Transversity GPDs in the large $N_c$ limit

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work in progress in collaboration with Christian Weiss

Overview:
- Limit of large number of colors $N_c$ in QCD
- What is it useful for? What can it predict?
- Applications to nucleon structure
- Chiral odd GPDs
- Conclusions
quark correlator: \[ \mathcal{M}(\Gamma)_{\lambda'\lambda} = \int \frac{dP^+ z^-}{2\pi} e^{ixP^+ z^-} \langle p', \lambda' | \bar{\psi}_q(-\frac{z}{2}) \Gamma [-\frac{z}{2}, \frac{z}{2}] \psi_q(\frac{z}{2}) | p, \lambda \rangle \bigg|_{z^+=z_T^+=0} \]

kinematic variables: \[ P = \frac{1}{2}(p' + p), \quad \Delta = (p' - p), \quad \xi = -\frac{\Delta^+}{P^+}, \quad t = \Delta^2 \]

quark GPDs

\[ \mathcal{M}(\gamma^+) = \bar{u}(p', \lambda') \left[ \gamma^+ H^q + \frac{i\sigma^{+\mu} \Delta^\mu}{2M_N} E^q \right] u(p, \lambda) \]

\[ \mathcal{M}(\gamma^+ \gamma_5) = \bar{u}(p', \lambda') \left[ \gamma^+ \gamma_5 \tilde{H}^q + \frac{i\gamma_5 \Delta^+}{2M_N} \tilde{E}^q \right] u(p, \lambda) \]

\[ \mathcal{M}(i\sigma^{+j}) = \bar{u}(p', \lambda') \left[ \sigma^{+j} H^q_T + \frac{\gamma^+ \Delta^j - \Delta^+ \gamma^j}{M_N^2} E^q_T \right. \right. \]

\[ + \left. \left. \frac{P^+ \Delta^j - \Delta^+ P^j}{M_N^2} \tilde{H}^q_T + \frac{\gamma^+ P^j - P^+ \gamma^j}{M_N^2} \tilde{E}^q_T \right] u(p, \lambda) \]

with \( \text{GPD}^q = \text{GPD}^q(x, \xi, t; \mu^2) \)
gluon GPDs analog (discussed elsewhere)
non-perturbative properties of GPDs

general constraints

• time-reversal & hermiticity: \( \text{GPD}(x, -\xi, t) = \pm \text{GPD}(x, \xi, t) = \begin{cases} H, E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T \\ \tilde{E}_T \end{cases} \)

• Lorentz-invariance: polynomiality \( \int dx \, x^N \text{GPD}(x, \xi, t) = a_0(t) + a_2(t) \xi^2 + \ldots + a_N \xi^N \)

• inequalities (\ldots, Pobylitsa, Pobylitsa & Kirch)

first principles

• lattice QCD (valuable information on Mellin moments, form factors)

• chiral perturbation theory (Diehl, Manashov, Schäfer; Kivel, Polyakov, Vladimirov)

• large \( N_c \) (\?) \( \leftarrow \) this talk

models

• bag (Ji et al), chiral quark soliton (Petrov et al; \ldots),
light-cone approaches (Pasquini et al; Lorcé), \ldots
What is large $N_c$ limit?

observation

- in nature $N_c = 3$ experimentally pretty well established

nevertheless

- in theory limit $N_c \to \infty$ not only fascinating, but also powerful and useful tool

statement

- $\frac{1}{N_c} =$ only small parameter in QCD at all energies” (Sidney Coleman, in “Aspects of Symmetry”)

technicalities

- $g_s^2 N_c =$ fixed as limit $N_c \to \infty$ is taken ’t Hooft limit
- planar diagrams dominate; in 1+1 possible to resum ’t Hooft NPB 72 & 75 (1974) 461
- 3+1 not solved. But . . .
conclusions in large $N_c$

- shown that meson & glueballs are free, stable, non-interacting particles, 
decay widths $\sim 1/\sqrt{N_c}$, coupling constants $\sim 1/N_c$ ('t Hooft; Veneziano; Callan et al; . . . 1970s)

- mesons = pure $\bar{q}q$ states  
  (and not $\bar{q}q\bar{q}q$ etc, which are hard to find in nature; implying that our real-world $N_c \gg 1$ ?)

- Zweig rule = exact, if $N_c \to \infty$ (i.e. Zweig rule explained in the large $N_c$ limit!?)

