# GPDs from meson leptoproduction

P. Kroll

Universitaet Wuppertal

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**Outline:**

- **The handbag, GPDs and power corrections**
- **Analysis of meson leptoproduction**
- **DVCS**
- **The GPD $E$ and Ji's sum rule**
- $\omega$ **production**
- **Summary**

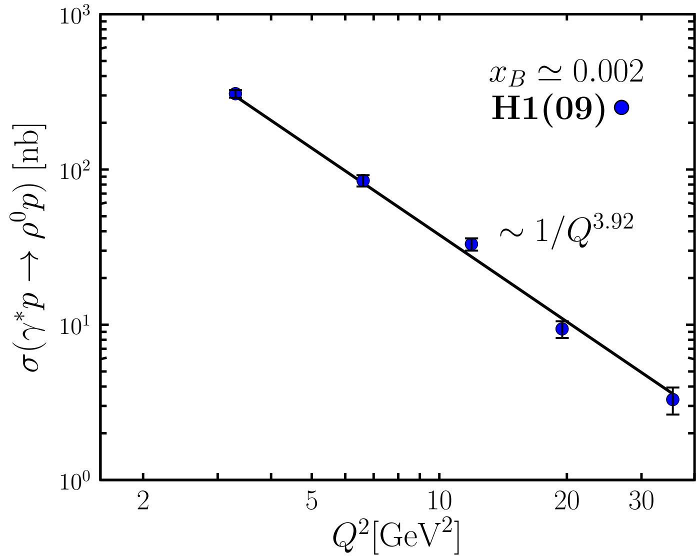
# Can we apply the asymp. fact. formula ?

rigorous proofs of collinear factorization in generalized Bjorken regime:
for $\gamma_L^* \to V_L(P)$ and $\gamma_T^* \to \gamma_T$ amplitudes $\big(Q^2, W \to \infty$, $x_{\rm B}$ fixed$\big)$
$\mathcal{K} = \int dx K(x,\xi,t)\mathcal{H}(x,\xi,Q^2)$ Radyushkin, Collins et al, Ji et al

possible power corrections not under control $\Longrightarrow$
unknown at which $Q^2$ asymptotic result can be applied

e.g. $\rho^0$ production: $\sigma_L/\sigma_T \propto Q^2$
experiment: $\simeq 2$ for $Q^2 \leq 10\,\mathrm{GeV}^2$
$\gamma_T^* \to V_T$ transitions substantial

$\sigma_L \propto 1/Q^6$ at fixed $x_{\rm B}$
modified by $ln^n(Q^2)$ experiment:



# Two concepts to solve problem with $\gamma_L^* \to V_L$ ampl.:

at small $x_B$(i.e. small $\xi$): only GPD $H$ relevant

Mueller et al (11,13): absorb effects into GPDs

$\Longrightarrow$ strong $\ln^n(Q^2)$ from evolution of GPDs

only shown for HERA data (i.e. at $W \simeq 90\,\mathrm{GeV}$) with $H_{g,sea}$

- can this be extended to lower $W$?

fits to only DVCS or to DVCS+DVMP data from HERA lead to different GPDs

Goloskokov-K (06): take into account transverse size of meson,

i.e. power corrections $1/Q^n$ to subprocess $\gamma_L^* q(g) \to V_L\, q(g)$

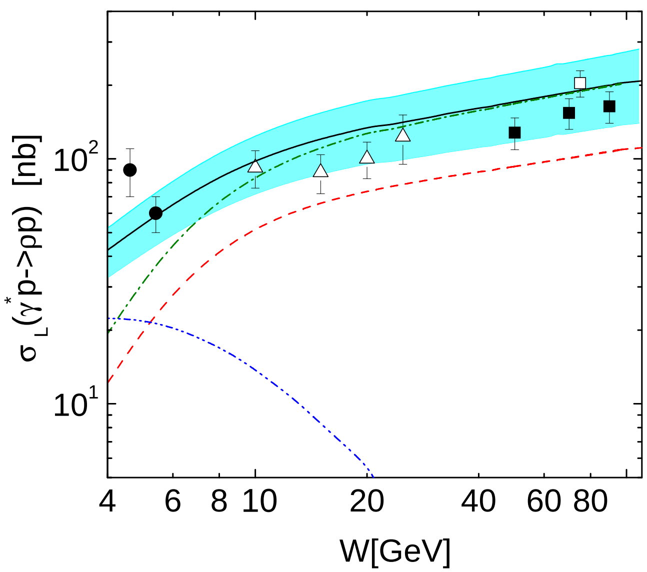
H for gluon, sea and valence

$W \gtrsim 4\,\mathrm{GeV} \qquad Q^2 = 4\,\mathrm{GeV}^2$

gluon + sea, gluon

valence + (gluon + sea)-valence

interference

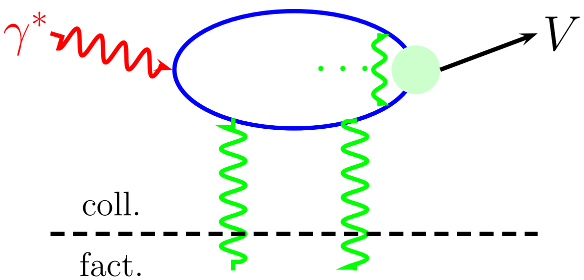


# The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources $\Longrightarrow$ gluon radiation




Sudakov factor  Sterman et al(93)

$$S(\tau, \mathbf{b}, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b\Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL $\Rightarrow \exp[-S]$

provides sharp cut-off at $b = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}^{M}_{0\lambda,0\lambda} = \int d\tau d^2b\, \hat{\Psi}_M(\tau, -\vec{b})\, e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \vec{b})$$

$\hat{\Psi}_M \sim \exp[\tau\bar{\tau}b^2/4a_M^2]$ LC wave fct of meson

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

LO pQCD

$+$ quark trans. mom.

$+$ Sudakov supp.

$\Rightarrow$ asymp. fact. formula

(lead. twist) for $Q^2 \to \infty$

Sudakov factor generates series of power corr. $\sim (\Lambda^2_{\rm QCD}/Q^2)^n$

(from region of soft quark momenta $\tau, \bar{\tau} \to 0$)

from intrinsic $k_\perp$ in wave fct: series $\sim (\langle b^2\rangle/Q^2)^n$ (from all $\tau$)

# Parametrizing the GPDs

double distribution representation Mueller *et al* (94), Radyushkin (99)

$$K^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x)\, K^i(\rho,\xi=0,t) w_i(\rho,\eta) + D_i\, \Theta(\xi^2-\bar{x}^2)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2-\eta^2]^{n_i}$ $(n_g = n_{\mathrm{sea}} = 2, n_{\mathrm{val}} = 1$, generates $\xi$ dep.)

zero-skewness GPD $K^i(\rho,\xi=0,t) = k^i(\rho)\exp\left[(b_{ki} - \alpha'_{ki}\ln(\rho))t\right]$

$k = q, \Delta q, \delta^q$ for $H, \widetilde{H}, H_T$ or $N_{ki}\rho^{-\alpha_{ki}(0)}(1-\rho)^{\beta_{ki}}$ for $E, \widetilde{E}, \bar{E}_T$

