Spacelike and Timelike Form Factors for Meson-Photon Transitions in the Light-Front Quark Model

Ho-Meoyng Choi, Hui-young Ryu, and Chueng-Ryong Ji

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Outline

1. Motivation
2. Why Light-Front?
3. $F_{P\gamma}(Q^2)$ for $P \rightarrow \gamma^*\gamma$ $(P = \pi^0, \eta, \eta')$ in Manifestly covariant Model
4. Application to Light-front quark model (LFQM)
5. Numerical Results

1. Motivation

- **Meson-photon transitions** $P(\pi^0, \eta, \eta') \to \gamma^* \gamma$ **:**

- Simplest exclusive processes involving the strong interaction
- Significant role for both the low- and high-energy precision tests of the SM

1. Motivation

- **Meson-photon transitions** $P(\pi^0, \eta, \eta') \to \gamma^* \gamma$ **:**

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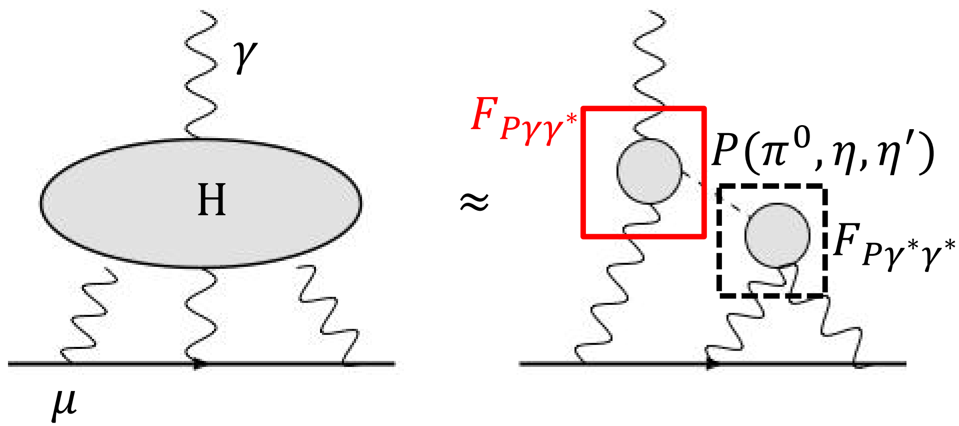
1) For the low-energy regime:

The transition form factors(TFFs) enter the prediction of important observables such as $P \to \ell\bar{\ell}(\ell = e, \mu)$ decays and the Hadronic Light by Light scattering (HLbL) contribution to the muon $(g-2)_\mu$:

e.g.) Pseudoscalar-pole contribution to HLbL.



$a_\mu = (g-2)/2$

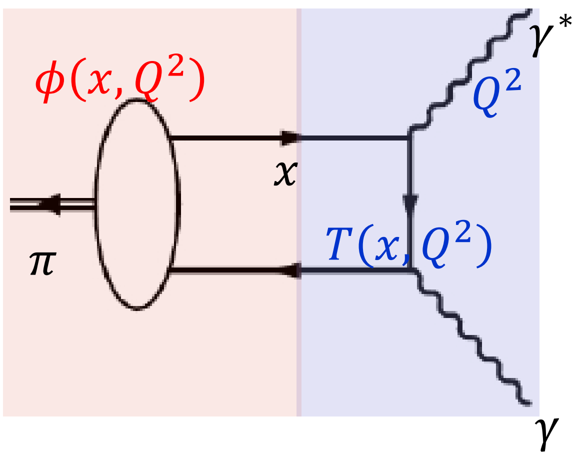
[Exp. –Th.(SM)]($\sim 3\sigma$)
$= (278 \pm 88) \times 10^{-11}$

HLbL
$= (116 \pm 40) \times 10^{-11}$

A. Nyffeler(2016)

2) For the high-energy regime: TFFs can be calculated from pQCD

e.g.) $\pi \to \gamma^* \gamma$ TFF



At leading twist:

$$F_{\pi\gamma}(Q^2) = \int T(x, Q^2)\phi(x, Q^2)dx + \cdots$$

T: Hard scattering amplitude for $\gamma^*\gamma \to q\bar{q}$ transition which is calculable in pQCD

ϕ: Nonperturbative meson DA describing $P \to q\bar{q}$ transition

$$\phi(x, \mu) \propto \int_{|\boldsymbol{k}_\perp|^2 \leq \mu^2} d^2\boldsymbol{k}_\perp \, \psi(x, \boldsymbol{k}_\perp)$$

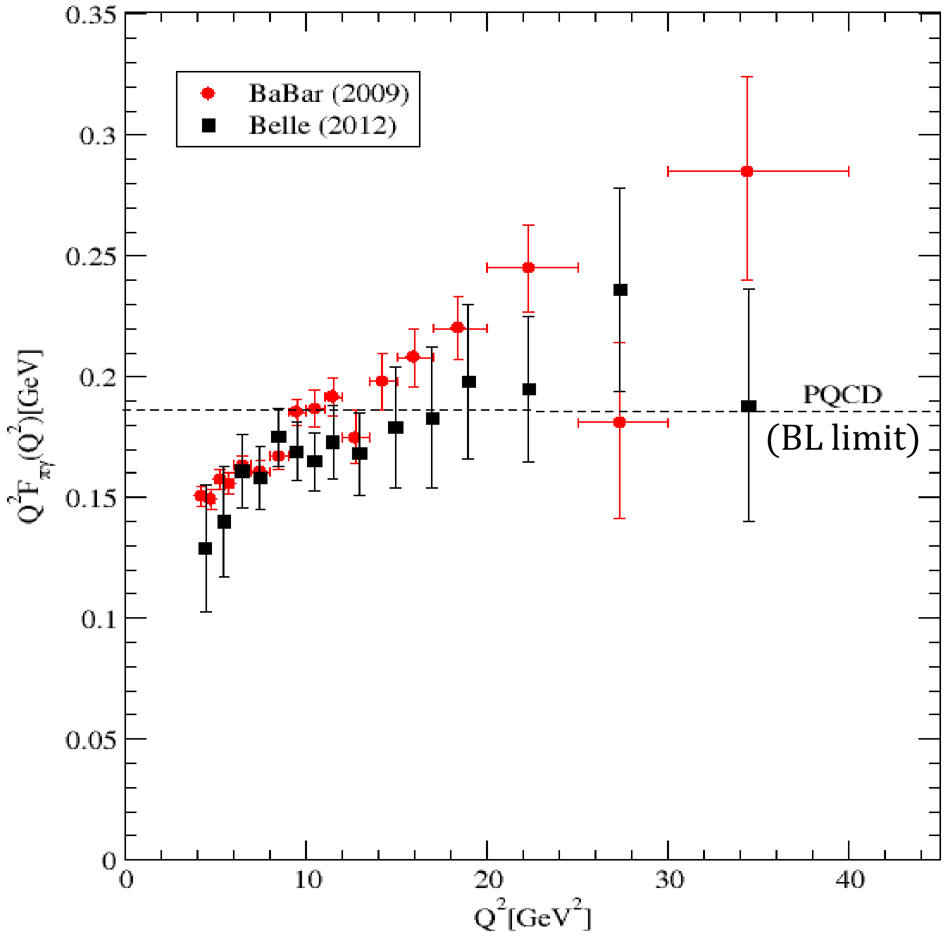
$$\xrightarrow{\mu \to \infty} \quad 6x(1-x): \text{"Asymptotic DA"}$$