- baryons = solitons of meson fields Witten, NPB 160 (1979) 57  
solitonic field not known, but it’s symmetries are known $\Leftarrow$ explore!
Important to keep in mind:
the mere “counting of colors $N_c$” is independent of dynamics!
many generic results are the same — in QCD gauge theory, Skyrme model, quark models.
In particular: quark-model-picture provides useful intuition (and supports rigorous derivations)

for $N_c = 3$: baryon = 3q in antisymmetric color state

![Proton](image1.png)

![Delta+](image2.png)
How would nucleon & Δ look for $N_c = 3, 5, 7, \ldots$?

\[ |\text{nucleon} \rangle = |S_N = \frac{1}{2}, \ T_N = \frac{1}{2} \rangle, \ |\Delta \rangle = |S_\Delta = \frac{3}{2}, \ T_\Delta = \frac{3}{2} \rangle \]

construction of nucleon wave-function in large $N_c$
(consideration in quark model, result generic)
must preserve spin and isospin(!) → add light quarks in scalar-isoscalar pairs
→ only in this way $S, T$ unchanged!
How would nucleon & $\Delta$-look for $N_c = 3, 5, 7, \ldots$?

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construction of nucleon wave-function in large $N_c$
(in quark model, but generic result)
must preserve spin and isospin(!) → add light quarks in scalar-isoscalar pairs → only in this way $S, T$ unchanged!
Observations:

• not at all surprising that we have a static mean field ("soliton")!
• total nucleon spin = result of enormous cancellations
• if for some reason $\exists$ tendency for spin of one flavor to be (anti-)aligned) with nucleon spin $\Rightarrow$ the other other flavor automatically exhibits opposite polarization

(does not explain why there should be a correlation between flavor- and nucleon-polarization, $\rightarrow$ need dynamics e.g. chiral dynamics)

technical description

• $\langle B', p' | \hat{O}_{\text{QCD}} | B, p \rangle = 2M_N \int d^3X \ e^{i(\vec{p} - \vec{p}') \cdot \vec{X}} \int dR \ \phi^*_B(R) \ O(R)_{\text{eff}} \ \phi_B(R) + \ldots$
  $R \in \text{SU}(2)$ describes collective coordinates in 3-space & flavor-space
  $\phi_B(R) =$ rotational wave-function (defined in terms of Wigner matrices)
  $O(R)_{\text{eff}}$ is the effective mean-field function associated with the operator $\hat{O}_{\text{QCD}}$

• $O(R)_{\text{eff}}$ is unknown (because 3+1 QCD in large $N_c$ not solved)
  but it’s symmetry properties are known!
  $\Rightarrow$ sufficient!
results (static properties):

- baryons are massive objects $M_B \sim N_c$
- nucleon and $\Delta$ different rotational states of the same soliton solution
- $M_\Delta - M_N \sim O(1/N_c)$

\[ \frac{M_\Delta - M_N}{M_\Delta + M_N} \sim O(1/N_c^2) \] vs in nature \[ \frac{M_\Delta - M_N}{M_\Delta + M_N} \sim \frac{1}{7} \] (reasonable)

- magnetic moments $\mu_p - \mu_n \sim O(N_c)$ vs $\mu_p + \mu_n \sim O(N_c^0)$

\[ \mu_p - \mu_n = 4.71 \quad \Rightarrow \quad \mu_p + \mu_n = 0.88 \quad \text{works!} \]

(∃ many more examples)
partonic (dynamical) properties:

- heavy nucleon moves slowly: $\vec{p}, \vec{p}' \sim N_c^0$ i.e. $|t| \sim N_c^0 \ll M_N^2 \sim N_c^2$

- $x N_c = \text{fixed in the limit } N_c \to \infty \text{ limit (address "non-exceptional" values of } x)$

- $\xi N_c = \text{fixed in the limit } N_c \to \infty \text{ limit (address "non-exceptional" values of } \xi)$

- normalization $\langle B', p' | B, p \rangle = 2 P^0 (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}')$ with $P^0 \sim N_c$

- operator is color-singlet: $\bar{\psi} \ldots \psi \sim N_c$

- generic expectation for correlator $\langle B', p' | \bar{\psi}_q \Gamma \psi_q | B, p, \rangle \sim N_c^2$ or (due to symmetry) less

- predictions $PDF_i(x) = N_{c}^{A_i} D_i(x N_c)$ with $A_i = 2$ or $1$ (Diakonov et al 1996, ...)

\[
(f_1^u + f_1^d)(x) = N_c^2 D_{f_1}^{u+d}(x N_c) \quad \Rightarrow \quad (f_1^u - f_1^d)(x) = N_c D_{f_1}^{u-d}(x N_c) \quad \checkmark
\]

\[
(g_1^u - g_1^d)(x) = N_c^2 D_{g_1}^{u-d}(x N_c) \quad \Rightarrow \quad (g_1^u + g_1^d)(x) = N_c D_{g_1}^{u+d}(x N_c) \quad \checkmark
\]

\[
(h_1^u - h_1^d)(x) = N_c^2 D_{h_1}^{u-1}(x N_c) \quad \Rightarrow \quad (h_1^u + h_1^d)(x) = N_c D_{h_1}^{u+d}(x N_c) \quad \checkmark \text{(prediction!)}
\]
helpful predictions:

