

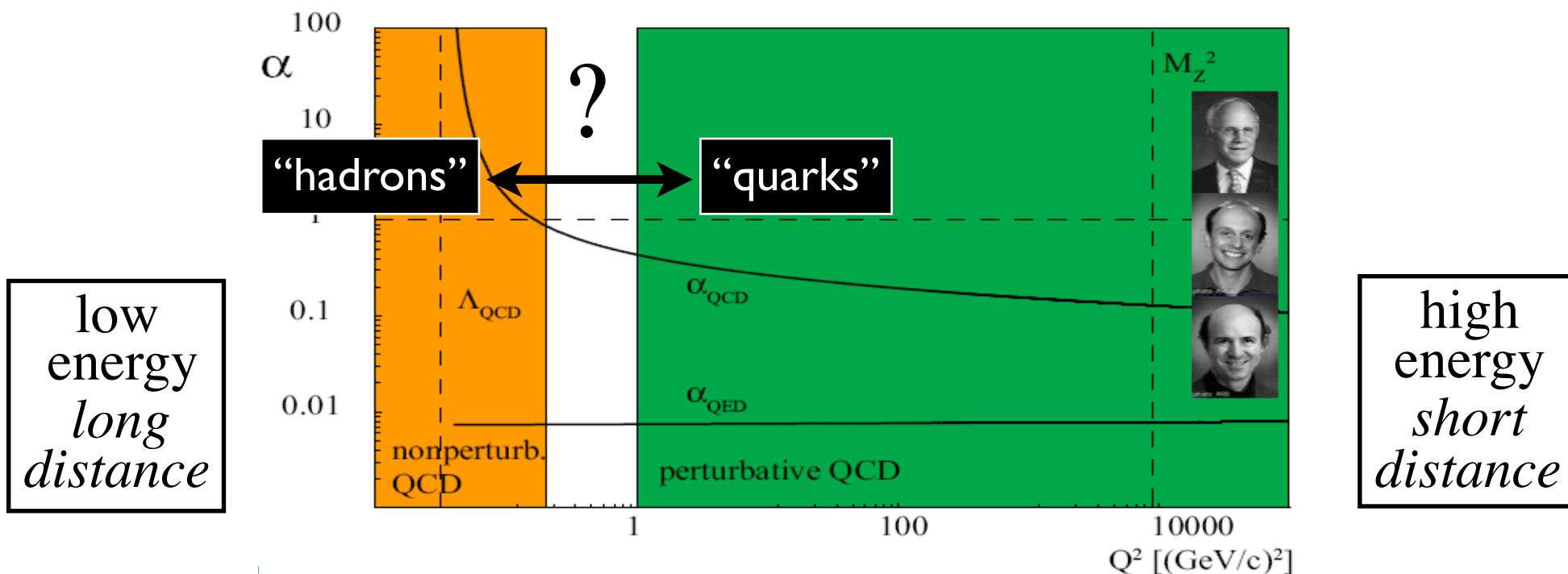
# Connecting resonance and deep-inelastic phenomena through quark-hadron duality

*Wally Melnitchouk*



# Outline

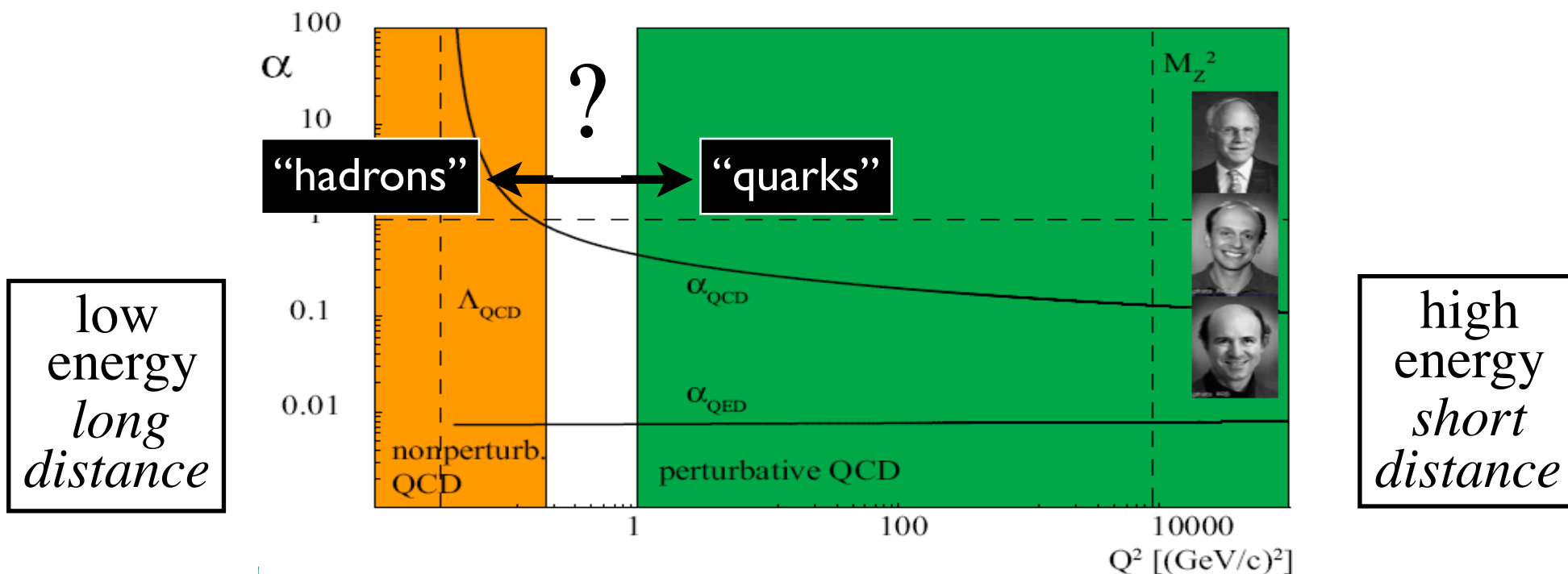
- Historical perspective
- Duality in QCD
  - resonances & higher twists
- Local duality
  - truncated moments
  - insights from models
- Applications of duality
  - global PDF analysis
  - single-hadron production
- Outlook



- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

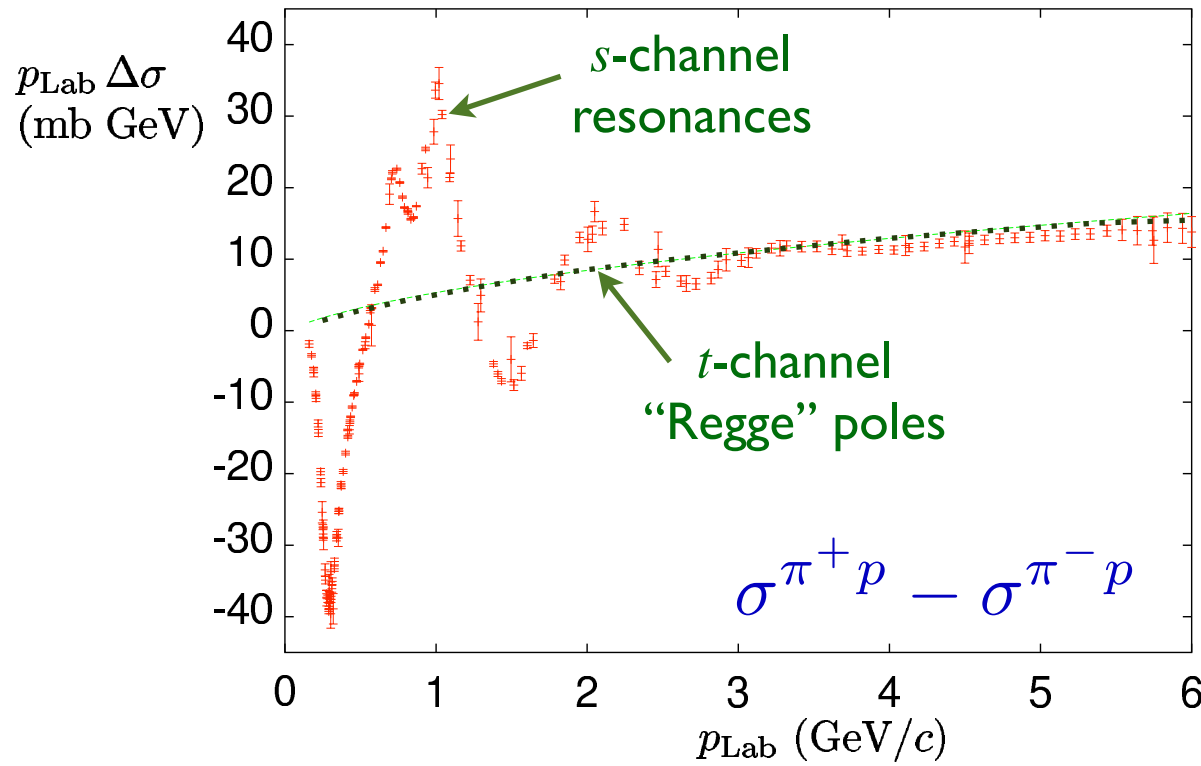
$$\sum_{\text{hadrons}} = \sum_{\text{quarks}}$$

