Tensor Charge and Transversity using Lattice QCD

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Outline

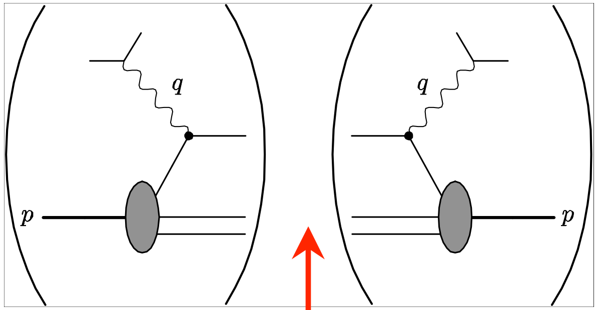
- *Transversity and Tensor Charge*
  - *Anatomy of a Calculation*
  - *Tensor Charge*
  - *Higher moments*
- *Prospects:*
  - *Flavor-singlet Hadron Structure*
  - *Calculations at physical light-quark masses*
  - *Direct calculations of x-dependence*
- *Outlook*

Transversity Distributions in LQCD

- **Describe distribution of longitudinal momentum and spin in proton**
- **Matrix elements of light-cone correlation functions**

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

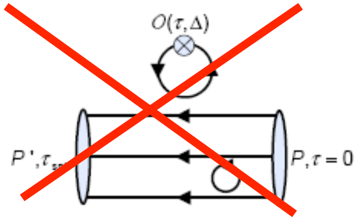


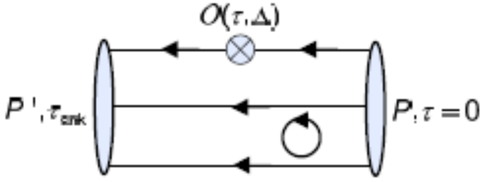
$$O^{\{\mu_1 \ldots \mu_n\}} = \bar{\psi}\sigma^{[\mu\{\nu]} iD^{\mu_1} \ldots iD^{\mu_{n-1}\}}\psi$$

- **Diagonal matrix element**

$$\langle x^n \rangle_{\delta_q} = \int_0^1 \left[\delta q(x) + (-1)^{n+1}\delta\bar{q}(x)\right]$$

Dominated by lightest state



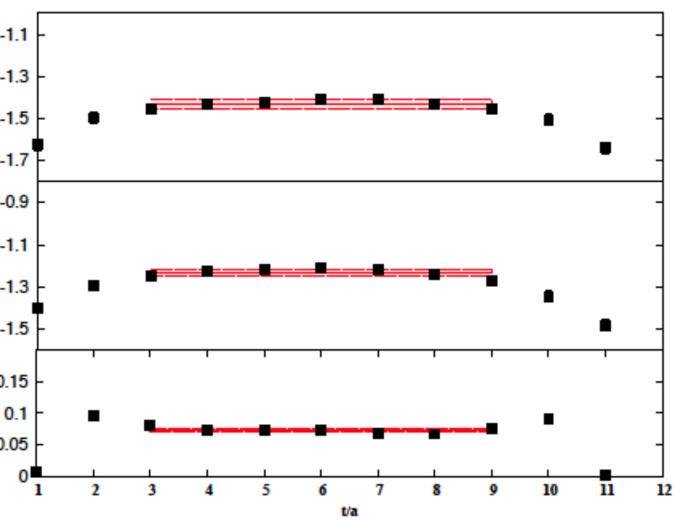


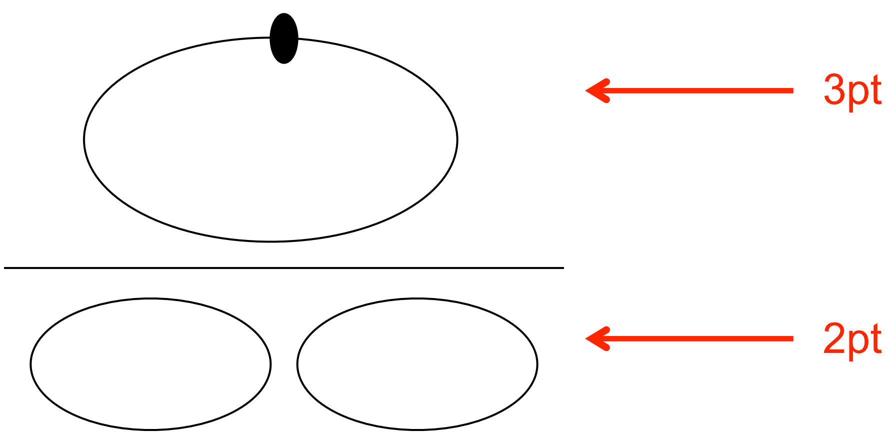
Anatomy of a Matrix Element Calculation

$$G^{2\mathrm{pt}}(q, t_f - t_i) = \sum_{\vec{x}} e^{-i\vec{q}\cdot\vec{x}} \langle N(t_f, \vec{x}) \bar{N}(t_i, 0) \rangle$$

$$G^{3\mathrm{pt}}(q, t_f - t_i, t) = \sum_{\vec{x}} e^{-i\vec{q}\cdot\vec{y}} \langle N(t_f, \vec{x}) \mathcal{O}(\vec{y}, t) \bar{N}(t_i, 0) \rangle$$