Regge-like $t$ dep. (for small $-t$ reasonable appr.), $D$-term neglected

advantage: polynomiality and reduction formulas automatically satisfied

positivity bounds respected (checked numerically)

sum rules for nucleon form factors respected

# Ansätze for zero-skewness GPDs

simplest ansatz: VGG(98), Freund et al (01)

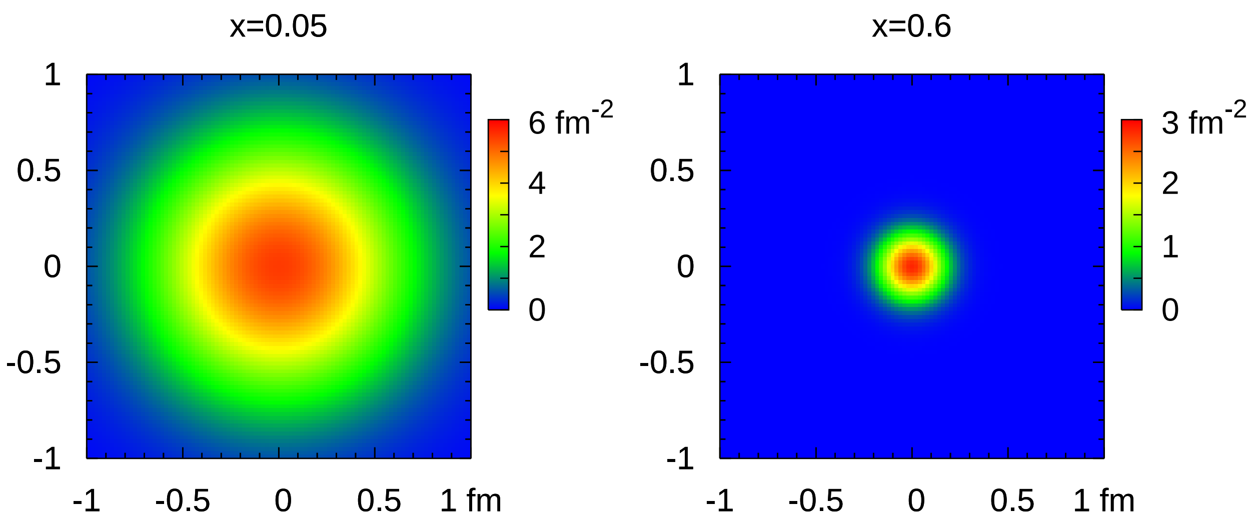
e.g. $H^q(x, \xi = 0, t) = c_q q(x) F_1^q(t)$ PDF times form factor

respects reduction formula and sum rules $\int dx H^q(x, \xi = 0, t) = F_1^q(t)$

Burkhardt(00,03): $q(x, \mathbf{b}_\perp) = \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}\boldsymbol{\Delta}_\perp} H^q(x, \xi = 0, t = -\Delta_\perp^2)$

(density interpretation) $\mathbf{b}_\perp$: transverse distance between struck parton and hadron's center of momentum $\sum x_i \mathbf{b}_{\perp i} = 0$;

partons with large (small) $x_i$ must (can) have small (large) $\mathbf{b}_{\perp i}$




$x - b$ $(x - t)$ correlation required

factorized ansatz obsolete

improvement: Regge-like ansatz (frequently used now)

$$H^q(x,\xi=0,t) = q(x)e^{tf_q(x)} \qquad f_q(x) = B_q - \alpha_q' \ln(x)$$

at small $x$: $q \sim x^{-\alpha_q(0)} \Longrightarrow H^q \sim x^{-\alpha_q(t)}$ $\qquad (\alpha_q(t) = \alpha_q(0) + \alpha_q' t)$

standard Regge trajectory - $H^q \sim 1/\sqrt{x}$ at $t \simeq 0$ $\qquad$ (fact. ansatz for all $t$)

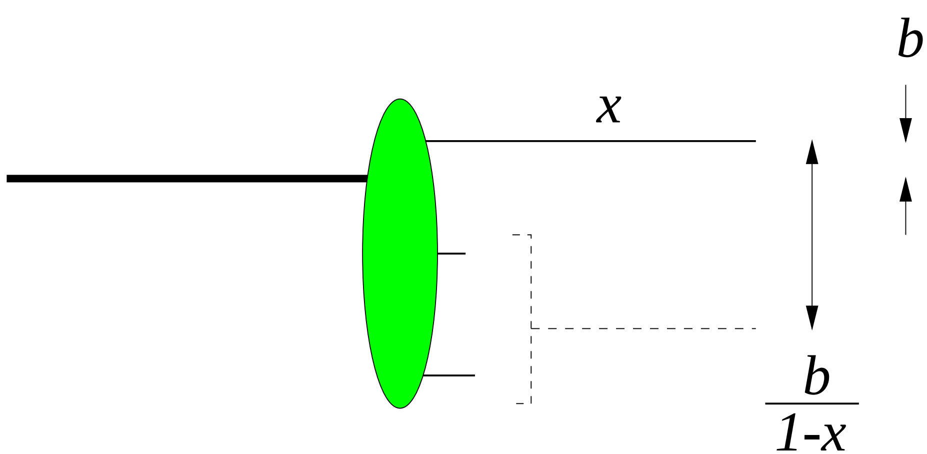
$\sim \sqrt{x}$ at $t \simeq -1\,\mathrm{GeV}^2$

FT: $q(x,\mathbf{b}_\perp) = \frac{1}{4\pi}\frac{q(x)}{f_q(x)} \exp\left[-b_\perp^2/4f_q(x)\right]$ and $< b_\perp^2 >_x^q = 4f_q(x)$

distance between active parton and cluster of spectators

(rough estimate of proton radius) $\qquad d_q(x) = \frac{\sqrt{< b_\perp^2 >_x^q}}{1-x} \sim 1/(1-x)$

for $x \to 1$!




Regge-like ansatz only suitable at small $x$, i.e. at small $-t$

unphysical at large $x$

further improvement: DFJK04, Diehl-K(13)

used in analysis of form factors for proton and neutron at $\xi = 0$

$$F_1^{p(n)} = e_{u(d)} \int_{-1}^{1} dx H_i^u(x, \xi = 0, t) + e_{d(u)} \int_{-1}^{1} dx H_i^d(x, \xi = 0, t)$$

Pauli form factor $\quad H \to E$

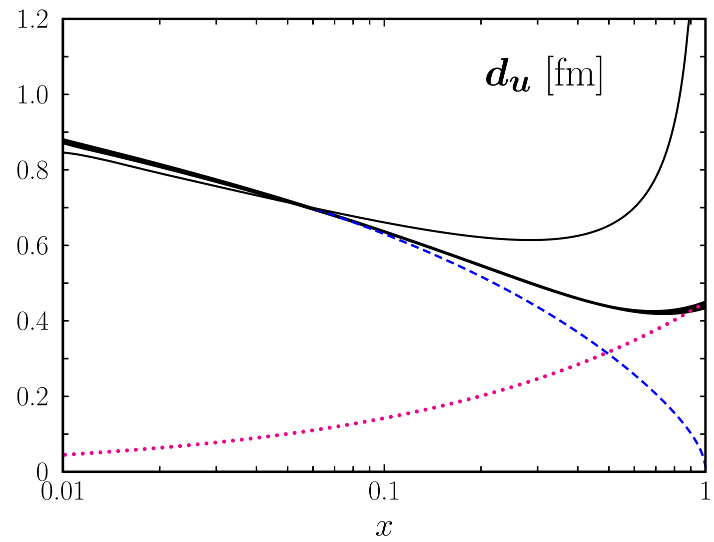
normalization fixed from $\kappa_q = \int_0^1 dx E_v^q(x, \xi = 0, t = 0)$

profile fct: $f_q = (B_q - \alpha_q' \ln x)(1-x)^3 + A_q x(1-x)^2$

$$d_q(x) = \frac{2\sqrt{f_q(x)}}{1-x} \to 2\sqrt{A_q}$$

for $x \to 1$

Regge-like profile fct can (only) be used at small $x$ (small $-t$)