→ can use either set of *complete* basis states to describe physical phenomena

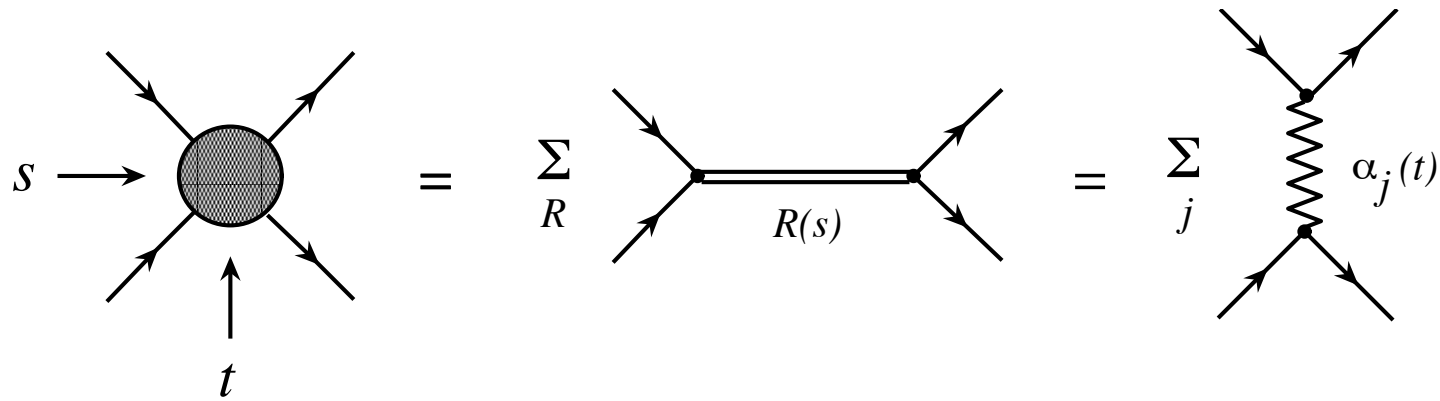


- In practice, at *finite energy* typically access only *limited* set of basis states
- Question is not “*why* duality exists”, but
  - *how* it arises?
  - how can we make *use* of it (in a controlled way)?

# Duality in hadron-hadron scattering



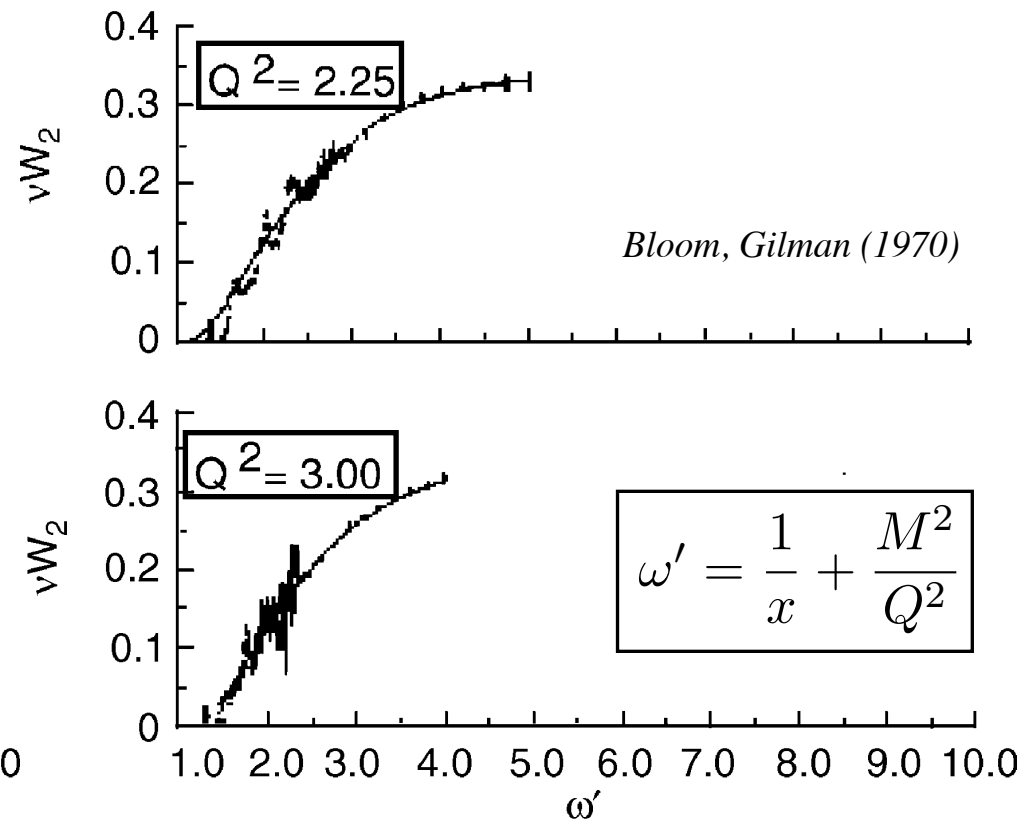
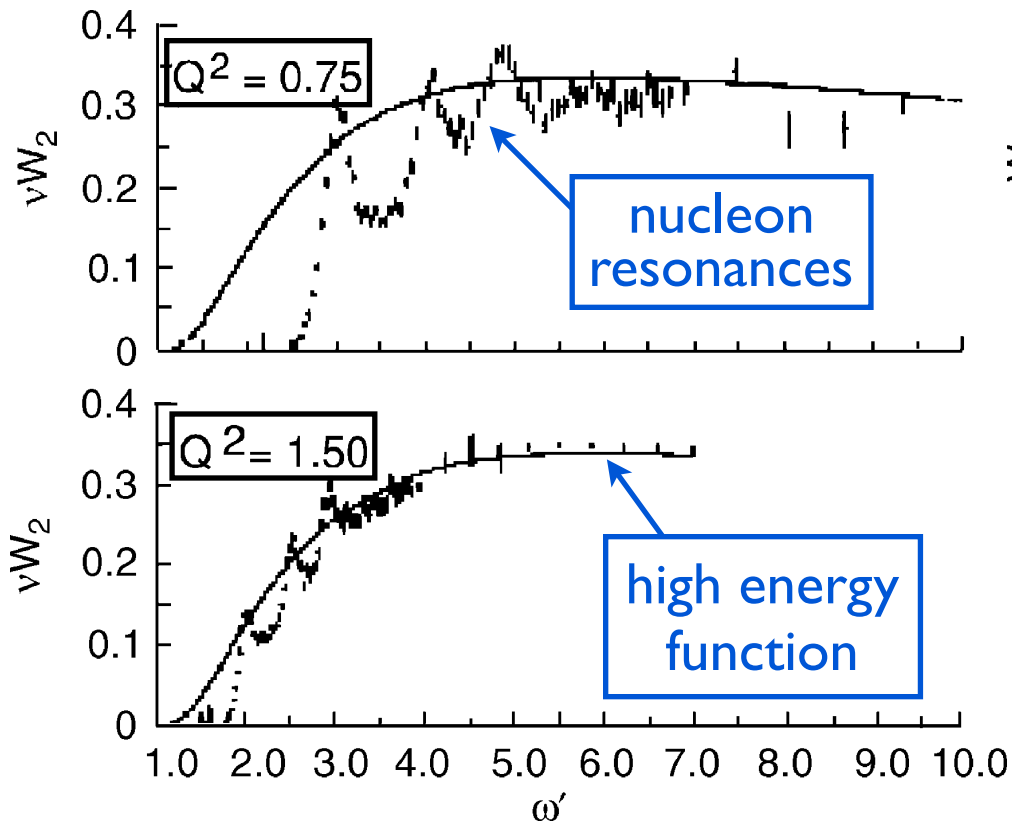
*Igi (1962)*  
*Dolen, Horn, Schmidt (1968)*



*s - t channel duality*

# Duality in electron-proton scattering

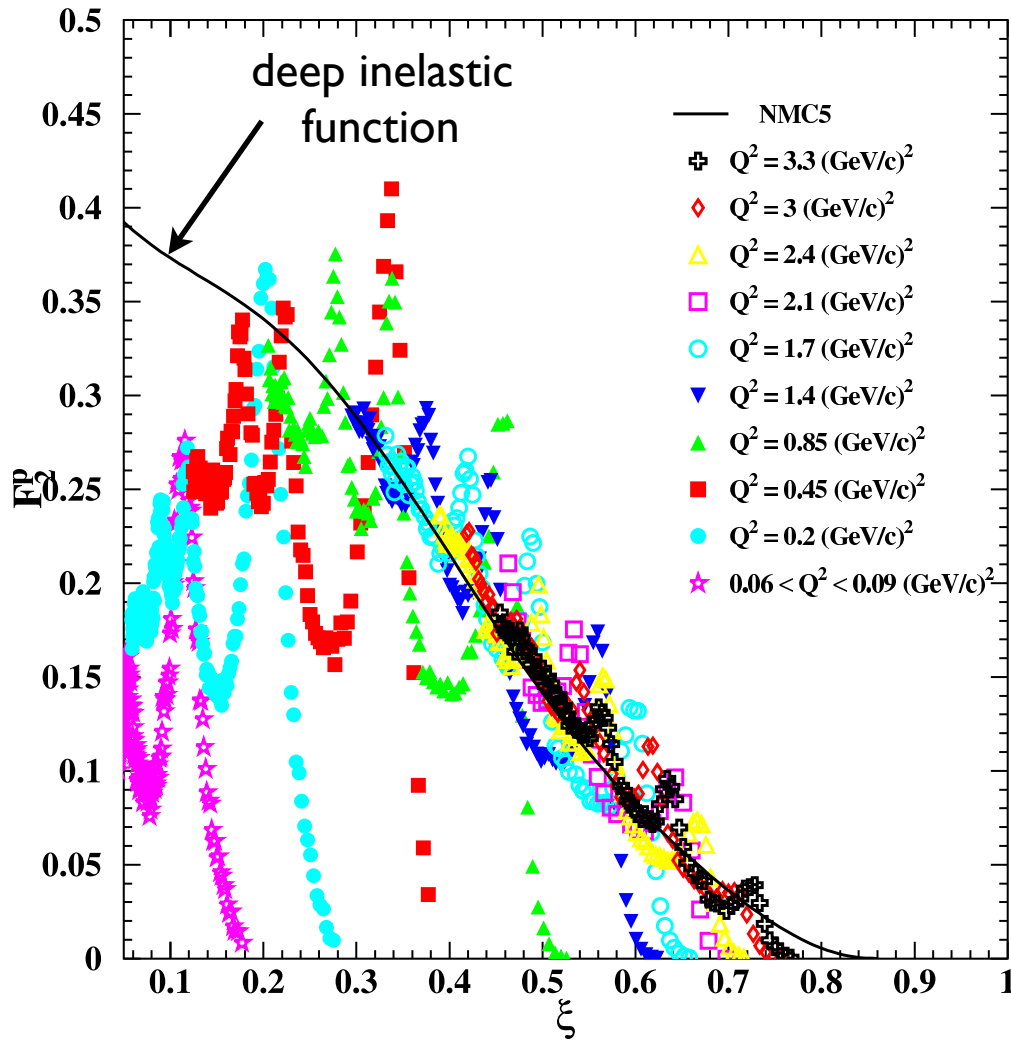
“Bloom-Gilman duality”



