# Precision Tests of the Standard Model and their Implications

## ElectroWeak Workshop
## Jefferson Lab, December 2006

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# Outline

Parity violating Møller asymmetry

Precise determinations of "ingredients":

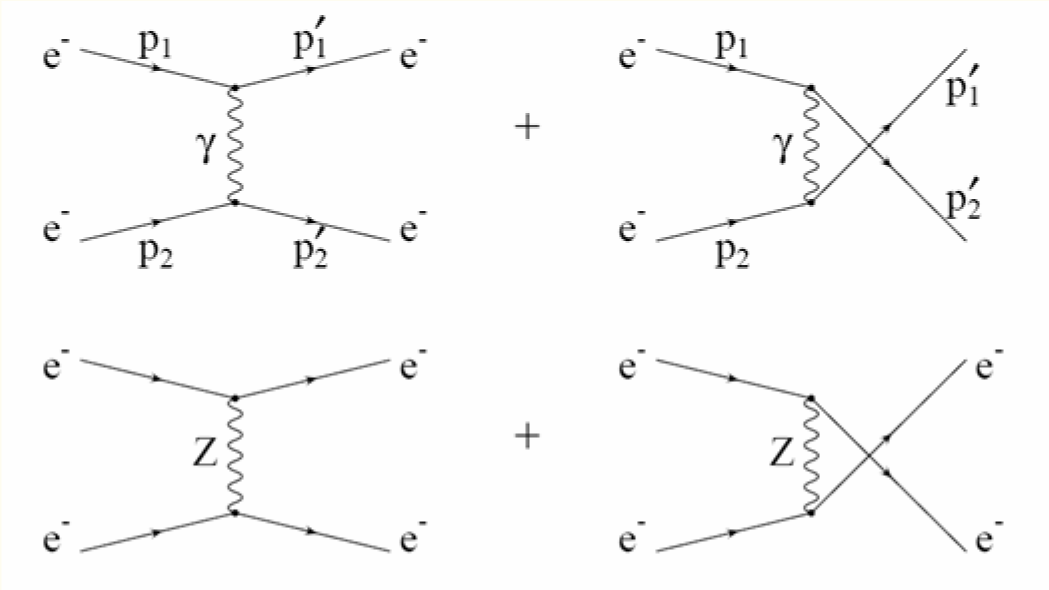
alpha, $G_F$, electron mass

Radiative corrections to the Møller scattering

* electroweak
* hadronic

Outlook

# Parity-violation in Møller scattering: tree level




$$A_{LR} \equiv \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

$$= \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \left(1 - 4\sin^2\theta_W\right)$$

$$Q^2 \sim m_e E_{\text{beam}}$$

$$A_{LR} \sim \frac{m_e E_{\text{beam}}}{M_Z^2}\left(1-4\sin^2\theta\right) \sim 5\cdot 10^{-8}\left(1-4\sin^2\theta\right)\frac{E_{\text{beam}}}{1\text{ GeV}}$$

*Note: interference of amplitudes with different $q^2$*

# Estimate of $A_{LR}$

$$\underbrace{\frac{G_\mu Q^2}{\sqrt{2}\pi\alpha}}_{O\left(10^{-6}\right)}\underbrace{\frac{1-y}{1+y^4+(1-y)^4}}_{O(1)}\underbrace{\left(1-4\sin^2\theta_W\right)}_{O(0.1)}$$

$$A_{LR} \simeq 6\times10^{-9}\,\frac{E_{\text{beam}}}{1\text{ GeV}}$$

$$A_{LR}\left(E_{\text{beam}}=12\,\text{GeV}\right)\simeq 10^{-7}$$

# Tests of the electroweak model

Three precisely measured quantities:

$G_\mu$

$\alpha$

$M_Z$

→ coupling constants of $SU(2)_L \otimes U(1)_Y$

→ electroweak symmetry breaking

determine $M_W$ and $\sin^2\theta_W$:

$$G_\mu \sim \frac{\alpha}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)} \quad \text{or} \quad \frac{\alpha}{M_Z^2\left(1-\sin^2\theta_W\right)\sin^2\theta_W}$$

