"Inclusive Meson Production and Short-Range Hadron Structure"

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Transversity GPDs in the large N_c limit

Peter Schweitzer
University of Connecticut

work in progress in collaboration with Christian Weiss

Overview:

- Limit of large number of colors N_c in QCD
- What is it useful for? What can it predict?
- Applications to nucleon structure
- Chiral odd GPDs
- Conclusions

quark correlator: $\mathcal{M}(\Gamma)_{\lambda'\lambda} = \int \frac{\mathrm{d}P^+ z^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \overline{\psi}_q(-\frac{z}{2})\, \Gamma\, [-\frac{z}{2}, \frac{z}{2}] \psi_q(\frac{z}{2})\, | p, \lambda \rangle \Big|_{z^+ = z_T^j = 0}$

kinematic variables: $P = \frac{1}{2}(p' + p), \quad \Delta = (p' - p), \quad \xi = -\frac{\Delta^+}{P^+}, \quad t = \Delta^2$

quark GPDs

$$\mathcal{M}(\gamma^+) = \overline{u}(p', \lambda') \left[\gamma^+ \; H^q + \frac{i\sigma^{+\mu}\Delta_\mu}{2M_N} \; E^q \right] u(p, \lambda)$$

$$\mathcal{M}(\gamma^+\gamma_5) = \overline{u}(p', \lambda') \left[\gamma^+ \gamma_5 \; \tilde{H}^q + \frac{i\gamma_5 \Delta^+}{2M_N} \; \tilde{E}^q \right] u(p, \lambda)$$

$$\mathcal{M}(i\sigma^{+j}) = \overline{u}(p', \lambda') \left[\sigma^{+j} \; H_T^q + \frac{\gamma^+\Delta^j - \Delta^+\gamma^j}{M_N^2} \; E_T^q + \frac{P^+\Delta^j - \Delta^+ P^j}{M_N^2} \; \tilde{H}_T^q + \frac{\gamma^+ P^j - P^+\gamma^j}{M_N^2} \; \tilde{E}_T^q \right] u(p, \lambda)$$

with $\mathrm{GPD}^q = \mathrm{GPD}^q(x, \xi, t; \mu^2)$

gluon GPDs analog (discussed elsewhere)

non-perturbative properties of GPDs

general constraints

- time-reversal & hermiticity: $\mathrm{GPD}(x,-\xi,t) = \pm\mathrm{GPD}(x,\xi,t) = \begin{cases} H, E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T \\ \\ \tilde{E}_T \end{cases}$

- Lorentz-invariance: polynomiality $\int \mathrm{d}x\, x^N \mathrm{GPD}(x,\xi,t) = a_0(t) + a_2(t)\,\xi^2 + \ldots + a_N\,\xi^N$

- inequalities (. . . , Pobylitsa, Pobylitsa & Kirch)

first principles

- lattice QCD (valuable information on Mellin moments, form factors)

- chiral perturbation theory (Diehl, Manashov, Schäfer; Kivel, Polyakov, Vladimirov)

- **large N_c** **(?)** $\Longleftarrow$ this talk

models

- bag (Ji et al), chiral quark soliton (Petrov et al; . . .),
  light-cone approaches (Pasquini et al; Lorcé), . . .

What is large N_c limit?

observation

- in nature $N_c = 3$ experimentally pretty well established

nevertheless

- in theory limit $N_c \to \infty$ not only fascinating, but also powerful and useful tool

statement

- "$\frac{1}{N_c}$ = only small parameter in QCD at *all* energies" (Sidney Coleman, in "Aspects of Symmetry")

technicalities

- $g_s^2 N_c =$ fixed as limit $N_c \to \infty$ is taken 't Hooft limit
- planar diagrams dominate; in 1+1 possible to resum 't Hooft NPB 72 & 75 (1974) 461
- 3+1 not solved. But . . .

conclusions in large N_c

- shown that meson & glueballs are free, stable, non-interacting particles, decay widths $\sim 1/\sqrt{N_c}$, coupling constants $\sim 1/N_c$ ('t Hooft; Veneziano; Callan et al; ... 1970s)

- mesons = pure $\bar{q}q$ states (and not $\bar{q}q\bar{q}q$ etc, which are hard to find in nature; implying that our real-world $N_c \gg 1$?)