“hadrons”  $\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \nu W_2(\omega')$  “quarks”

finite-energy sum rules

# Duality in electron-proton scattering



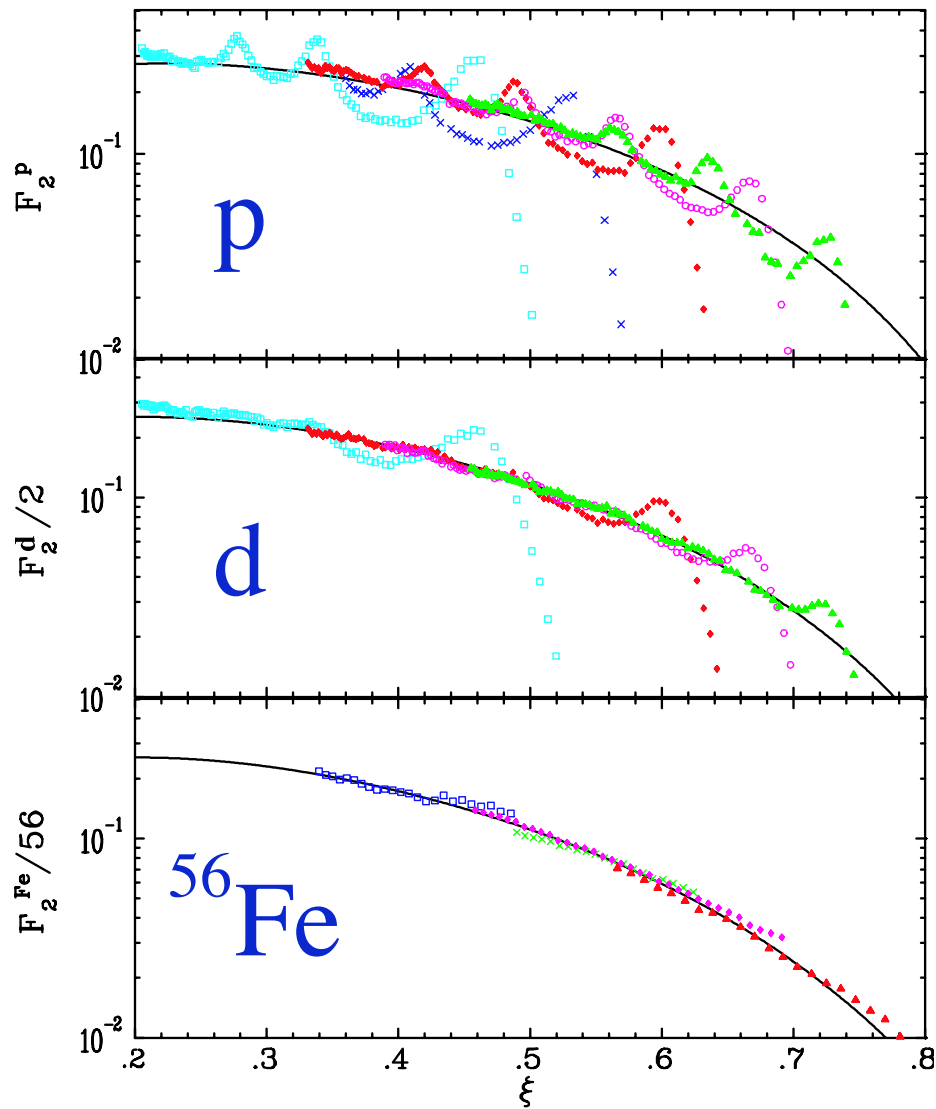
Niculescu et al. (2000)

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

■ average over resonances  
(strongly  $Q^2$  dependent)

$\approx Q^2$  independent  
scaling function

# Duality in electron-nucleus scattering



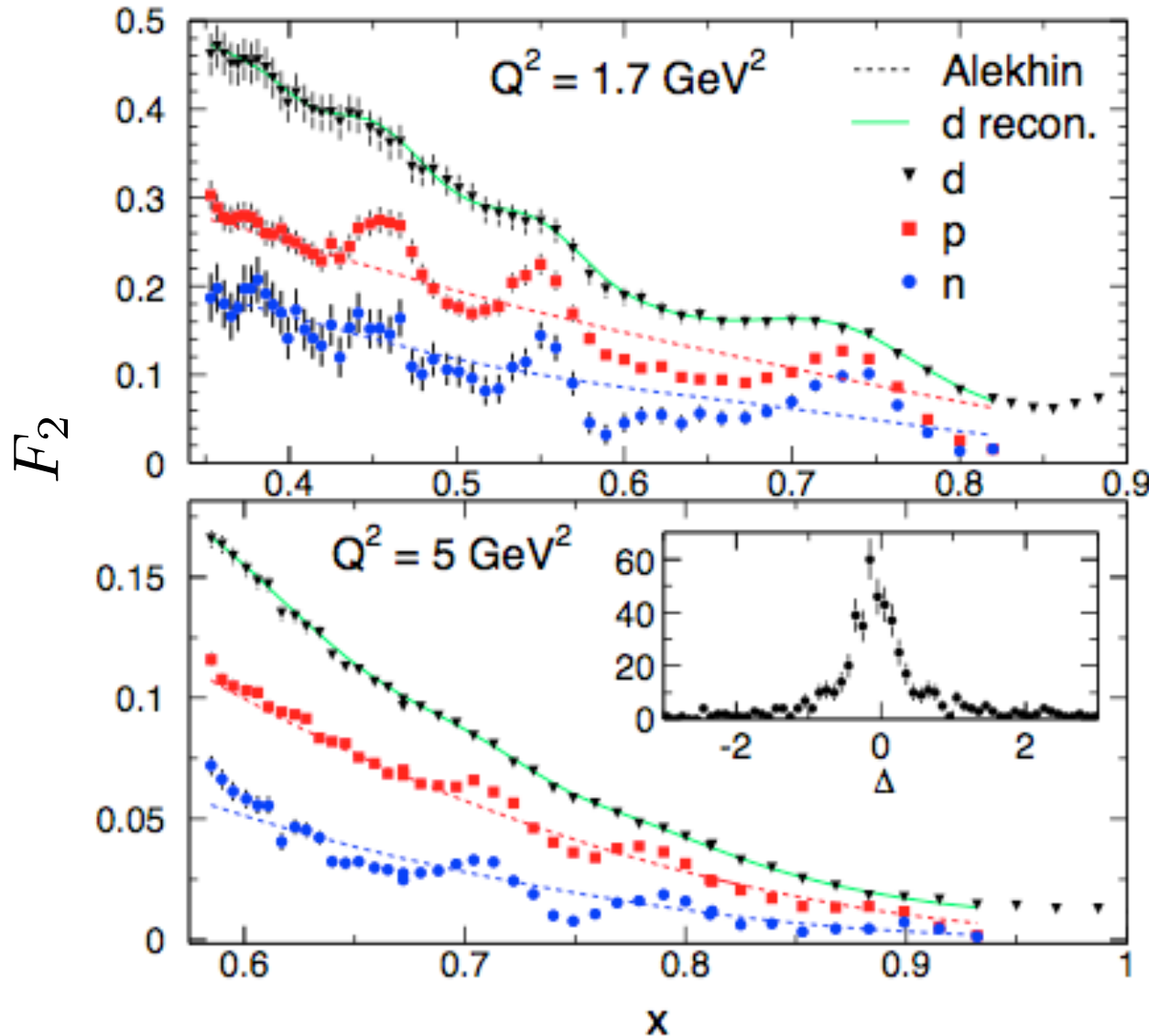
- further resonance averaging from Fermi smearing in *nuclear* structure functions

WM, Ent, Keppel (2005)