Resolution of unity – insert states

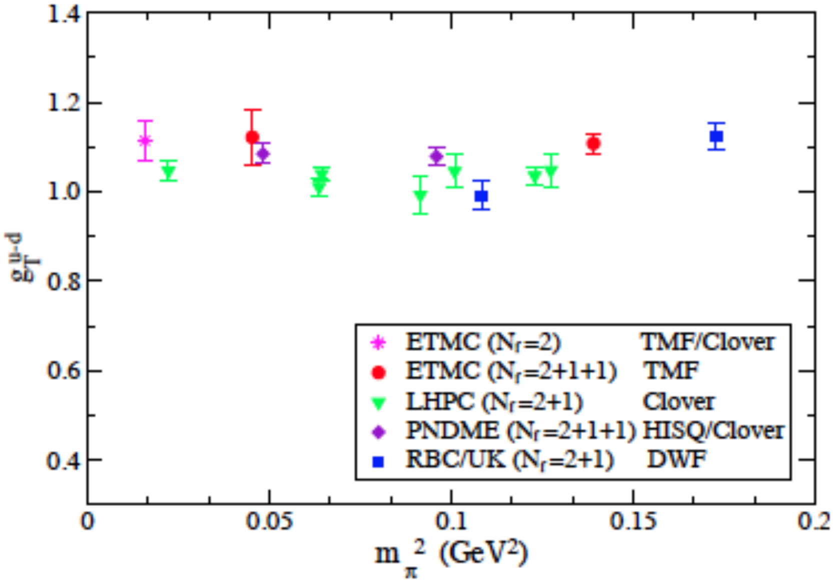




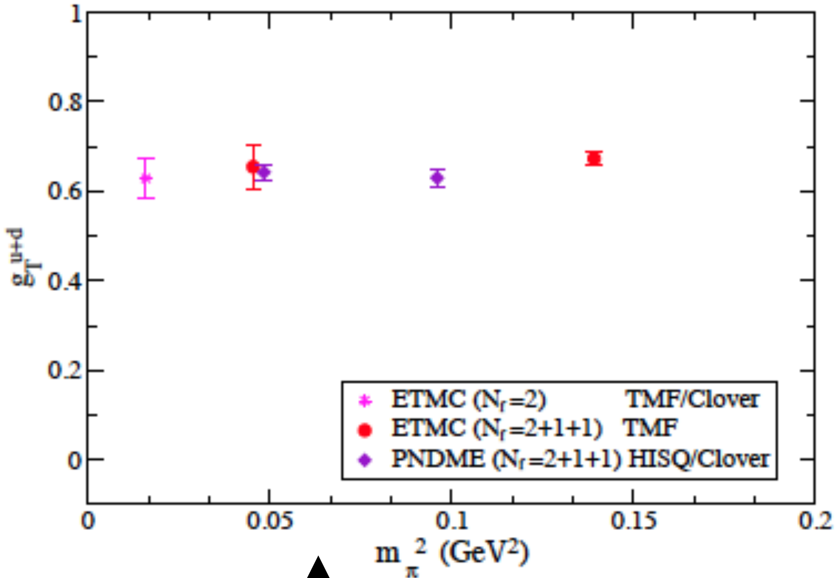
Tensor Charge

Constantinou et al, arXiv:1311.4670

Simply lowest moment of transversity distributions



Isovector



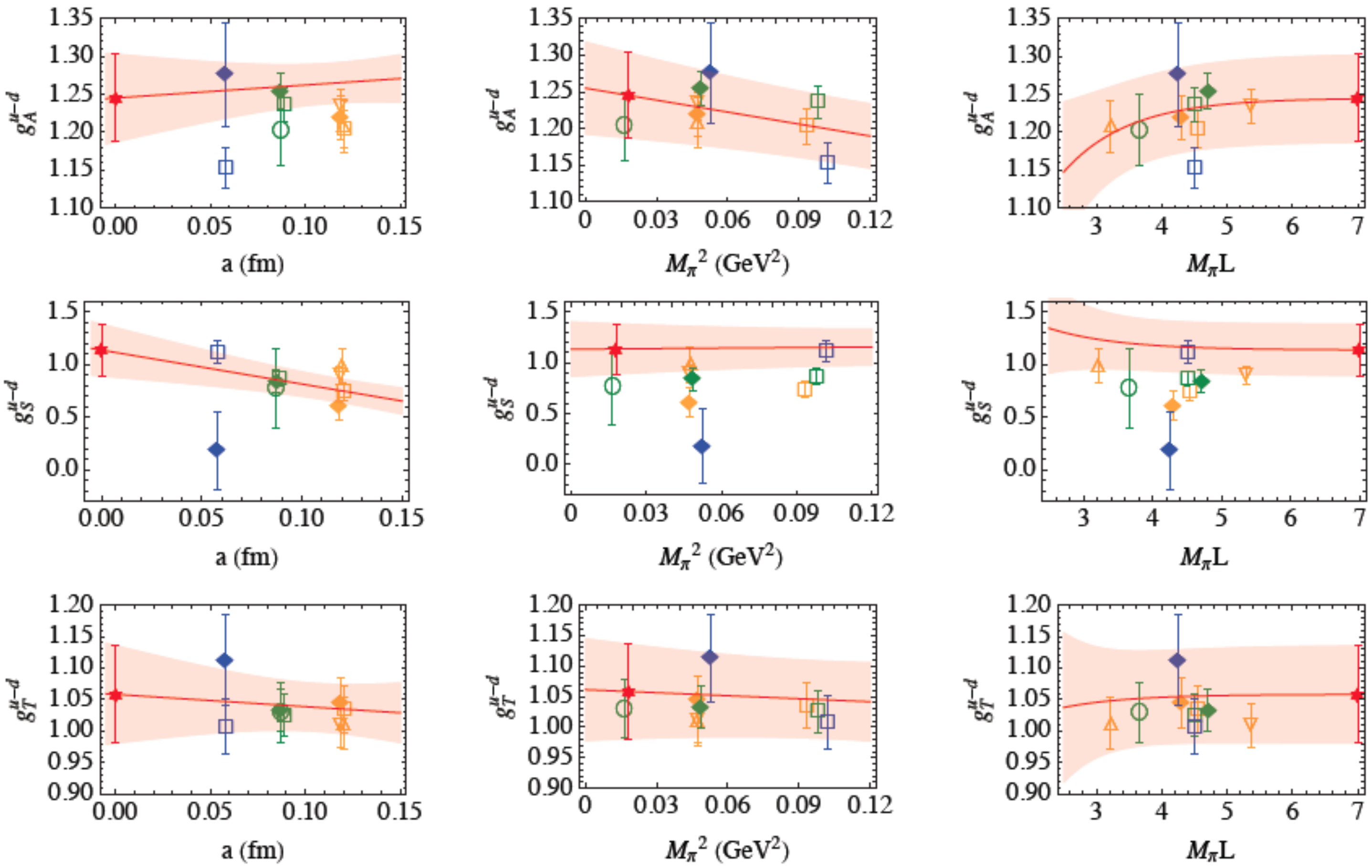
Isoscalar

Consistency between different discretisations

…but with disconnected contributions neglected…

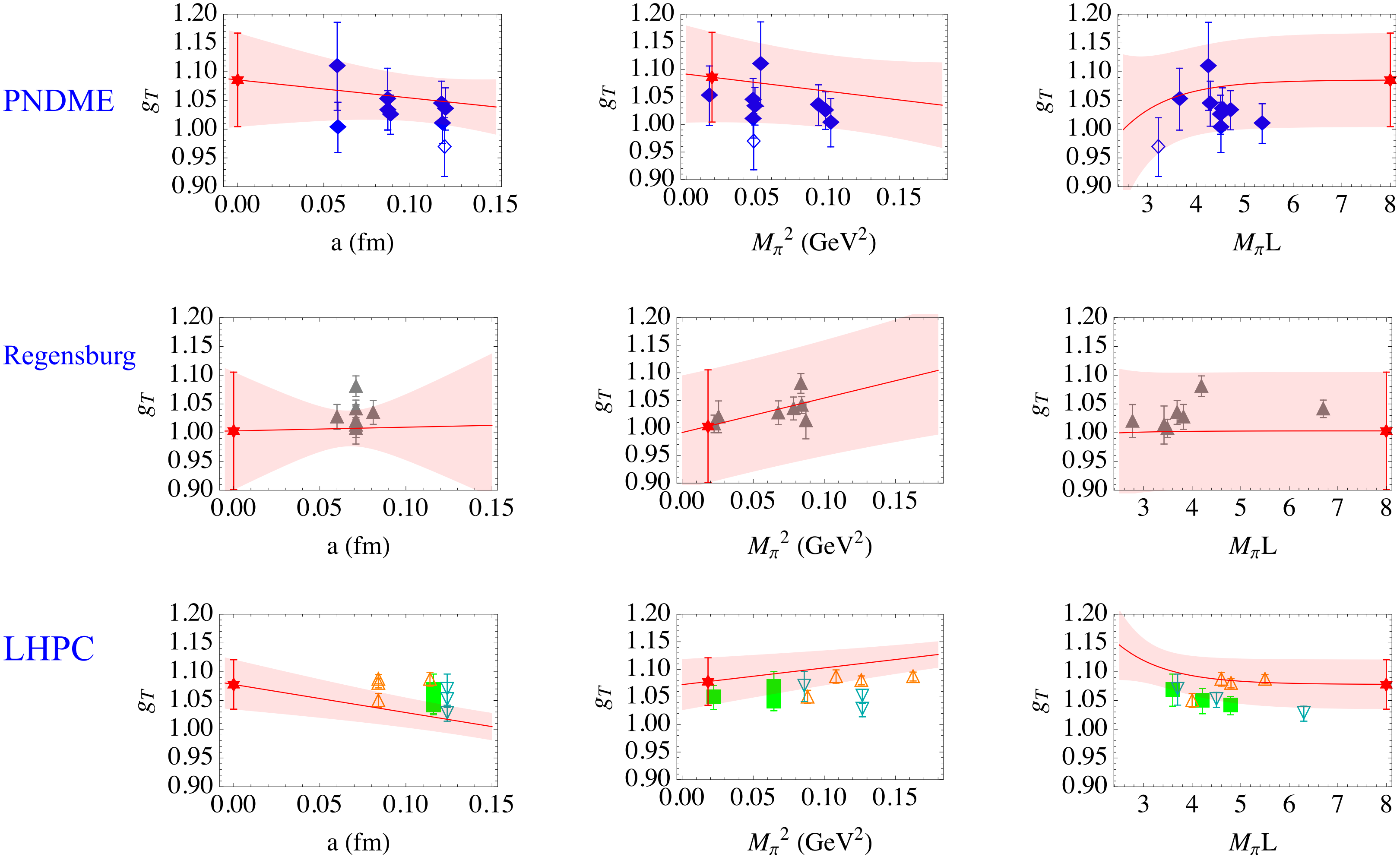
Precision Nucleon Matrix Elements

- LANL Group: *precision calculations of g_S, g_T and g_A.*
  Needed for BSM studies of ultra-cold neutron decays