*Recently improved ingredients:*

*fine structure constant,*

*electron mass,*

*Fermi constant.*

# Fine structure constant: free-electron g-2

$$a_e^{\mathrm{QED}} = \frac{\alpha}{2\pi} - 0.328478966\left(\frac{\alpha}{\pi}\right)^2 + 1.1812415\left(\frac{\alpha}{\pi}\right)^3 - 1.7283(35)\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$

Schwinger Sommerfield Laporta+Remiddi Kinoshita+Nio

Result: the most precise value of the Fine Structure Constant

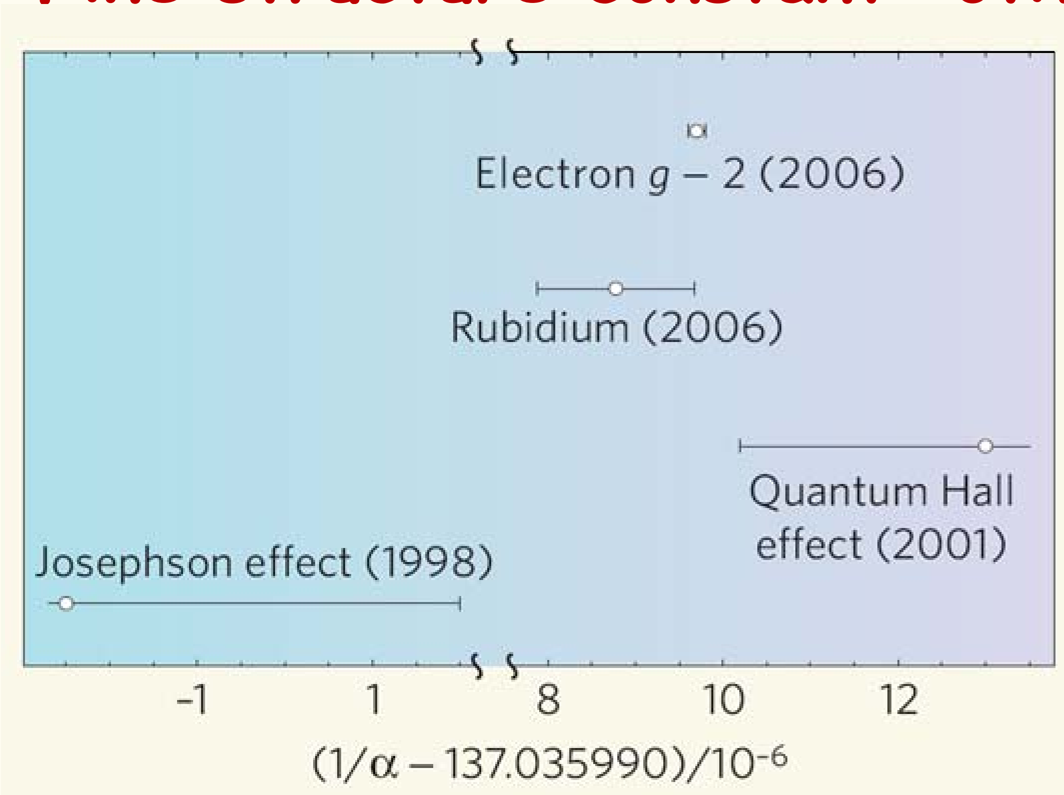
Previous: $\alpha = 1/137.035\,998\,834(12)_{\mathrm{th4}}(31)_{\mathrm{th5}}(502)_{\mathrm{exp}}$ (3.7 ppb)

Kinoshita & Nio, hep-ph/0507249

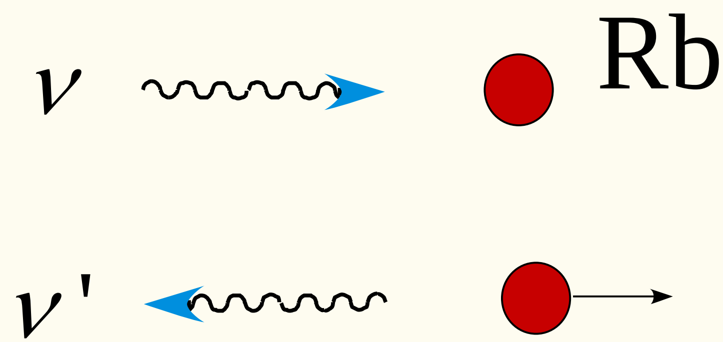
**NEW:** $\alpha = 1/137.035\,999\,710(12)_{\mathrm{th4}}(30)_{\mathrm{th5}}(90)_{\mathrm{exp}}$ (0.7 ppb)