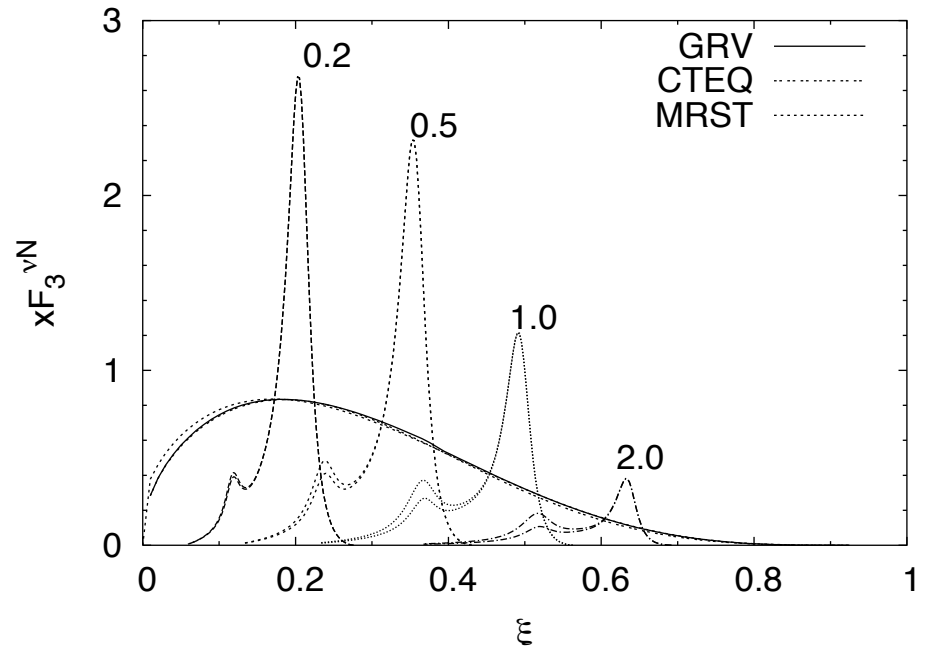
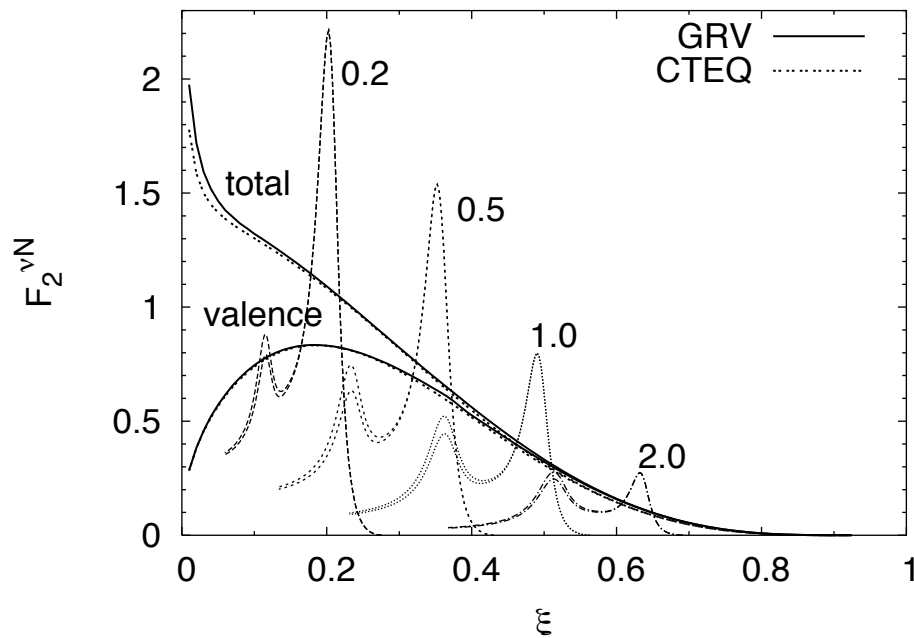
# Duality in electron-neutron scattering

No free neutron targets, but (new) iterative method allows neutron resonance structure function to be extracted



■ evidence for duality also in neutron!

# Duality in neutrino-nucleon scattering



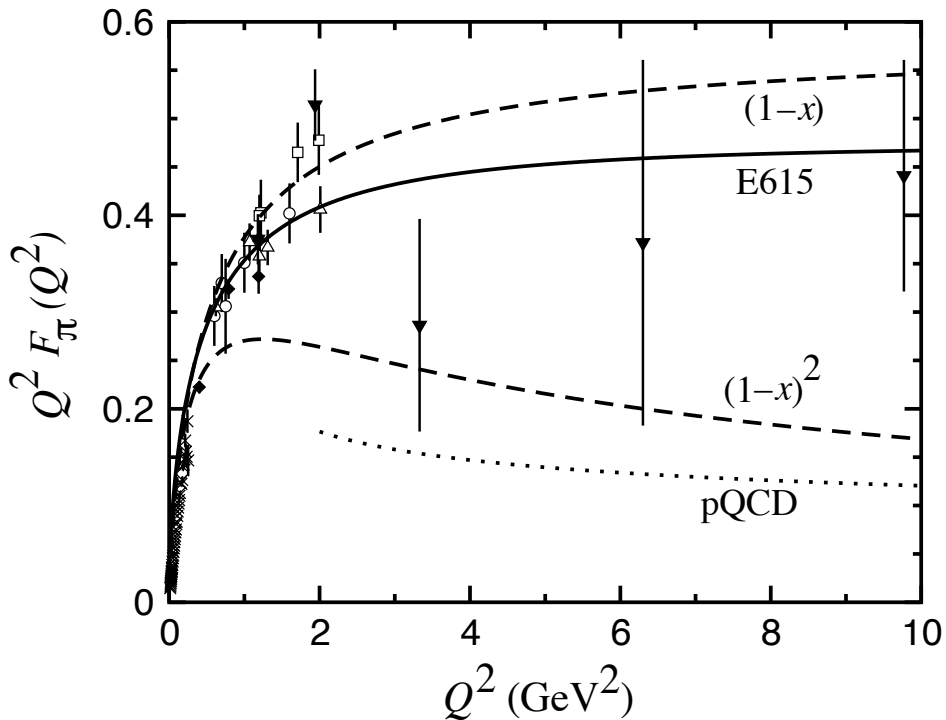
*Lalakulich, WM, Paschos (2007)*

- indications of duality in neutrino structure functions  
from models of weak transition matrix elements  
from resonance neutrino-production data (FNAL, ANL)

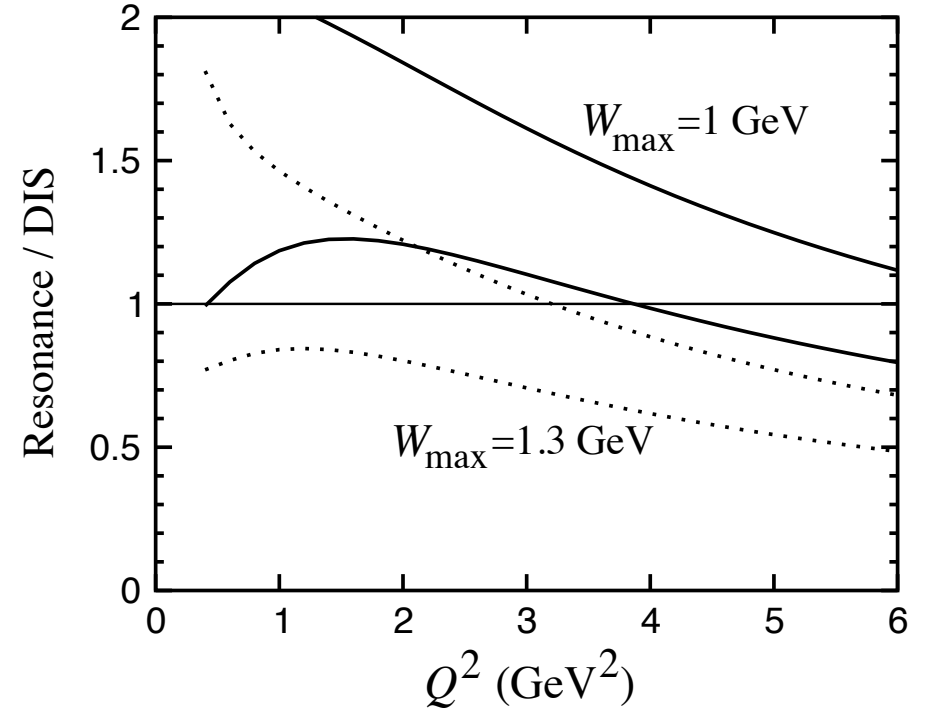
# Duality in electron-pion scattering (?)

Extend finite-energy sum rule to threshold region

$$[F_\pi(Q^2)]^2 \approx \int_1^{\omega_{\max}} d\omega \nu W_2^\pi(\omega) \quad \omega = 1/x, \quad \omega_{\max} = 1 + (W_{\max}^2 - m_\pi^2)/Q^2$$



→  $Q^2$  dependence of FF correlated with  $x \rightarrow 1$  behavior of SF



→ nontrivial relation between  $L$  and  $T$  cross sections?