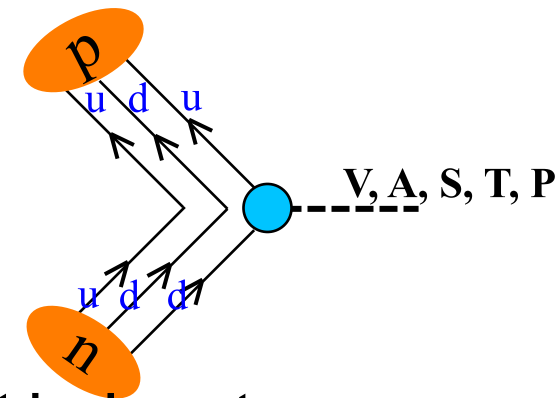
Comparison of gT

Slide: R. Gupta



Nucleon Charges

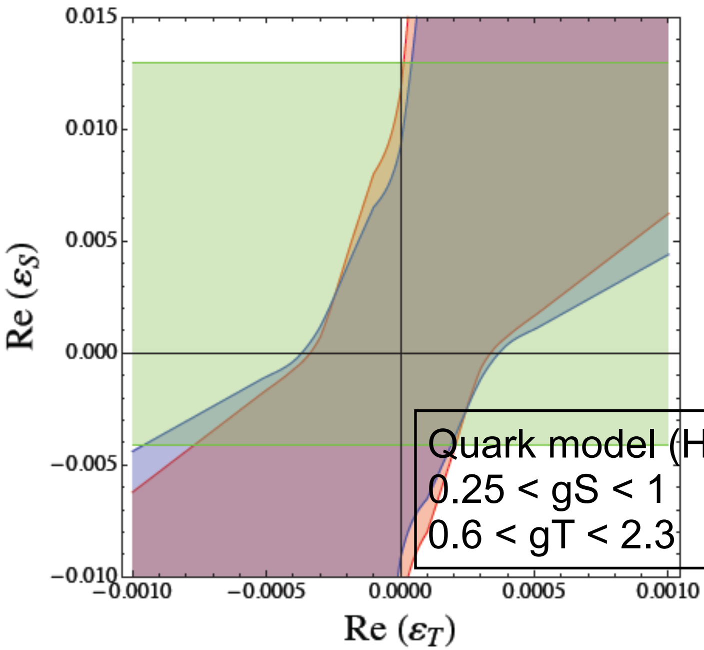
e.g. novel interactions probed in ultra-cold neutron decay



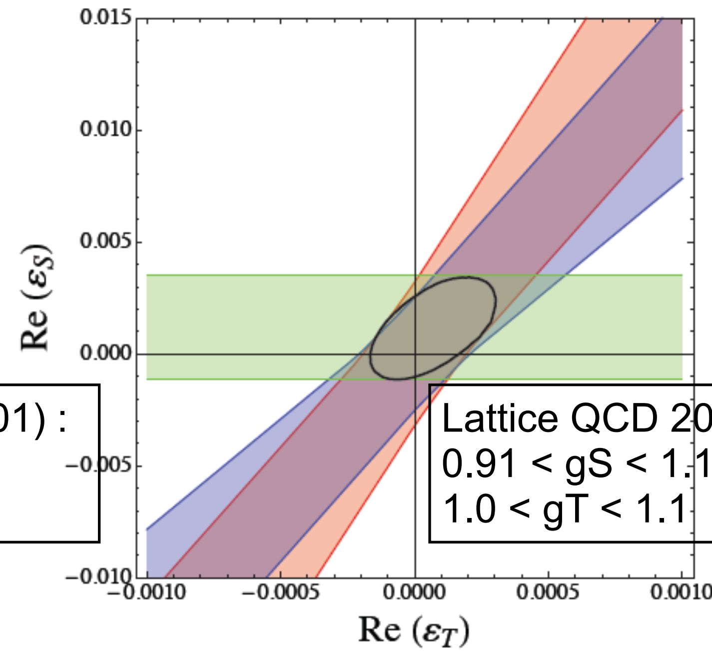
$$H_{eff} \supset G_F\left[\varepsilon_S\, \bar{u} d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T\, \bar{u}\sigma_{\mu\nu} d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e\right]$$

$$g_S = Z_S \langle p|\bar{u} d|n\rangle \quad g_T = Z_T \langle p|\bar{u}\sigma_{\mu\nu} d|n\rangle$$