Gabrielse et al, 2006

# Fine structure constant: other methods




Nature 442 (2006) 516.




$$\alpha^2 = \frac{2h\,\mathrm{Ry}}{m_e c}$$

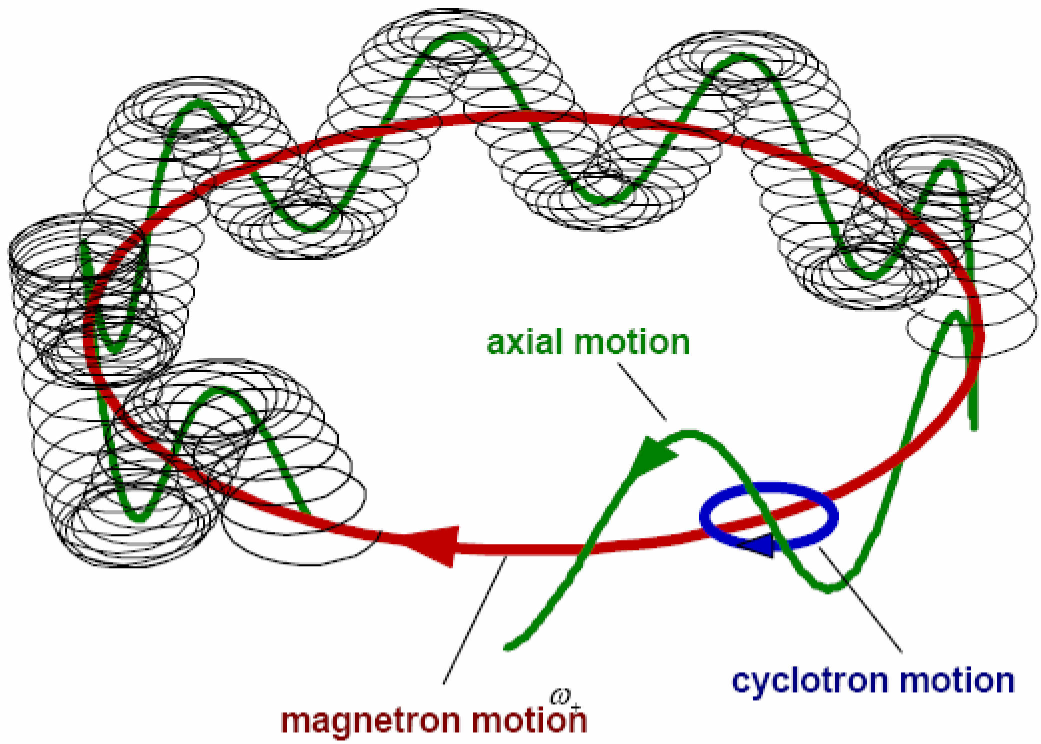
$$= \frac{2\mathrm{Ry}}{c}\frac{m_p}{m_e}\frac{m_{\mathrm{Rb}}}{m_p}\frac{h}{m_{\mathrm{Rb}}}$$

$$h\cdot(\nu-\nu') \simeq \frac{2h^2\nu^2}{m_{\mathrm{Rb}}c^2}$$

$$\frac{h}{m_{\mathrm{Rb}}} = \frac{\Delta\nu}{2\nu^2}c^2$$

# Electron mass: recent progress

Motion in a Penning trap:




Spin precession (Larmor) frequency

$$h\nu_L = g \cdot \mu_B \cdot B$$

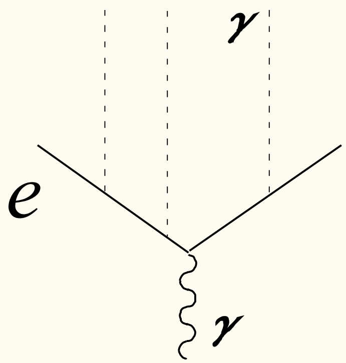
Cyclotron frequency:

$$h\nu_C = \frac{q}{M} B$$

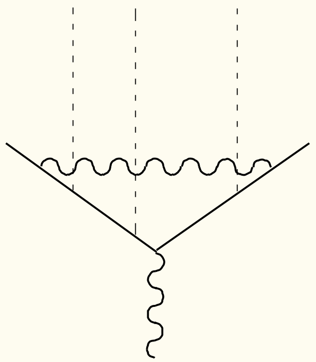
$$g = 2\frac{\nu_L}{\nu_C}\frac{q}{e}\frac{m}{M}$$

From Werth

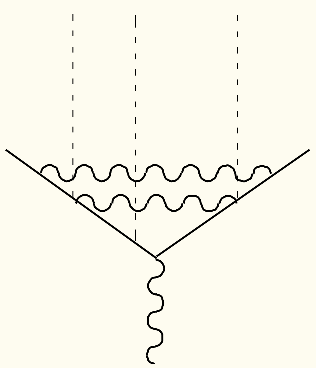
# Electron mass: bound-electron g-2




$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$



$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2} + a_{40}\right) + O(Z\alpha)^5\right]$$



$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\left(b_{41}\ln\frac{1}{(Z\alpha)^2} + b_{40}\right) + ..\right]$$

$$b_{40} = -16.4$$

Pachucki, AC, Jentschura, Yerokhin , 2005

# Electron mass: two-loop bound-state effect

Using the Mainz group measurements of $\nu_L / \nu_C$ we get

$$m_e\left({}^{12}C^{5+}\right) = 0.000\,548\,579\,909\,31(29)_{\text{exp}}\,(1)_{\text{th}}\;u$$