In theory (pQCD):

$$Q^2 F_{\pi\gamma} = f_\pi \sqrt{2} \sim 0.185 \text{ GeV}$$

: Brodsky-Lepage(BL) limit

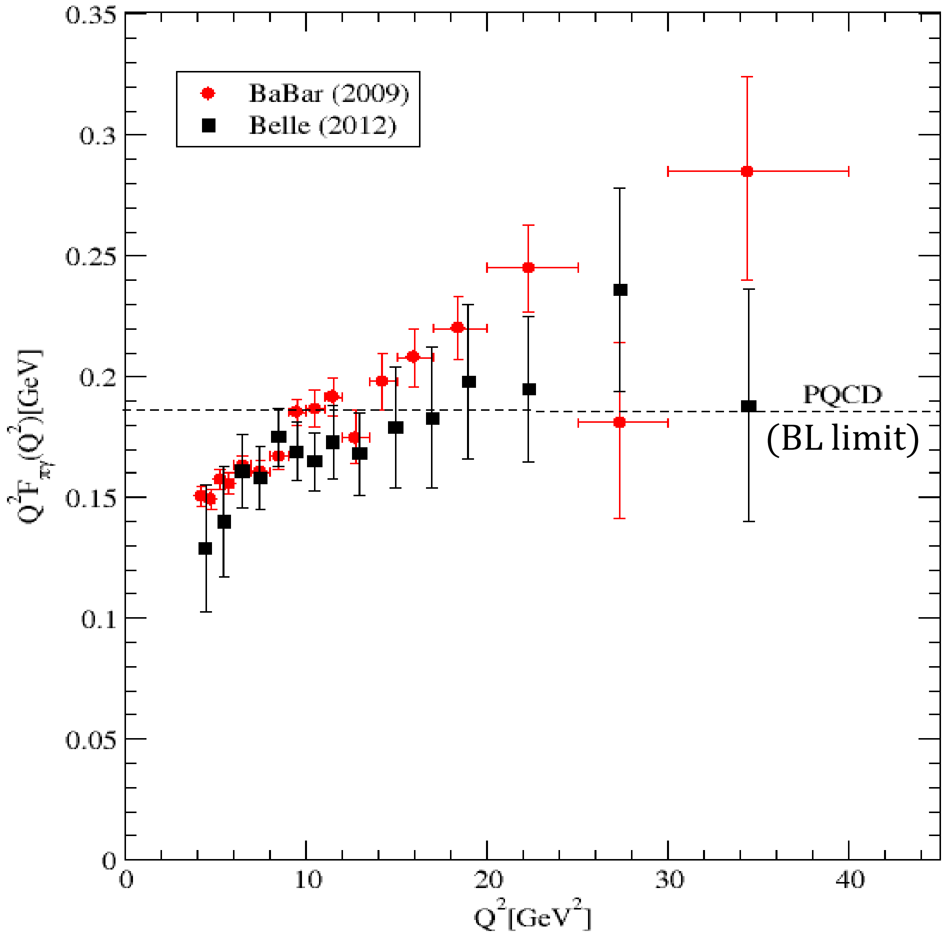
- **Experimental status for $F_{\pi\gamma}(Q^2)$** from $e^+e^- \to e^+e^-\pi^0$



1) BarBar vs. Belle?

2) What about timelike region?

- **Experimental status for** $F_{\pi\gamma}(Q^2)$ from $e^+e^- \to e^+e^-\pi^0$



1) BarBar vs. Belle?

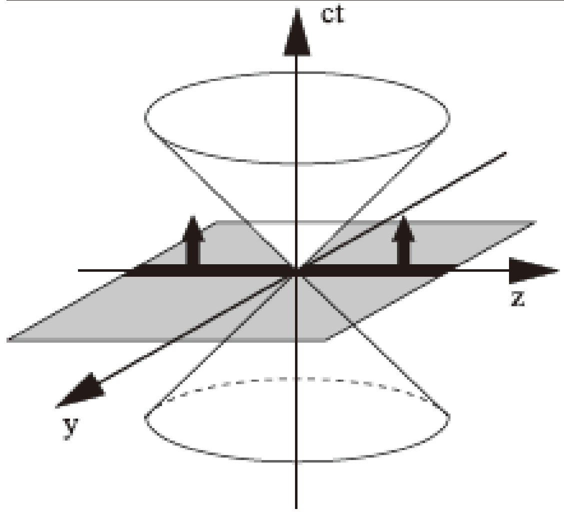
2) What about timelike region?

Analyzing both the spacelike region and the timelike region appears important to examine the issue of scaling behavior of $Q^2F_{\pi\gamma}$.

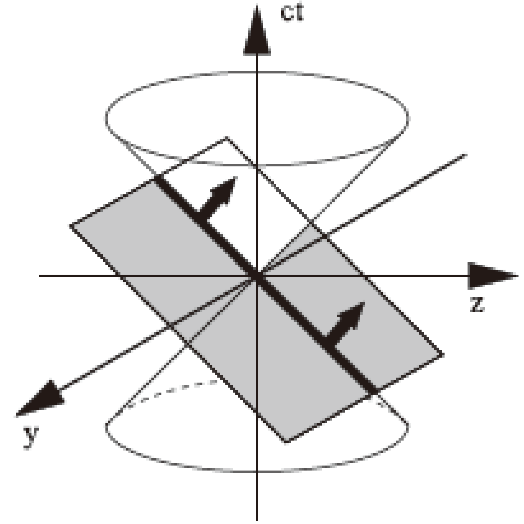
We shall explore both timelike and spacelike regions using the light-front quark model (LFQM)!

2. Why Light-Front?

Instant form ($x^0 = 0$)



Front form ($x^+ = x^0 + x^3 = 0$)



# of Kinematic generators: 6 (P^i, J^i) 7 (P^+, $\boldsymbol{P}_\perp, J_3, K_3, K_{1(2)} \pm J_{2(1)}$)

Hamiltonian	P^0	$P^- = P^0 - P^3$
Momentum	$\boldsymbol{P}_\perp = (P^1,\ P^2)$ P^3	$\boldsymbol{P}_\perp$ $P^+ = P^0 + P^3$
E-P dispersion Relation	$P^0 = \sqrt{M^2 + \vec{P}^2}$	$P^- = \dfrac{M^2 + \boldsymbol{P}_\perp^2}{P^+}$

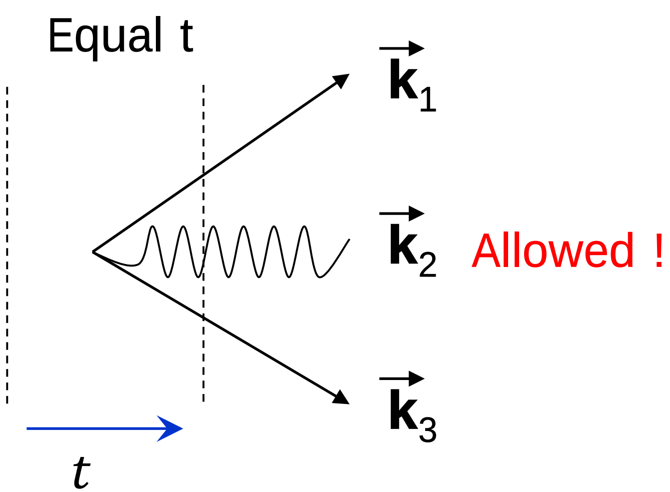
Irrational vs. Rational

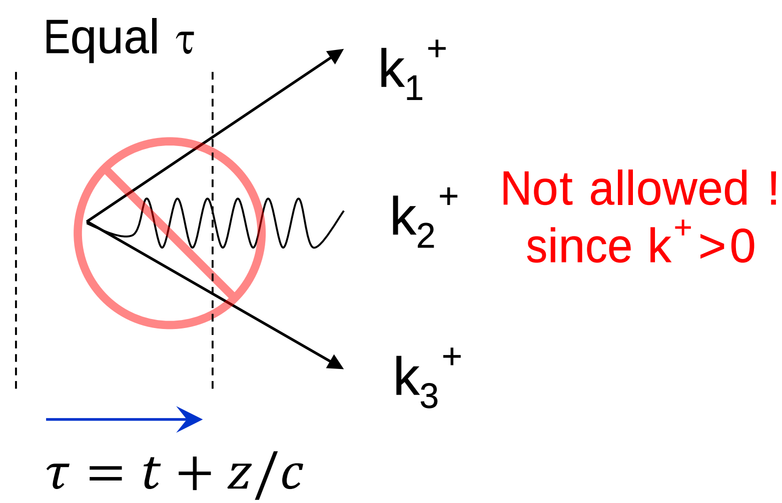
- Distinguished Features in LFD: Advantages in hadron phenomenology

(1) Construct boost invariant LF wave function!