# What has been done?

- analysis of FF (DFJK04, update: Diehl-K 1302.4604)
  using CTEQ6 (ABM11) PDFs, fixes $H, E, (\widetilde{H})$ for valence quarks

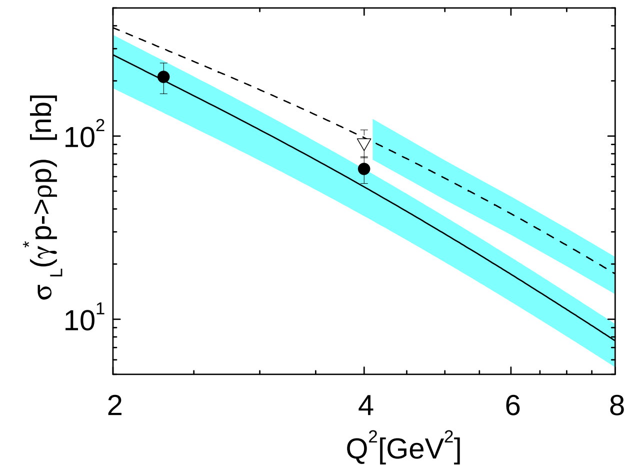
- analysis of $d\sigma_L/dt$ for $\rho^0$ and $\phi$ production Goloskokov-K, hep-ph/0611290
  data from H1, ZEUS, E665, HERMES
  for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$ and $W \gtrsim 4\,\mathrm{GeV}$ ($\xi \lesssim 0.1$, $-t \lesssim 0.5\,\mathrm{GeV}^2$)
  fixes $H$ for sea quarks and gluons for given $H^{\mathrm{val}}$
  ($E$ negligible, others don't contr.) update with ABM11 required

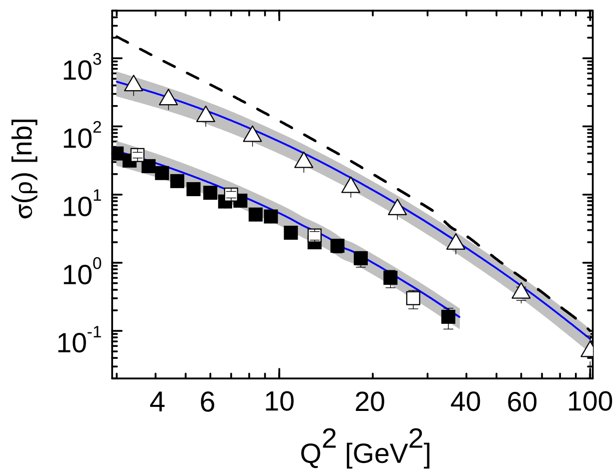
- analysis of $\pi^+$ production, Goloskokov-K, 0906.0460
  $d\sigma/dt$ and $A_{UT}$ data from HERMES ($W \simeq 4\,\mathrm{GeV}$, $Q^2 \simeq 2-5\,\mathrm{GeV}^2$)
  evidence for strong contr. from $\gamma^*_T$ ($H_T$)
  fixes $\widetilde{H}$, pion pole and $H_T$ (no clear signal for $\widetilde{E}_{\mathrm{non-pole}}$)

- SDME and $A_{UT}$ for $\rho^0$ production HERMES,
  $\pi^0$ cross section and $\eta/\pi^0$ cross section ratio from CLAS (large skewness!),
  and lattice QCD QCDSF and UKQCD, hep-lat/0612032
  hints at strong contributions from $\bar{E}_T = 2\widetilde{H}_T + E_T$

# Results on DVMP

long. cross sections fix $H$ for gluons and sea quarks for given $H_{\rm val}$ GK(06)



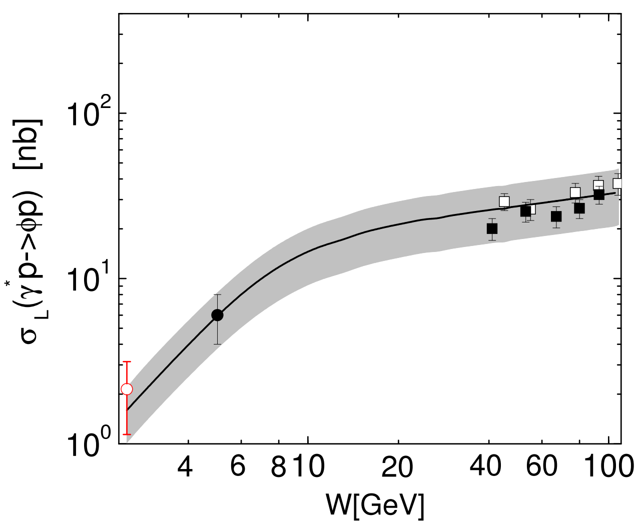

left: $W = 5, 10\,\mathrm{GeV}$

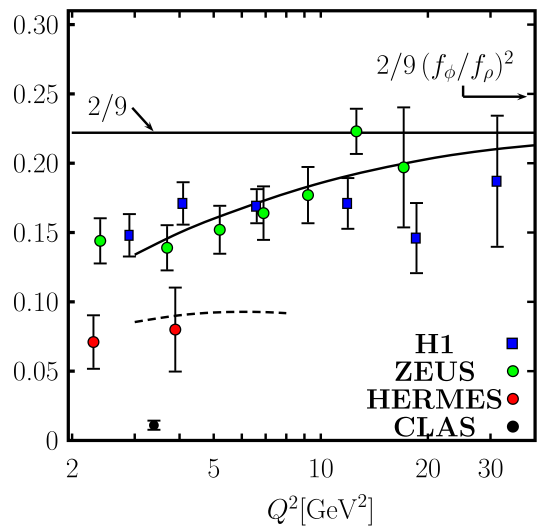
HERMES, E665

right: H1, ZEUS

$W = 75, 90\,\mathrm{GeV}$

dashed: collinear




left: $Q^2 = 3.8\,\mathrm{GeV}^2$

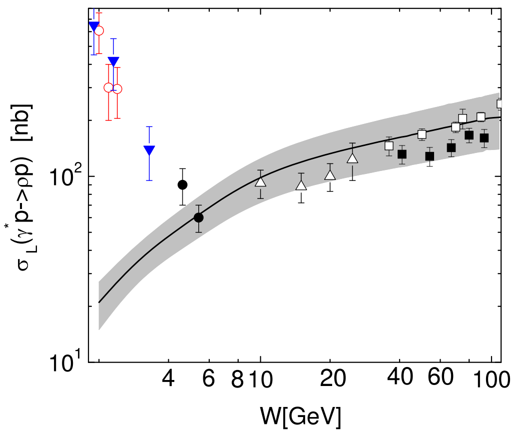
CLAS, HERMES, H1, ZEUS

right: $\sigma_L(\phi)/\sigma_L(\rho)$

$\langle H^s \rangle < \langle H^{\bar{u}} \rangle$

(and $a_\phi \neq a_\rho$)