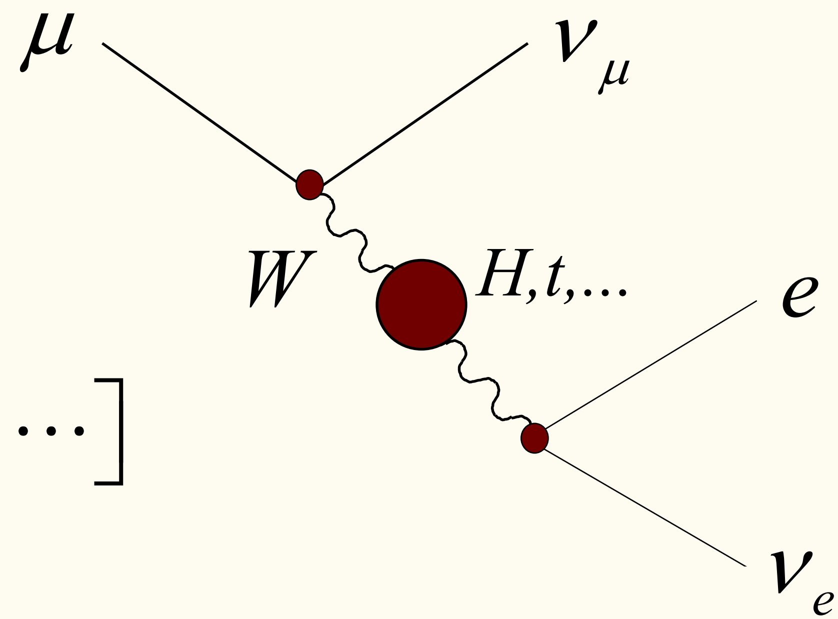
The free-electron mass determination,

$$m_e\left(\text{free}\right) = 0.000\;548\;579\;911\,10(120)_{\text{exp}}\;u$$

van Dyck, Farnham, Schwinberg 1995

# Fermi constant: progress in muon lifetime measurements

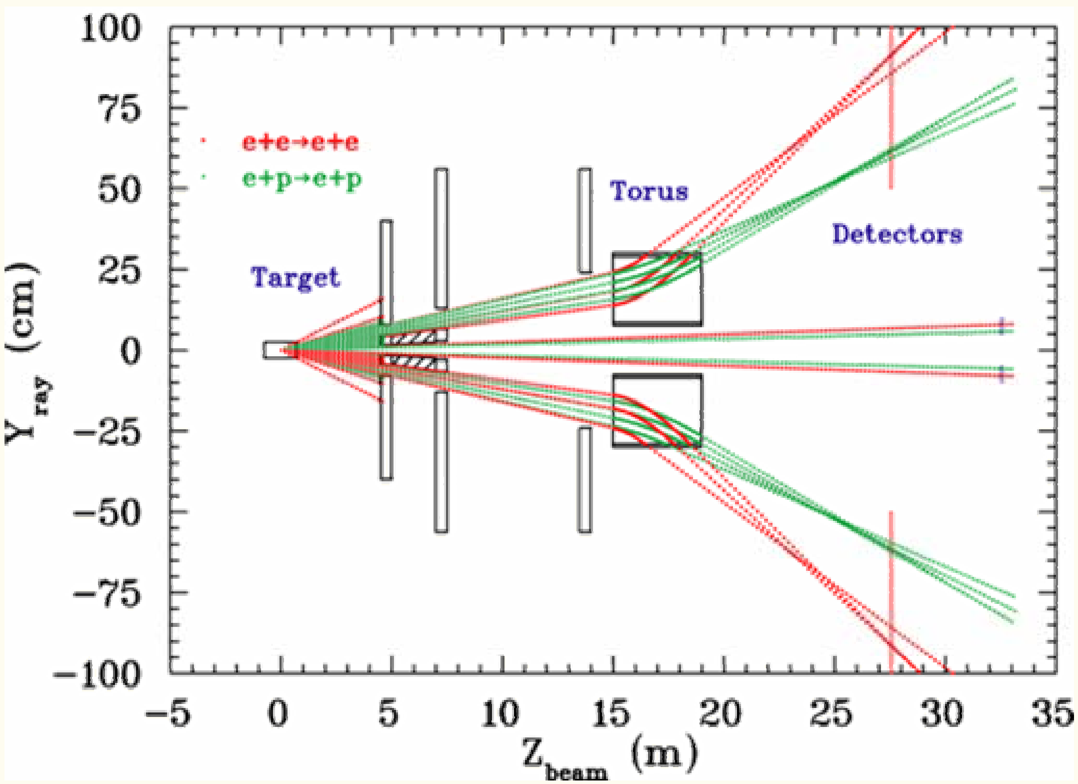
$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1+\cdots\right]$$




**muLan:**
**New number (within a week from now!): 11 ppm level, from 2004 run.**
**2006 run: $10^{12}$ events → 1ppm.**

From Hertzog

With the e2e @ 12 GeV,
Møller parity-violating amplitude
joins the club of "beyond one-loop"
electroweak observables.




From Paschke

# Electroweak radiative corrections to $A_{LR}$

$$A_{LR}(e^-e^-) = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4}$$
$$\times \Big\{ 1 - 4\kappa(0)\sin^2\theta_W(m_Z)_{\overline{\rm MS}} + \frac{\alpha(m_Z)}{4\pi s^2}$$
$$- \frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2]$$
$$+ F_1(y,Q^2) + F_2(y,Q^2) \Big\}$$

AC, W. J. Marciano
Ferroglia, Ossola, Sirlin
Erler, Ramsey-Musolf
Petriello

# Electroweak radiative corrections to $A_{LR}$

neutral current normalized in terms of the muon decay;
$\rho$ = 1.00122

$$A_{LR}(e^-e^-) = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha}\frac{1-y}{1+y^4+(1-y)^4}$$
$$\times\Big\{1 - 4\kappa(0)\sin^2\theta_W(m_Z)_{\overline{\rm MS}} + \frac{\alpha(m_Z)}{4\pi s^2}$$
$$- \frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2]$$
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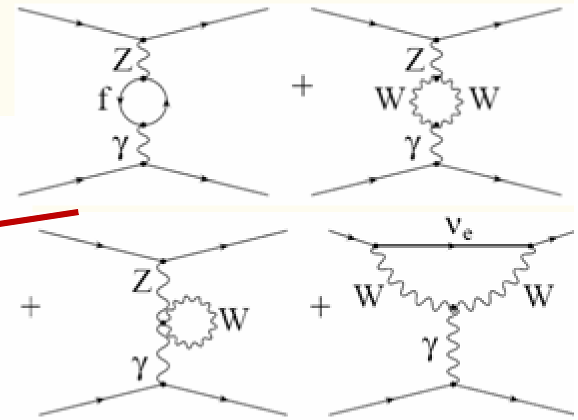
# Electroweak radiative corrections to $A_{LR}$