Require precise knowledge of QCD nucleon matrix elements, g_A, g_T, g_S



Quark model (Herczeg 2001) :
0.25 < gS < 1
0.6 < gT < 2.3



Lattice QCD 2014 (R. Gupta)
0.91 < gS < 1.13
1.0 < gT < 1.1

Higher Moments

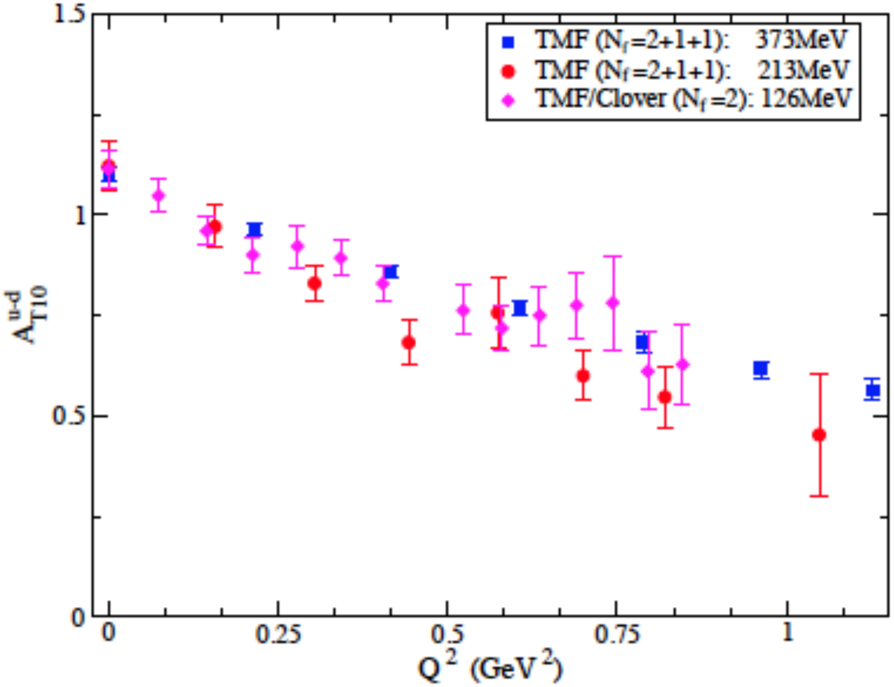
$$\langle\langle \overline{q}(0) i\sigma^{\mu\nu} q(0) \rangle\rangle = \langle\langle i\sigma^{\mu\nu} \rangle\rangle A_{T10}(q^2) + \langle\langle \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N} \rangle\rangle B_{T10}(q^2) + \langle\langle \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2} \rangle\rangle \widetilde{A}_{T10}(q^2),$$

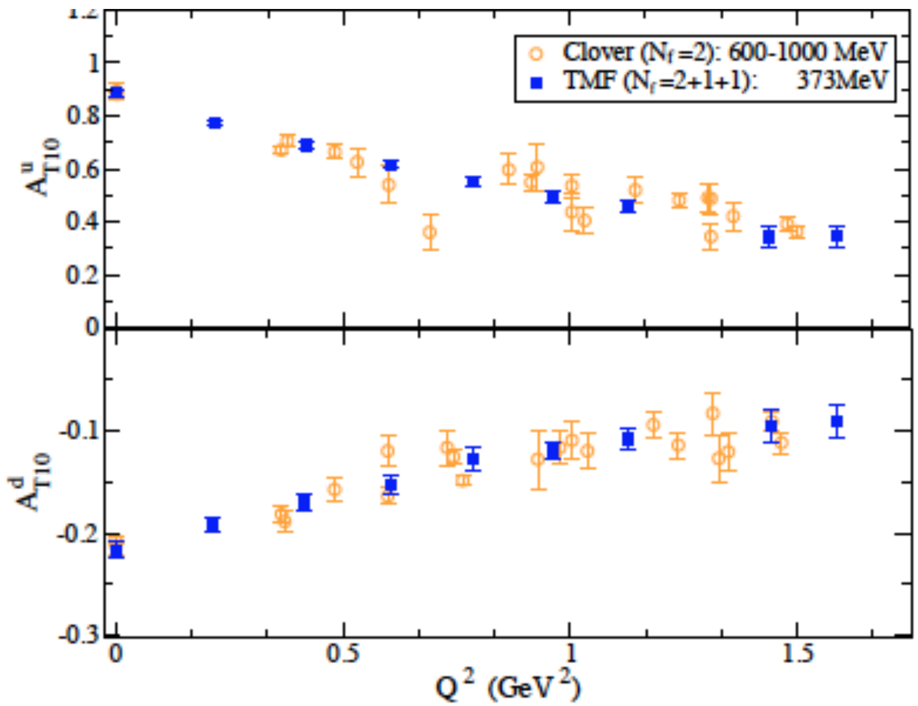
$$\langle\langle \overline{q}(0) \mathcal{O}_T^{\mu\nu\mu_1}(0) q(0) \rangle\rangle = \mathcal{A}_{\mu\nu}\mathcal{S}_{\nu\mu_1} \Big\{ \langle\langle i\sigma^{\mu\nu}\overline{P}^{\mu_1} \rangle\rangle A_{T20}(q^2) + \langle\langle \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\overline{P}^{\mu_1} \rangle\rangle B_{T20}(q^2) + \langle\langle \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\overline{P}^{\mu_1} \rangle\rangle \widetilde{A}_{T20}(q^2) + \langle\langle \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\Delta^{\mu_1} \rangle\rangle \widetilde{B}_{T21}(q^2) \Big\}.$$

Can directly obtain in forward limit

First moment of Transversity

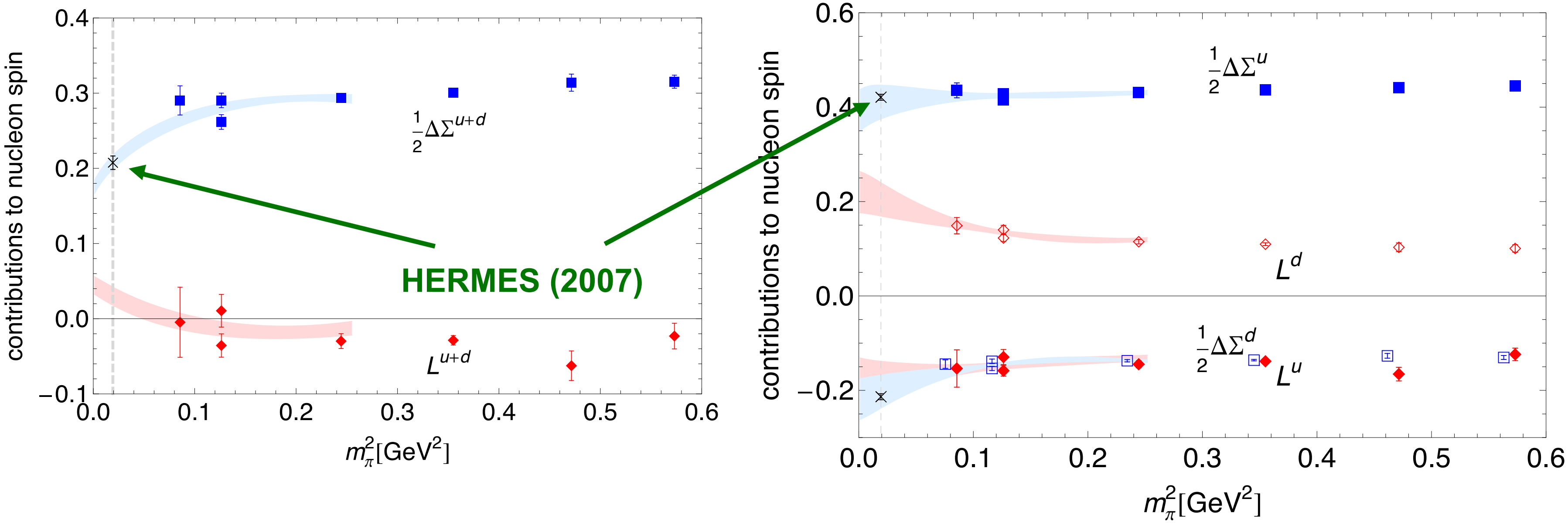
First calculations in chiral regime - only perturbative renormalization





