Effective Lagrangian for interactions of high-spin baryons

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Exploring Hadrons and Electromagnetic Probes:
Structure, Excitations, Interactions
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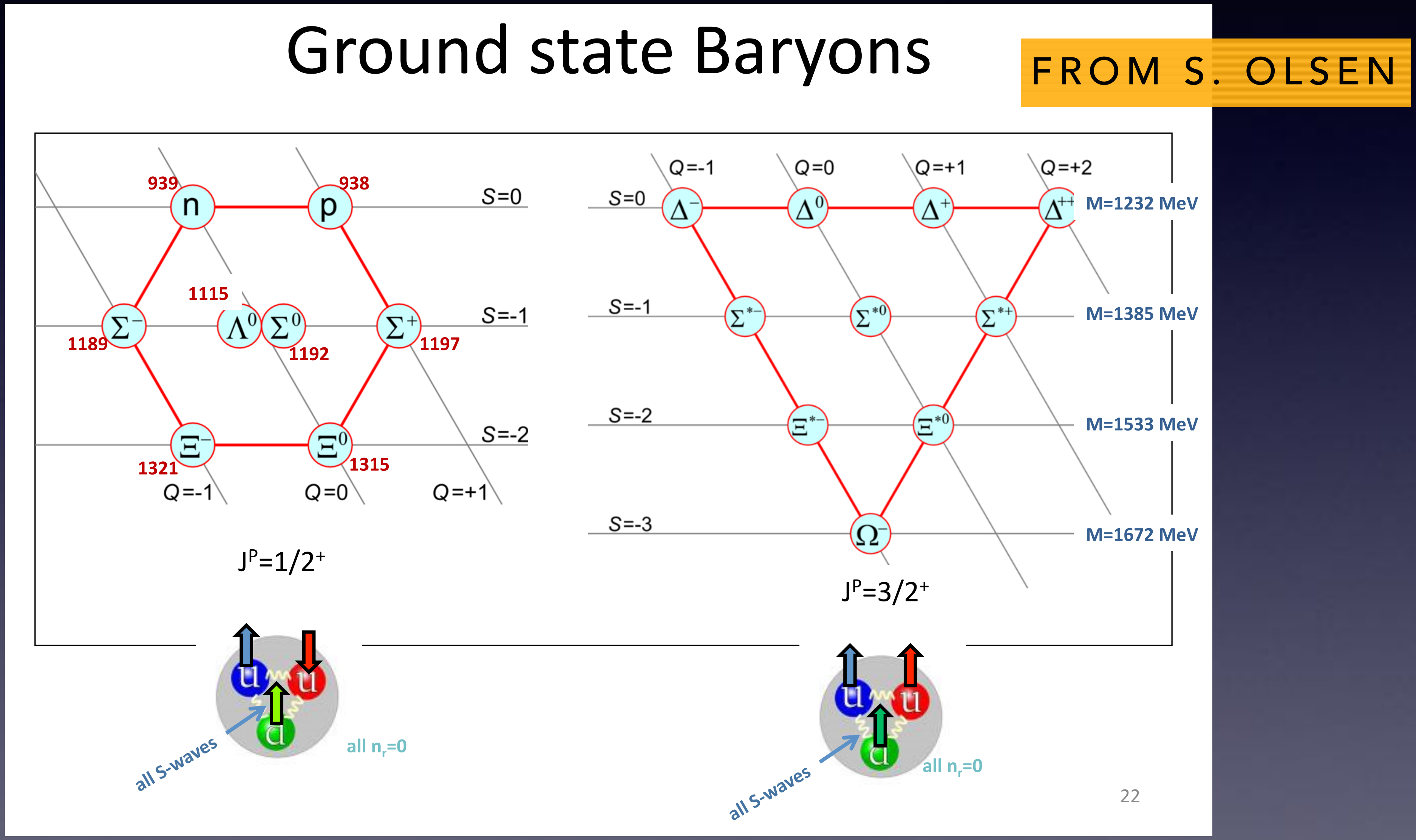
- Introduction & Motivation
- Formalism
- Decay of baryons: $R \longrightarrow \pi\, N$, $R \longrightarrow V\, N$ or $\gamma\, N$
- Summary and Outlook

Introduction & Motivation

- Ground state baryons



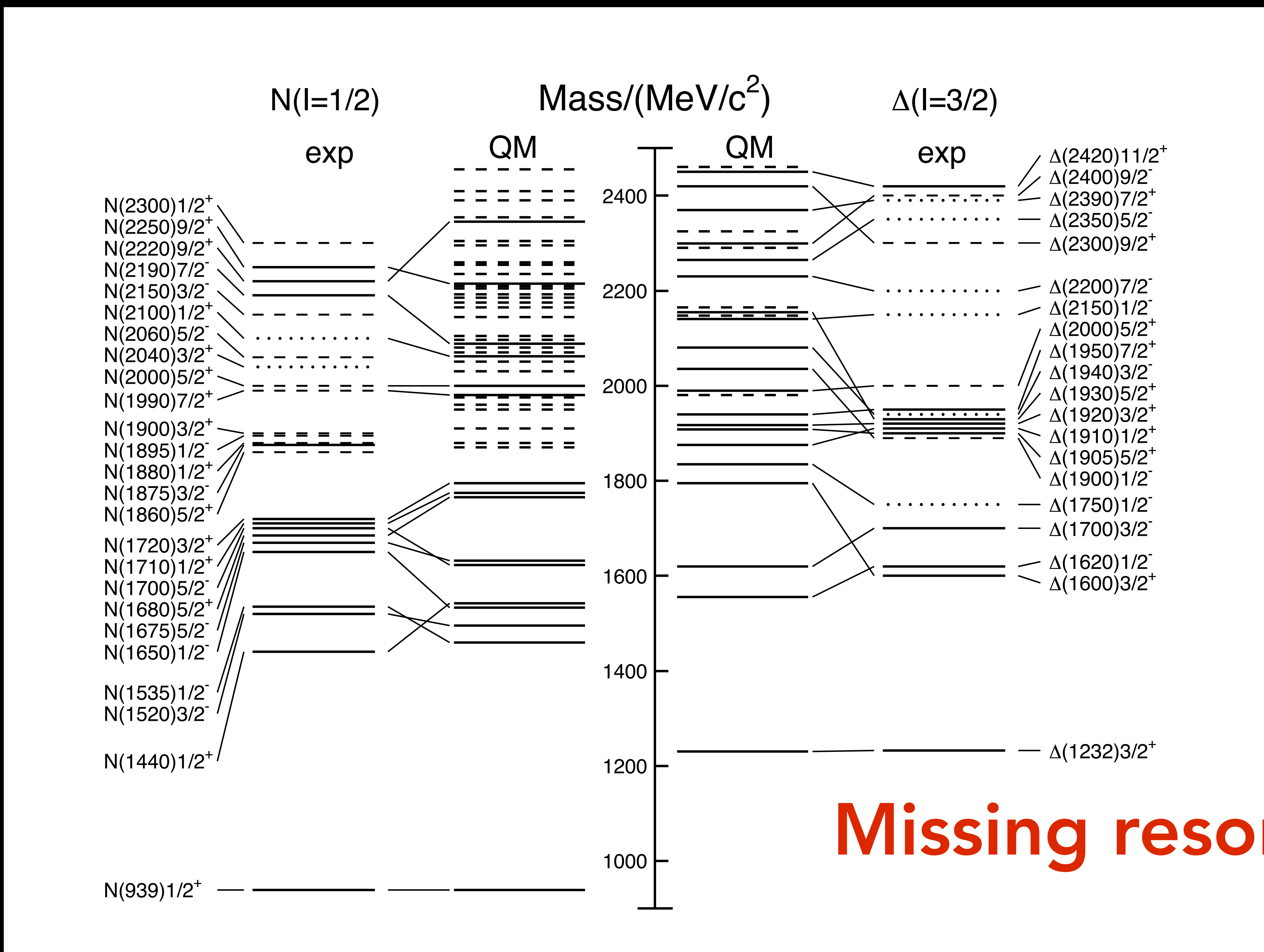
- Most static properties of the ground state baryons are governed by the group structure.
- How can we get information on the dynamics of the constituents of hadrons?