$$A_{LR}(e^-e^-) = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha}\frac{1-y}{1+y^4+(1-y)^4}$$
$$\times \left\{ 1 - 4\kappa(0)\sin^2\theta_W(m_Z)_{\overline{\rm MS}} + \frac{\alpha(m_Z)}{4\pi s^2} - \frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2] + F_1(y,Q^2) + F_2(y,Q^2) \right\}$$

Important effect: reduces $A_{LR}$ by ~38%

$$\kappa(0) = 1.0301 \pm 0.0025$$

Uncertainty from hadronic contributions;
recently reduced (Erler & Ramsey-Musolf)

# Electroweak radiative corrections to $A_{LR}$

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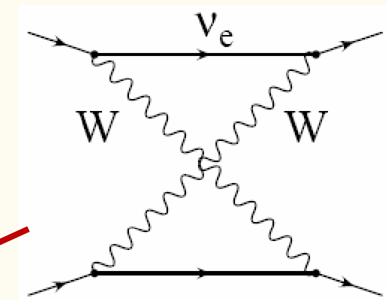
$$\times\Big\{1 - 4\kappa(0)\sin^2\theta_W(m_Z)_{\overline{\mathrm{MS}}} + \frac{\alpha(m_Z)}{4\pi s^2}$$

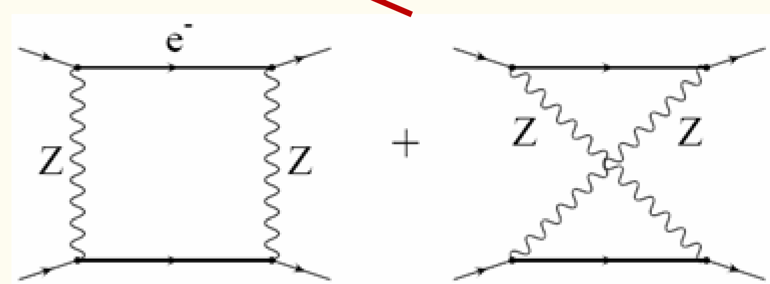
$$-\frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2]$$

$$+F_1(y,Q^2)+F_2(y,Q^2)\Big\}$$



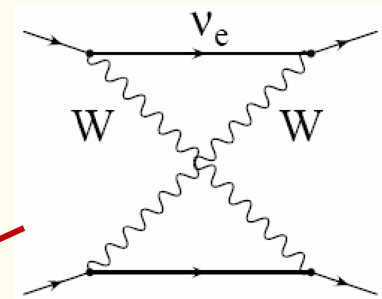

*+0.0027*
in 't Hooft-Feynman gauge

*-0.0001*



# Electroweak radiative corrections to $A_{LR}$

$$A_{LR}(e^-e^-) = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \times \left\{ 1 - 4\kappa(0)\sin^2\theta_W(m_Z)_{\overline{\rm MS}} + \frac{\alpha(m_Z)}{4\pi s^2} - \frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2] + F_1(y,Q^2) + F_2(y,Q^2) \right\}$$




*+0.0027*
in 't Hooft-Feynman gauge

*Note: four times larger in APV and Qweak; cancels much of the kappa effect there.*