# Why restriction to small skewness data?




at $Q^2 = 4\,\mathrm{GeV}^2$

data: E665, HERMES, CORNELL, H1, ZEUS, CLAS

breakdown of handbag physics?

at large $x_{\rm B}$ (small $W$)

- power corrections are strong at least in some cases
- kinematic corrections strong, e.g.

$$\xi \simeq \frac{x_{\rm B}}{2-x_{\rm B}}\Big[1+\frac{1}{(1-x_{\rm B}/2)Q^2}\Big(m_M^2 - x_{\rm B}(1-x_{\rm B})(x_{\rm B}m^2+t)\Big)\Big]$$

(see also Braun et al(14))

($t/Q^2$ correct. neglected in general)

$t_0 = -4m^2\xi^2/(1-\xi^2)$ large

probes GPD in different regions of $t$

(e.g. $W = 2\,\mathrm{GeV}$, $Q^2 = 4\,\mathrm{GeV}^2$: $t_0 = -0.86\,\mathrm{GeV}^2$)

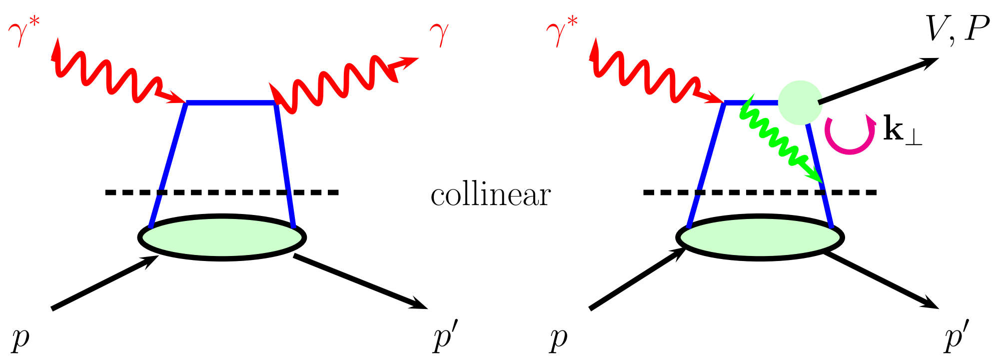
$\boxed{\text{Our GPDs may be used in large skewness region but success is not guaranteed}}$

# Applications

exploiting universality: our set of GPDs allows for parameter free calculations of other hard exclusive reactions (except of possible wave fct effects)

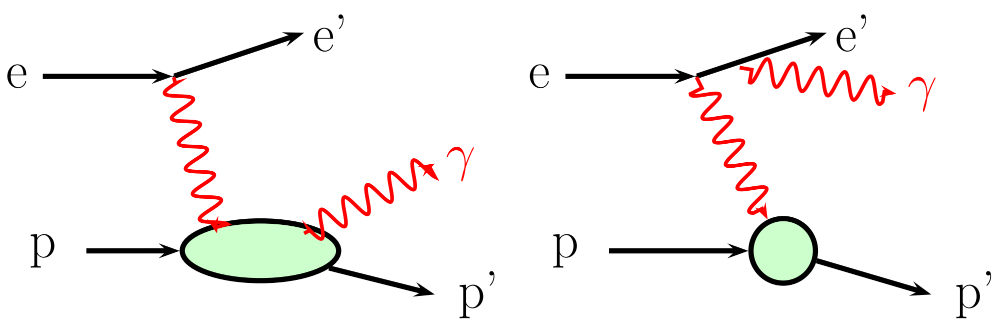
- $\nu_l p \to l P p$ Kopeliovich et al (13)
  V-A structure leads to different combinations of GPDs no data
- timelike DVCS Pire et al (13) no data
- $\gamma^* p \to \omega p$ Goloskokov-K(14)
  compared with SDMEs from HERMES(14) (asymmetries will come)
  prominent role of pion pole
- DVCS K-Moutarde-Sabatie(13)
  compared to data from Jlab, HERMES, H1, ZEUS
  good agreement with small skewness data, less good with Jlab data

# DVCS




collinear (leading-twist) calculation to LO accuracy for consistency

NLO: gluon GPDs contribute as well




$$d\sigma(lp \to lp\gamma) = d\sigma_{BH} + d\sigma_I + d\sigma_{DVCS}$$

$$d\sigma_i \propto \sum_{n=0}^{3} \left[c_n^i \cos(n\phi) + s_n^i \sin(n\phi)\right]$$

