac{i}{f_\pi}\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-i(2u-1)p_z z}\langle\pi(p)|\bar{q}(z)\gamma_z\gamma_5[z,-z]q(-z)|0\rangle$$

and match to pion DA using Large momentum effective theory (LaMET)

$$\tilde{\phi}_\pi(u,a^{-1},p_z) = \int_0^1 dv\, Z_\phi(u,v,a^{-1},\mu,p_z)\phi_\pi(v,\mu) + \mathcal{O}\left(\frac{\Lambda^2}{p_z^2},\frac{M_\pi^2}{p_z^2}\right).$$

? higher twists $\mathcal{O}\left(\frac{\Lambda^2}{p_z^2}\right)$ or $\mathcal{O}\left(\frac{\Lambda^2}{u^2 p_z^2}\right)$; perturbative expansion in $\alpha_s(p_z)$ or $\alpha_s(up_z)$

? Fourier transform numerically unstable and involves large-z region

? Integration over z does not allow to cancel the nonlocal RG factor

K Orginos, A Radyushkin, J Karpie, S Safeiropoulos, 1706.05373

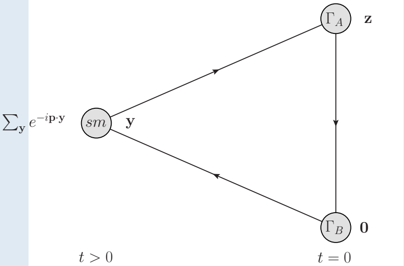
- first calculation using this approach *J-H Zhang, J-W Chen, X Ji, L Jin, H-W Lin, 1702.00008*

Work in progress: G. Bali et. al., [RQCD Collaboration], 1709.04325

- We follow *Braun, Müller, EPJC,* **55***, 349 (2008)* with generic spinor structure

$$\langle 0|T\{\bar{q}(z/2)\Gamma_A q(z/2)\bar{q}(-z/2)\Gamma_B q(-z/2)\}\pi^0(p)\rangle \propto T(p\cdot z, z^2)$$

- so far $\Gamma_A \otimes \Gamma_B \to I \otimes \gamma_5$



Tree level result:

$$T(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4}\Phi_\pi(p\cdot z),$$

with the position-space DA

$$\Phi_\pi(p\cdot z) = \int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u).$$

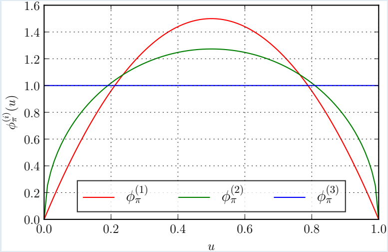
- QCD factorization

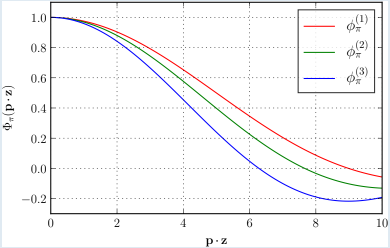
$$T(p\cdot z, z^2) \;=\; \int_0^1 du\, e^{i(2u-1)p\cdot z} H(u, \mu_F^2 z^2, \alpha_s(\mu_F))\,\phi_\pi(u,\mu_F) \;+\; \mathcal{O}(z^2)$$

- Take into account $H(u,\ldots)$ to NLO and twist-4 corrections $\mathcal{O}(z^2)$

- Three illustrative models of the DA (taken at a scale $\mu_0 = 1\,\text{GeV}$)

$$\phi_\pi^{(1)}(u) = 6u(1-u), \qquad \phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}, \qquad \phi_\pi^{(3)}(u) = 1$$





- left: momentum space, right: position space ("Ioffe time")

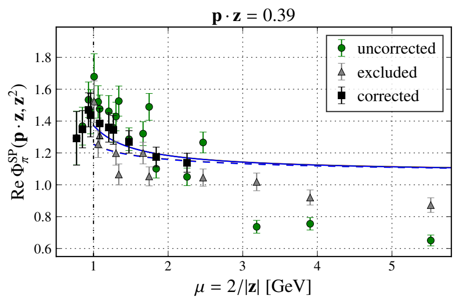
- In practice we compute for large t

$$\frac{T(p\cdot z, z^2)}{F_\pi} = \frac{Z_S Z_P}{Z_A}\,\frac{\langle 0|[\bar{d}\mathbb{I}q](0,\vec{z}/2)\;[\bar{q}\gamma_5 u](0,-\vec{z}/2)\;O_\pi^\dagger(-t,\vec{p})|0\rangle}{\langle 0|[\bar{d}\gamma_0\gamma_5 u](0,\vec{0})\;O_\pi^\dagger(-t,\vec{p})|0\rangle}\,E(\vec{p})$$

with RG factors $Z_S(\mu_R, g^2)$, $Z_P(\mu_R, g^2)$ and $Z_A(g^2)$ and $\mu_R = 2/|z|$