Disconnected diagrams not included

Origin of Nucleon Spin



$$J^q = 1/2\left(A_{20}^q(t=0) + B_{20}^q(t-0)\right)$$

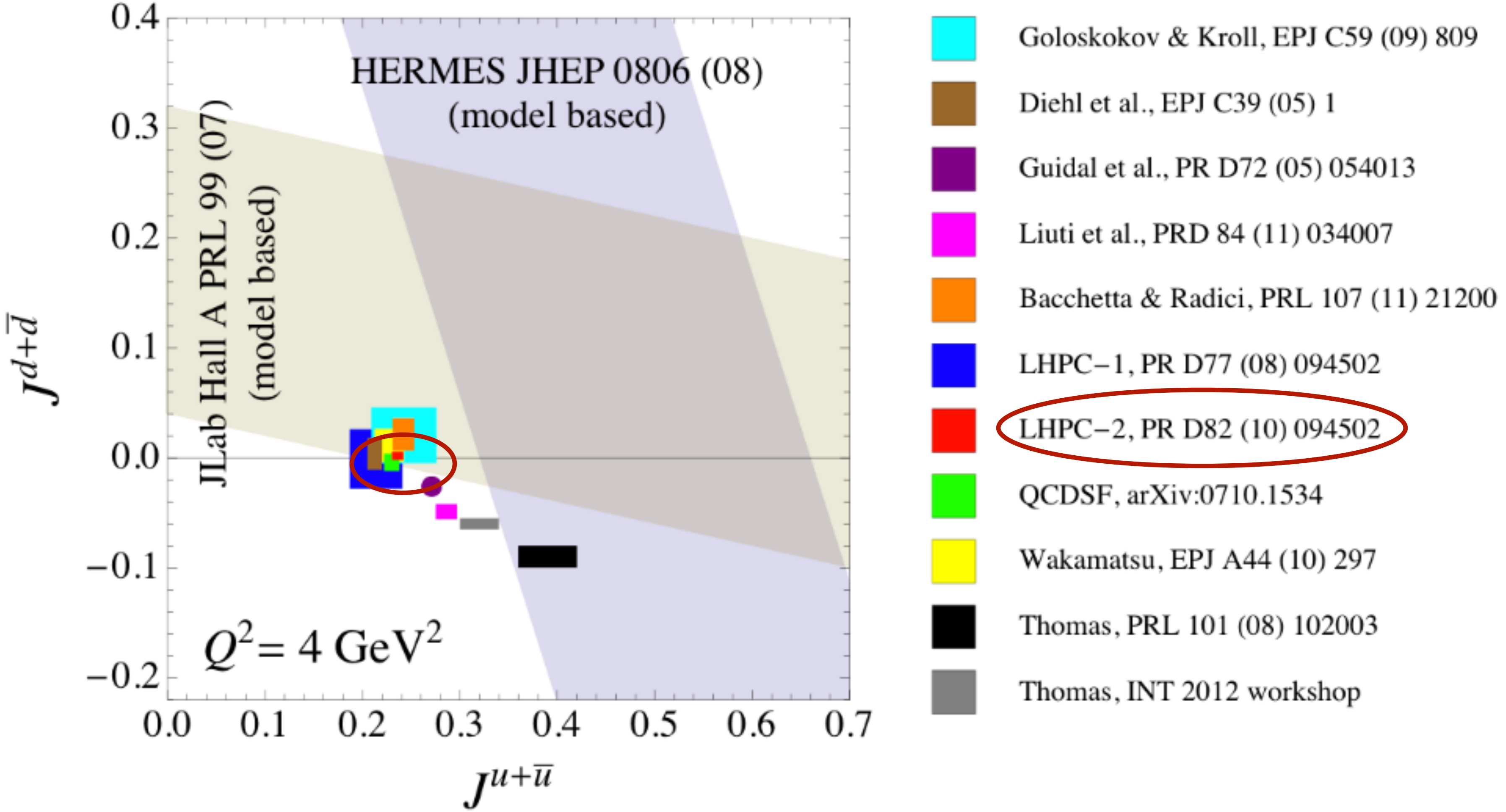
$$\Delta\Sigma^q/2 = \tilde{A}_{10}^q(t=0)/2$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

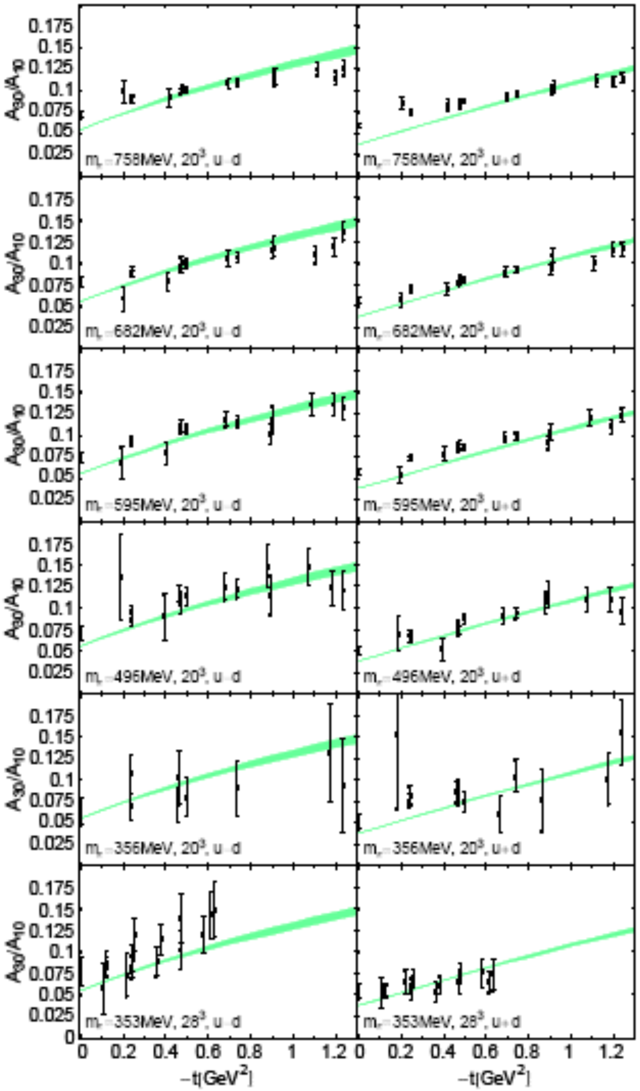
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); D82, 094502 (2010)