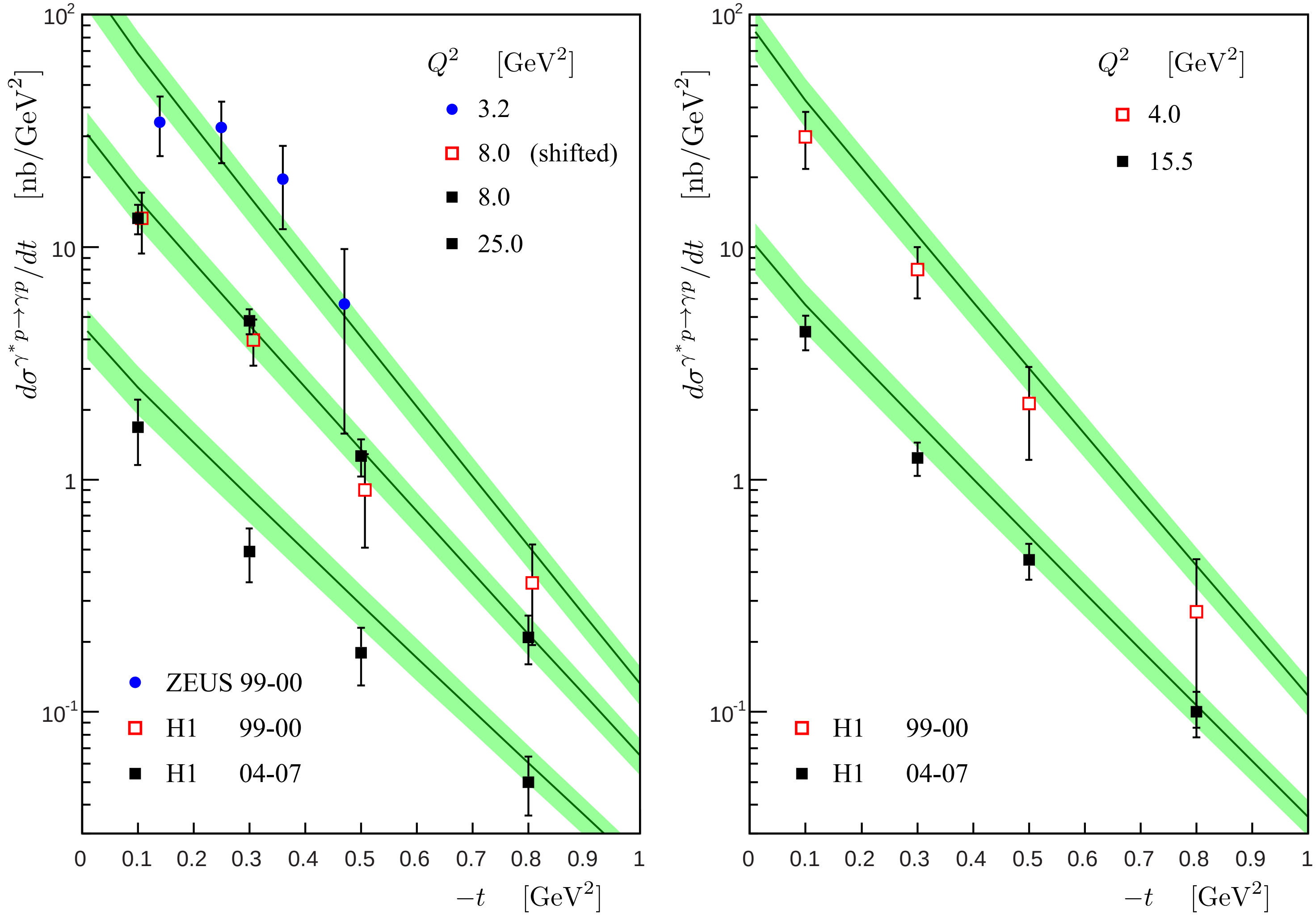
DVCS convolutions

$$\langle K \rangle = e_u^2 \langle K^u \rangle + e_d^2 \langle K^d \rangle + e_s^2 \langle K^s \rangle$$

**most of the DVCS observables can be evaluated from $H$**

($\phi$ azimuthal angle between lepton and hadron plane)

# DVCS at HERA




$W \simeq 90\,\mathrm{GeV}$ data from ZEUS, H1 K-Moutarde-Sabatie(13)

# $E$ for gluons and sea quarks

$E$ for valence quarks from FF analysis Diehl-K(13)

Teryaev(99): sum rule (Ji's s.r. and momentum s.r. of DIS) at $t = \xi = 0$

$$\int_0^1 dx x e_g(x) = e_{20}^g = -\sum e_{20}^{a_v} - 2\sum e_{20}^{\bar{a}}$$

valence term very small

$\Rightarrow$ 2nd moments of gluon and sea quarks cancel each other almost completely (holds approximately for other moments too provided GPDs don't have nodes)

positivity bound for FTs forbids large sea $\Longrightarrow$ gluon small too

$\frac{b_\perp^2}{m^2}\left(\frac{\partial e_s(x,b_\perp)}{\partial b_\perp^2}\right)^2 \leq s^2(x,b_\perp) - \Delta s^2(x,b_\perp)$

parameterization as described: $\beta_e^s = 7$, $\beta_e^g = 6$ Regge-like parameters as for $H$

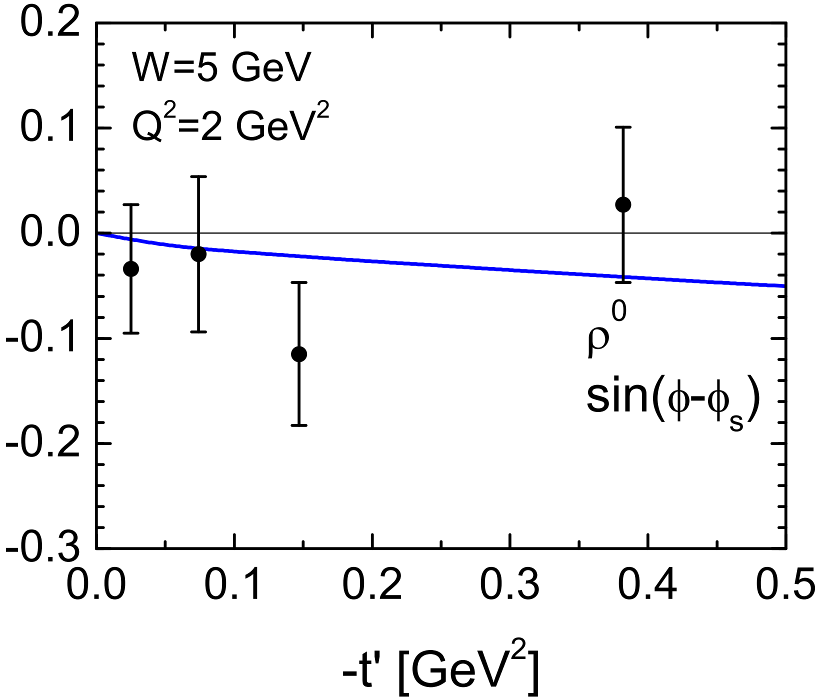
$e_i = N_i x^{-\alpha_g(0)}(1-x)^{\beta_i}$ flavor symm. sea for $E$ assumed

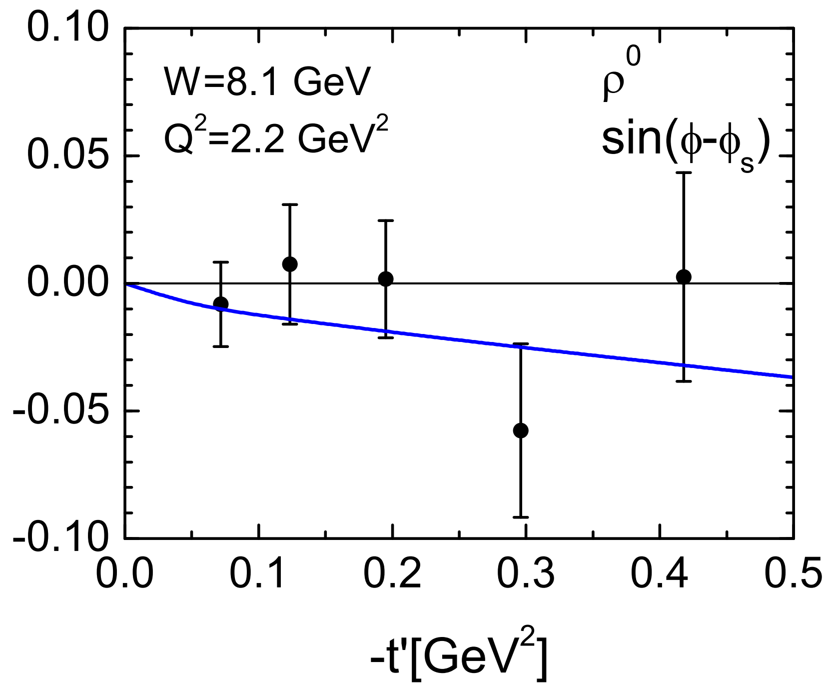
$N_s$ fixed by saturating bound ($N_s = \pm 0.155$), $N_g$ from sum rules

$(e_{20}^s = \pm 0.026)$

for $\xi \neq 0$ input to double distribution ansatz

# $A_{UT}^{\sin(\phi-\phi_s)}$ for $\rho^0$ production




data: HERMES(08) COMPASS(12)

theor. result: Goloskokov-K(09)