(2) Vacuum fluctuations are suppressed!

$P^0 = \sqrt{M^2 + P_\perp^2 + P_3^2}$ (Instant Form)	$P^- = \frac{M^2 + P_\perp^2}{P^+}$ (Front form)

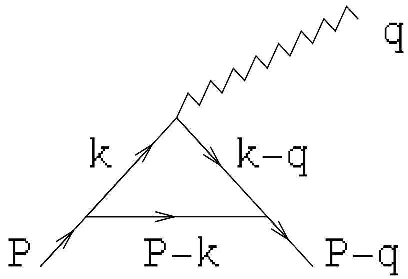




$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$

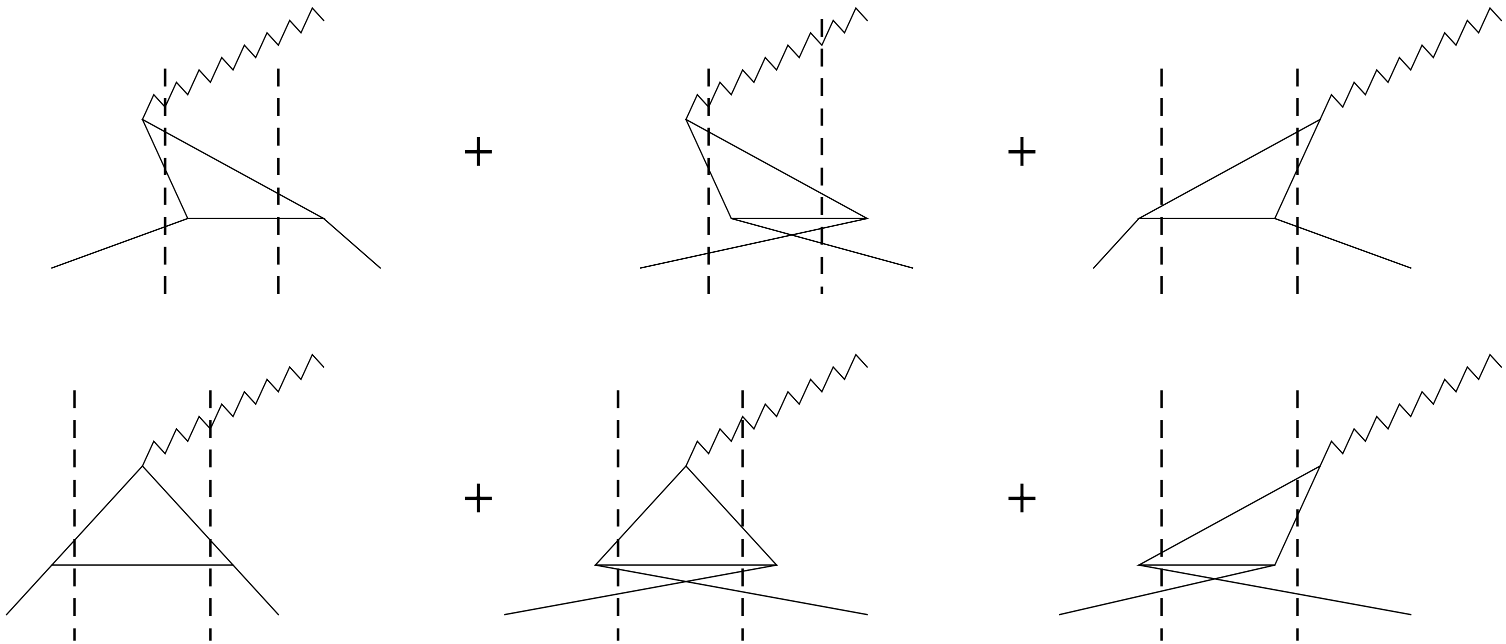
$k_1^+ + k_2^+ + k_3^+ = 0$

- Advantage of LFD in the calculation of Form Factors :
  Equal-t Theory vs. Light-Front Theory



Equal t (Instant form)

=



Need to calculate 6 time-ordered diagrams!

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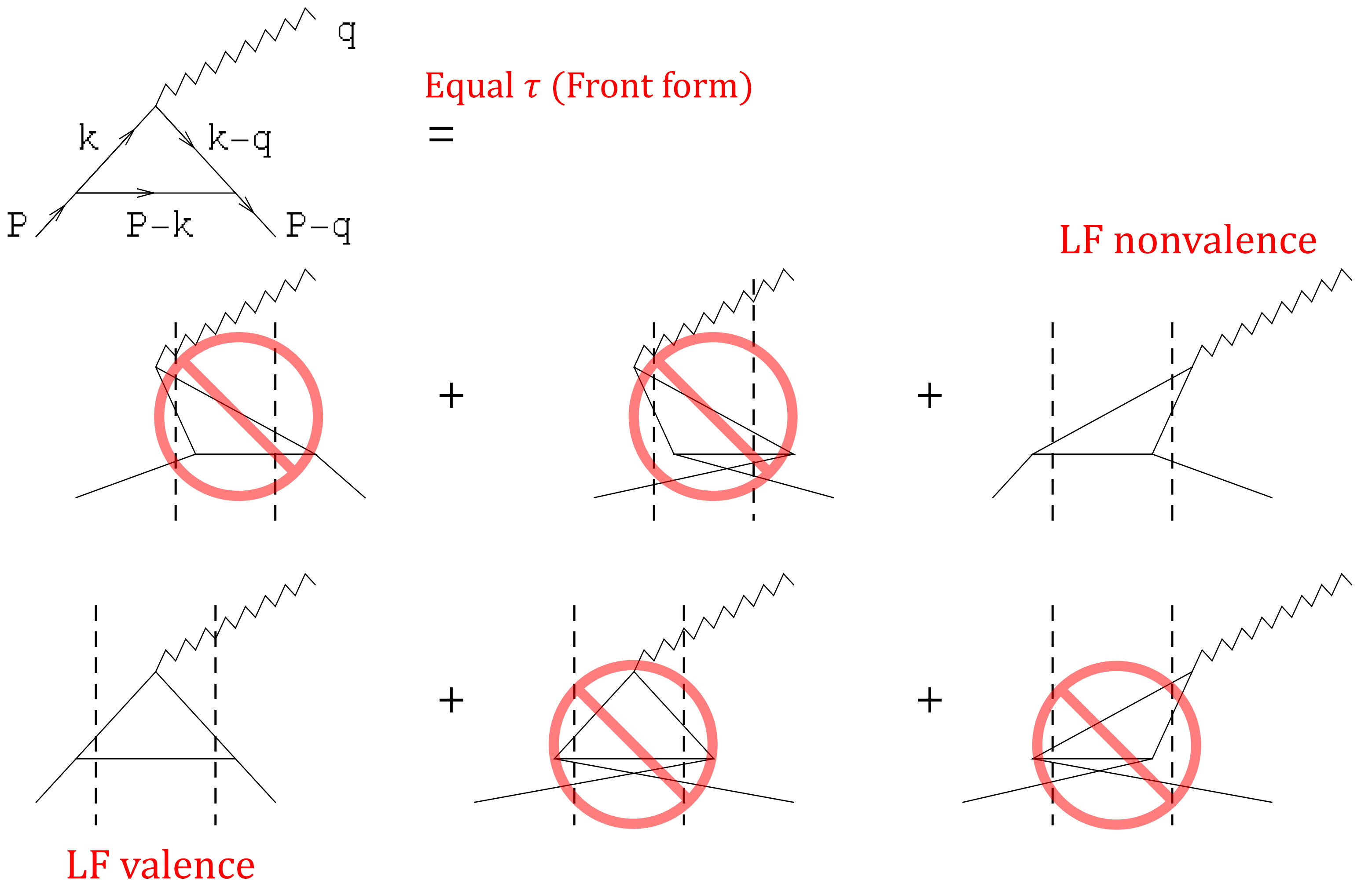
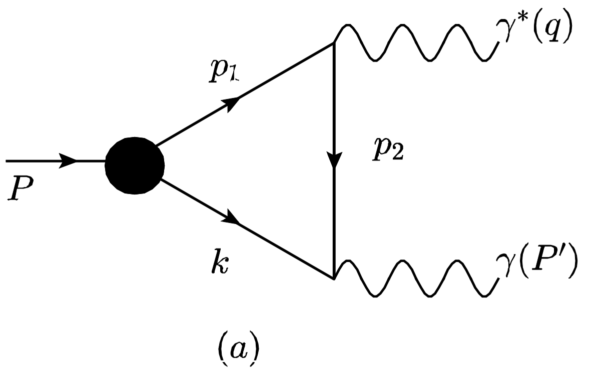