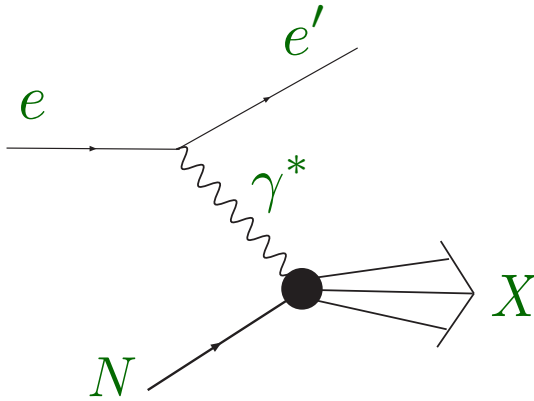
$$F_\pi^2 + \left(1 + \frac{m_\rho^2 - m_\pi^2}{Q^2}\right) F_{\pi\rho}^2$$

# Duality in QCD

— *global duality* —

# Duality and QCD

## ■ Kinematics of inclusive deep-inelastic scattering (DIS)

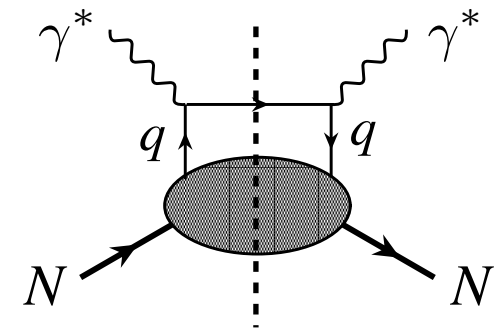


$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left( 2 \tan^2 \frac{\theta}{2} \frac{F_1}{2M} + \frac{F_2}{\nu} \right)$$

$$\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^2 - \nu^2 \\ W^2 &= M^2 + Q^2 \frac{(1-x)}{x} \end{aligned} \quad x = \frac{Q^2}{2M\nu}$$

## ■ In *deep-inelastic* region ( $W \gtrsim 2 \text{ GeV}$ , $Q^2 \gtrsim 1 \text{ GeV}^2$ ) structure functions given by parton distributions

$$F_2(x, Q^2) \stackrel{\text{LO}}{=} x \sum_q e_q^2 q(x, Q^2)$$



# Duality and QCD

## ■ Operator product expansion in QCD

→ expand *moments* of structure functions in powers of  $1/Q^2$

$$\begin{aligned}
 M_n(Q^2) &= \int_0^1 dx x^{n-2} F_2(x, Q^2) \\
 &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots
 \end{aligned}$$

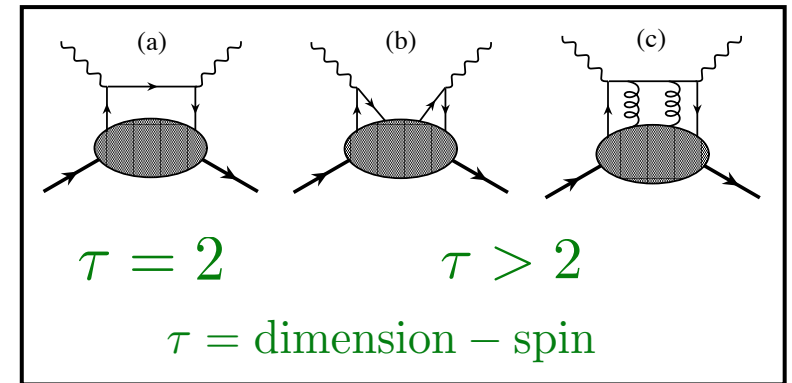


matrix elements of operators  
with specific “twist”  $\tau$

e.g.  $\langle N | \bar{\psi} \gamma^+ \psi | N \rangle$

$\langle N | \bar{\psi} \tilde{G}^{+\nu} \gamma_\nu \psi | N \rangle$

etc.



# Duality and QCD

## ■ Operator product expansion in QCD

→ expand *moments* of structure functions in powers of  $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

## ■ If moment $\approx$ independent of $Q^2$

→ “higher twist” terms  $A_n^{(\tau>2)}$  small

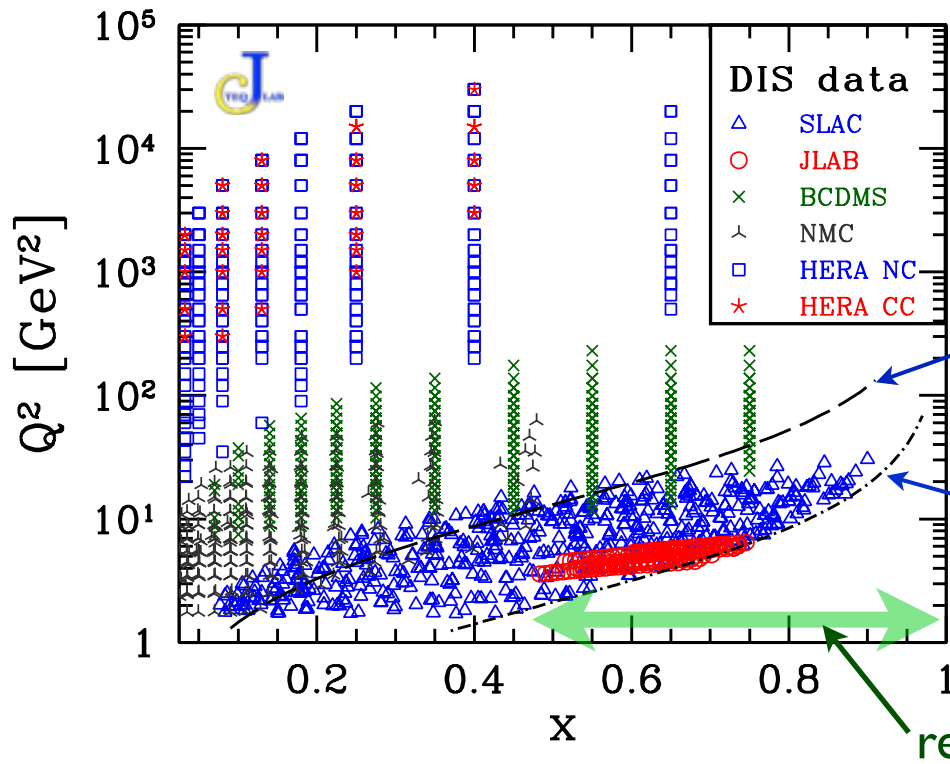
## ■ Duality $\longleftrightarrow$ suppression of higher twists

# Duality and QCD

- Note: at finite  $Q^2$ , from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

$$W^2 = M^2 + Q^2 \frac{(1-x)}{x} \quad \longrightarrow \quad x_{\text{res}} = \frac{Q^2}{W_{\text{res}}^2 - M^2 + Q^2}$$

$$W_{\text{res}} = 2 \text{ GeV} \quad \Longrightarrow \quad x_{\text{res}} \approx 0.24 \text{ at } Q^2 = 1 \text{ GeV}^2$$



$$W^2 > 12.25 \text{ GeV}^2$$

$$W^2 > 3 \text{ GeV}^2$$

resonances



# Duality and QCD

- Note: at finite  $Q^2$ , from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region
  
- Resonance and DIS regions intimately connected
  - resonances an *integral* part of scaling structure function
  - e.g.* in large- $N_c$  limit, spectrum of zero-width resonances is “maximally dual” to quark-level (smooth) structure function

# Local Duality

— *truncated moments* —

# Truncated moments

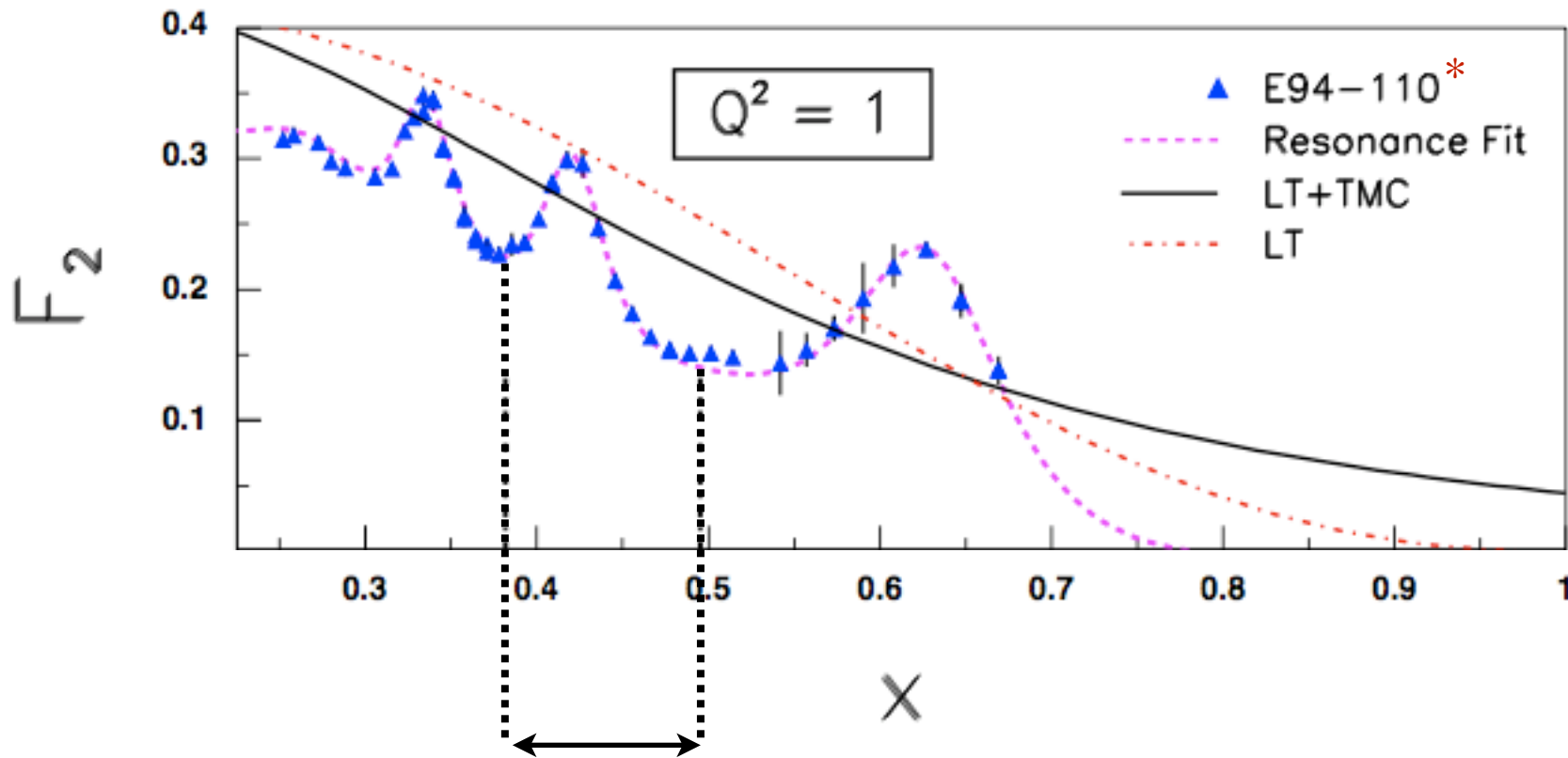
- Complete moments can be studied via twist expansion
  - Bloom–Gilman duality has a precise meaning  
(*i.e.*, duality violation = higher twists)
- Rigorous connection between local duality & QCD difficult
  - need prescription for how to average over resonances
- *Truncated* moments allow study of restricted regions in  $x$  (or  $W$ ) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx x^{n-2} F_2(x, Q^2)$$