- **Total orbital angular momentum carried by quarks small**
- **Orbital angular momentum carried by individual quark flavors substantial.**

Origin of Nucleon Spin - II



Parametrizations of GPDs

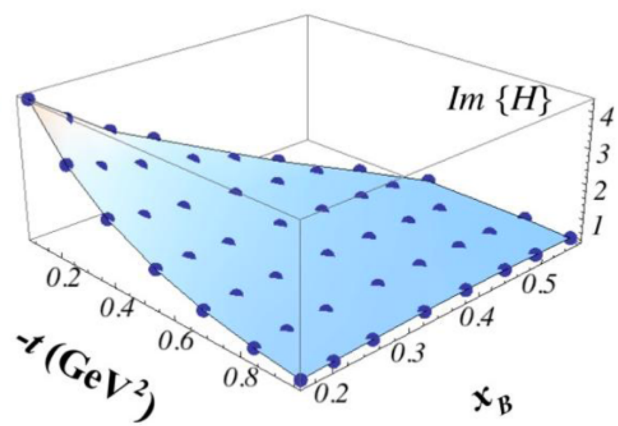


Provide phenomenological guidance for GPD's

- *CTEQ, Nucleon Form Factors, Regge*

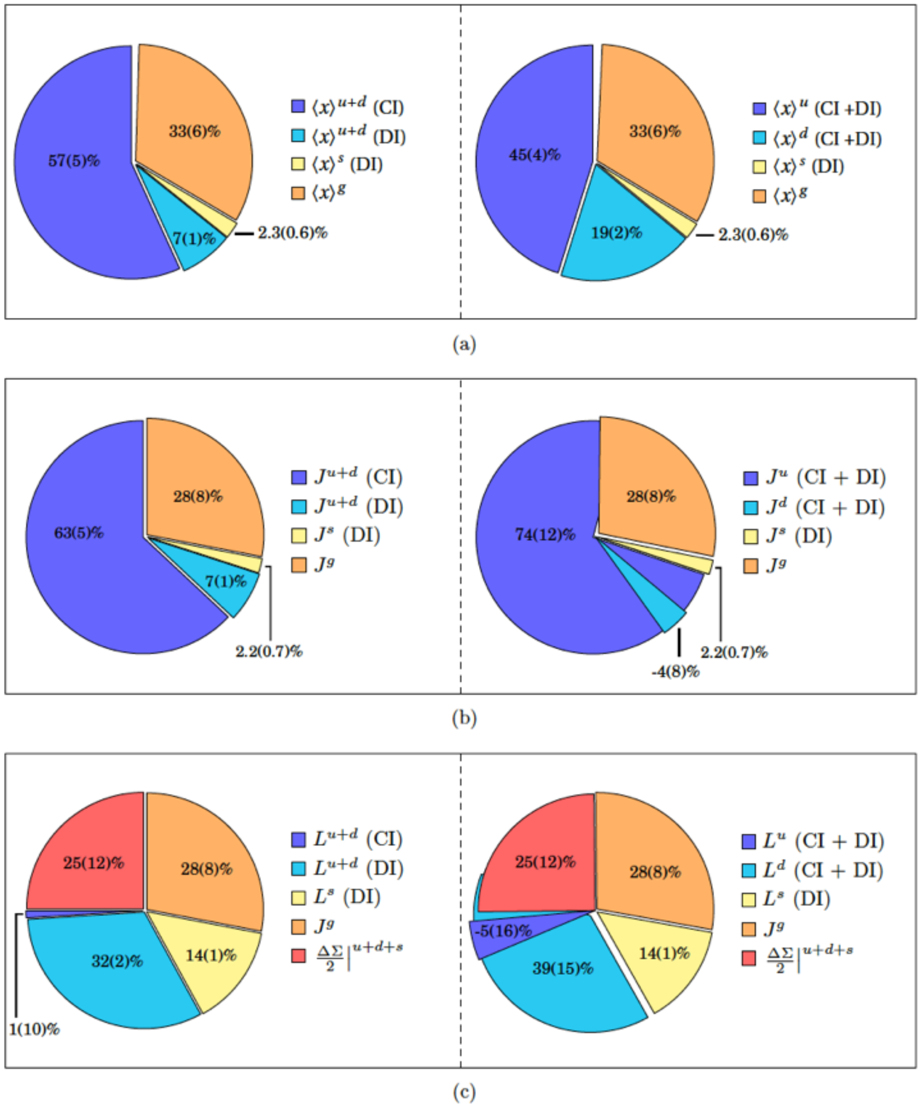
Comparison with *Diehl et al*, hep-ph/0408173