3. $F_{P\gamma}(Q^2)$ for $P \to \gamma^*\gamma$ Manifestly Covariant Model



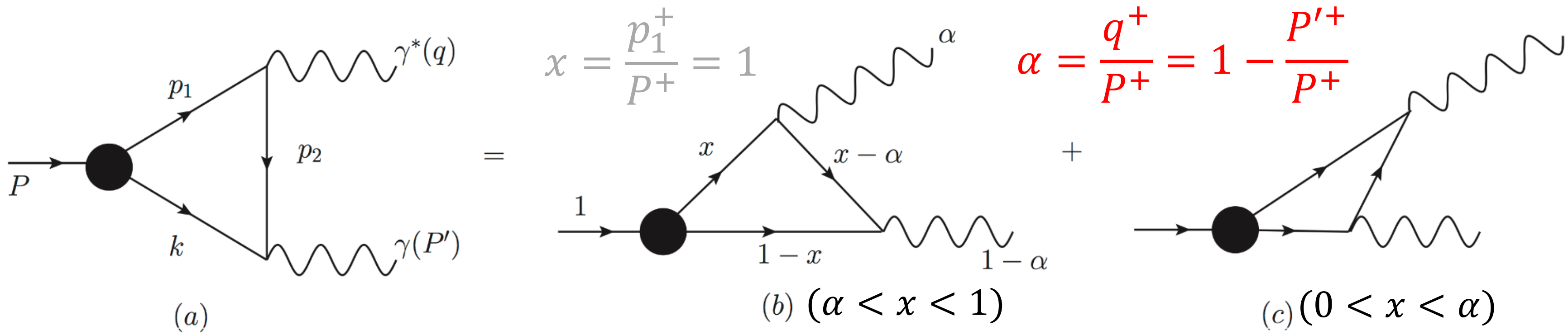
$$\Gamma^\mu = \langle \gamma(P-q)|J^\mu_{em}|P(P)\rangle = ie^2 F_{P\gamma}(Q^2)\epsilon^{\mu\nu\varrho\sigma}P_\nu\epsilon_\rho q_\sigma$$

$$\Gamma^\mu_{(a)} = ie_Q e_{\bar{Q}} N_c \int \frac{d^4k}{(2\pi)^4}\frac{H_0}{N_{p_1}N_k N_{p_2}}\, S^\mu$$

$$N_{p_j} = p_j^2 - m_Q^2 + i\epsilon\ (j=1,2) \qquad N_k = k^2 - m_{\bar{Q}}^2 + i\epsilon \quad (m_Q = m_{\bar{Q}})$$

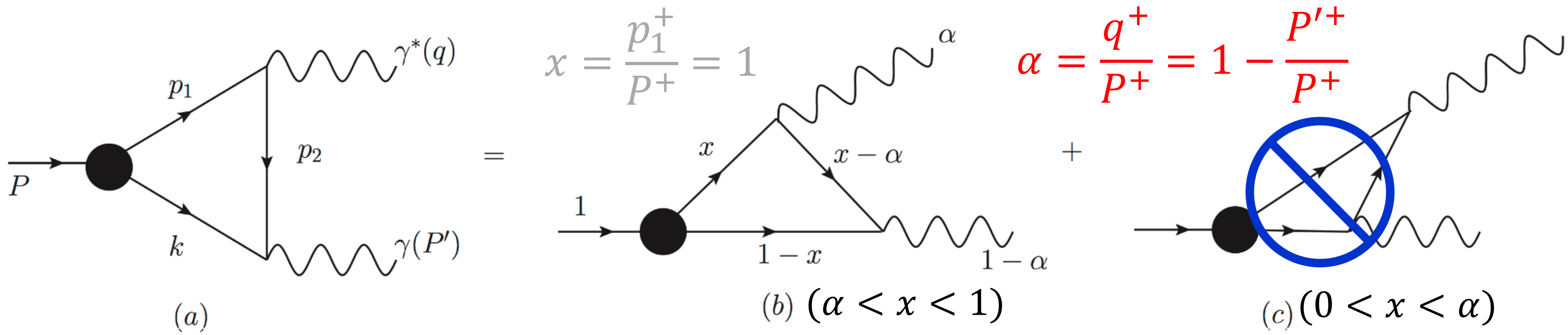
$$S^\mu = \mathrm{Tr}[\gamma_5(\not{p}_1 + m_Q)\gamma^\mu(\not{p}_2 + m_Q)\not{\epsilon}(-\not{k} + m_Q)]$$

$$H_0(p^2, k^2) = g$$



1. Equivalence between Covariant Calculation and Light-Front Calculation

Covariant Calculation	LF Calculations in different reference frames
Diagram (a) =	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)



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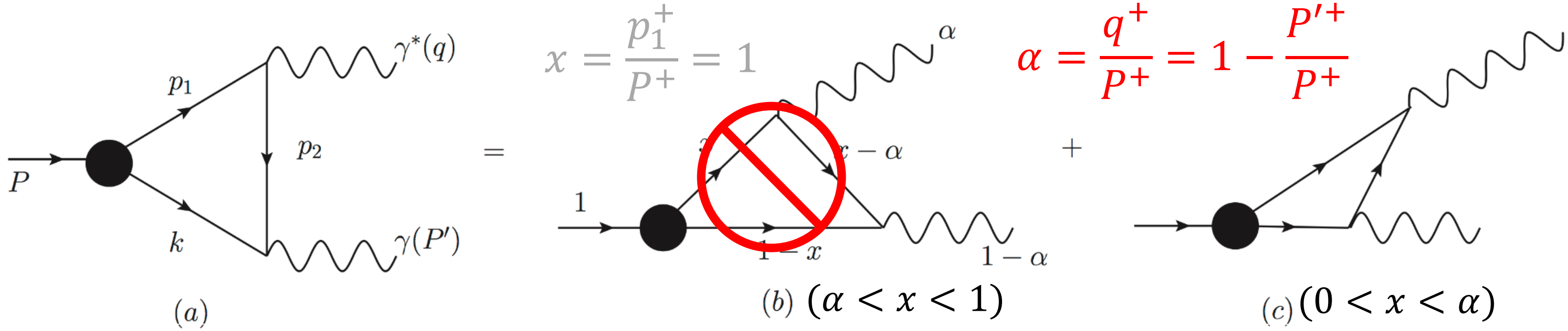

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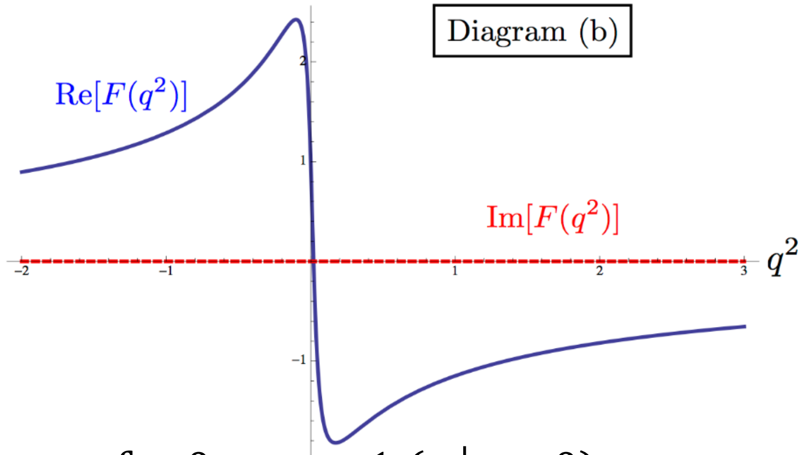
Covariant Calculation	**LF Calculations in different reference frames**
Diagram (a)=	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)
	(b) for $\alpha = 0$ ($q^+ = 0$) : defined in $q^2 < 0$
	(c) for $\alpha = 1$ ($q^+ \neq 0$) : defined in $q^2 > 0$
$F^{Cov}_{(a)}(q^2) = [F^{LF}_{(b)} + F^{LF}_{(c)}]_{0<\alpha<1} = [F^{LF}_{(b)}]_{\alpha=0} = [F^{LF}_{(c)}]_{\alpha=1}$	

$$[F^{LF}_{(c)}]_{\alpha=1}(Q^2) \propto \int_0^1 \frac{dx}{(1-x)^2} \int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2 - q^2} \chi(x, \boldsymbol{k}_\perp)$$