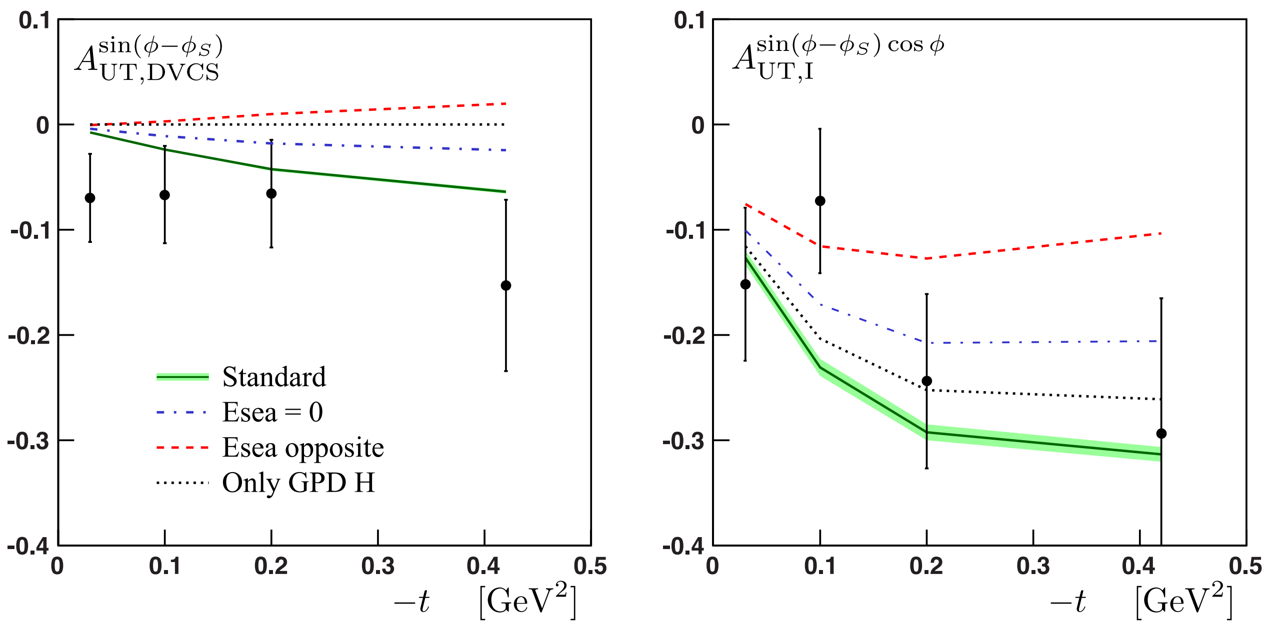
$$A_{UT}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\langle E\rangle^* \langle H\rangle\big]$$

gluon and sea contr. from $E$ cancel to a large extent

dominated by valence quark contr. from $E$

($\phi_s$ orientation of target spin vector)

# Transverse target spin asymmetry in DVCS




data: HERMES(08)

$\langle Q^2 \rangle \simeq 2.5\,\mathrm{GeV}^2$

$\langle x_{\mathrm{B}} \rangle \simeq 0.09$

theory: KMS(12)

$A_{UT,DVCS}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\langle E \rangle^* \langle H \rangle\big]$

no cancellation between

sea and gluon

$\Rightarrow \langle E^{\mathrm{sea}} \rangle$ seen

from BH-DVCS interference

separate contr. from

$\mathrm{Im}\,\langle H \rangle$ and $\mathrm{Im}\,\langle E \rangle$

negative $\langle E^{\mathrm{sea}} \rangle$ favored in both cases

$\langle E^g \rangle \geq 0$ Koempel et al(11) transverse target polarisation in $J/\Psi$ photo- and electroproduction, dominated by gluonic GPDs

# Application: Angular momenta of partons

$$J^a = \frac{1}{2}\Big[q^a_{20} + e^a_{20}\Big] \qquad J^g = \frac{1}{2}\Big[g_{20} + e^g_{20}\Big] \qquad\qquad (\xi = t = 0)$$

$q^a_{20}, g_{20}$ from ABM11 (NLO) PDFs $\qquad (a = u, d, s, \bar{u}, \bar{d}, \bar{s})$

$e^{a_v}_{20}$ from form factor analysis Diehl-K. (13):

$e^s_{20} \simeq 0 \ldots -0.026$ from analysis of $A_{UT}$ in DVCS and pos. bound

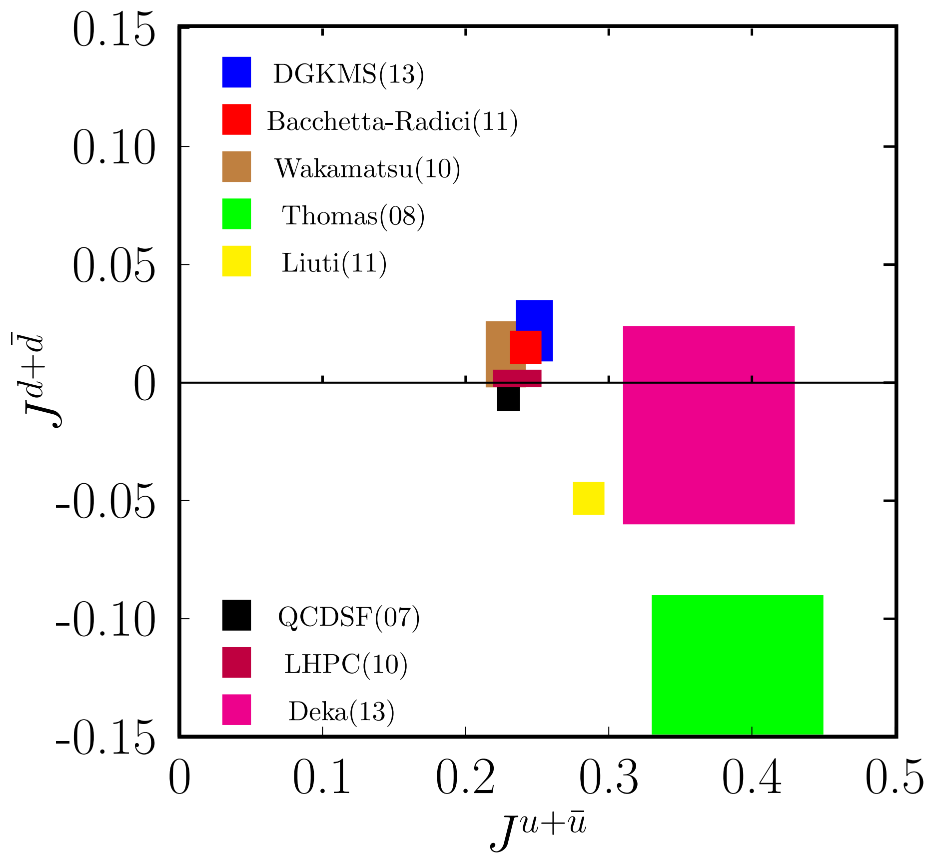
$e^g_{20}$ from sum rule for $e_{20}$

$$\begin{aligned} J^{u+\bar{u}} &= 0.261\ldots 0.235\,; \qquad J^{d+\bar{d}} = 0.035\ldots 0.009\,; \\ J^g &= 0.187\ldots 0.265\,; \qquad J^{s+\bar{s}} = 0.017\ldots -0.009\,; \end{aligned}$$