*Forte, Magnea (1999)*

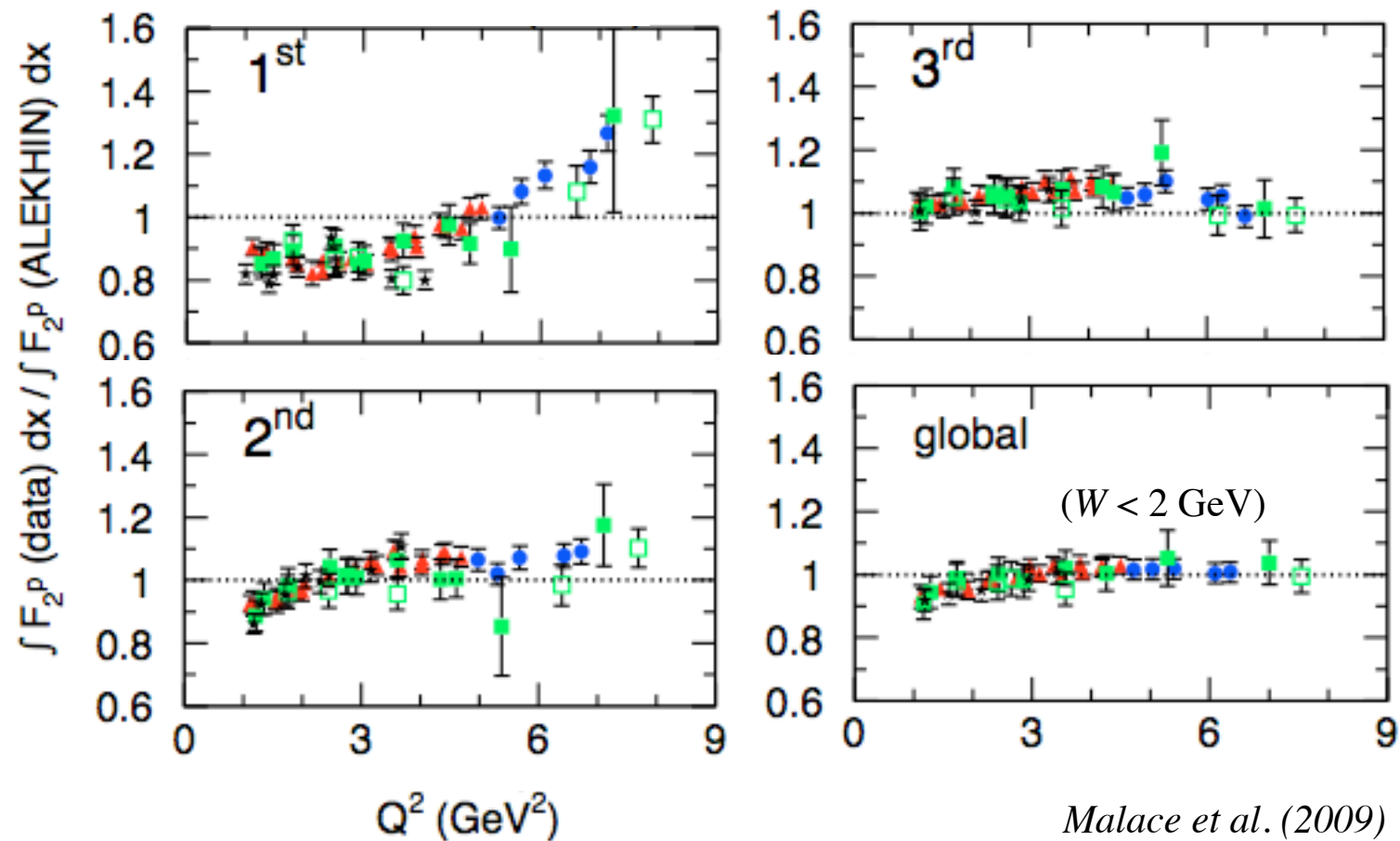
*Psaker, Malace, Keppel, WM (2008)*

# Truncated moments



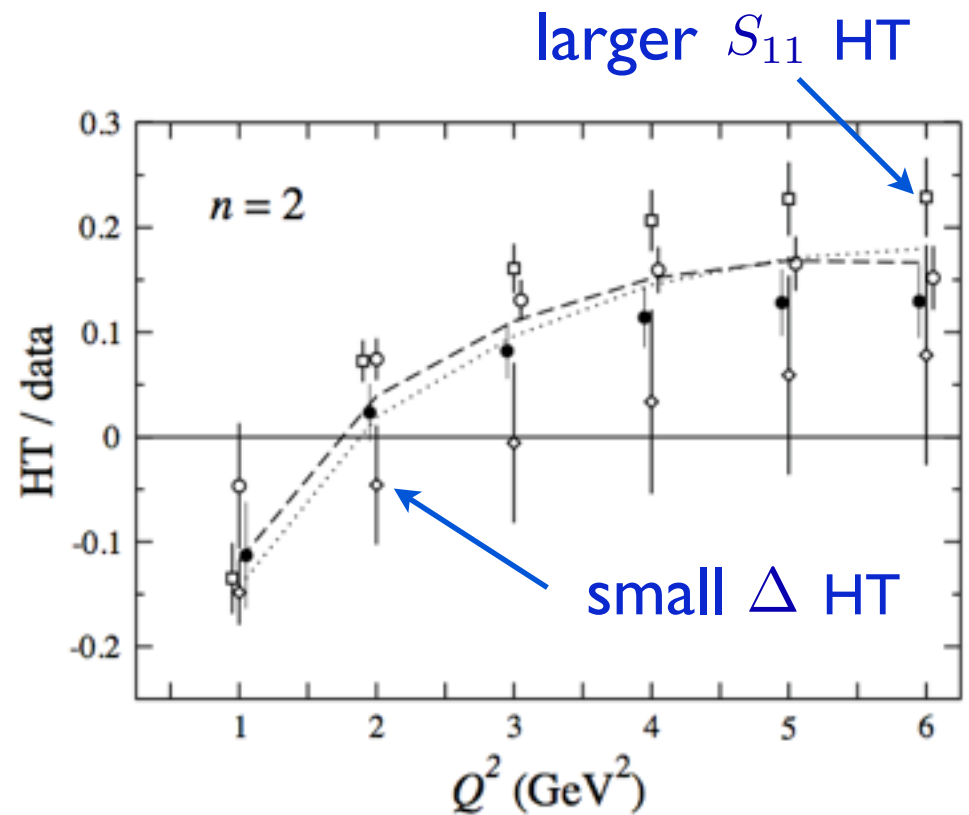
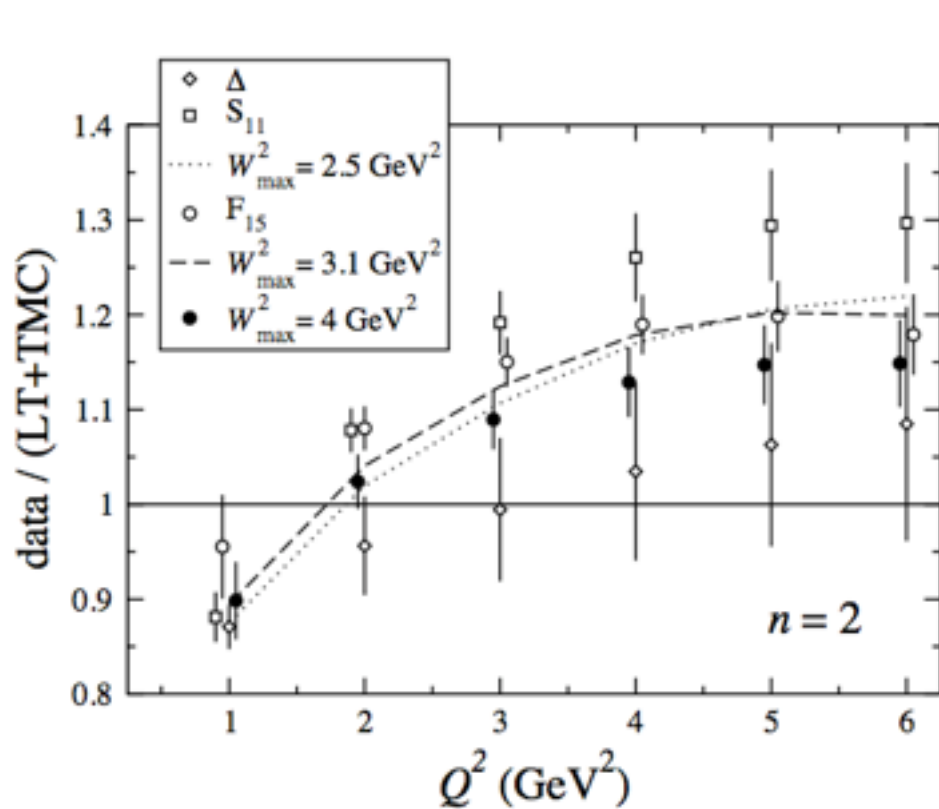
how much of this region is leading twist ?