# Electroweak radiative corrections to $A_{LR}$

$$A_{LR}(e^-e^-) = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4}$$
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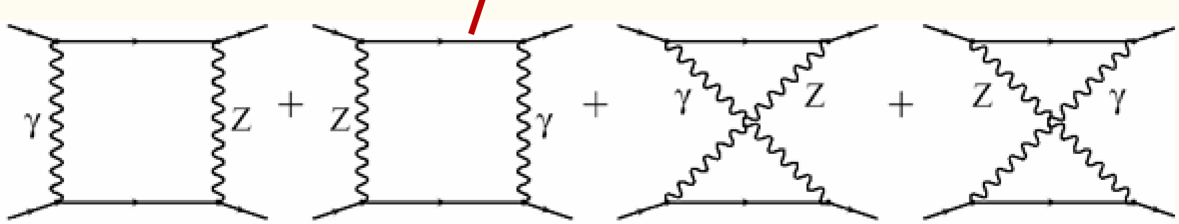
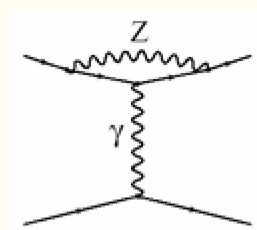
$$+ F_1(y,Q^2) + F_2(y,Q^2) \Big\}$$

This is the second most significant shift, -0.0041(10)



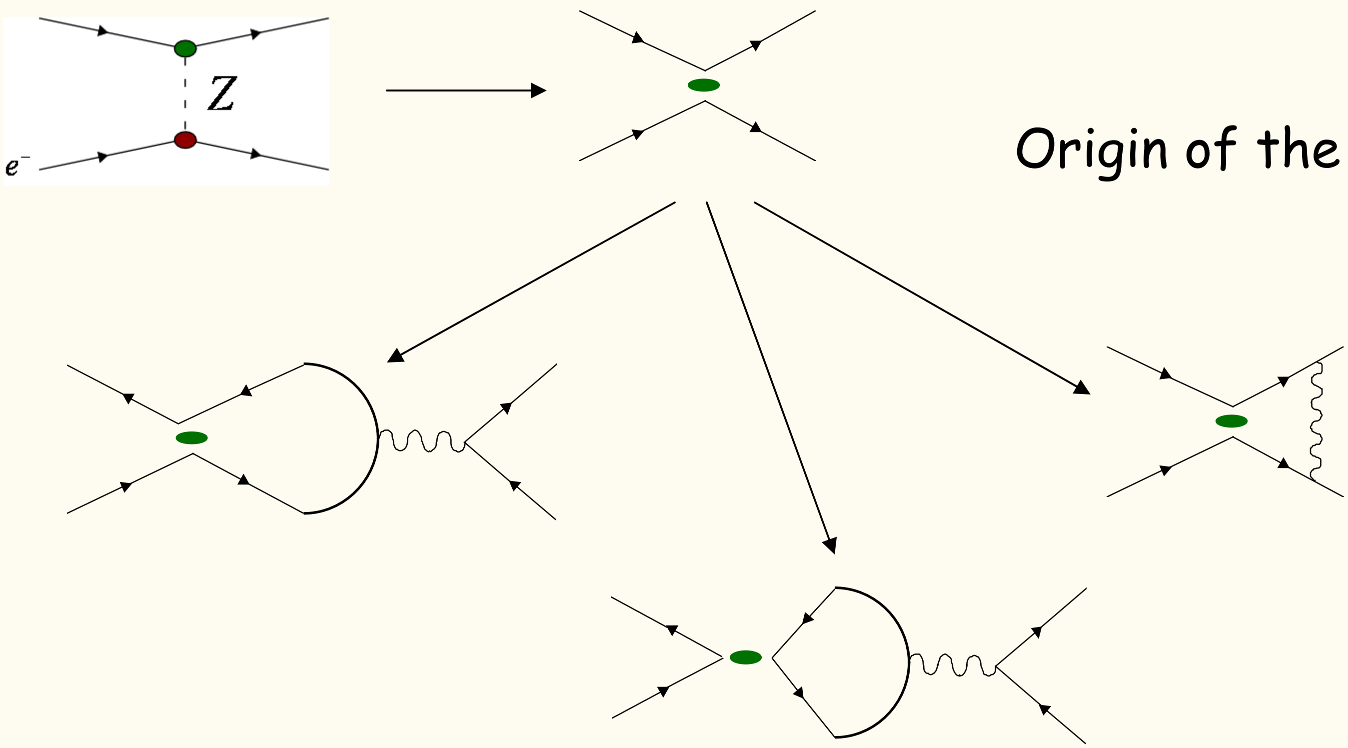

*and*



# Photon-Z boxes and the anapole moment

$$F_1(y,Q^2) = -\frac{\alpha}{4\pi}(1-4s^2)\left\{\frac{22}{3}\ln\frac{ym_Z^2}{Q^2}+\frac{85}{9}+f(y)\right\}$$

$$f\left(\frac{1}{2}\right) = \frac{17}{12}\pi^2+\frac{70}{9}\ln 2-\frac{8}{3}\ln^2 2 \approx 18.09$$