- analog predictions for $TMD_i(x, p_T) = N_c^{B_i} D_i(xN_c, p_T)$ (Pobylitsa hep-ph/0301236)
- Sivers function $(u - d) \gg (u + d)$ (used in early fits PLB 612 (2005) 233, PRD 73 (2006) 014021)
- other TMDs to be tested
- also for gluon distributions large $N_c$ counting possible

$$\Delta G(x)/G(x) = \mathcal{O}(1/N_c)$$ Efremov, Goeke, Pobylitsa PLB 488 (2000) 182

$$\frac{\text{gluon Sivers}}{\text{quark Sivers}} = \mathcal{O}(1/N_c)$$ Efremov et al, PLB 612 (2005) 233

- extremely helpful, if we do not know anything otherwise!

Remark: Sivers function $\sim N_c^{2+1}$
while $f_1 \sim N_c^2$ (?), does Sivers asymmetry grow with $N_c$?

No! Trivial additional power of $N_c$ due to correlator $i \sim \frac{p_T^i}{M_N} f_{1T}^i(x, p_T)$
correlators of PDFs, TMDs, GPDs $\sim N_c^2$ at most (same story with GPDs below)
chiral-even GPDs (Petrov et al 1998; Pentinnen et al; Goeke, Polyakov, Vanderhaeghen)

- \((H^u + H^d)(x, \xi, t) = N_c^2 \, D_{H}^{u+d}(xN_c, \xi N_c, t) \gg (u - d) = \text{smaller}\)

- \((E^u - E^d)(x, \xi, t) = N_c^3 \, D_{E}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller}\)

- \((\tilde{H}^u - \tilde{H}^d)(x, \xi, t) = N_c^2 \, D_{\tilde{H}}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller}\)

- \((\tilde{E}^u - \tilde{E}^d)(x, \xi, t) = N_c^4 \, D_{\tilde{E}}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller}\)

predictions being tested (so far, so good)

chiral-odd GPDs

- marginally mentioned in literature (works & proceedings by Matthias Burkardt)

- no systematic study so far ← ongoing study
chiral-odd GPDs (first results of ongoing study)

- \((H_T^u - H_T^d)(x, \xi, t) = N_c^2 D_{HT}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller} \) (cf. transversity \(\checkmark\))

- \((E_T^u - E_T^d)(x, \xi, t) = N_c^3 D_{ET}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller}\)

- \((\tilde{H}_T^u + \tilde{H}_T^d)(x, \xi, t) = N_c^3 D_{HT}^{u-d}(xN_c, \xi N_c, t) \gg (u - d) = \text{smaller}\)

- \((\tilde{E}_T^u - \tilde{E}_T^d)(x, \xi, t) = N_c^3 D_{ET}^{u-d}(xN_c, \xi N_c, t) \gg (u + d) = \text{smaller}\)

compatible with lattice, models

experiment (?) \(\rightarrow\) JLab, PRC 90 (2014) 039901 + preliminary, see talk by V. Kubarovsky
**data indicate** (important: at small $|t| \sim N_c^0 \ll M_N^2 \sim N_c^2$)

(i) $\langle H^u_T \rangle \approx - \langle H^d_T \rangle$

compatible with large $N_c$ ✓ (work in progress)

(ii) $\langle \bar{E}^u_T \rangle \approx \langle \bar{E}^d_T \rangle$ where $\bar{E}_T = 2\tilde{H}_T + E_T$

in large $N_c$ we found $\tilde{H}^u_T \approx \tilde{H}^d_T$ ✓ but opposite for $E_T$ (?) is $E_T$ small (?) need studies in dynamical approaches

- in any case: same-sign flavors for chiral-odd GPDs must involve $\tilde{H}_T$ (and does! ✓)
- and it even enters with factor 2 enhancement (this is numerically almost factor $N_c \ldots$)
- overall claim (with all reservations): it’s encouraging! ✓
- more work needed! (much motivated!!)
Conclusions

- **large $N_c$:** powerful, model-independent theoretical tool
- it provides insights & guidelines, **works** phenomenologically
- report from study in progress of **chiral odd GPDs** in large $N_c$
- first results are in **encouraging agreement** with first data
- more work needed, underway, impact of **chiral dynamics**
- looking forward to **exciting results**
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Thank you !!!