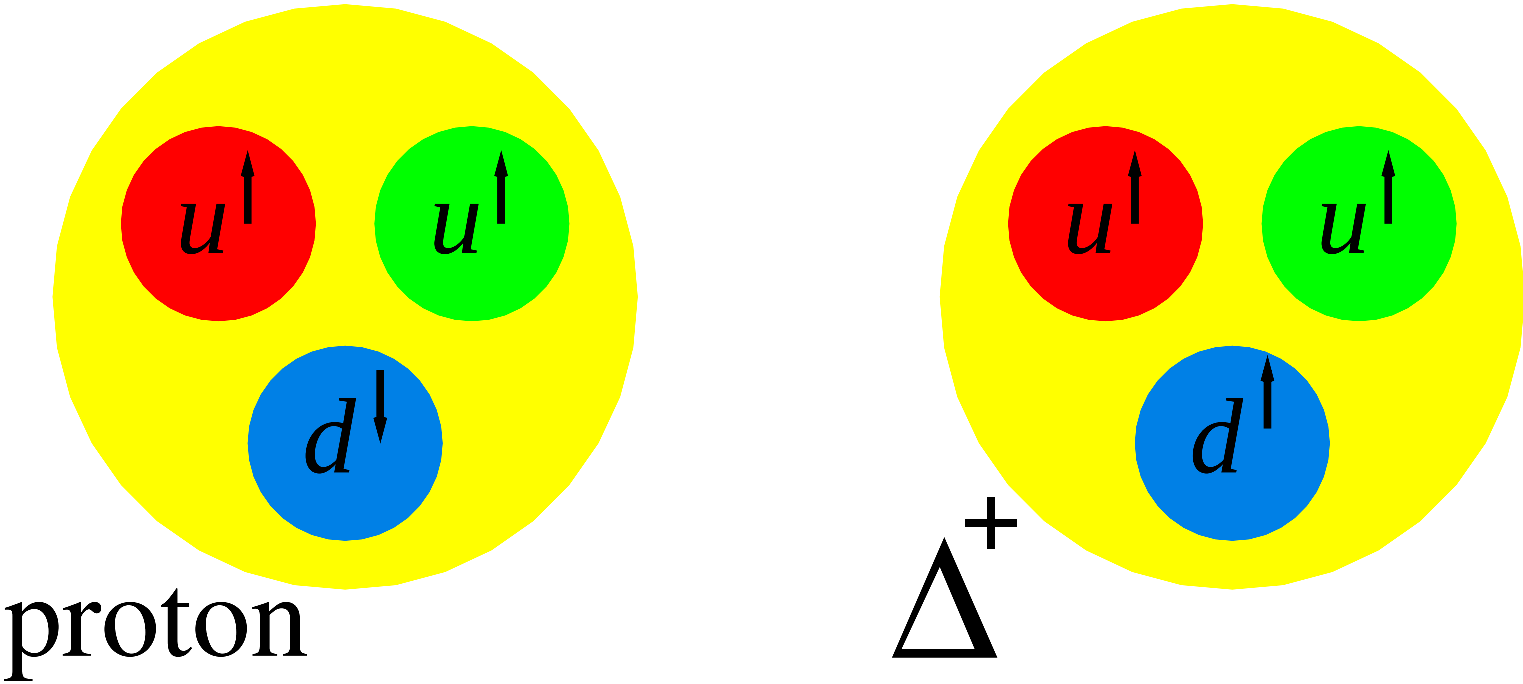
- Zweig rule = exact, if $N_c \to \infty$ (i.e. Zweig rule explained in the large N_c limit!?)

- baryons = solitons of meson fields Witten, NPB 160 (1979) 57
solitonic field not known, but it's symmetries are known $\Longleftarrow$ **explore!**

Important to keep in mind:

the mere "counting of colors N_c" is independnet of dynamics!
many generic results are the same — in QCD gauge theory, Skyrme model, quark models.
In particular: quark-model-picture provides useful intuition (and supports rigorous derivations)

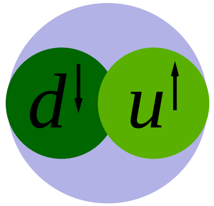
for $N_c = 3$: baryon = 3q in antisymmetric color state



How would nucleon & Δ look for N_c =3, 5, 7, . . . ?

$|\mathrm{nucleon}\rangle = |S_N = \frac{1}{2}, T_N = \frac{1}{2}\rangle$, $|\Delta\rangle = |S_\Delta = \frac{3}{2}, T_\Delta = \frac{3}{2}\rangle$

construction of nucleon wave-function in large N_c
(considertion in quark model, result generic)
must preserve spin and isospin(!) $\rightarrow$ add light quarks in scalar-isoscalar pairs
$\rightarrow$ only in this way S, T unchanged!