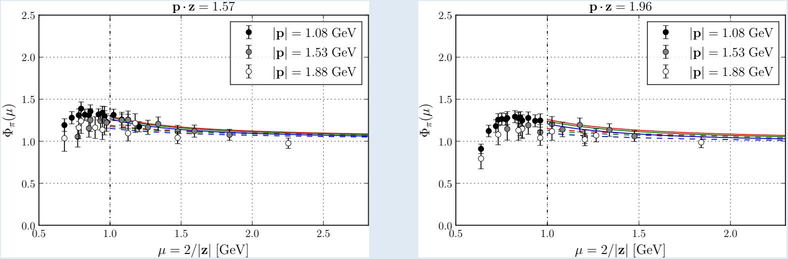
- We tree-level correct for $\vec{z}$-dependent lattice artefacts

$$T(p\cdot z, z^2) \mapsto T(p\cdot z, z^2)\,\frac{\mathrm{tr}\,[\not{z}\,G^{\mathrm{tree}}_{\mathrm{cont}}(z)]}{\mathrm{tr}\,[\not{z}\,G^{\mathrm{tree}}_{\mathrm{latt}}(z,a)]} \qquad \text{(for the chiral even part)}$$



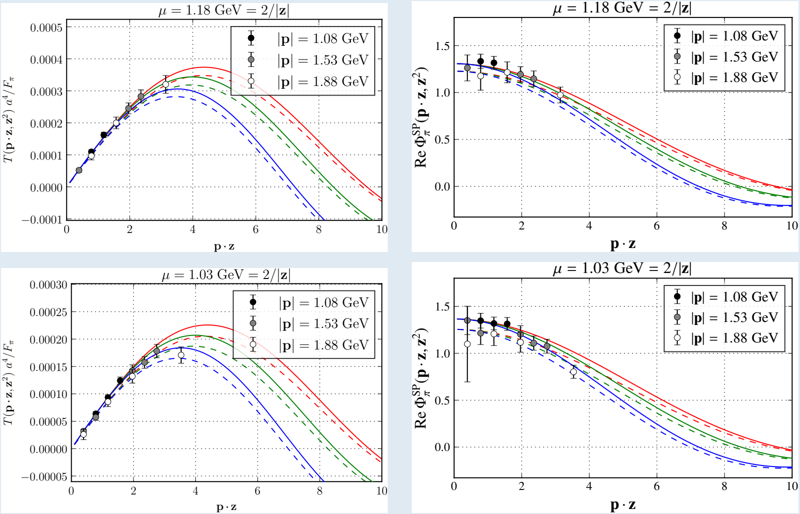
Results I: Renormalization/factorization scale dependence

$N_f = 2$ NP improved Wilson-clover quarks (old QCDSF ensemble);
$a^{-1} \approx 2.76\,\text{GeV}$, $m_\pi \approx 290\,\text{MeV}$, $L = 32a \approx 3.4/m_\pi$.



Results II: "Ioffe time " $p \cdot z$ dependence

- Solid/dashed curves are with/without higher twist corrections



- The future: other Dirac structures, smaller a, larger $|\vec{p}|$.

Outlook

- Momentum smearing is a leap forward

- Second moment
  - — Accuracy goal: 3% for $\langle\xi^2\rangle^{\overline{\mathrm{MS}}}$; 15% for $a_2^{\overline{\mathrm{MS}}}$
  - — Work in progress: continuum extrapolation

- DA shapes from customized Euclidean correlation functions
  - — We presented a proof of concept
  - — For $2/|z| \gtrsim 1$ GeV need $|\vec{p}| \gtrsim 4$ GeV to reach “Ioffe times” $|p\cdot z|$ large enough to discriminate between different shapes

- In future
  - — a new algorithm that enables smaller statistical errors
  - — other current-current combinations to minimize higher-twists and for cross-checks
  - — may expand to K-meson and pion PDF
  - — ? discretisation errors $\sim ap$