Introduction & Motivation

Excitation Spectrum of the nucleon

orbital excitations, radial excitations

$$\boldsymbol{J} = \boldsymbol{S} + \boldsymbol{L}$$

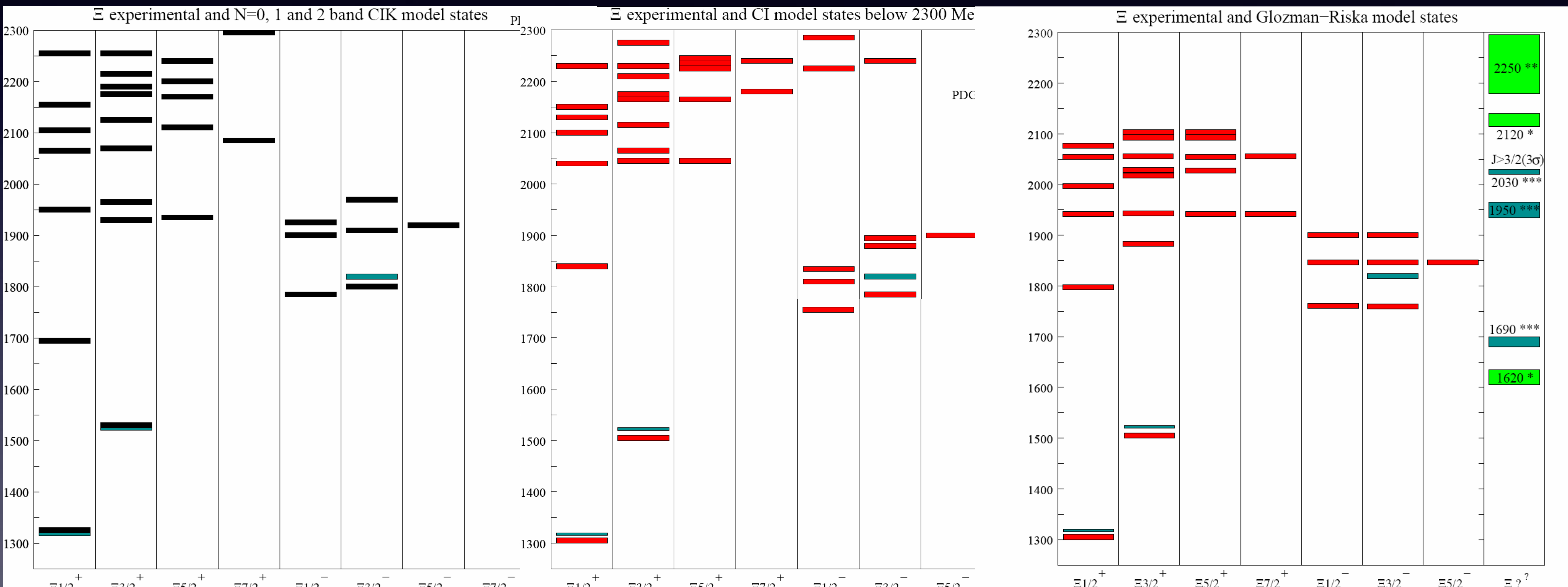


Particle Data Group

Missing resonances problem

Introduction & Motivation

Sensitivity of baryon spectrum on dynamics



NRQM

Chao, Isgur, Karl

RQM

Isgur, Karl

OBEM

Glozman, Riska

Introduction & Motivation

Highly model-dependent and sensitive to dynamics

Table 1. Low-lying Ξ and Ω baryon spectrum of spin 1/2 and 3/2 predicted by the non-relativistic quark model of Chao *et al.* (CIK), relativized quark model of Capstick and Isgur (CI), Glozman-Riska model (GR), large N_c analysis, algebraic model (BIL), and QCD sum rules (SR). The recent quark model prediction (QM) and the Skyrme model results (SK) are given as well. The mass is given in the unit of MeV.

State	CIK [4]	CI [5]	GR [6]	Large-N_c [7–11]	BIL [12]	SR [13, 14]	QM [15]	SK [1]
$\Xi(\frac{1}{2}^+)$	1325	1305	1320		1334	1320 (1320)	1325	1318
	1695	1840	1798	1825	1727		1891	1932
	1950	2040	1947	1839	1932		2014	
$\Xi(\frac{3}{2}^+)$	1530	1505	1516		1524		1520	1539
	1930	2045	1886	1854	1878		1934	2120
	1965	2065	1947	1859	1979		2020	
$\Xi(\frac{1}{2}^-)$	1785	1755	1758	1780	1869	1550 (1630)	1725	1614
	1890	1810	1849	1922	1932		1811	1660
	1925	1835	1889	1927	2076			
$\Xi(\frac{3}{2}^-)$	1800	1785	1758	1815	1828	1840	1759	1820
	1910	1880	1849	1973	1869		1826	
	1970	1895	1889	1980	1932			
$\Omega(\frac{1}{2}^+)$	2190	2220	2068	2408	2085		2175	2140
	2210	2255	2166		2219		2191	
$\Omega(\frac{3}{2}^+)$	1675	1635	1651		1670		1656	1694
	2065	2165	2020	1922	1998		2170	2282
	2215	2280	2068	2120	2219		2182	
$\Omega(\frac{1}{2}^-)$	2020	1950	1991	2061	1989		1923	1837
$\Omega(\frac{3}{2}^-)$	2020	2000	1991	2100	1989		1953	1978

Introduction & Motivation

Particle	$L_{2I\cdot 2J}$	Overall status
$N(939)$	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1\,11}$	***
$N(2700)$	$K_{1\,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3\,11}$	****
$\Delta(2750)$	$I_{3\,13}$	**
$\Delta(2950)$	$K_{3\,15}$	**

Λ states					Σ states				
State	J^P	Γ (MeV)	Rating	$\lvert g_{N\Lambda K}\rvert$	State	J^P	Γ (MeV)	Rating	$\lvert g_{N\Sigma K}\rvert$
$\Lambda(1116)$	$1/2^+$		****		$\Sigma(1193)$	$1/2^+$		****	
$\Lambda(1405)$	$1/2^-$	≈ 50	****		$\Sigma(1385)$	$3/2^+$	≈ 37	****	
$\Lambda(1520)$	$3/2^-$	≈ 16	****						
$\Lambda(1600)$	$1/2^+$	≈ 150	***	4.2	$\Sigma(1660)$	$1/2^+$	≈ 100	***	2.5
$\Lambda(1670)$	$1/2^-$	≈ 35	****	0.3	$\Sigma(1670)$	$3/2^-$	≈ 60	****	2.8
$\Lambda(1690)$	$3/2^-$	≈ 60	****	4.0	$\Sigma(1750)$	$1/2^-$	≈ 90	***	0.5
$\Lambda(1800)$	$1/2^-$	≈ 300	***	1.0	$\Sigma(1775)$	$5/2^-$	≈ 120	****	
$\Lambda(1810)$	$1/2^+$	≈ 150	***	2.8	$\Sigma(1915)$	$5/2^+$	≈ 120	****	
$\Lambda(1820)$	$5/2^+$	≈ 80	****		$\Sigma(1940)$	$3/2^-$	≈ 220	***	< 2.8
$\Lambda(1830)$	$5/2^-$	≈ 95	****		$\Sigma(2030)$	$7/2^+$	≈ 180	****	
$\Lambda(1890)$	$3/2^+$	≈ 100	****	0.8	$\Sigma(2250)$	$?^?$	≈ 100	***	
$\Lambda(2100)$	$7/2^-$	≈ 200	****						
$\Lambda(2110)$	$5/2^+$	≈ 200	***						
$\Lambda(2350)$	$9/2^+$	≈ 150	***						

At the mass region of > 1.8 GeV, many resonances are high spin states

$j \geq 5/2$

Introduction & Motivation

- Missing resonances – couple weakly to πN ?
- Search for resonances in the reactions other than the πN channel
- $\gamma N \rightarrow \omega N$
  with A.I. Titov and T.-S.H. Lee (2001) *
- $\gamma N \rightarrow \rho N$
  with T.-S.H. Lee (2004)
- $\gamma N \rightarrow \phi N$
  with A.I. Titov, S.N. Yang, T. Morii, H.-C. Bhang (1997,1999,2001)
- $\gamma N \rightarrow K^* \Lambda, K^* \Sigma$
  with Hungchong Kim (2006), with S.-H. Kim, S.-I. Nam, H.-Ch. Kim (2012) *
  B.-G. You and K.-J. Kong (2017)