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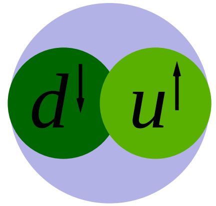
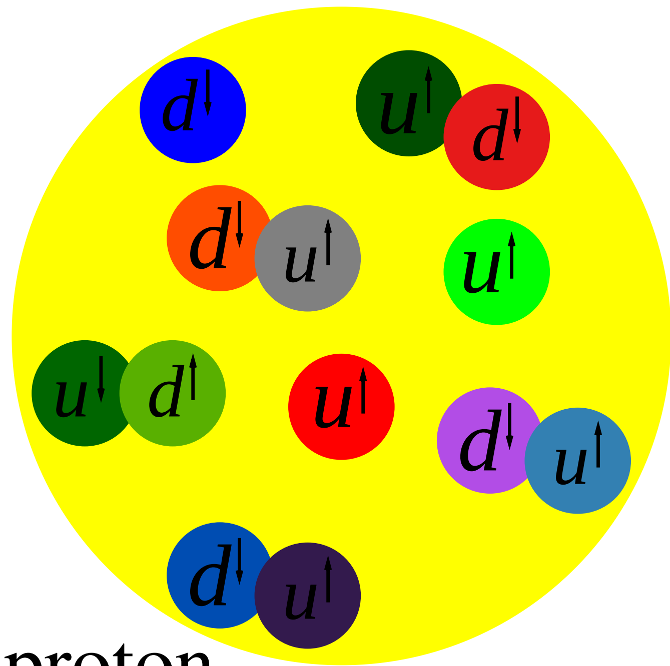
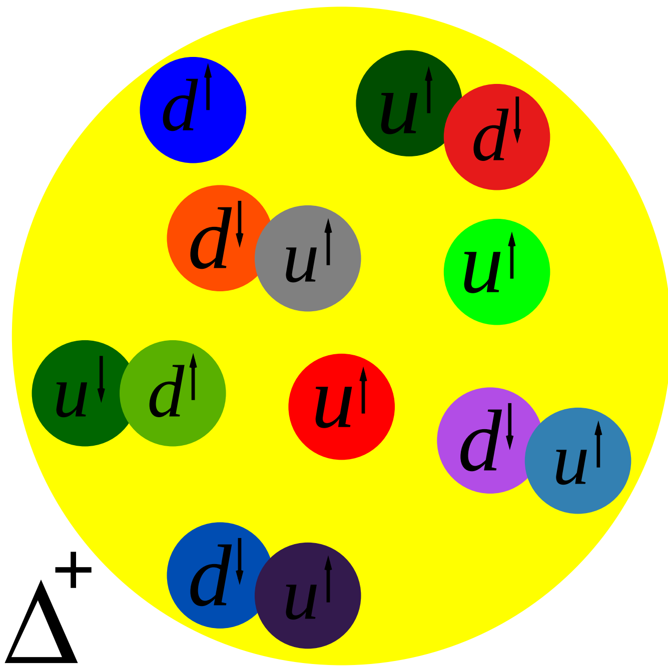




proton



Δ^+

Observations:

- not at all surprising that we have a static mean field ("soliton")!
- total nucleon spin $=$ result of enormous cancellations
- if for some reason $\exists$ tendency for spin of one flavor to be (anti-)aligned) with nucleon spin
  $\Rightarrow$ the other other flavor automatically exhibits opposite polarization

  (does not explain *why* there should be a correlation between flavor- and nucleon-polarization,
  $\rightarrow$ need dynamics e.g. chiral dynamics)

technical description

- $\boldsymbol{\langle B', p' |\, \hat{O}_{\rm QCD}\, | B, p\rangle = 2M_N \int d^3X\, e^{i(\vec{p}-\vec{p}\,')\vec{X}} \int dR\, \phi^*_{B'}(R)\, O(R)_{\rm eff}\, \phi_B(R) + \dots}$
  $R \in$ SU(2) describes collective coordinates in 3-space & flavor-space
  $\phi_B(R) =$ rotational wave-function (defined in terms of Wigner matrices)
  $O(R)_{\rm eff}$ is the effective mean-field function associated with the operator $\hat{O}_{\rm QCD}$
- $O(R)_{\rm eff}$ is unknown (because 3+1 QCD in large N_c not solved)
  but it's symmetry properties are known!
  $\Rightarrow$ sufficient!

results (static properties):

- baryons are massive objects $M_B \sim N_c$
- nucleon and Δ different rotational states of the $same$ soliton solution
- $M_\Delta - M_N \sim \mathcal{O}(1/N_c)$