# Truncated moments



→ duality appears in various resonance regions

# Truncated moments



→ higher twists < 10–15% for  $Q^2 > 1 \text{ GeV}^2$

## Resonances & twists

- Total “higher twist” is *small* at scales  $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
  - nontrivial interference between resonances
- Can we understand this dynamically, at quark level?
- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?
  - expanded data set has potentially significant implications for global quark distribution studies

# Local Duality

— *insights from models* —



# Scaling functions from resonances

## ■ Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum  $M_n^2 = (n + 1)\Lambda^2$   
including states with spin = 1/2, ...,  $n+1/2$

( $n$  even:  $I = 1/2$ ,  $n$  odd:  $I = 3/2$ )

*Domokos et al. (1971)*

→ at large  $Q^2$  magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2 / M_n^2)^2} \quad r^2 \approx 1.41$$

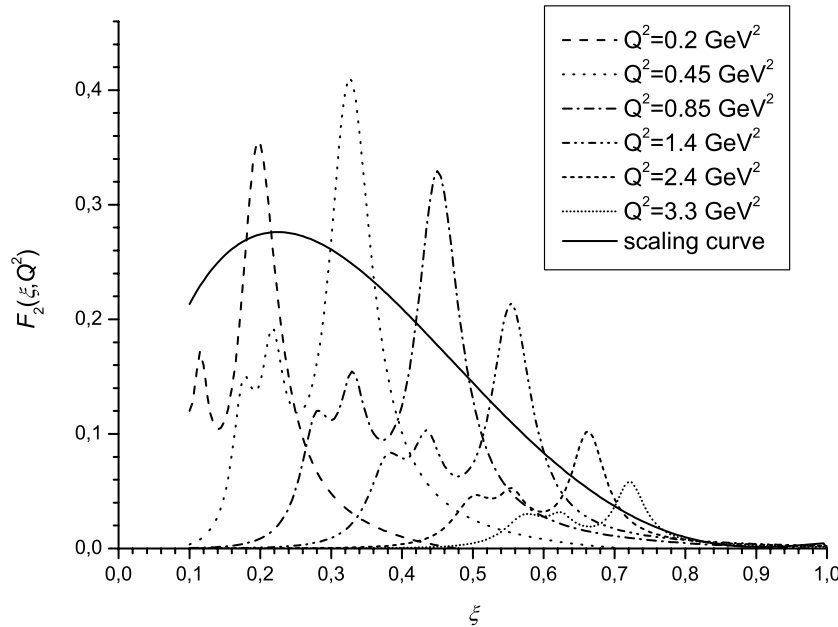
→ in Bjorken limit,  $\sum_n \rightarrow \int dz$ ,  $z \equiv M_n^2 / Q^2$

$$F_2 \sim (\omega' - 1)^{1/2} (\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2} (1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

→ scaling function of  $\omega' = \omega + M^2 / Q^2$  ( $\omega = 1/x$ )

# Scaling functions from resonances

## ■ Phenomenological analyses at finite $Q^2$



21 isospin-1/2 & 3/2  
resonances (mass < 2 GeV)

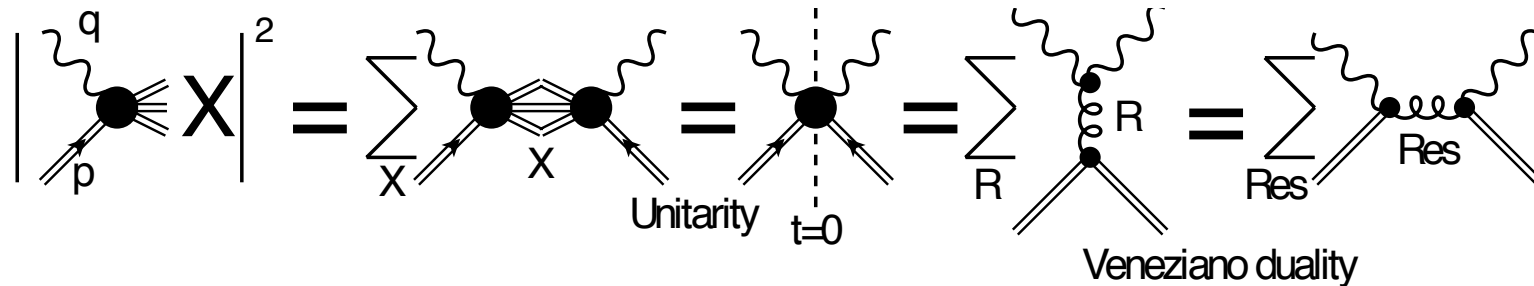
*Davidovsky, Struminsky (2003)*

→ valence-like structure of dual function suggests  
“two-component duality”:

- valence (Reggeon exchange) dual to resonances  $F_2^{(\text{val})} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background  $F_2^{(\text{sea})} \sim x^{-0.08}$

# Scaling functions from resonances

## ■ Explicit realization of Veneziano & Bloom-Gilman duality



$$V(s, t) = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(2 - \alpha(s) - \alpha(t))}$$

$$\rightarrow s^{\alpha(t)} \quad \text{high } s, \text{ low } |t|$$

→ Veneziano model not unitary,  
has no imaginary parts

→ generalization of narrow-resonance approximation,  
with nonlinear, complex Regge trajectories

$$D(s, t) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s(1-z))-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(tz)-1}$$

“dual amplitude with Mandelstam analyticity” (DAMA) model

*Jenkovszky et al.*

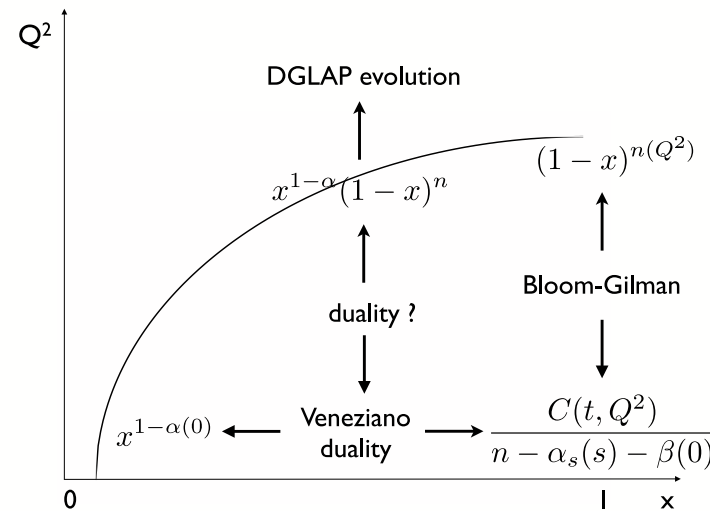
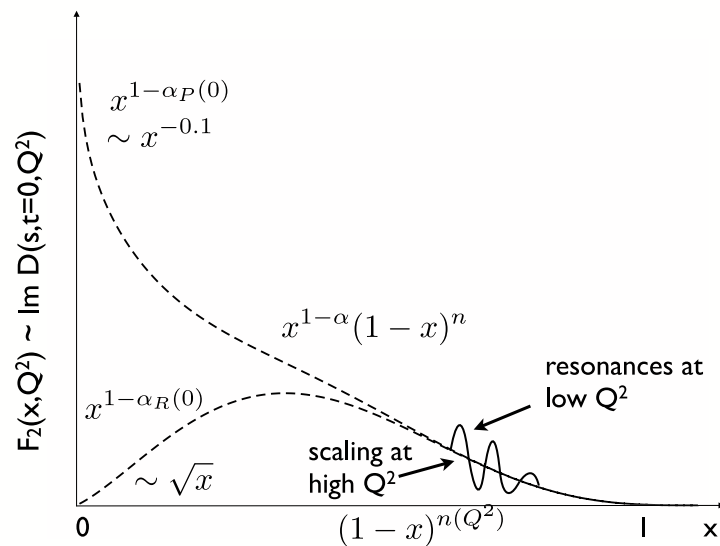
# Scaling functions from resonances