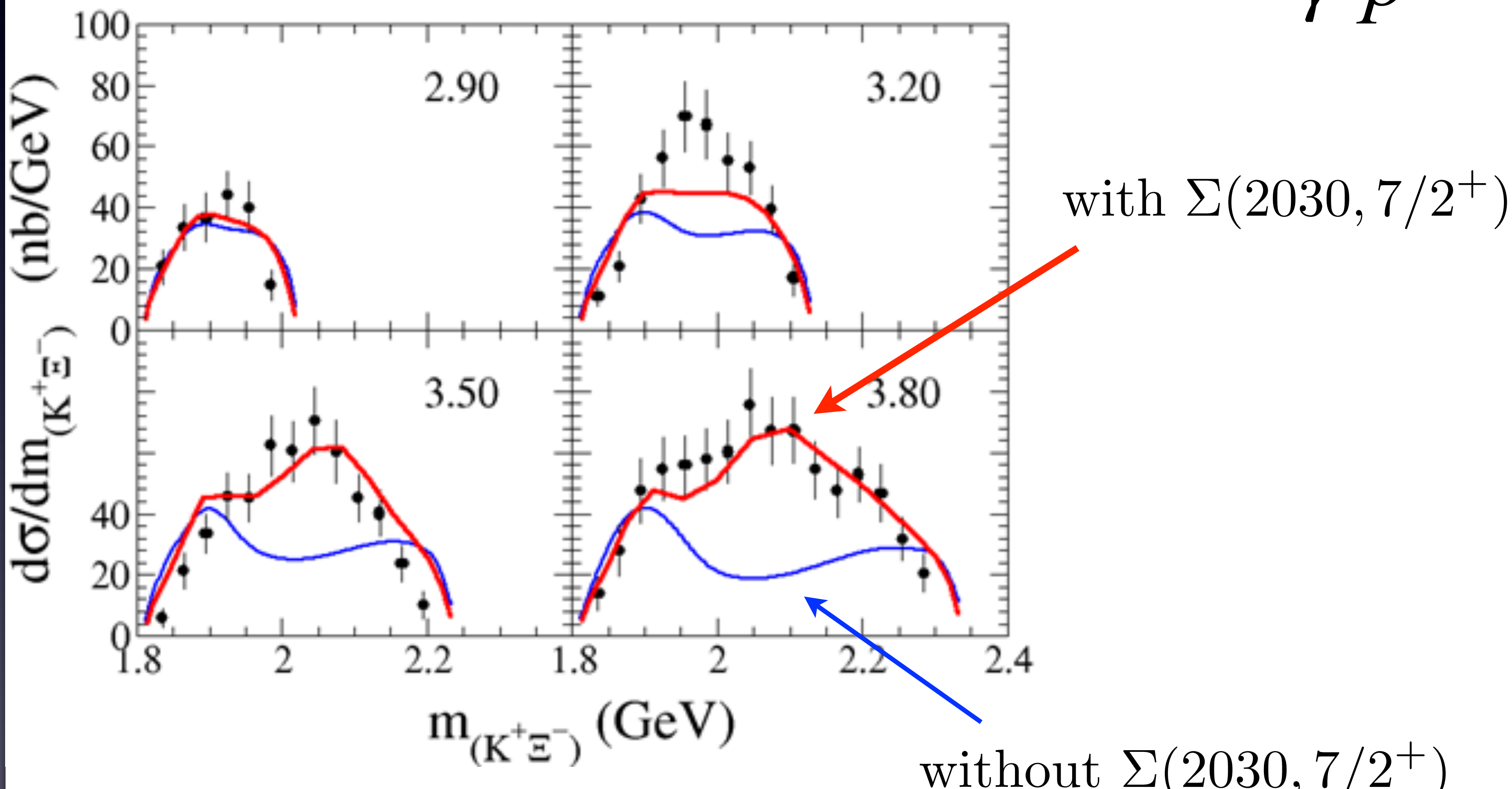
Introduction & Motivation

- $\gamma\, N \rightarrow K\, K\, \Xi$
  with K. Nakayama, H. Haberzettl (2006,2011) *
- $Kbar\, N \rightarrow K\, \Xi$
  with B. Jackson, K. Nakayama, H. Haberzettl (2012,2015) *
- $\gamma\, N \rightarrow K\, \Sigma^*(1385)$
  with K. Nakayama and C.M. Ko (2008)
- $\pi\, N \rightarrow \omega\, N$
  YO (2011) *
- What we need
  - vertices of $J^{\pm} \to 0^- + \frac{1}{2}^+, \quad J^{\pm} \to 1^- + \frac{1}{2}^+, J^{\pm} \to 0^- + \frac{3}{2}^+, \quad J^{\pm} \to 1^- + \frac{3}{2}^+$
    for an arbitrary value of J

Introduction & Motivation

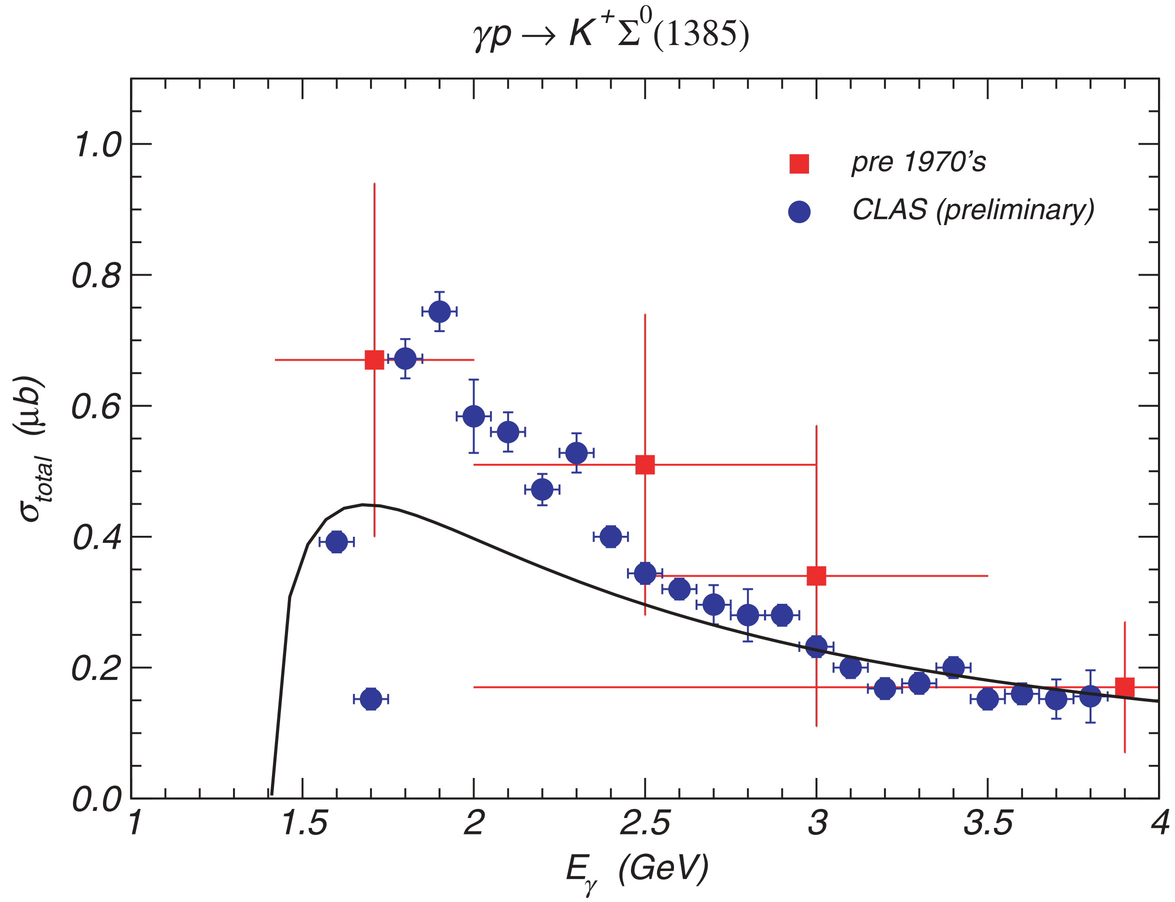
Importance of high-spin resonances

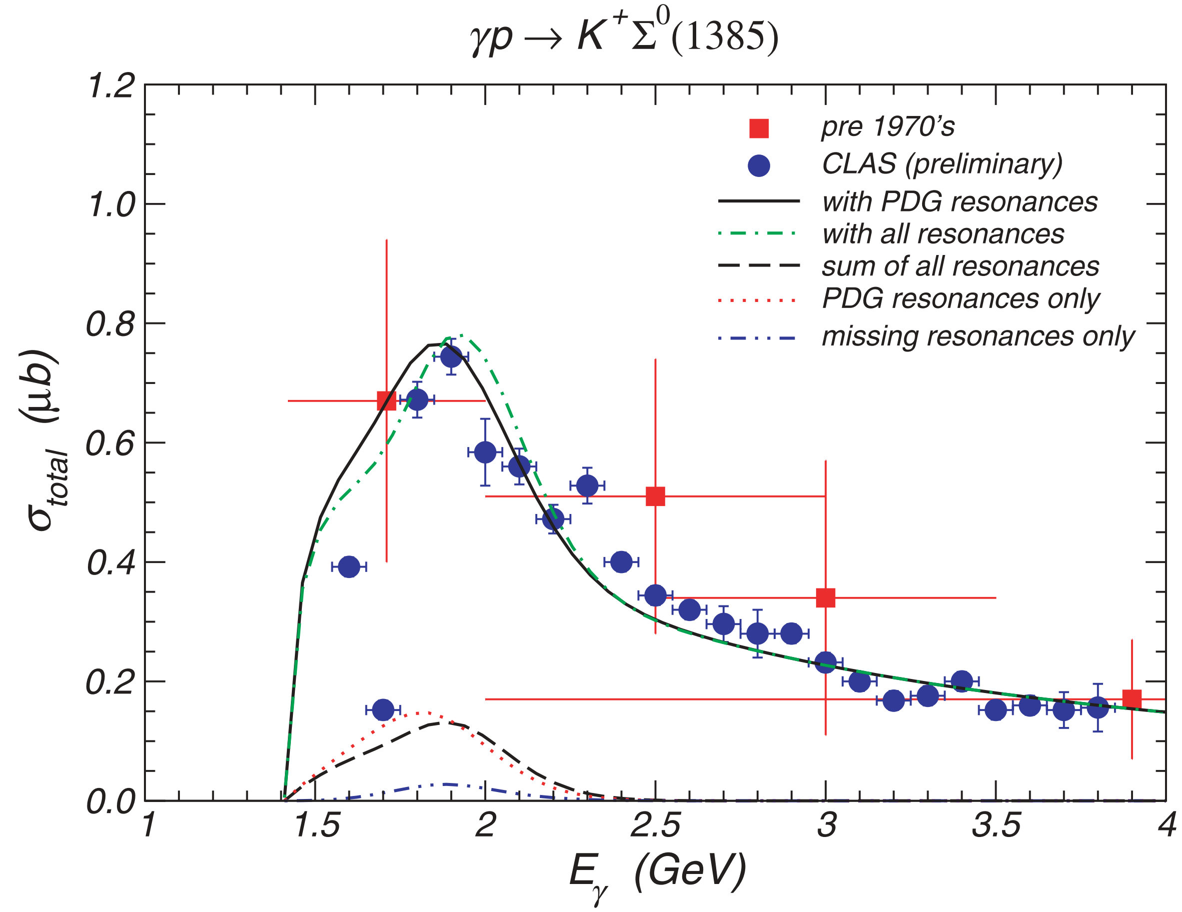
$\gamma\, p \longrightarrow K^+\, K^+\, \Xi^-$