$J^i$ quoted at scale $2\,\mathrm{GeV}$ $\qquad \sum J^i = 1/2 \qquad$ (spin of the proton)

need better determ. of $E^s$ (smaller errors of $A_{UT}$ in DVCS)

# Comparison with other results




from DVCS exp:

CLAS $J^{d+\bar{d}} + J^{u+\bar{u}}/5 = 0.18 \pm 0.14$ HERMES $J^{d+\bar{d}}/2.9 + J^{u+\bar{u}} = 0.42 \pm 0.22$

strongly model dependent assumption: $e_{q_v}(x) \sim q_v(x)$

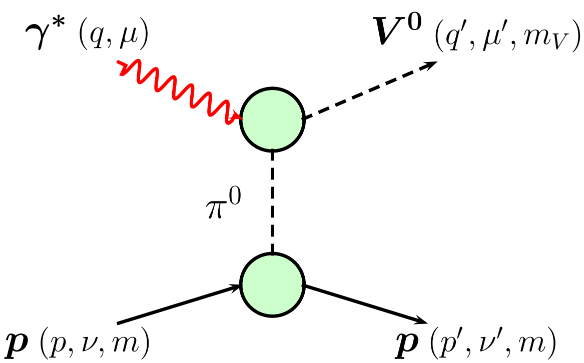
in conflict with FF analysis and with pert.QCD Yuan(04)

# $\omega$ production

important ingredient: pion pole (as for $\pi^+$ production)

$$\widetilde{E}^u_{\rm pole} = \widetilde{E}^d_{\rm pole} = \Theta(|x| \le \xi) \frac{m f_\pi g_{\pi NN}}{\sqrt{2}\xi} \frac{F_{\pi NN}(t)}{m_\pi^2 - t} \Phi_\pi\left(\frac{x+\xi}{2\xi}\right)$$

underestimates this contribution – treat pole as OPE Goloskokov-K(14)




$$\langle\omega|j^{\rm el}_\kappa(0)|\pi\rangle = e_0 g_{\gamma^*\pi\omega}(Q^2,t)\varepsilon(\kappa,q,\epsilon_\omega,q')$$

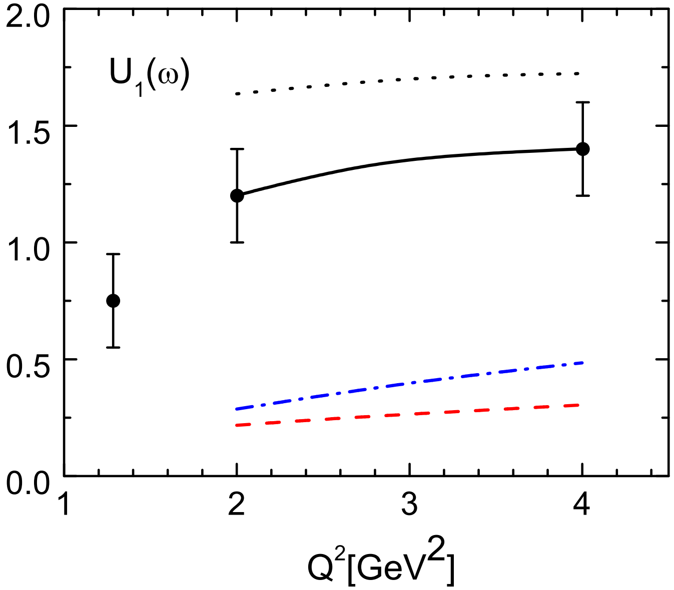
large $Q^2$, small $-t$: $g_{\gamma^*\pi\omega}(Q^2,t) \simeq g_{\pi\omega}(Q^2)$
dominant $T \to T$ transitions
(suppressed by $1/Q$ as compared to $L \to L$)
subdominant $L \to T$ (suppressed by $1/Q^2$)

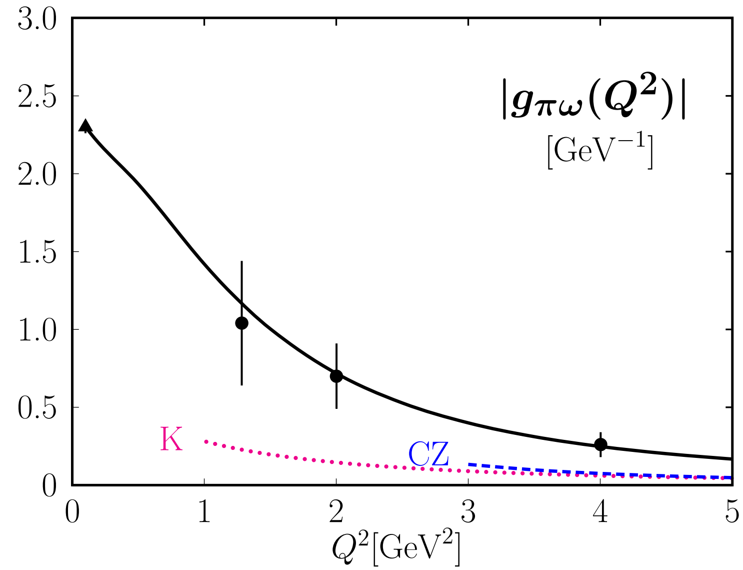
HERMES(14) $\omega$ SDMEs at $W = 4.8\,\mathrm{GeV}$:

$$U_1 = 1 - r^{04}_{00} + 2r^{04}_{1-1} - 2r^1_{11} - 2r^1_{1-1} = 2\frac{d\sigma_U}{d\sigma} \qquad d\sigma_U = \frac{U_1}{2-U_1} d\sigma_N$$

$d\sigma_N$: from our GPDs $H$ and $E$ (like $\rho^0$) and small contr. from $H_T$ and $\bar{E}_T$
$d\sigma_U$: pion pole ($\sim g_{\pi\omega}(Q^2)$) and small background ($\widetilde{H}$)

# $\pi - \omega$ form factor




$g_{\pi\omega}$ unknown exp. in space-like region, except at $Q^2 = 0$ from $\omega \to \pi\gamma$ decay