Important Role for LQCD

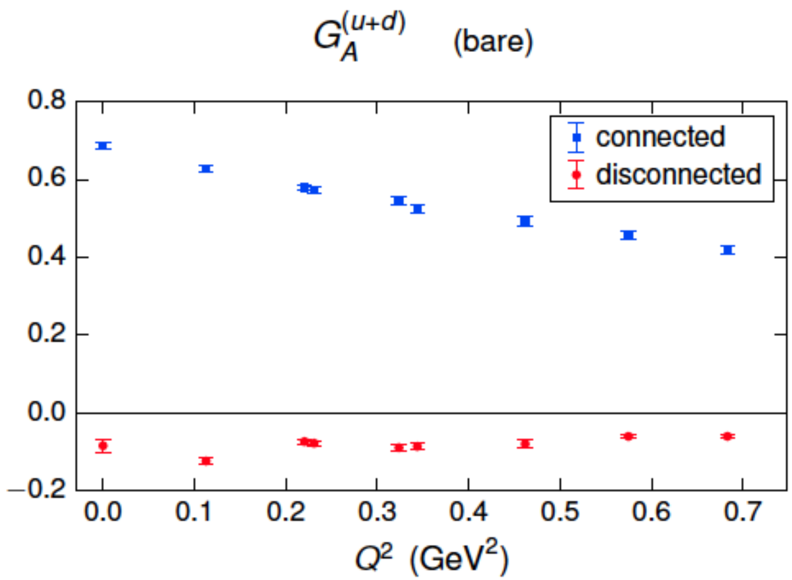


Future Prospects

Flavor-separated and Gluon Contributions



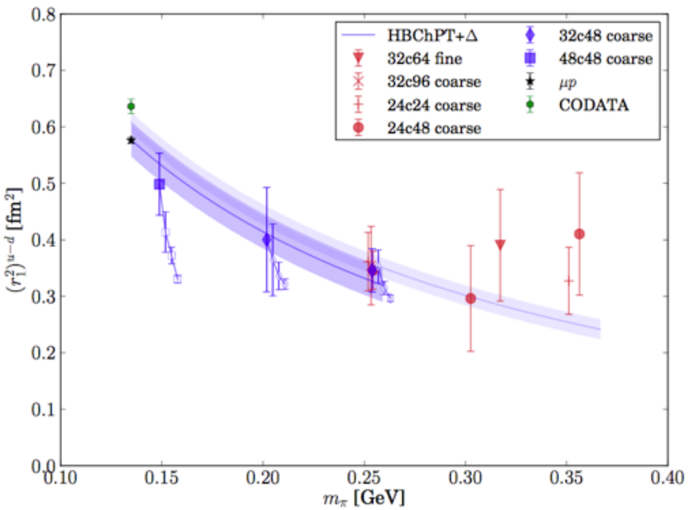
PRELIMINARY: S. Meinel, Lattice 2014



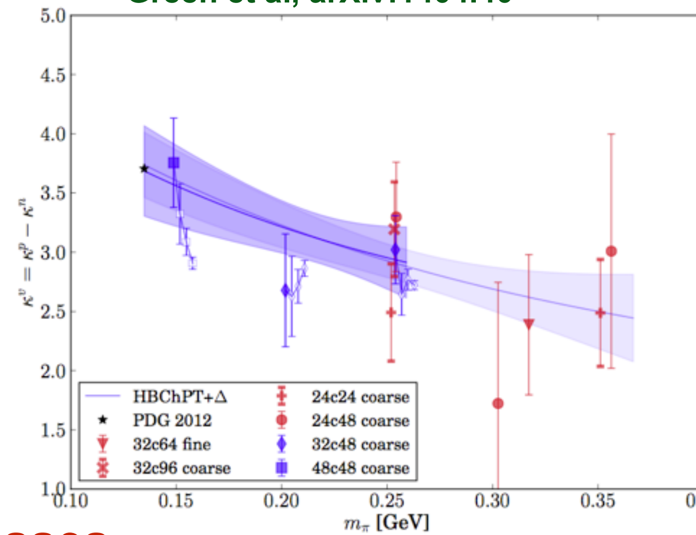
Complete calculation of flavor-separated and gluonic contributions to nucleon spin

Deka et al, arXiv:1312.4816

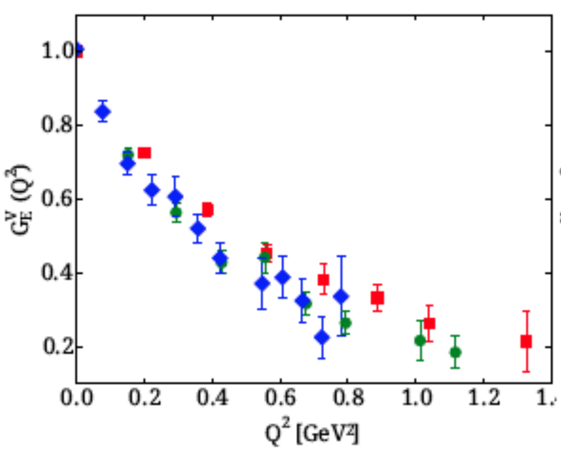
Calculations at Physics Quark Masses

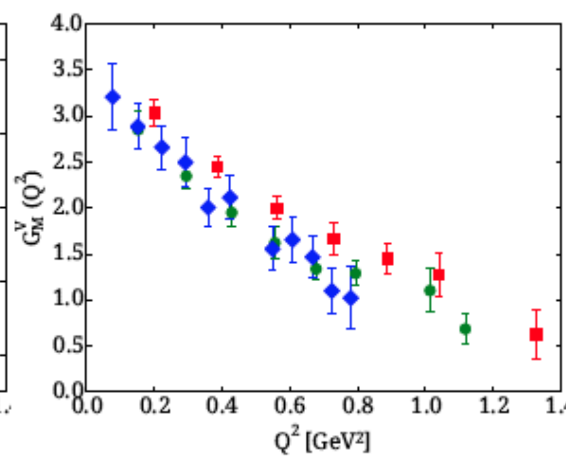


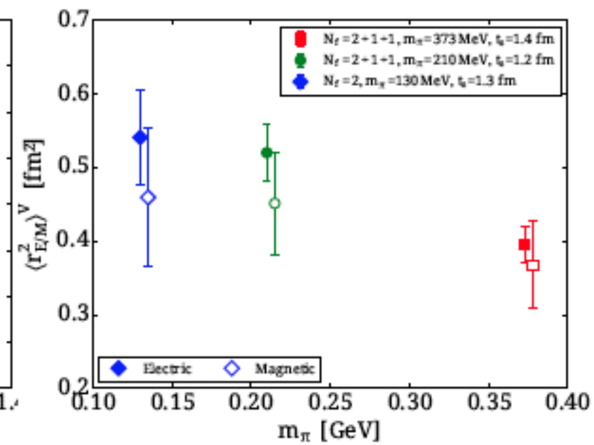
Green et al, arXiv:1404.40



Precision Calculations at physical quark masses







Parton Distributions - I

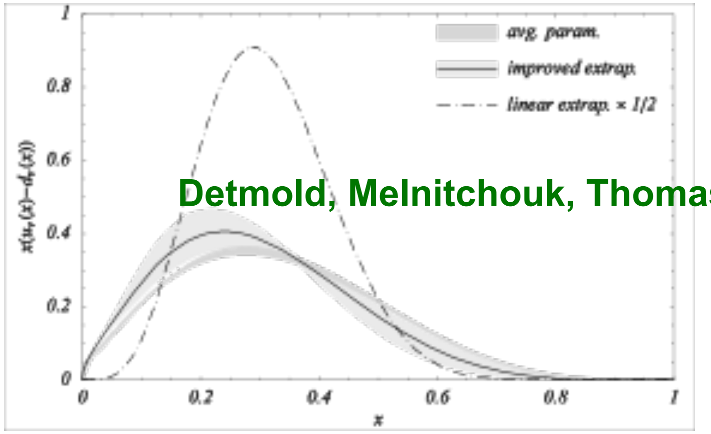
- x-dependence of distributions related to matrix elements of operators separated on the light-cone

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

LQCD formulated in Euclidean space to ensure the action is real - importance sampling

Compute instead x-moments of distributions. Hyper cubic symmetry restricts calculation to lowest few moments

Can you recover the distributions?