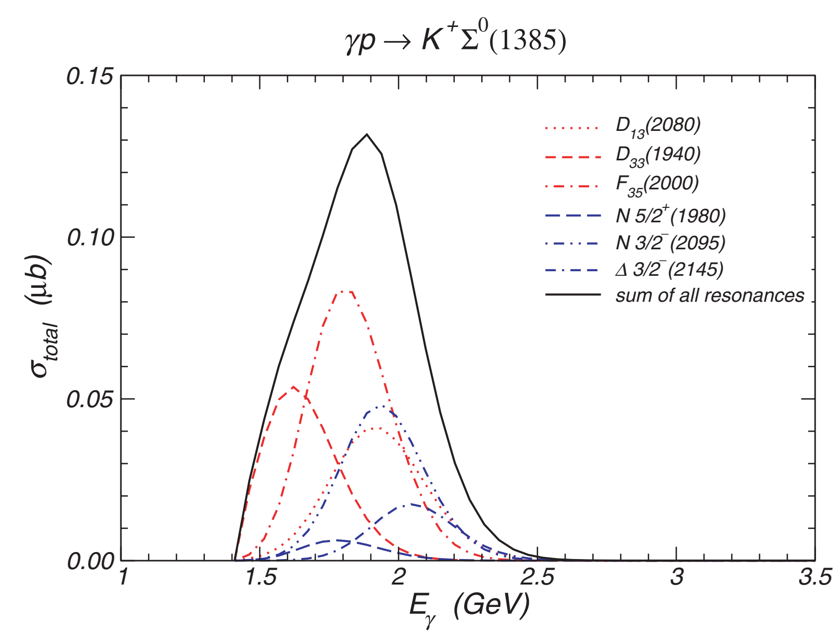
Nakayama, YO, Haberzettl, PRC **74** (2006) 035205
Man, YO, Nakayama, PRC **83** (2011) 055201

Introduction & Motivation





YO, C.M. Ko, K. Nakayama, PRC **77** (2008) 045204



Introduction & Motivation

- Testing hadron models (such as quark models)
  - Data analyses: coupled-channels analyses
    - > extract coupling constants of effective interactions
    - > meson cloud effects (e.g. E2/M1 transition of $\Delta \to N$)
  - Quark models can give predictions on the decay amplitudes.
  - Decay width cannot determine the sign of the coupling constant (sign ambiguity)
    - need to work with decay amplitudes
    - need the relationship between coupling constants and the decay amplitudes predicted by baryon structure models
    - Tabel for $J^{\pm} \to 0^- + \frac{1}{2}^+, \quad J^{\pm} \to 1^- + \frac{1}{2}^+, J^{\pm} \to 0^- + \frac{3}{2}^+, \quad J^{\pm} \to 1^- + \frac{3}{2}^+$

Formalism

- Rarita-Schwinger fields
  - boson of spin-*j*: tensor of rank *n=j* $R_{\alpha_1\alpha_2\cdots\alpha_n}$

$$(\partial_\mu\partial^\mu + M^2)R_{\alpha_1\alpha_2\cdots\alpha_n} = 0 \quad \text{with} \quad R_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_n} = R_{\alpha_1\cdots\alpha_j\cdots\alpha_i\cdots\alpha_n}$$

subsidiary conditions

$$p^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad g^{\alpha_1\alpha_2}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0$$

  - fermion of spin-*j*: tensor of rank *n=j-1/2*

$$(i\partial\!\!\!/ - M)R_{\alpha_1\alpha_2\cdots\alpha_n} = 0. \quad \text{with} \quad R_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_n} = R_{\alpha_1\cdots\alpha_j\cdots\alpha_i\cdots\alpha_n}$$

subsidiary conditions

$$p^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad g^{\alpha_1\alpha_2}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad \gamma^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0$$

Propagators

$$S(p) = \frac{1}{p^2 - M^2}\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n} \quad \text{for a boson}$$

$$S(p) = \frac{1}{p^2 - M^2}(\not{p} + M)\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n} \quad \text{for a fermion}$$

with the projection operator $\displaystyle\sum_{\text{spin}} R_{\alpha_1\cdots\alpha_n} R^{\beta_1\cdots\beta_n} = \Lambda_\pm \Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n}$

where $\Lambda_\pm = \begin{cases} 1 & \text{for a boson} \\ (M \pm \not{p})/2M & \text{for a fermion} \end{cases}$

General form

Rushbrooke, PR 143 (66')
Behrends and Fronsdal, PR 106 (57')
Chang, PR 161 (67')

Boson $n = j$

$$\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n}(j,p) = \left(\frac{1}{n!}\right)^2 \sum_{P(\alpha),P(\beta)} \left[\prod_{i=1}^{n} \bar{g}^{\beta_i}_{\alpha_i} + a_1^{(n)} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2} \prod_{i=3}^{n} \bar{g}^{\beta_i}_{\alpha_i} + \cdots + a^{(n)}_{n/2} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\cdots \bar{g}_{\alpha_{n-1}\alpha_n}\bar{g}^{\beta_{n-1}\beta_n}\right]$$

for even *j*

$$\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n}(j,p) = -\left(\frac{1}{n!}\right)^2 \sum_{P(\alpha),P(\beta)} \left[\prod_{i=1}^{n} \bar{g}^{\beta_i}_{\alpha_i} + a_1^{(n)} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2} \prod_{i=3}^{n} \bar{g}^{\beta_i}_{\alpha_i} + \cdots \right.$$

for odd *j*

$$\left. + a^{(n)}_{(n-1)/2} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\cdots \bar{g}_{\alpha_{n-2}\alpha_{n-1}}\bar{g}^{\beta_{n-2}\beta_{n-1}}\bar{g}^{\beta_n}_{\alpha_n}\right]$$

Fermion $n = j - 1/2$

$$\Delta^{\beta_1\cdots\beta_{n-1}}_{\alpha_1\cdots\alpha_{n-1}}(j,p) = \frac{n}{2n+1}\gamma^{\alpha}\gamma_{\beta}\Delta^{\beta\beta_1\cdots\beta_{n-1}}_{\alpha\alpha_1\cdots\alpha_{n-1}}(j+\tfrac{1}{2},p)$$

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{M^2}p_\mu p_\nu.$$

$$a_r^{(n)} = \left(-\frac{1}{2}\right)^r \frac{n!}{r!(n-2r)!}\frac{1}{(2n-1)(2n-3)\cdots(2n-2r+1)}$$

Explicitly,

- **spin-1**

$$\Delta_\alpha^\beta(1,p) = -\bar{g}_\alpha^\beta = -\left(g_\alpha^\beta - \frac{1}{M^2}p_\alpha p^\beta\right).$$

- **spin-1/2**

$$\Delta(\tfrac{1}{2},p) = \tfrac{1}{3}\gamma^\alpha\gamma_\beta\Delta_\alpha^\beta(1,p) = 1.$$

- **spin-2**

$$\Delta_{\alpha_1\alpha_2}^{\beta_1\beta_2}(2,p) = \frac{1}{2}\left(\bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_1}^{\beta_2} + \bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_2}^{\beta_1} - \frac{2}{3}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\right).$$

- **spin-3/2**

$$\begin{aligned}\Delta_{\alpha_1}^{\beta_1}(\tfrac{3}{2},p) &= \frac{5}{2}\gamma^\alpha\gamma_\beta\Delta_{\alpha\alpha_1}^{\beta\beta_1}(2,p)\\ &= -\left(\bar{g}_{\alpha_1}^{\beta_1} - \frac{1}{3}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\right)\\ &= -g_{\alpha_1}^{\beta_1} + \frac{1}{3}\gamma_{\alpha_1}\gamma^{\beta_1} + \frac{1}{3M}\left(\gamma_{\alpha_1}p^{\beta_1} - p_{\alpha_1}\gamma^{\beta_1}\right) + \frac{2}{3M^2}p_{\alpha_1}p^{\beta_1}.\end{aligned}$$

- **spin-3**

$$\Delta_{\alpha_1\alpha_2\alpha_3}^{\beta_1\beta_2\beta_3}(3,p) = -\frac{1}{36}\sum_{P(\alpha),P(\beta)}\left[\bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_2}^{\beta_2}\bar{g}_{\alpha_3}^{\beta_3} - \frac{3}{5}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\bar{g}_{\alpha_3}^{\beta_3}\right].$$

Explicitly,

- **spin-5/2**

$$\Delta^{\beta_1\beta_2}_{\alpha_1\alpha_2}(\tfrac{5}{2},p) = \frac{3}{7}\gamma^{\alpha}\gamma_{\beta}\Delta^{\beta\beta_1\beta_2}_{\alpha\alpha_1\alpha_2}(3,p)$$
$$= \frac{1}{2}\left(\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2} + \bar{g}^{\beta_2}_{\alpha_1}\bar{g}^{\beta_1}_{\alpha_2}\right) - \frac{1}{5}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2} - \frac{1}{10}\left(\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_2} + \bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_2}\bar{g}^{\beta_1}_{\alpha_2} + \bar{\gamma}_{\alpha_2}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_1} + \bar{\gamma}_{\alpha_2}\bar{\gamma}^{\beta_2}\bar{g}^{\beta_1}_{\alpha_1}\right). \tag{A10}$$

- **spin-4**

$$\Delta^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}(4,p) = -\frac{1}{576}\sum_{P(\alpha),P(\beta)}\left[\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3}\bar{g}^{\beta_4}_{\alpha_4} - \frac{6}{7}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\bar{g}^{\beta_3}_{\alpha_3}\bar{g}^{\beta_4}_{\alpha_4} + \frac{3}{35}\bar{g}_{\alpha_1\alpha_2}\bar{g}_{\alpha_3\alpha_4}\bar{g}^{\beta_1\beta_2}\bar{g}^{\beta_3\beta_4}\right]. \tag{A11}$$