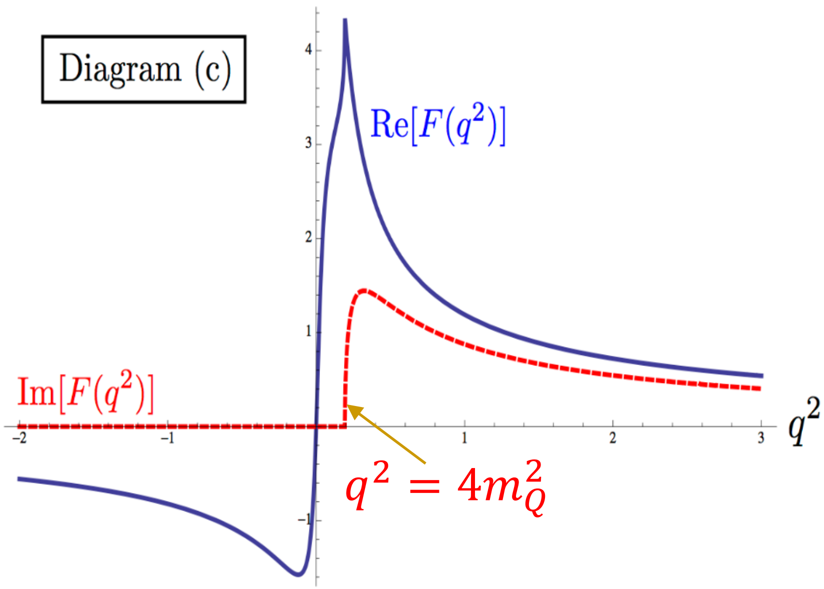
$$\chi(x, \boldsymbol{k}_\perp) = \frac{g}{x(M^2 - M_0^2)}$$

$$\text{with } M_0^2 = \frac{\boldsymbol{k}_\perp^2 + m_Q^2}{x(1-x)}$$

2. Toy model Calculation of $F_{\pi\gamma}(q^2)$:



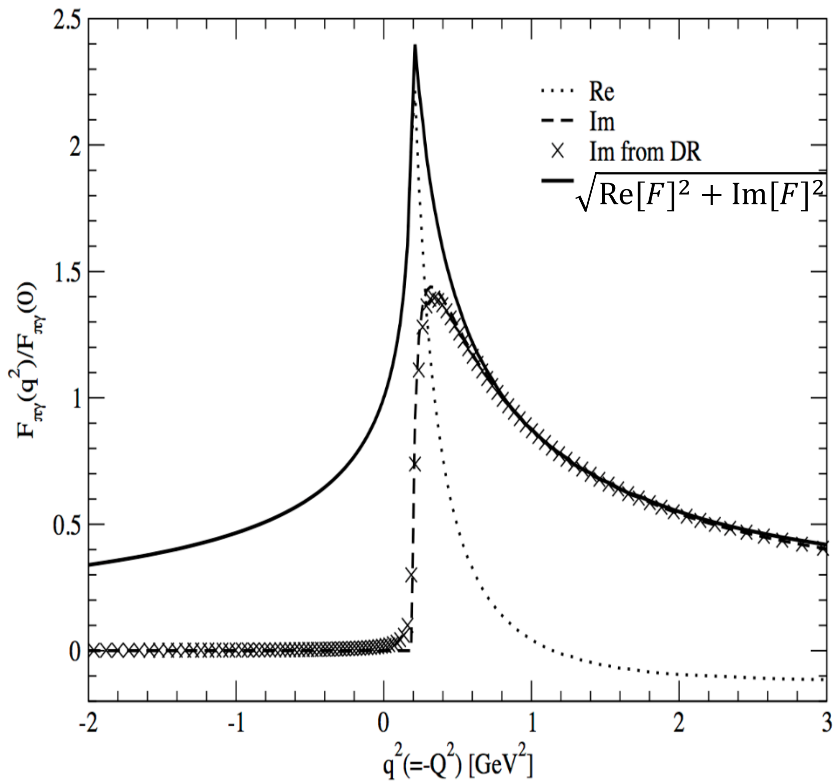
for $0 < \alpha < 1$ ($q^+ \neq 0$)



Dispersion Relation(DR) for
$F(q^2) = \text{Re}\, F(q^2) + i\text{Im}\, F(q^2)$:

$$\text{Re}\, F(q^2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Im}\, F(q'^2)}{q'^2 - q^2} dq'^2$$

$$\text{Im}\, F(q^2) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Re}\, F(q'^2)}{q'^2 - q^2} dq'^2$$



4. Application to Light-Front Quark Model (LFQM)

Manifestly Covariant Model :

$$\chi(x, \boldsymbol{k}_\perp) = \frac{g}{x\,(M^2 - M_0^2)}$$

LFQM : $\Psi^{SS_z}_{\lambda_1\lambda_2}(x, \boldsymbol{k}_\perp) = R^{SS_z}_{\lambda_1\lambda_2}(x, \boldsymbol{k}_\perp)\phi_R(x, \boldsymbol{k}_\perp)$

$$\phi_R(x, \boldsymbol{k}_\perp) = C(x, \boldsymbol{k}_\perp)\exp(-\frac{M_0^2}{8\beta^2})$$

$$1 = \int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} |\phi_R(x, \boldsymbol{k}_\perp)|^2$$

Process-independent correspondence

$$\sqrt{2N_c}\frac{\chi}{1-x} = \frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

CJ: PRD91,014018(15) , PRD89,033011(14)

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2 - q^2}\frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 0} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0'^2}\frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

$$M_0'^2 = M_0^2(\boldsymbol{k}_\perp \to \boldsymbol{k}_\perp + (1-x)\boldsymbol{q}_\perp)$$

• Summary for the LFQM Calculation of $F_{\pi\gamma}(q^2)$

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2 - q^2}\frac{\phi_R}{\sqrt{m_Q^2+\boldsymbol{k}_\perp^2}}$$

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 0} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0'^2}\frac{\phi_R}{\sqrt{m_Q^2+\boldsymbol{k}_\perp^2}}$$

At sufficiently high Q^2

$$F_{\pi\gamma}(q^2) \approx \frac{f_\pi\sqrt{2}}{3}\int_0^1 \frac{dx}{(1-x)Q^2}\phi_{2;\pi}(x) + O(1/Q^4) \qquad (q^2 = -q_\perp^2 = -Q^2)$$

where

$$\phi_{2;\pi}(x) = \frac{\sqrt{2N_c}}{f_P 8\pi^3}\int d^2\boldsymbol{k}_\perp \frac{\phi_R(x,\boldsymbol{k}_\perp)}{\sqrt{\boldsymbol{k}_\perp^2+m_Q^2}} m_Q$$

For the $(\boldsymbol{\eta}, \boldsymbol{\eta}') \to \boldsymbol{\gamma\gamma^*}$ transitions:

Use $\eta - \eta'$ mixing scheme in the quark-flavor basis

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \qquad \eta_q = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$$