## ■ Explicit realization of Veneziano & Bloom-Gilman duality

→ for large  $x$  and  $Q^2$ , have power-law behavior

$$F_2 \sim (1-x)^{2\alpha_t(0) \ln 2g / \ln g}$$

where parameter  $g$  can be  $Q^2$  dependent



*Jenkowszky, Magas, Londergan, Szczepaniak (2012)*

# Applications of Duality

# CTEQ-JLab (CJ) global PDF analysis

- Global QCD analysis of high-energy scattering data, including large- $x$ , low- $Q^2$  region
- Systematically study effects of  $Q^2$  &  $W$  cuts

cut0:  $Q^2 > 4 \text{ GeV}^2$ ,  $W^2 > 12.25 \text{ GeV}^2$

cut1:  $Q^2 > 3 \text{ GeV}^2$ ,  $W^2 > 8 \text{ GeV}^2$

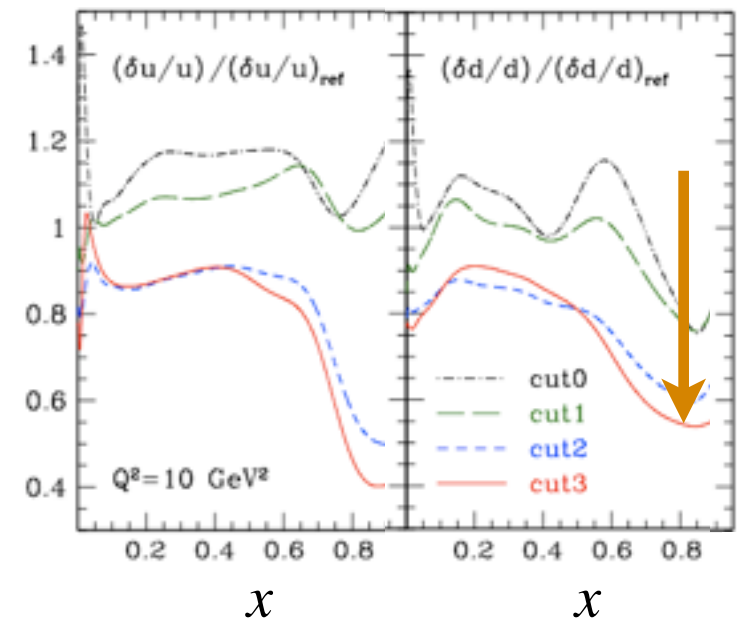
cut2:  $Q^2 > 2 \text{ GeV}^2$ ,  $W^2 > 4 \text{ GeV}^2$

cut3:  $Q^2 > m_c^2$ ,  $W^2 > 3 \text{ GeV}^2$

factor 2 increase  
in DIS data from  
cut0  $\rightarrow$  cut3

$\rightarrow$  larger database with weaker cuts significantly reduced errors, especially at large  $x$

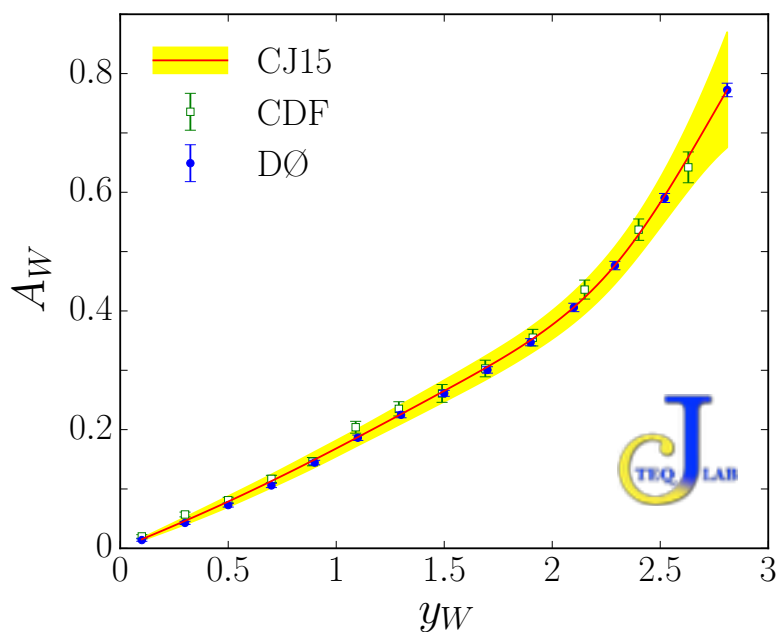
$\rightarrow$  up to  $\sim 40\text{--}60\%$  error reduction when cuts extended into near-resonance region



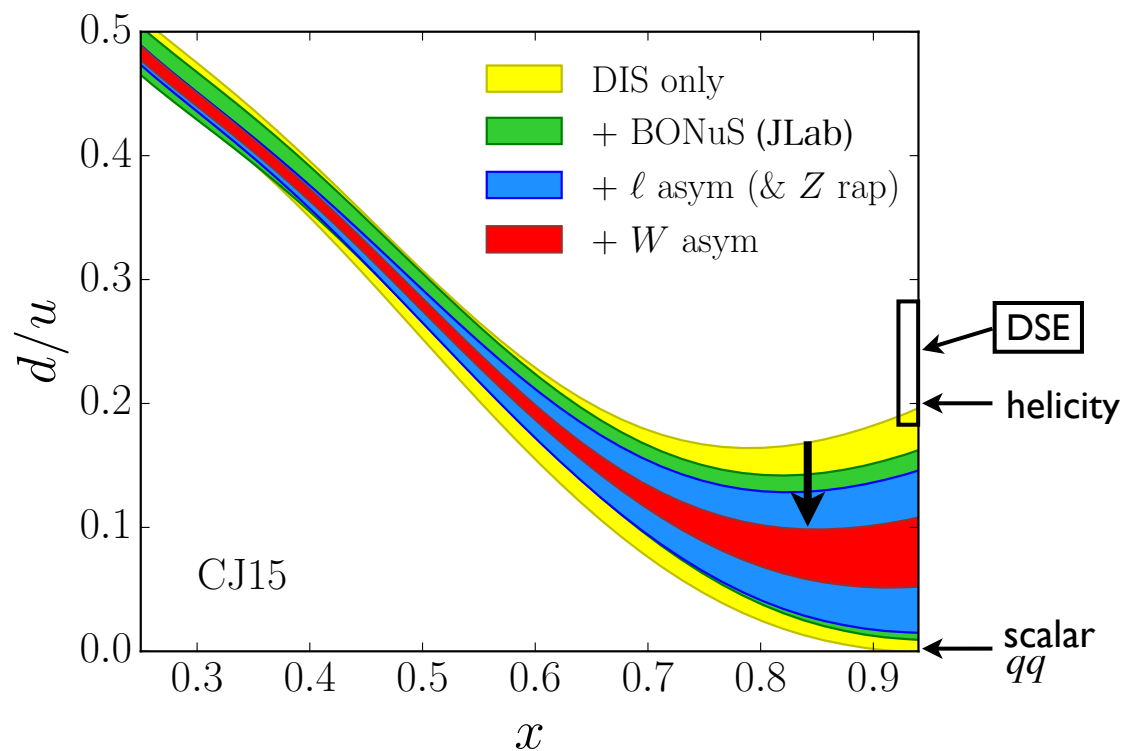
# CTEQ-JLab (CJ) global PDF analysis

## Valence $d/u$ ratio at high $x$

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL  $W$ -asymmetry data



Accardi, WM, Owens (2016)



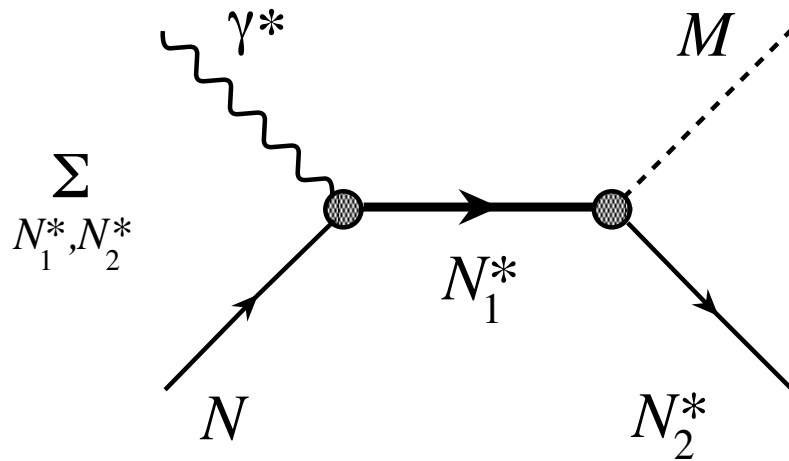
→ extrapolated ratio at  $x = 1$

$$d/u \rightarrow 0.09 \pm 0.03$$

→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine  $d/u$  up to  $x \sim 0.85$

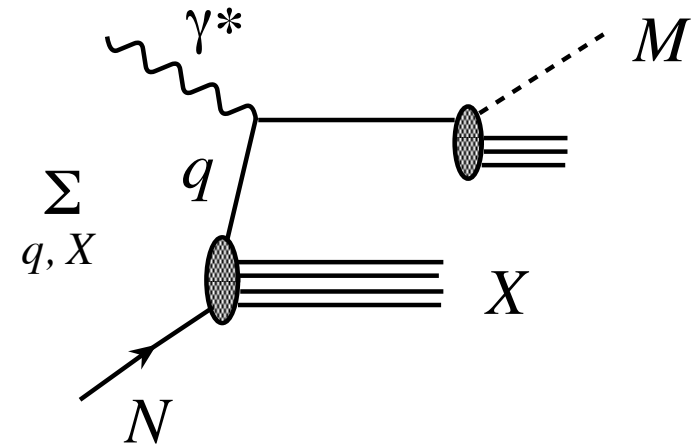
# Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction



$s$ -channel resonance  
excitation and decay

=



parton level scattering  
and fragmentation

$$\sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \rightarrow N_1^*}(Q^2, M_1^*) \mathcal{D}_{N_1^* \rightarrow N_2^* M}(M_1^*, M_2^*) \right|^2 = \sum_q e_q^2 q(x, Q^2) D_q^M(z, Q^2)$$

*Afanasev, Carlson, Wahlquist (2000)*  
*Hoyer (2002)*  
*Close, WM (2009)*



# Outlook

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances
  - explore more realistic descriptions based on phenomenological  $\gamma^* NN^*$  form factors (→ CLAS  $N^*$  program)
  - incorporate nonresonant background in same framework
- Practical application of duality
  - use resonance region data to constrain PDFs at high  $x$
  - application to other processes, *e.g.*, semi-inclusive DIS, DVCS / GPDs, ...