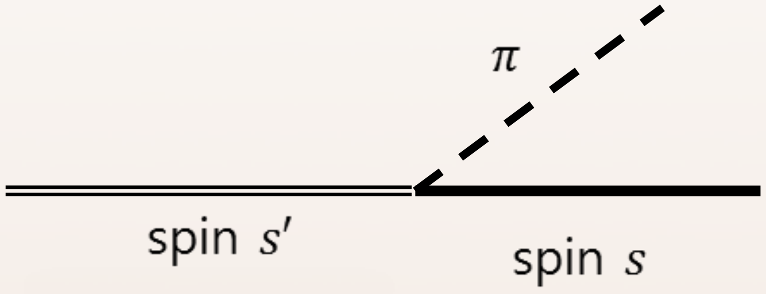
- **spin-7/2**

$$\Delta^{\beta_1\beta_2\beta_3}_{\alpha_1\alpha_2\alpha_3}(\tfrac{7}{2},p) = \frac{4}{9}\gamma^{\alpha}\gamma_{\beta}\Delta^{\beta\beta_1\beta_2\beta_3}_{\alpha\alpha_1\alpha_2\alpha_3}(4,p)$$
$$= -\frac{1}{36}\sum_{P(\alpha),P(\beta)}\left\{\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3} - \frac{3}{7}\bar{g}^{\beta_1}_{\alpha_1}\bar{g}_{\alpha_2\alpha_3}\bar{g}^{\beta_2\beta_3} - \frac{3}{7}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3} + \frac{3}{35}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}_{\alpha_2\alpha_3}\bar{g}^{\beta_2\beta_3}\right\}. \tag{A12}$$

$$\bar{\gamma}^{\mu} = \gamma^{\nu}\bar{g}^{\mu}_{\nu} = \gamma^{\mu} - \frac{1}{M^2}\not{p}p^{\mu}.$$

Interactions

- Number of independent couplings
  - Angular momentum conservation
  - P and T invariance

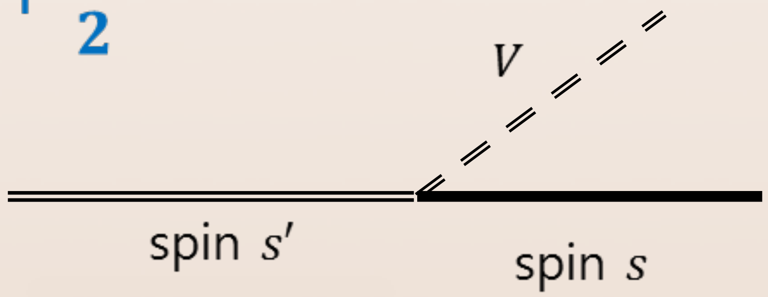


- Pion ($J^P = 0^-$) vertex: $\boldsymbol{J^{\pm} \rightarrow 0^- + \frac{1}{2}^+}$
  - $RN\pi$ vertex where N: nucleon $(1/2^+)$
    R: nucleon resonance of J^P
    ⇨ only ***one*** interaction term

- Vector meson ($J^P = 1^-$) vertex: $\boldsymbol{J^{\pm} \rightarrow 1^- + \frac{1}{2}^+}$
  - RNV vertex

    ⇨ ***two*** interaction terms for R with $\frac{1}{2}^{\pm}$

    ⇨ ***three*** interaction terms for R with $J^{\pm}$ $(J \geq \frac{3}{2})$



  - If V is the photon, the numbers are reduced by one. $(\because q^2 = 0)$

$RN\pi$ Lagrangian

- $J^P = \frac{1}{2}^{\pm}$ case

$$\mathcal{L}_{1/2} = g_{\pi NR}\bar{N}\left[i\lambda\Gamma^{(\pm)}\pi \mp \frac{1-\lambda}{M_R \pm M_N}\Gamma_\mu^{(\pm)}\partial^\mu\pi\right]R + \mathrm{H.c.}$$

$$\Gamma^{(\pm)} = \begin{pmatrix}\gamma_5 \\ 1\end{pmatrix}, \quad \Gamma_\mu^{(\pm)} = \begin{pmatrix}\gamma_\mu\gamma_5 \\ \gamma_\mu\end{pmatrix}$$

- $J^P = \frac{3}{2}^{\pm}$ case

$$\mathcal{L}_{3/2} = \frac{g_{\pi NR}}{M_\pi}\bar{N}\Gamma^{(\mp)}\partial^\mu\pi R_\mu + \mathrm{H.c.}$$

$$\mathcal{L}_{RN\pi} = \frac{g_{RN\pi}}{M_\pi^{n-1}}\bar{N}\partial_{\mu_1}\cdots\partial_{\mu_{n-1}}\pi\left[i\Gamma^{(\pm)}\right]R^{\mu_1\cdots\mu_{n-1}} + \mathrm{H.c.},$$

- $J^P = \frac{5}{2}^{\pm}$ case

$$\mathcal{L}_{5/2} = i\frac{g_{\pi NR}}{M_\pi^2}\bar{N}\Gamma^{(\pm)}\partial^\mu\partial^\nu\pi R_{\mu\nu} + \mathrm{H.c.}$$

- $J^P = \frac{7}{2}^{\pm}$ case

$$\mathcal{L}_{7/2} = \frac{g_{\pi NR}}{M_\pi^3}\bar{N}\Gamma^{(\mp)}\partial^\mu\partial^\nu\partial^\alpha\pi R_{\mu\nu\alpha} + \mathrm{H.c.}$$

Pion interaction Lagrangian

- The general expression for the decay widths

$$\Gamma(R \to N\pi) = \frac{3g_{\pi NR}^2}{4\pi}\frac{2^n(n!)^2}{n(2n)!}\frac{k_\pi^{2n-1}}{M_R M_\pi^{2(n-1)}}\left(E_N \pm M_N\right)$$

for $(-1)^n P_s = \pm 1$ with P_s being the parity of the spin - s resonance R

- Examples

$$\Gamma\left(\frac{1}{2}^{\pm} \to N\pi\right) = \frac{3g_{\pi NR}^2}{4\pi}\frac{k_\pi}{M_R}\left(E_N \mp M_N\right)$$

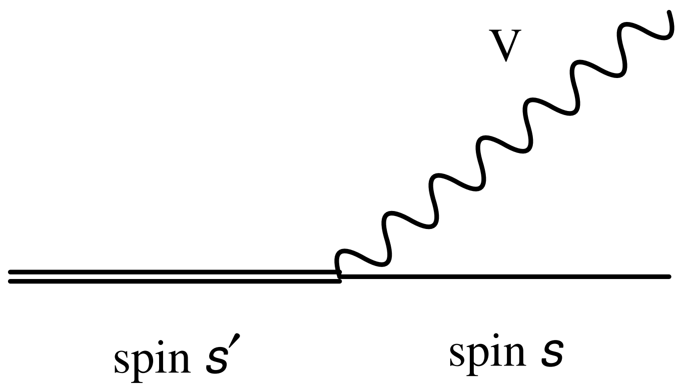
Isospin factor 3 is included.

$$\Gamma\left(\frac{3}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{k_\pi^3}{M_R M_\pi^2}\left(E_N \pm M_N\right)$$

$$\Gamma\left(\frac{5}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{2}{5}\frac{k_\pi^5}{M_R M_\pi^4}\left(E_N \mp M_N\right)$$

$$\Gamma\left(\frac{7}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{6}{35}\frac{k_\pi^7}{M_R M_\pi^6}\left(E_N \pm M_N\right)$$

RNV interactions



	$(-1)^\gamma = +1$		$(-1)^\gamma = -1$	
	J_0	J_{+1}	J_0	J_{+1}
Nonidentical fermions $(s' = s)$	$j_{\rm min} + \frac{1}{2}$	$2j_{\rm min}$	$j_{\rm min} + \frac{1}{2}$	$2j_{\rm min}$
$(s' \neq s)$	$j_{\rm min} + \frac{1}{2}$	$2j_{\rm min} + 1$	$j_{\rm min} + \frac{1}{2}$	$2j_{\rm min} + 1$
Nonidentical bosons $(s' = s)$	$j_{\rm min} + 1$	$2j_{\rm min}$	$j_{\rm min}$	$2j_{\rm min}$
$(s' \neq s)$	$j_{\rm min} + 1$	$2j_{\rm min} + 1$	$j_{\rm min}$	$2j_{\rm min} + 1$
Identical fermions$(s' = s)$	$j_{\rm min} + \frac{1}{2}$	$j_{\rm min} + \frac{1}{2}$	—	—
Identical bosons $(s' = s)$	$j_{\rm min} + 1$	$j_{\rm min}$	—	—

TABLE II. Number of independent form factors for the vector current of a spin-s' particle into a spin-s particle transition [25]. Here $\gamma = s + s' + P$, where $(-1)^P$ is the relative parity of the initial and final states whose spins are s' and s, respectively, and $j_{\rm min} = \min(s, s')$. See Ref. [25] for the details.