⇩

Transition form factor $\boldsymbol{F_{P\gamma}}$ mixing scheme for $\boldsymbol{P \to \gamma\gamma^*}$ $(\boldsymbol{P = \pi^0, \eta, \eta'})$

$$F_{\pi\gamma}(q^2) = \frac{(e_u^2 - e_d^2)}{\sqrt{2}} I_{\text{tot}}^{m_{u(d)}}$$

$$F_{\eta\gamma}(q^2) = \cos\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\text{tot}}^{m_{u(d)}} - \sin\phi \, e_s^2 \, I_{\text{tot}}^{m_s}$$

$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\text{tot}}^{m_{u(d)}} + \cos\phi \, e_s^2 \, I_{\text{tot}}^{m_s}$$

For the $(\boldsymbol{\eta}, \boldsymbol{\eta}') \to \boldsymbol{\gamma\gamma}^*$ transitions:

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Quadratic(linear)
Gell-Mann-Okubo
mass formula: $\phi = [44.7^{\mathrm{o}}, 31.7^{\mathrm{o}}]$

Transition form factor $\boldsymbol{F_{P\gamma}}$ mixing scheme for $\boldsymbol{P \to \gamma\gamma^*}$ $(\boldsymbol{P = \pi^0, \eta, \eta'})$

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$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\mathrm{tot}}^{m_{u(d)}} + \cos\phi\, e_s^2\, I_{\mathrm{tot}}^{m_s}$$

We shall use
$\phi = (37 \pm 5)^{\mathrm{o}}$

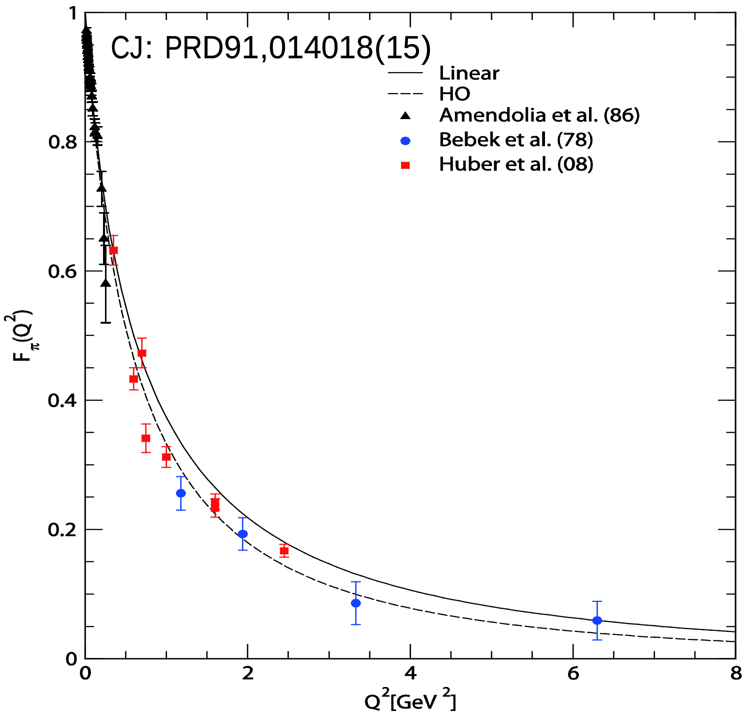
4. Numerical Results

(in unit of GeV)

Model	m_q	m_s	β_{qq}	β_{ss}
Linear	0.22	0.45	0.3659	0.4128

CJ: PRD59, 074015(99); PLB460, 461(99)

1) Pion E&M form factor



2) Pion Charge Radius

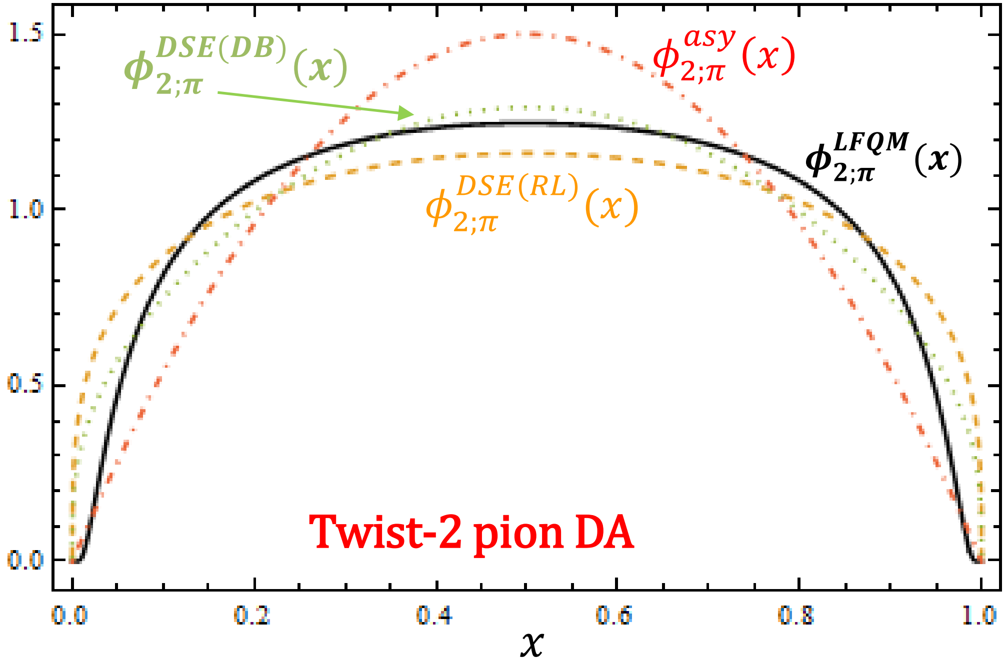
	LFQM	Exp.
$\langle r_\pi^2 \rangle^{1/2}$ [fm]	0.652	0.672(8)

3) Decay Constants

	LFQM	Exp.
f_π [MeV]	130	130.41(23)
f_0	$1.16 f_\pi$	$1.17 f_\pi$[1] $1.25 f_\pi$[2]
f_8	$1.32 f_\pi$	$1.26 f_\pi$[1] $1.28 f_\pi$[2]

[1]Feldmann,Kroll, Stech, PRD58,114006(98)

[2] Leutwyler, Nucl. Phys. B (Proc. Suppl.) 64,223(98)



$$\langle \xi^n \rangle = \int_0^1 dx\, \xi^n \phi_{2;\pi}(x)$$

where $\xi = x - (1 - x)$

$\langle \xi^2 \rangle$; measure of the width of the DA

$\boldsymbol{\langle \xi^2 \rangle_\pi^{LFQM} = 0.24}$

$\langle \xi^2 \rangle_\pi^{RL(DB)} = 0.28\ (0.25)$ [Chang et al. 13]

$\langle \xi^2 \rangle_\pi^{asy} = 0.20\ (\phi_{2;\pi}^{asy} = 6x(1-x))$

$\langle \xi^2 \rangle_\pi^{LAT} = 0.27 \pm 0.04$ [Braun et al. 06]