Detmold, Melnitchouk, Thomas

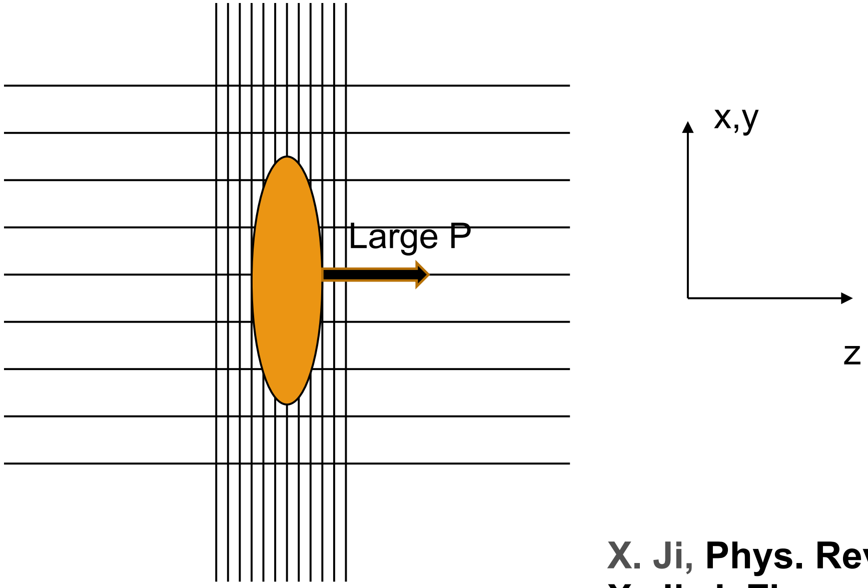
$$x(u_v(x) - d_v(x)) = a\, x^b (1-x)^c (1 + \varepsilon\sqrt{x} + \gamma\, x)$$

Parton Distributions - II

Formulation of LQCD in Euclidean space precludes direct calculation of light-cone correlation functions

→ LQCD computes Moments of parton distributions

New ideas: calculations of QUASI-distributions in *infinite-momentum frame*



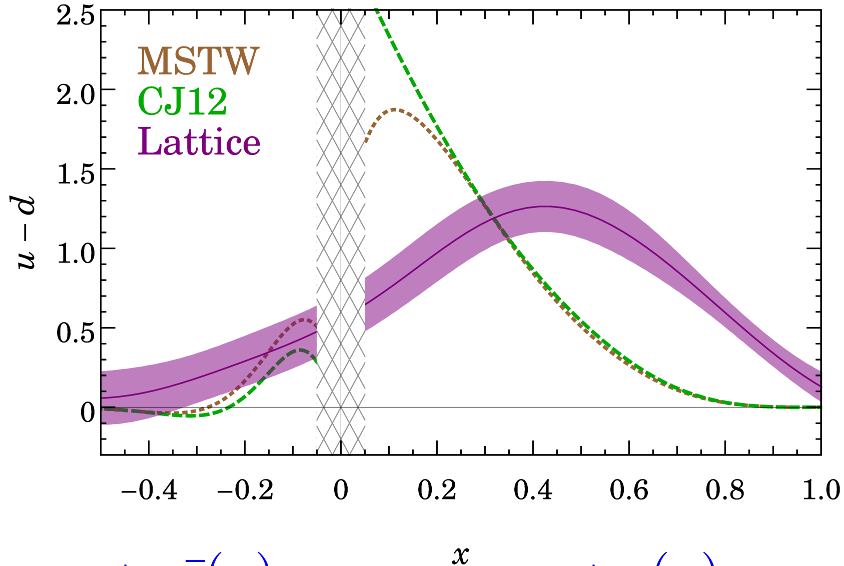
X. Ji, **Phys. Rev. Lett. 110, 262002 (2013).**
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk} \times \left\langle \vec{P} \right| \bar{\psi}(z)\gamma_z e^{ig\int_0^z A_z(z')dz'} \psi(0) \left| \vec{P} \right\rangle$$

"Equal time" correlator

…Flavor Structure

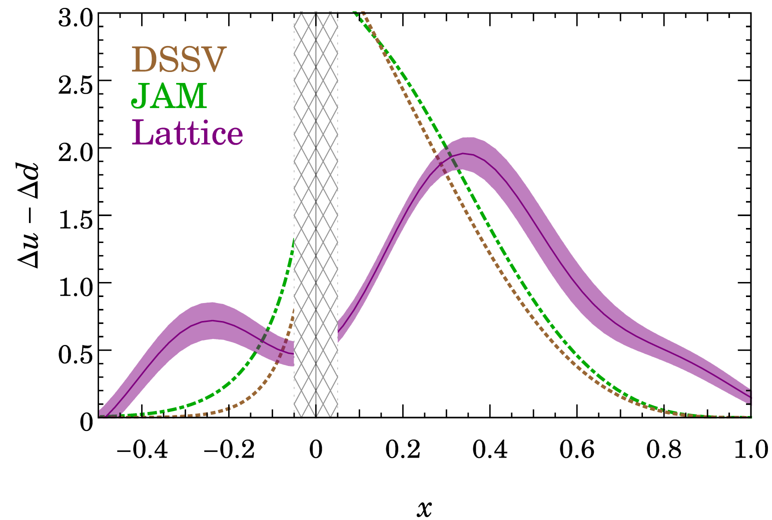
$$\tilde{q}(x,\mu,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \frac{M_N^2}{P_z^2}\right) + \ldots$$



$\leftarrow \bar{q}(x)$ $\rightarrow q(x)$

H.W. Lin et al, arXiv:1402.1462

First lattice calculations of Quasi Distributions



smallest $x \simeq 1/a$

12 GeV; Future EIC

Violation of Gottfried sum rule $\bar{d}(x) > \bar{u}(x)$

Summary

- **Lattice Calculations of the simplest quantities are now appearing at physical values of the quark masses**
- ***High-precision calculations of local matrix elements - relevant for searches for new physics in, e.g. UCN.***
  - **To directly explore x distributions, there are now a slew of new ideas… Ji et al, Qiu et al.**
- **Major effort underway in US in generating lattices designed for hadron structure calculations.**

TABLE I. Resources, in Tflops-years (TF-yrs), required to calculate the connected diagrams for a complete set of form factors and the lowest three moments of structure functions and generalized parton distributions for the nucleon at the physical pion mass. Lattice Generation is the cost of 10^4 trajectories, Measurements A reduce the error for g_A to 3 % , and Measurements B reduce the error for $\langle x \rangle$ to 3 %. The actions used are Wilson-clover fermions (W) and Domain-wall fermions (DW); measurements for both use the EigCG algorithm[32], and for the latter we also employ low-mode averaging[31].

$N_s^3 \times N_t$	Action	a_s (fm)	$m_\pi L$	$m_\pi T$	Lattice Generation (TF-yrs)	Measurements A (TF-yrs)	Measurements B (TF-yrs)
$64^3 \times 128$	W	0.11	5.0	10.0	45	120	1800
	DW	0.11	5.2	10.3	700	100	1400
$96^3 \times 192$	W	0.09	6.1	12.3	380	210	3100
	DW	0.09	5.9	11.8	4000	390	5100
$128^3 \times 256$	W	0.06	5.4	10.9	1940	340	5000
	DW	0.07	5.9	11.8	20000	1000	13000

“Lattice QCD for Cold Nuclear Physics”: USQCD Whitepaper