Durand III, DeCelles, Marr, PR (1962)

RNV interactions

$$\Gamma(R \to NV) = \frac{q^2}{\pi}\frac{2M_N}{(2j+1)M_R} \times \left\{ \left|A_{1/2}\right|^2 + \left|A_{3/2}\right|^2 + \left|S_{1/2}\right|^2 \right\}$$

q: three-momentum of the vector meson in the rest frame of R

$$q = \frac{1}{2M_R}\sqrt{[M_R^2 - (M_N + M_V)^2][M_R^2 - (M_N - M_V)^2]}.$$

Helicity amplitude

$$A_\lambda(j) = \frac{1}{\sqrt{8M_N M_R q}}\frac{2j+1}{4\pi} \times \int d\cos\theta d\phi e^{-i(m-\lambda)\phi} d^j_{\lambda m}(\theta) \times \langle \mathbf{k}_\gamma, \lambda_\gamma, \lambda_N \mid -i\mathcal{M} \mid jm\rangle,$$

RNV Lagrangian

- $J^P = \frac{1}{2}^{\pm}$ case

$$\mathcal{L}_{1/2} = -\frac{1}{2M_N}\bar{N}\left[g_3\left(\pm\frac{\Gamma_\mu^{(\pm)}\partial^2}{M_R \mp M_N} - i\Gamma^{(\pm)}\partial_\mu\right)V^\mu - g_1\Gamma^{(\pm)}\sigma_{\mu\nu}\partial^\nu V^\mu\right]R + \text{H.c.}$$

- $J^P = \frac{3}{2}^{\pm}$ case

$$\mathcal{L}_{3/2} = -i\frac{g_1}{2M_N}\bar{N}\Gamma_\nu^{(\pm)}V^{\mu\nu}R_\mu - \frac{g_2}{(2M_N)^2}\partial_\nu\bar{N}\Gamma^{(\pm)}V^{\mu\nu}R_\mu + \frac{g_3}{(2M_N)^2}\bar{N}\Gamma^{(\pm)}\partial_\nu V^{\mu\nu}R_\mu + \text{H.c.}$$

- $J^P = \frac{5}{2}^{\pm}$ case

$$\mathcal{L}_{5/2} = \frac{g_1}{(2M_N)^2}\bar{N}\Gamma_\nu^{(\mp)}\partial^\alpha V^{\mu\nu}R_{\mu\alpha} - \frac{ig_2}{(2M_N)^3}\partial_\nu\bar{N}\Gamma^{(\mp)}\partial^\alpha V^{\mu\nu}R_{\mu\alpha} + \frac{ig_3}{(2M_N)^3}\bar{N}\Gamma^{(\mp)}\partial^\alpha\partial_\nu V^{\mu\nu}R_{\mu\alpha} + \text{H.c.}$$

- $J^P = \frac{7}{2}^{\pm}$ case

$$\mathcal{L}_{7/2} = \frac{ig_1}{(2M_N)^3}\bar{N}\Gamma_\nu^{(\pm)}\partial^\alpha\partial^\beta V^{\mu\nu}R_{\mu\alpha\beta} + \frac{g_2}{(2M_N)^4}\partial_\nu\bar{N}\Gamma^{(\pm)}\partial^\alpha\partial^\beta V^{\mu\nu}R_{\mu\alpha\beta}$$
$$- \frac{g_3}{(2M_N)^4}\bar{N}\Gamma^{(\pm)}\partial^\alpha\partial^\beta\partial_\nu V^{\mu\nu}R_{\mu\alpha\beta} + \text{H.c.}$$

RNV interactions

$$\Gamma(\tfrac{1}{2}^{\pm} \to NV) = \frac{1}{16\pi}\frac{q(E_N \mp M_N)}{M_R M_N^2}\Bigg\{g_1^2\left[2(M_R \pm M_N)^2 + M_V^2\right] - 6g_1g_3\frac{M_V^2}{(M_R \mp M_N)^2}(M_R^2 - M_N^2)$$

$$+ g_3^2\frac{M_V^2}{(M_R \mp M_N)^2}\left[(M_R \pm M_N)^2 + 2M_V^2\right]\Bigg\},$$

$$A_{1/2}^{\pm} = \mp\frac{1}{2\sqrt{2}}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}$$

$$\times\left[(M_R \pm M_N)g_1 - \frac{M_V^2}{M_R \mp M_N}g_3\right],$$

$$S_{1/2}^{\pm} = \mp\frac{M_V}{4M_N}\sqrt{\frac{E_N \mp M_N}{qM_N}}\left[g_1 - \frac{M_R \pm M_N}{M_R \mp M_N}g_3\right].$$

$$\bar{g}_1 = \frac{g_1}{(2M_N)}, \quad \bar{g}_2 = \frac{g_3}{(2M_N)^2}, \quad \bar{g}_3 = \frac{g_3}{(2M_N)^2}.$$

$$\Gamma(\tfrac{3}{2}^{\pm} \to NV) = \frac{1}{12\pi}\frac{q}{M_R}(E_N \mp M_N)$$

$$\times\Bigg\{\bar{g}_1^2\left[2E_N(E_N \pm M_N) + (M_R \pm M_N)^2 + 2M_V^2\right] + \bar{g}_2^2\left[E_N^2(2M_R^2 + M_V^2) - 2M_N^2(M_R^2 - M_V^2)\right]$$

$$+ \bar{g}_3^2 M_V^2(E_N^2 - M_N^2 + 3M_V^2) \mp 2\bar{g}_1\bar{g}_2\left[\frac{E_N}{2}(3M_R^2 + M_N^2 \pm 2M_NM_R + 3M_V^2) - M_N^2(2M_R \pm M_N)\right]$$

$$+ 2\bar{g}_2\bar{g}_3M_V^2(E_N^2 + 2M_N^2 - 3E_NM_R) \pm 2\bar{g}_3\bar{g}_1M_V^2(3M_R - 2E_N \pm M_N)\Bigg\}, \tag{35}$$

$$A_{3/2}^{\pm} = \mp\frac{1}{4}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{g_1(M_R \pm M_N) \mp \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) \pm \frac{g_3}{2M_N}M_V^2\right\},$$

$$A_{1/2}^{\pm} = \mp\frac{1}{4\sqrt{3}}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{\frac{g_1}{M_R}\left[M_N(M_R \pm M_N) \mp M_V^2\right] + \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) - \frac{g_3}{2M_N}M_V^2\right\},$$

$$S_{1/2}^{\pm} = \mp\frac{1}{2\sqrt{6}}\frac{M_V\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{g_1 \mp \frac{g_2}{4M_NM_R}(M_R^2 + M_N^2 - M_V^2) \pm \frac{g_3}{4M_NM_R}(M_R^2 - M_N^2 + M_V^2)\right\}.$$

RNV interactions

$$\Gamma(\tfrac{5}{2}^\pm \to NV) = \frac{1}{60\pi}\frac{q^3}{M_R}(E_N \pm M_N)$$
$$\times \Big\{ \bar{g}_1^2 \left[4E_N(E_N \mp M_N) + (M_R \mp M_N)^2 + 4M_V^2\right]$$
$$+ \bar{g}_2^2 \left[E_N^2(3M_R^2 + 2M_V^2) - 3M_N^2(M_R^2 - M_V^2)\right] + \bar{g}_3^2 M_V^2(2E_N^2 - 2M_N^2 + 5M_V^2)$$
$$\pm 2\bar{g}_1\bar{g}_2 \left[E_N(2M_R^2 + M_N^2 \mp M_N M_R + 3M_V^2) - M_N^2(3M_R \mp M_N)\right]$$
$$+ 2\bar{g}_2\bar{g}_3 M_V^2(2E_N^2 + 3M_N^2 - 5E_N M_R) \mp 2\bar{g}_3\bar{g}_1 M_V^2(5M_R - 4E_N \mp M_N)\Big\},$$

$$\bar{g}_1 = \frac{g_1}{(2M_N)^2}, \quad \bar{g}_2 = \frac{g_3}{(2M_N)^3}, \quad \bar{g}_3 = \frac{g_3}{(2M_N)^3}.$$

$$A_{3/2}^\pm = \pm\frac{1}{4\sqrt{10}}\frac{\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\left\{g_1(M_R \mp M_N) \pm \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) \mp \frac{g_3}{2M_N}M_V^2\right\},$$

$$A_{1/2}^\pm = \pm\frac{1}{8\sqrt{5}}\frac{\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\Big\{\frac{g_1}{M_R}\left[M_N(M_R \mp M_N) \pm M_V^2\right]$$
$$+ \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) - \frac{g_3}{2M_N}M_V^2\Big\},$$

$$S_{1/2}^\pm = \pm\frac{1}{4\sqrt{10}}\frac{M_V\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\Big\{g_1 \pm \frac{g_2}{4M_R M_N}(M_R^2 + M_N^2 - M_V^2)$$
$$\mp \frac{g_3}{4M_R M_N}(M_R^2 - M_N^2 + M_V^2)\Big\}.$$

and so on …

$RN\gamma$ interaction

- Similar to RNV interaction
  - But with $g_3 = 0$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{1}{2}^{\pm}\right) = \frac{ef_1}{2M_N}\bar{N}\Gamma^{(\mp)}\sigma_{\mu\nu}\partial^{\nu}A^{\mu}R + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{3}{2}^{\pm}\right) = -\frac{ief_1}{2M_N}\overline{N}\Gamma_{\nu}^{(\pm)}F^{\mu\nu}R_{\mu} - \frac{ef_2}{(2M_N)^2}\partial_{\nu}\bar{N}\Gamma^{(\pm)}F^{\mu\nu}R_{\mu} + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{5}{2}^{\pm}\right) = \frac{ef_1}{(2M_N)^2}\bar{N}\Gamma_{\nu}^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} - \frac{ief_2}{(2M_N)^3}\partial_{\nu}\bar{N}\Gamma^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} + \text{H.c.},$$