Origin of the large logs

# Electroweak radiative corrections to $A_{LR}$

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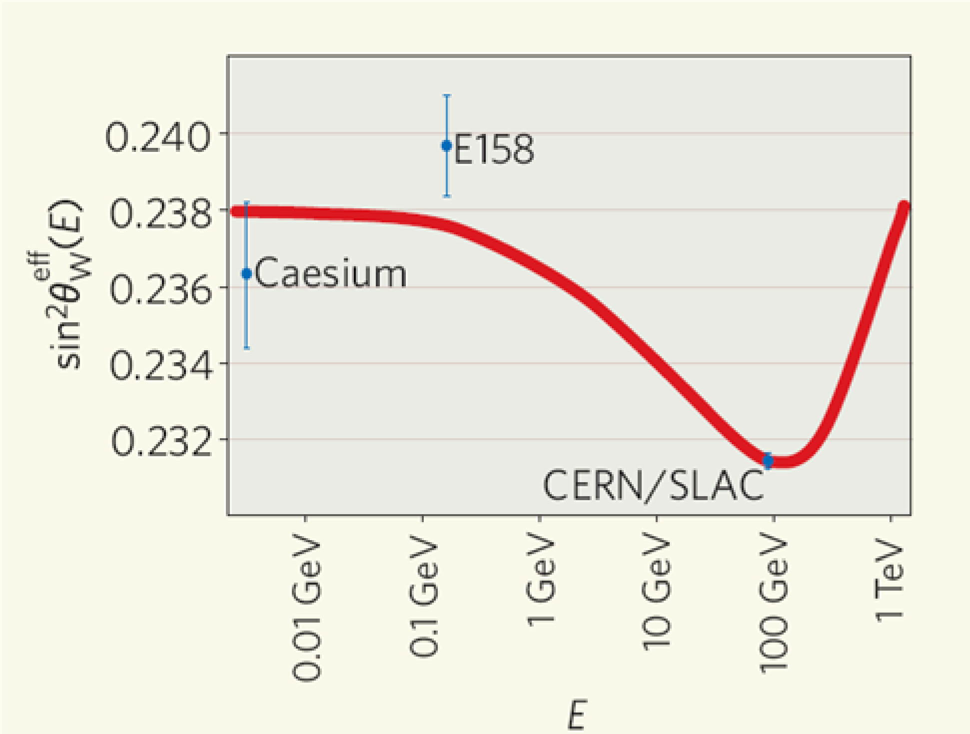
$$- \frac{3\alpha(m_Z)}{32\pi s^2 c^2}(1-4s^2)[1+(1-4s^2)^2]$$
$$\left. + F_1(y,Q^2) + F_2(y,Q^2) \right\}$$

$Q^2$ dependence in $\kappa$

# Running sin²θ




$$\kappa_b(Q^2) = 1 - \frac{\alpha}{2\pi \sin^2\theta_W}\left\{-\frac{42\cos^2\theta_W + 1}{12}\ln\cos^2\theta_W + \frac{1}{18}\right.$$

$$-\left(\frac{p}{2}\ln\frac{p+1}{p-1} - 1\right)\left[(7-4z)\cos^2\theta_W + \frac{1}{6}(1+4z)\right]$$

$$\left. -z\left[\frac{3}{4} - z + \left(z - \frac{3}{2}\right)p\ln\frac{p+1}{p-1} + z\,(2-z)\ln^2\frac{p+1}{p-1}\right]\right\}$$

$$z \equiv \frac{m_W^2}{Q^2}, \qquad p \equiv \sqrt{1+4z}.$$

AC, W. J. Marciano

## Conclusions

Radiative corrections significantly influence $A_{LR}$.

The e2e experiment at 12 GeV will challenge theorists to determine leading two-loop effects: difficult but feasible.

Theoretical and experimental efforts very worthwhile – address the largest discrepancy inherited from the LEP/SLC era.