  $\to \frac{M_\Delta - M_N}{M_\Delta + M_N} \sim \mathcal{O}(1/N_c^2)$ vs in nature $\frac{M_\Delta - M_N}{M_\Delta + M_N} \sim \frac{1}{7}$ (reasonable)
- magnetic moments $\mu_p - \mu_n \sim \mathcal{O}(N_c)$ vs $\mu_p + \mu_n \sim \mathcal{O}(N_c^0)$
  vs in nature $\mu_p - \mu_n = 4.71 \;\gg \mu_p + \mu_n = 0.88$ works!
  ($\exists$ many more examples)

partonic (dynamical) properties:

- heavy nucleon moves slowly: $\vec{p},\ \vec{p}^{\,\prime} \sim N_c^0$ i.e. $|t| \sim N_c^0 \ll M_N^2 \sim N_c^2$
- $xN_c =$ fixed in the limit $N_c \to \infty$ limit (address "non-exceptional" values of x)
- $\xi N_c =$ fixed in the limit $N_c \to \infty$ limit (address "non-exceptional" values of ξ)
- normalization $\langle B', p' |\, B, p\rangle = 2P^0(2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}^{\,\prime})$ with $P^0 \sim N_c$
- operator is color-singlet: $\bar{\psi} \ldots \psi \sim N_c$
- generic expectation for correlator $\langle B', p' |\overline{\psi}_q\, \Gamma \psi_q\, | B, p, \rangle \sim N_c^2$ or (due to symmetry) less
- predictions $\mathrm{PDF}_i(x) = N_c^{A_i} D_i(xN_c)$ with $A_i = 2$ or 1 (Diakonov et al 1996, ...)

$$(f_1^u + f_1^d)(x) = N_c^2 D_{f_1}^{u+d}(xN_c) \quad \gg \quad (f_1^u - f_1^d)(x) = N_c D_{f_1}^{u-d}(xN_c) \quad \checkmark$$

$$(g_1^u - g_1^d)(x) = N_c^2 D_{g_1}^{u-d}(xN_c) \quad \gg \quad (g_1^u + g_1^d)(x) = N_c D_{g_1}^{u+d}(xN_c) \quad \checkmark$$

$$(h_1^u - h_1^d)(x) = N_c^2 D_{h_1}^{u-1}(xN_c) \quad \gg \quad (h_1^u + h_1^d)(x) = N_c D_{h_1}^{u+d}(xN_c) \quad \checkmark$$ (prediction!)

helpful predictions:

- analog predictions for $\mathrm{TMD}_i(x, p_T) = N_c^{Bi} D_i(xN_c, p_T)$ (Pobylitsa hep-ph/0301236)
- Sivers function $(u-d) \gg (u+d)$ $\checkmark$(used in early fits PLB 612 (2005) 233, PRD 73 (2006) 014021)
- other TMDs to be tested
- also for gluon distributions large N_c counting possible

  $\Delta G(x)/G(x) = \mathcal{O}(1/N_c)$ Efremov, Goeke, Pobylitsa PLB 488 (2000) 182

  (gluon Sivers)/(quark Sivers) $= \mathcal{O}(1/N_c)$ Efremov et al, PLB 612 (2005) 233

- extremely helpful, if we do not know anything otherwise!

Remark: Sivers function $\sim N_c^{2+1}$
while $f_1 \sim N_c^2$ (?), does Sivers asymmetry grow with N_c ?

No! Trivial additional power of N_c due to correlator$^i \sim \dfrac{p_T^i}{M_N} f_{1T}^{\perp}(x, p_T)$
correlators of PDFs, TMDs, GPDs $\sim N_c^2$ at most (same story with GPDs below)

chiral-even GPDs (Petrov et al 1998; Pentinnen et al; Goeke, Polyakov, Vanderhaeghen)

- $(H^u + H^d)(x, \xi, t) = N_c^2 \; D_H^{u+d}(xN_c, \xi N_c, t) \quad \gg \quad (u - d) =$ smaller

- $(E^u - E^d)(x, \xi, t) = N_c^3 \; D_E^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u + d) =$ smaller

- $(\tilde{H}^u - \tilde{H}^d)(x, \xi, t) = N_c^2 \; D_{\tilde{H}}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u + d) =$ smaller

- $(\tilde{E}^u - \tilde{E}^d)(x, \xi, t) = N_c^4 \; D_{\tilde{E}}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u + d) =$ smaller

predictions being tested (so far, so good)

chiral-odd GPDs

- marginally mentioned in literature (works & proceedings by Matthias Burkardt)
- no systematic study so far $\leftarrow$ **ongoing study**

chiral-odd GPDs (first results of ongoing study)

- $(H_T^u - H_T^d)(x,\xi,t) = N_c^2\, D_{H_T}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u+d) =$ smaller (cf. transversity $\checkmark$)