Helicity amplitudes

$$\Gamma(R \to N\gamma) = \frac{k_{\gamma}^2}{\pi}\frac{2M_N}{(2j+1)M_R}[|A_{1/2}|^2 + |A_{3/2}|^2],$$

$$A_{1/2}\left(\tfrac{1}{2}^{\pm}\right) = \mp\frac{ef_1}{2M_N}\sqrt{\frac{k_{\gamma}M_R}{M_N}},$$

$$A_{1/2}\left(\tfrac{3}{2}^{\pm}\right) = \mp\frac{e\sqrt{6}}{12}\sqrt{\frac{k_{\gamma}}{M_N M_R}}\left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \mp M_N)\right],$$

$$A_{3/2}\left(\tfrac{3}{2}^{\pm}\right) = \mp\frac{e\sqrt{2}}{4M_N}\sqrt{\frac{k_{\gamma}M_R}{M_N}}\left[f_1 \mp \frac{f_2}{4M_N}(M_R \mp M_N)\right],$$

$$A_{1/2}\left(\tfrac{5}{2}^{\pm}\right) = \pm\frac{e}{4\sqrt{10}}\frac{k_{\gamma}}{M_N}\sqrt{\frac{k_{\gamma}}{M_N M_R}} \times \left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \pm M_N)\right],$$

$$A_{3/2}\left(\tfrac{5}{2}^{\pm}\right) = \pm\frac{e}{4\sqrt{5}}\frac{k_{\gamma}}{M_N^2}\sqrt{\frac{k_{\gamma}M_R}{M_N}}\left[f_1 \pm \frac{f_2}{4M_N}(M_R \pm M_N)\right],$$

Interactions of R with spin-3/2 baryons

- $J^{\pm} \to 0^{-} + \frac{3}{2}^{+}$

Lagrangian

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{1}{2}^{\pm}) = \frac{h_1}{M_K}\partial_\mu K\,\bar{\Sigma}^{*\mu}\Gamma^{(\mp)}R + \text{H.c.},$$

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{3}{2}^{\pm}) = \frac{h_1}{M_K}\partial^\alpha K\,\bar{\Sigma}^{*\mu}\Gamma^{(\pm)}_\alpha R_\mu + \frac{ih_2}{M_K^2}\partial^\mu\partial^\alpha K\,\bar{\Sigma}^*_\alpha\Gamma^{(\pm)}R_\mu + \text{H.c.},$$

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{5}{2}^{\pm}) = \frac{ih_1}{M_K^2}\partial^\mu\partial^\beta K\,\bar{\Sigma}^{*\alpha}\Gamma^{(\mp)}_\mu R_{\alpha\beta} - \frac{h_2}{M_K^3}\partial^\mu\partial^\alpha\partial^\beta K\,\bar{\Sigma}^*_\mu\Gamma^{(\mp)}R_{\alpha\beta} + \text{H.c.}.$$

By angular momentum and parity conservation,

1 coupling for the resonance with $j = 1/2$
2 couplings for the resonance with $j \geq 3/2$

Decay widths

$$\Gamma(\tfrac{1}{2}^{\pm} \to K\Sigma^*) = \frac{h_1^2}{2\pi}\frac{q^3 M_R}{M_K^2 M_{\Sigma^*}^2}(E_{\Sigma^*} \pm M_{\Sigma^*}),$$

$$\begin{aligned}\Gamma(\tfrac{3}{2}^{\pm} \to K\Sigma^*) = {}& \frac{1}{24\pi}\frac{q}{M_R M_{\Sigma^*}^2}(E_{\Sigma^*} \mp M_{\Sigma^*}) \\ & \times\left\{\frac{h_1^2}{M_K^2}(M_R \pm M_{\Sigma^*})^2\right. \\ & \times(2E_{\Sigma^*}^2 \mp 2E_{\Sigma^*}M_{\Sigma^*} + 5M_{\Sigma^*}^2) \\ & \mp 2\frac{h_1h_2}{M_K^3}M_R q^2(M_R \pm M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*}) \\ & \left. + 2\frac{h_2^2}{M_K^4}M_R^2 q^4\right\},\end{aligned}$$

$$\begin{aligned}\Gamma(\tfrac{5}{2}^{\pm} \to K\Sigma^*) = {}& \frac{1}{60\pi}\frac{q^3}{M_R M_{\Sigma^*}^2}(E_{\Sigma^*} \pm M_{\Sigma^*}) \\ & \times\left\{\frac{h_1^2}{M_K^4}(M_R \mp M_{\Sigma^*})^2\right. \\ & \times(4E_{\Sigma^*}^2 \pm 4E_{\Sigma^*}M_{\Sigma^*} + 7M_{\Sigma^*}^2) \\ & \mp 4\frac{h_1h_2}{M_K^5}M_R q^2(M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \pm M_{\Sigma^*}) \\ & \left. + 4\frac{h_2^2}{M_K^6}M_R^2 q^4\right\},\end{aligned}$$

Matching with quark model predictions

- Decay amplitude

$$\langle K(q)\Sigma^*(-q, m_f)| - i\mathcal{H}_{\rm int}|R(\mathbf{0}, m_j)\rangle = 2\pi M_R\sqrt{\frac{2}{q}}\sum_{\ell,m_\ell}\langle \ell m_\ell \tfrac{3}{2} m_f | j m_j\rangle Y_{\ell m_\ell}(\hat{q})G(\ell),$$

$$\Gamma(R\to K\Sigma^*) = \sum_\ell |G(\ell)|^2,$$

Quark model predictions on G(l)

For $\frac{1}{2}^+$
$$G(1) = -\frac{1}{\sqrt{2\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{qM_R}\sqrt{E_{\Sigma^*}+M_{\Sigma^*}}\frac{h_1}{M_K},$$

For $\frac{1}{2}^-$
$$G(2) = -\frac{1}{\sqrt{2\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{qM_R}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}}\frac{h_1}{M_K},$$

G's can be related to cc

For $\frac{3}{2}^+$
$$G(1) = G_{11}^{(3/2)}\frac{h_1}{M_K} + G_{12}^{(3/2)}\frac{h_2}{M_K^2},$$
$$G(3) = G_{31}^{(3/2)}\frac{h_1}{M_K} + G_{32}^{(3/2)}\frac{h_2}{M_K^2},$$

$$G_{11}^{(3/2)} = \frac{\sqrt{30}}{60\sqrt{\pi}}\frac{1}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}}(M_R+M_{\Sigma^*}) \times (E_{\Sigma^*}+4M_{\Sigma^*}),$$
$$G_{12}^{(3/2)} = -\frac{\sqrt{30}}{60\sqrt{\pi}}\frac{q^2\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}},$$
$$G_{31}^{(3/2)} = -\frac{\sqrt{30}}{20\sqrt{\pi}}\frac{1}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}} \times (M_R+M_{\Sigma^*})(E_{\Sigma^*}-M_{\Sigma^*}),$$
$$G_{32}^{(3/2)} = \frac{\sqrt{30}}{20\sqrt{\pi}}\frac{q^2\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}},$$

and so on …

Application

TABLE I. Resonances listed in the review of PDG [29] and their decay amplitudes of $R \to K\Sigma(1385)$ and of $R \to N\gamma$ predicted in Refs. [10,30]. The coupling constants are calculated using the resonance masses of PDG.

Resonance	PDG [29]	Amplitudes of $R \to K\Sigma(1385)$[a] $G(\ell_1)$	$G(\ell_2)$	h_1	h_2	Amplitudes of $R \to N\gamma$[b] $A^p_{1/2}$	$A^p_{3/2}$	f_1	f_2
$N\frac{1}{2}^-(1945)$	$S^*_{11}(2090)$	$G(2) = +1.7$	–	−9.8	–	+12	–	−0.055	–
$N\frac{3}{2}^-(1960)$	$D^{**}_{13}(2080)$	$G(0) = +1.3$	$G(2) = +1.4$	0.24	−0.54	+36	−43	−1.25	1.21
$N\frac{5}{2}^-(2095)$	$D^{**}_{15}(2200)$	$G(2) = -2.0$	$G(4) = 0.0$	0.29	−0.08	−9	−14	0.37	−0.57
$\Delta\frac{3}{2}^-(2080)$	$D^*_{33}(1940)$	$G(0) = -4.1$	$G(3) = -0.5$	−0.68	1.00	−20	−6	0.39	−0.57
$\Delta\frac{5}{2}^+(1990)$	$F^{**}_{35}(2000)$	$G(1) = +4.0$	$G(3) = -0.1$	−0.87	0.11	−10	−28	−0.68	−0.062

[a] In $\sqrt{\text{GeV}}$.
[b] In $10^{-3}/\sqrt{\text{GeV}}$.