$\langle \xi^2 \rangle_\pi^{flat} = 1/3 (\phi_{2;\pi}^{flat} = 1)$

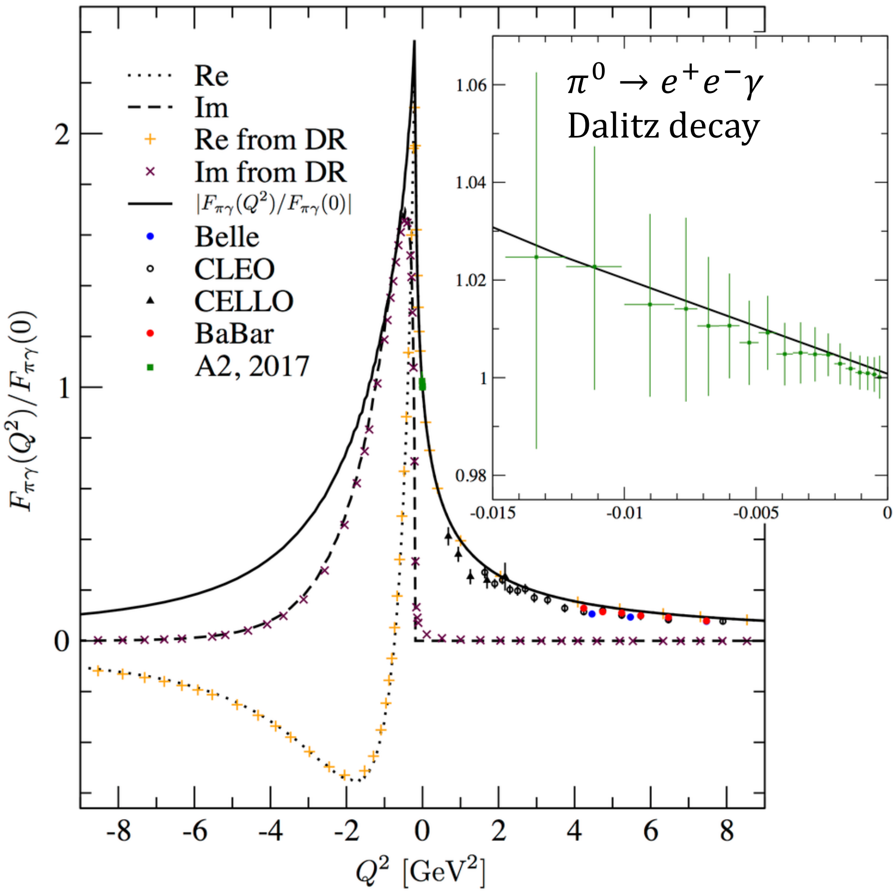
$\langle \xi^2 \rangle_\pi^{Ads/QCD} = 0.25 (\phi_{2;\pi}^{Ads/QCD} = \frac{8}{\pi}\sqrt{x(1-x)})$

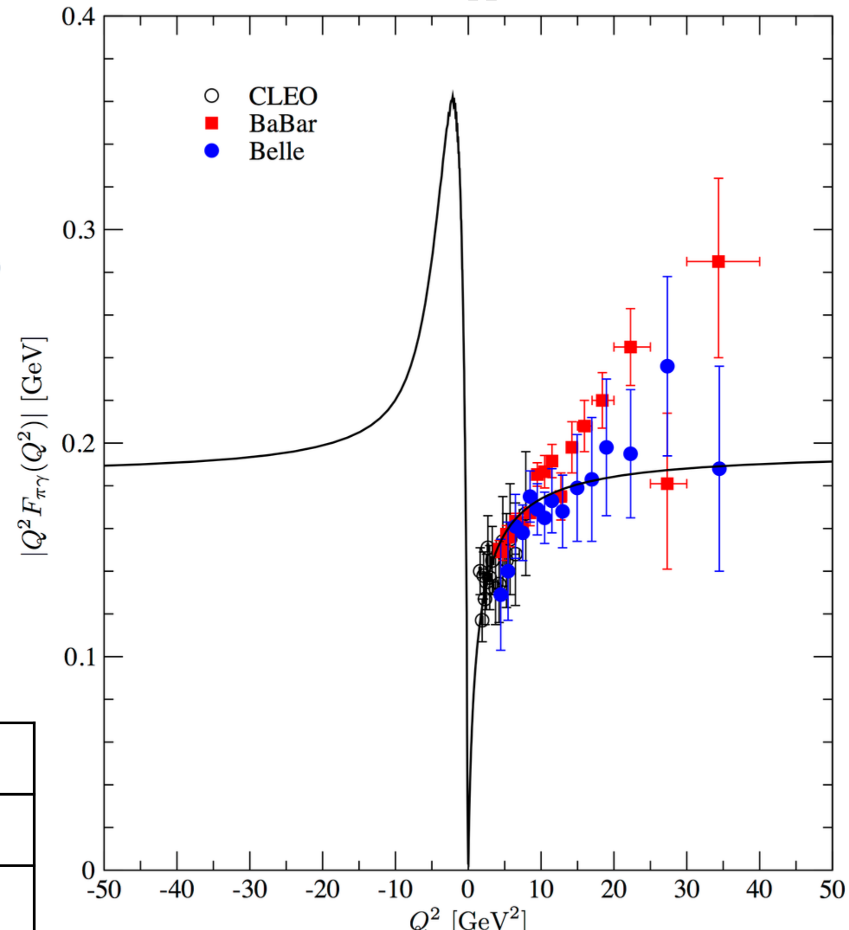
[Brodsky et al. 11]

$\langle \xi^2 \rangle_\pi^{delta} = 0\ (\phi_{2;\pi}^{delta} = \delta(x - \frac{1}{2}))$

Results for $F_{\pi\gamma}(q^2)$

$$F(m_{ll} = q) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}} \approx 1 + a_\pi \frac{m_{ll}^2}{m_\pi^2}$$