- $(E_T^u - E_T^d)(x,\xi,t) = N_c^3\, D_{E_T}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u+d) =$ smaller

- $(\tilde{H}_T^u + \tilde{H}_T^d)(x,\xi,t) = N_c^3\, D_{\tilde{H}_T}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u-d) =$ smaller

- $(\tilde{E}_T^u - \tilde{E}_T^d)(x,\xi,t) = N_c^3\, D_{\tilde{E}_T}^{u-d}(xN_c, \xi N_c, t) \quad \gg \quad (u+d) =$ smaller

compatible with lattice, models

experiment (?) $\rightarrow$ JLab, PRC 90 (2014) 039901 + preliminary, see talk by V. Kubarovsky

data indicate (important: at small $|t| \sim N_c^0 \quad \ll \; M_N^2 \sim N_c^2$)

(i) $\langle \boldsymbol{H_T^u} \rangle \approx - \langle \boldsymbol{H_T^d} \rangle$

compatible with large N_c $\surd$ (work in progress)

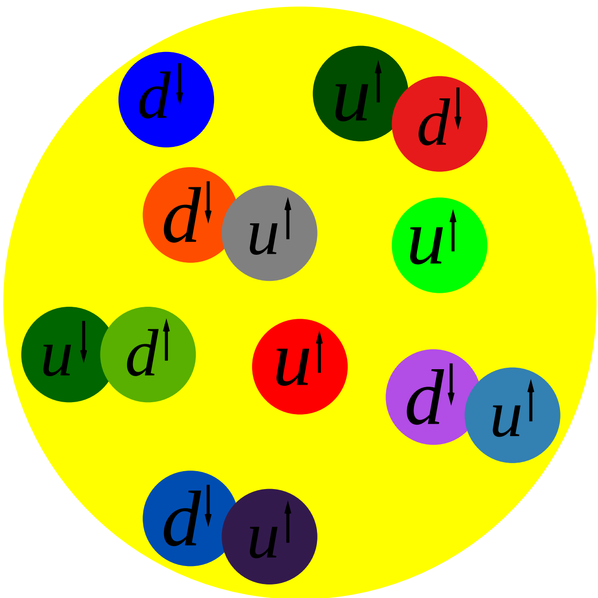
(ii) $\langle \overline{\boldsymbol{E}}_{\boldsymbol{T}}^{\boldsymbol{u}} \rangle \approx \langle \overline{\boldsymbol{E}}_{\boldsymbol{T}}^{\boldsymbol{d}} \rangle$ where $\overline{E}_T = 2\tilde{H}_T + E_T$

in large N_c we found $\tilde{H}_T^u \approx \tilde{H}_T^d$ $\surd$ but opposite for E_T (?)
is E_T small (?) need studies in dynamical approaches

- in any case: same-sign flavors for chiral-odd GPDs must involve $\tilde{H}_T$ (and does! $\surd$)
- and it even enters with factor 2 enhancement (this is numerically almost factor N_c . . .)
- overall claim (with all reservations): it's encouraging! $\surd$
- more work needed! (much motivated!!)

Conclusions

- **large N_c**: powerful, model-independent theoretical tool
- it provides insights & guidelines, **works** phenomenologically
- report from study in progress of **chiral odd GPDs** in large N_c
- first results are in **encouraging agreement** with first data
- more work needed, underway, impact of **chiral dynamics**
- looking forward to **exciting results**



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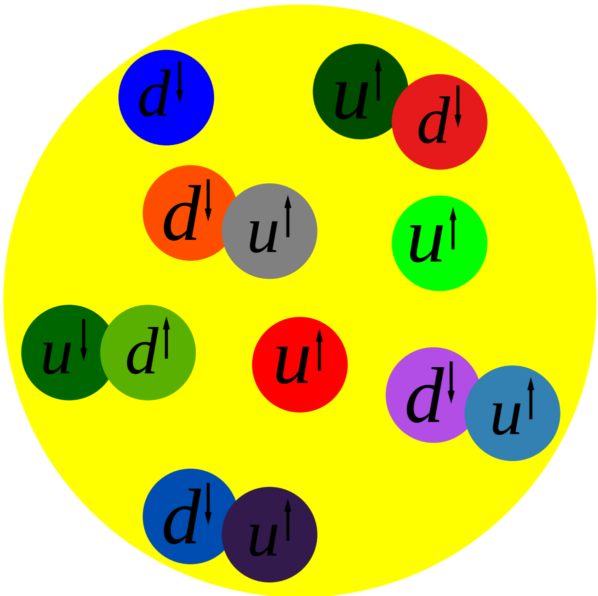

Thank you !!!