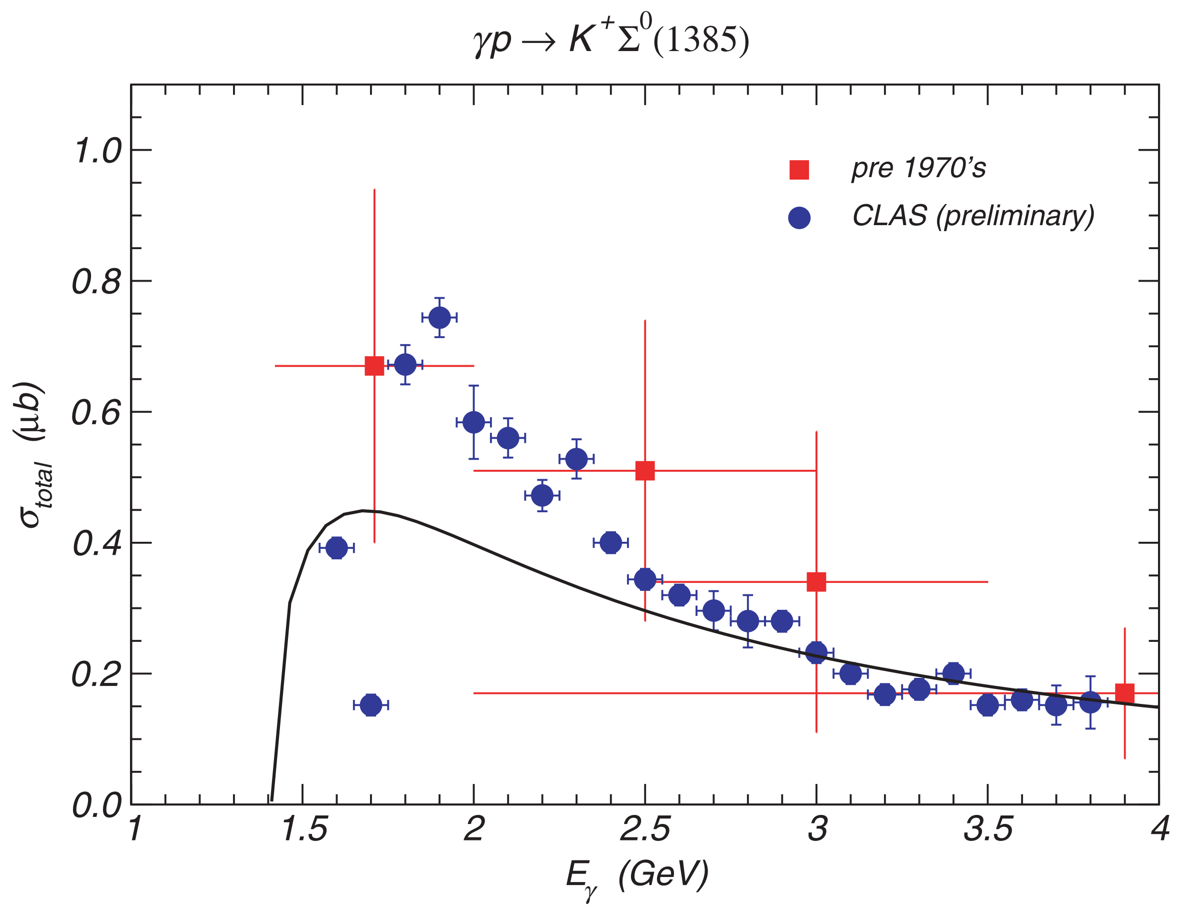
TABLE II. Missing resonances and their decay amplitudes predicted in Refs. [10,30].

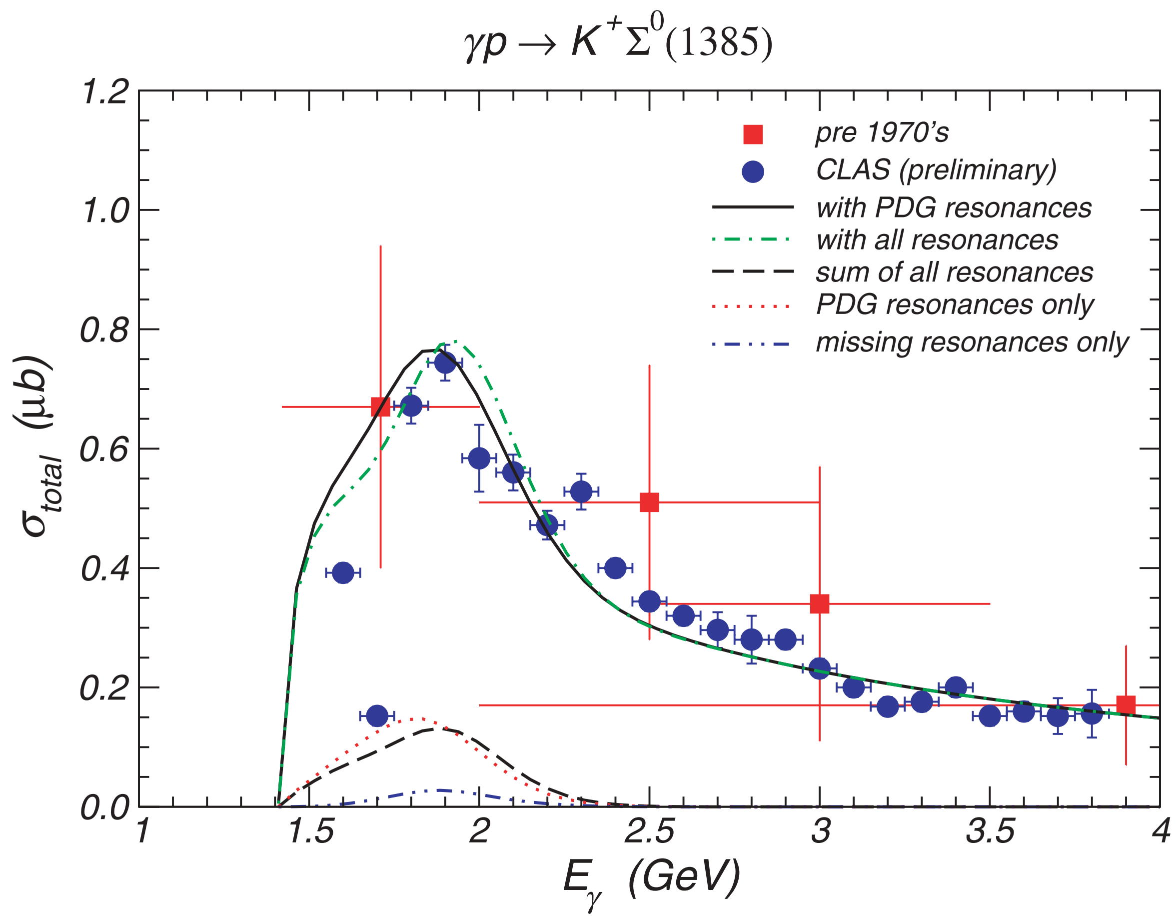
Resonance	Amplitudes of $R \to K\Sigma(1385)$[a] $G(\ell_1)$	$G(\ell_2)$	h_1	h_2	Amplitudes of $R \to N\gamma$[b] $A^p_{1/2}$	$A^p_{3/2}$	f_1	f_2
$N\frac{3}{2}^-(2095)$	$G(0) = +7.7$	$G(2) = -0.8$	0.99	0.27	−9	−14	0.49	−0.83
$N\frac{5}{2}^+(1980)$	$G(1) = -3.6$	$G(3) = -0.1$	0.59	0.24	−11	−6	0.019	−0.13
$\Delta\frac{3}{2}^-(2145)$	$G(0) = +5.2$	$G(2) = -1.9$	0.25	0.46	0	+10	0.11	−0.059

[a] In $\sqrt{\text{GeV}}$.
[b] In $10^{-3}/\sqrt{\text{GeV}}$.

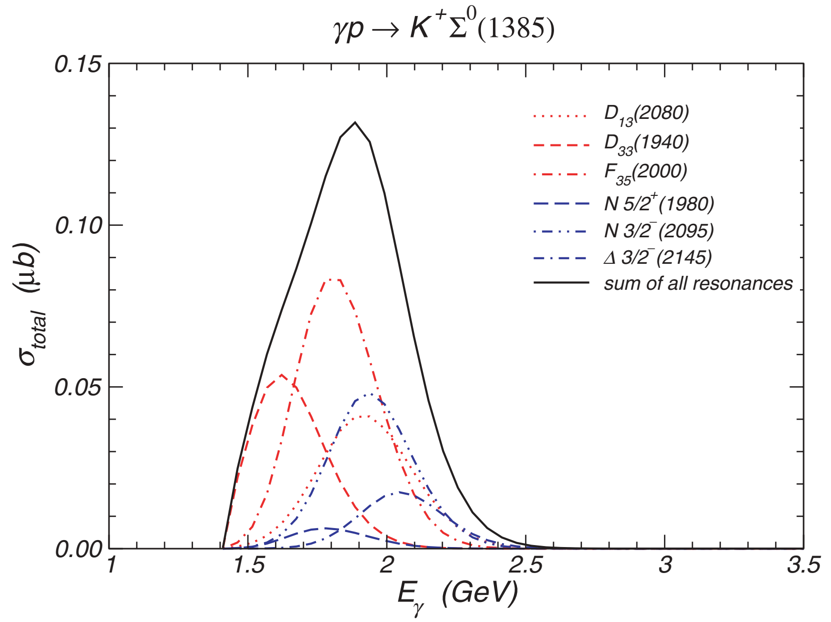
Based on the quark model of Capstick & Roberts

Application





YO, C.M. Ko, K. Nakayama, PRC **77** (2008) 045204



Summary and Outlook

- Needs for High-spin baryon resonances
  - to understand the production mechanisms of various reactions in the resonance region ~ 2 GeV
  - To search for the missing resonances
  - To test various models of baryon structure
- Further works
  - Extraction of coupling constants from various baryon models: A complete list for the coupling constants for various models
  - Issues on gauge invariance, off-shell parameters, etc (GWU group, V. Pascalutsa, T. Mart etc)
  - More results to come.