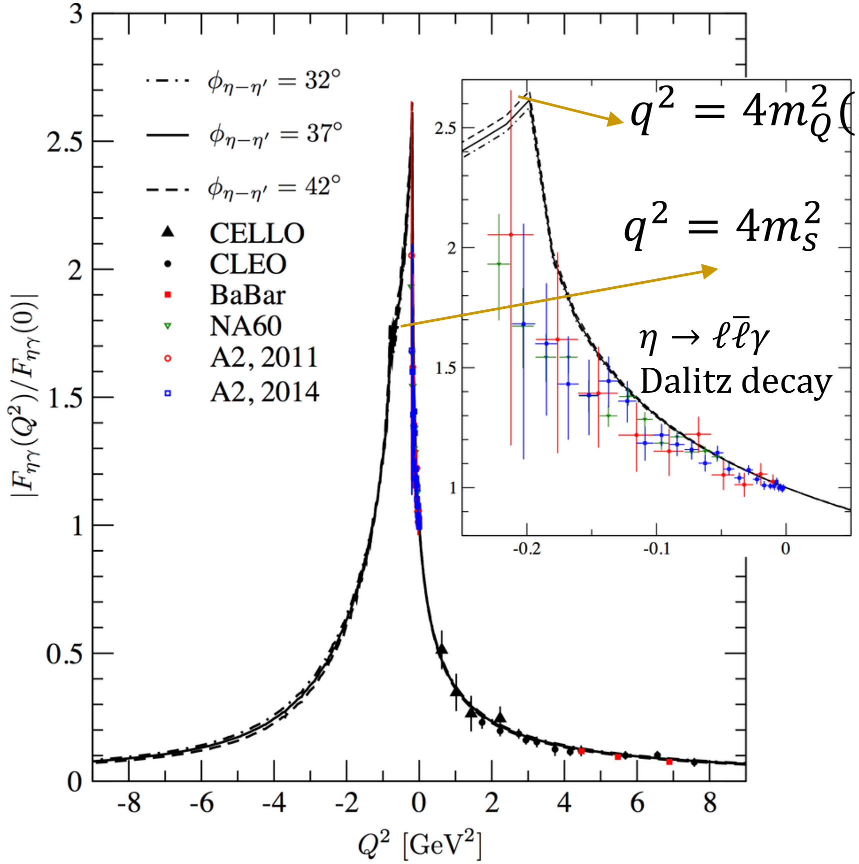


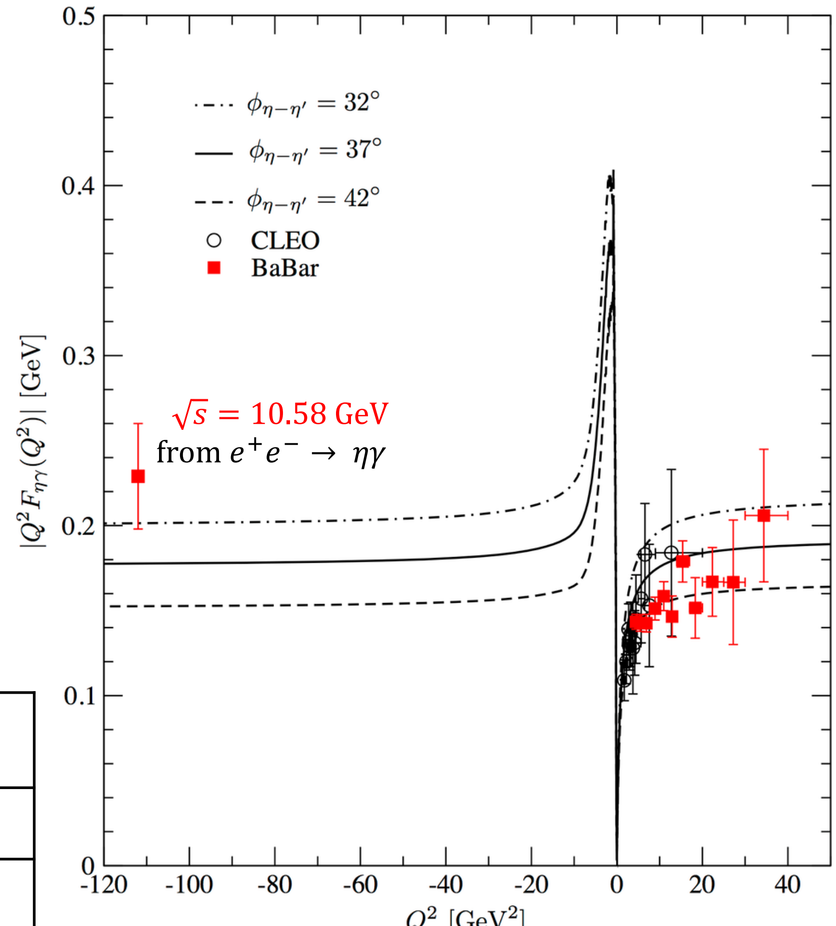
Slope parameter a_π:

Ours	0.0355
A2 at MAMI(16)	0.030 ± 0.010
World average(PDG)	0.032 ± 0.004

Results for $F_{\eta\gamma}(q^2)$

$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$



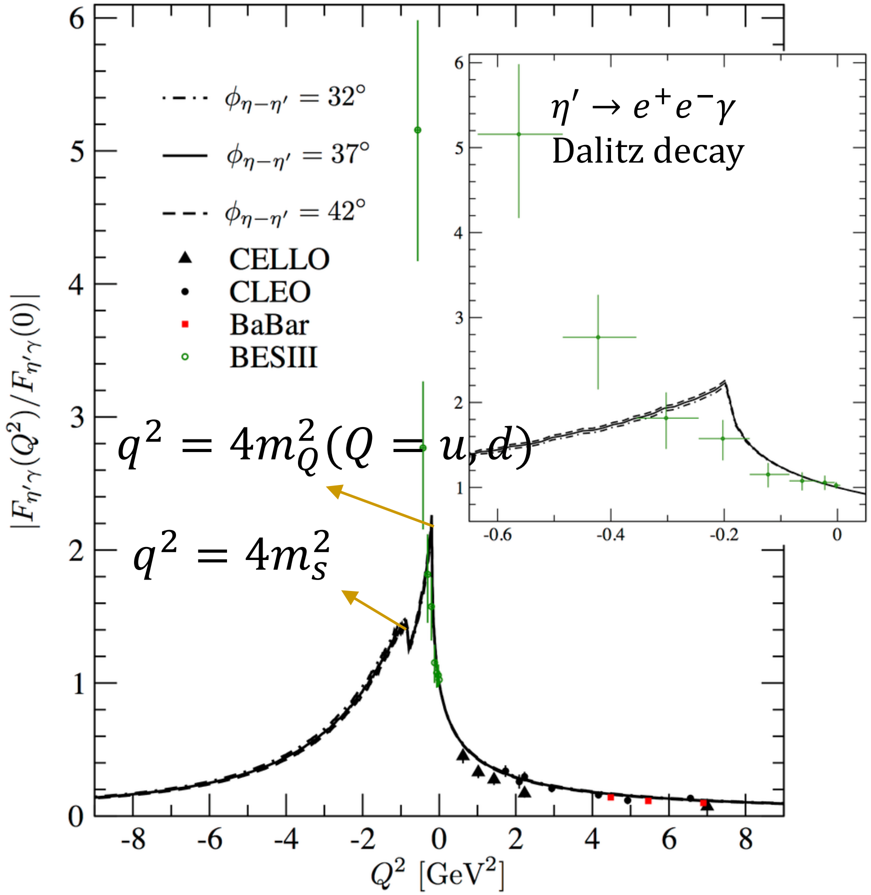


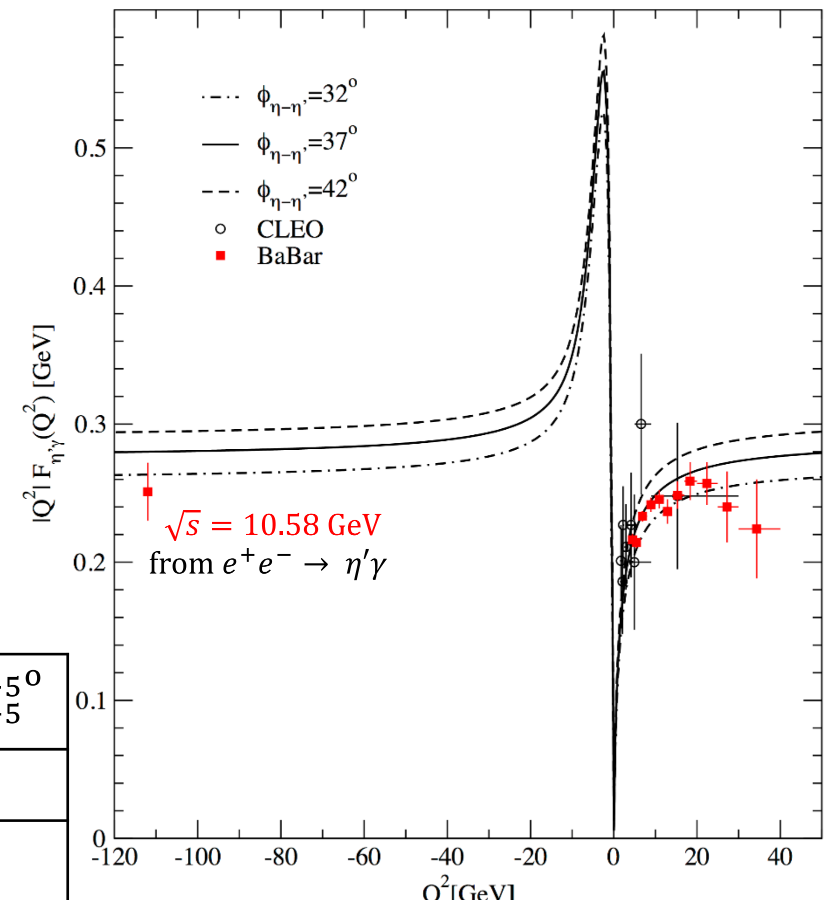
Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$2.112^{-0.031}_{+0.038}$ for $\phi = 37^{-5}_{+5}{}^{o}$
A2 at MAMI	$1.95 \pm 0.15 \pm 0.10$
NA2 at CERN	$1.95 \pm 0.17 \pm 0.05$

Results for $F_{\eta'\gamma}(q^2)$

$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$





Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$1.732^{-0.035}_{+0.031}$ for $\phi = 37^{-5}_{+5}{}^{o}$
BESIII (2015)	1.60 ± 0.25
Lepton-Col.(1979)	1.7 ± 0.4

5. Conclusion

- We investigate $(\pi^0, \eta, \eta') \to \gamma\gamma^*$ transitions both for the spacelike and timelike regions using the LFQM.

- Present the new direct method to explore the timelike region and show the agreement with the result from the DR.

- For the low energy regime:

- Our results of the TFFs and a slope parameters at $q^2 = 0$ show a good agreement with the available exp. Data.
- Observe the resonance peaks corresponding to ρ-type pole for $F_{\pi\gamma}$ and (ρ, ϕ) meson type poles for $F_{(\eta,\eta')\gamma}$ respectively.

- For the high energy regime:

- Our results for $Q^2 F_{\pi\gamma}(Q^2)$ are consistent with the PQCD prediction showing a scaling behavior for both timelike and spacelike regions.