fit $g_{\pi\omega}$ to $U_1$ results consistent with $Q^2 = 0$ value

interpolation: $(a_1 = 3.1\,\mathrm{GeV} \qquad a_2 = 1.2\,\mathrm{GeV})$

$$|g_{\pi\omega}| = \frac{2.3\mathrm{GeV}^{-1}}{1 + Q^2/a_1^2 + Q^4/a_2^4}$$

sign cannot be fixed from SDME; need spin asymmetries

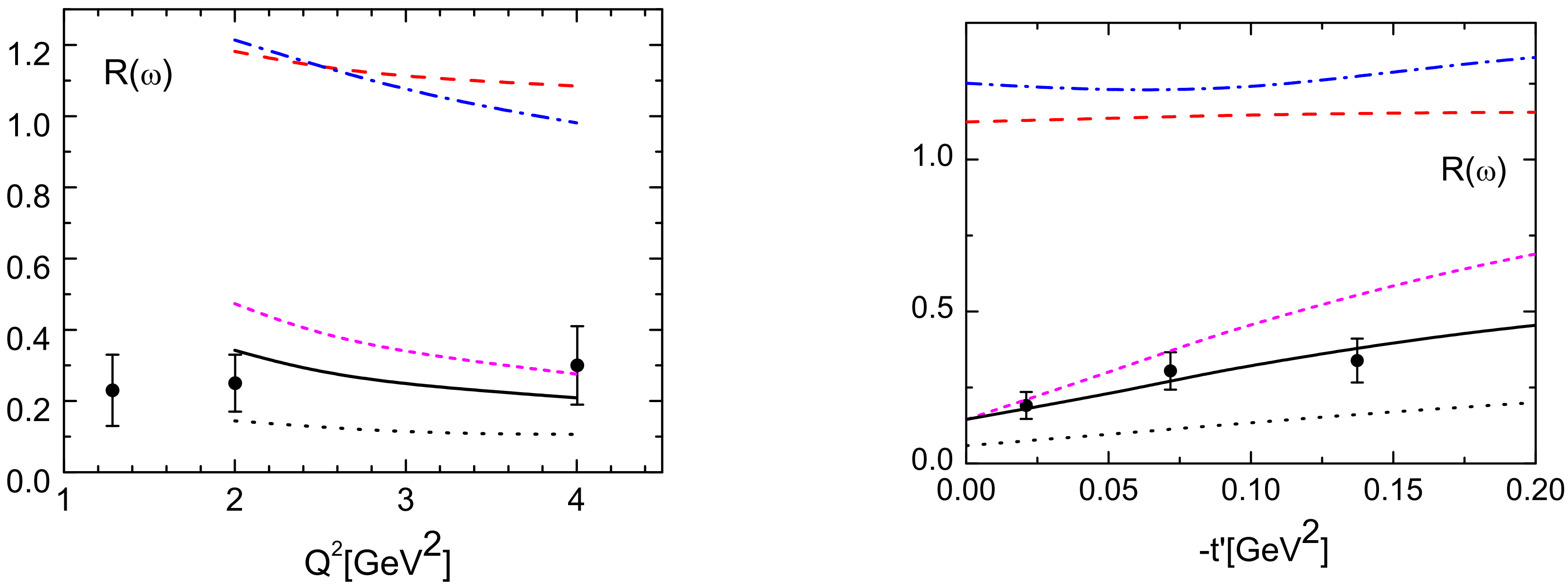
theory: QCD sum rules (only soft) Khodjamirian(99), Braun-Halperin(94)

perturbative QCD to twist-3 accuracy Chernyak-Zhitnitsky(84)

$(W = 4.8\,\mathrm{GeV}$, without pion pole, 8 $\mathrm{GeV}$, dotted 3.5 $\mathrm{GeV}$, $t' = -0.08\,\mathrm{GeV}^2)$

# Longitudinal/transversal separation

$$R = \frac{1}{\epsilon}\frac{r_{00}^{04}}{1-r_{00}^{04}} = \frac{d\sigma(L\to L) + d\sigma(T\to L)/\epsilon}{d\sigma(T\to T) + \epsilon d\sigma(L\to T)}$$

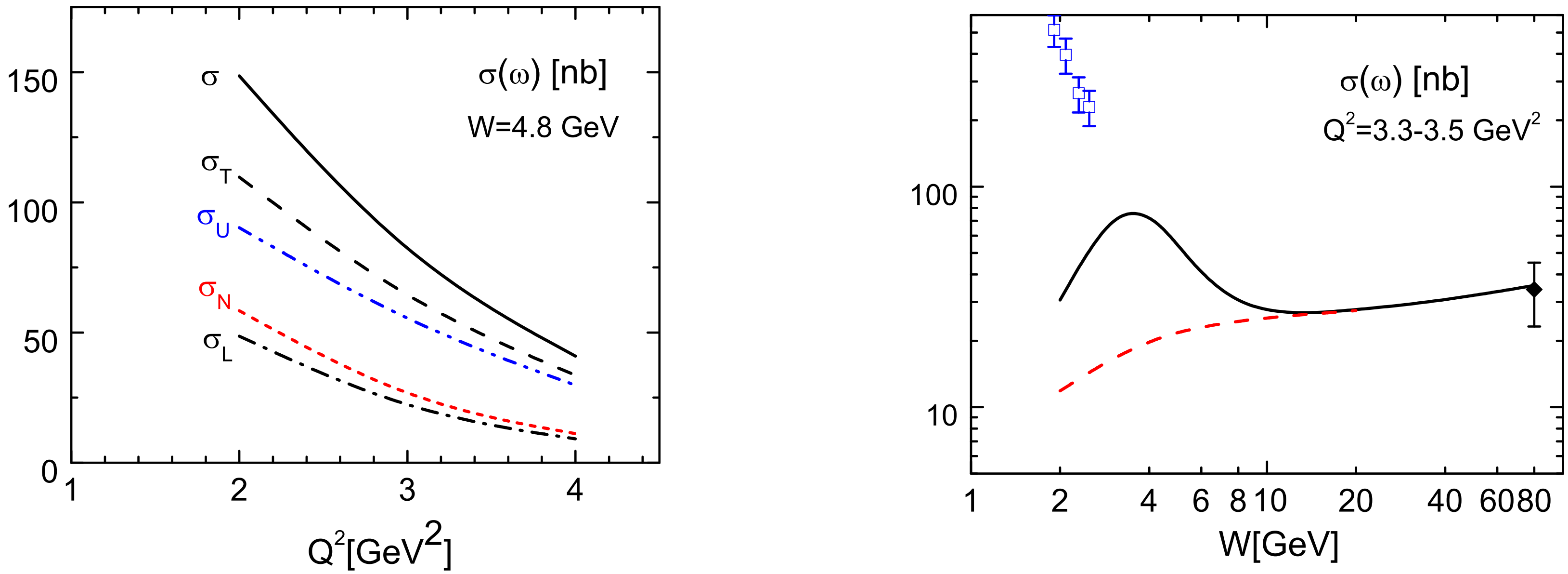



$W = 4.8\,\mathrm{GeV}$, $t' = -0.08\,\mathrm{GeV}^2$ $\qquad$ $Q^2 = 2.42\,\mathrm{GeV}^2$

differs from $d\sigma_L/d\sigma_T$ (dashed line)

by $L \to T$ (pion pole) and $T \to L$ $(H_T, \bar{E}_T)$ transitions

# Integrated cross sections




data: CLAS, ZEUS

integrated on $0 < -t' < 0.5\,\mathrm{GeV}^2$

very different from expectations for $Q^2 \to \infty$

without pion pole: $\sigma_U < \sigma_N$ and $\sigma_T < \sigma_L$ as for $\rho^0$ production

# Summary

- The handbag approach, generalized to transverse photons and with meson size corrections, describes all DVMP and DVCS data at small skewness
- From the combined analysis of nucleon form factors, DVMP and DVCS a set of GPDs is extracted ($H, E, \widetilde{H}$ and a bit of information on $H_T$ and $\bar{E}_T$).
- This set of GPDs allows to calculate other hard exclusive processes, to evaluate Ji's sum rule and to study the transverse localization of partons in the proton (at least for valence quarks).
- Nothing is perfect - the GPDs need improvements:
  use of new PDFs, more complicated profile fcts. for all GPDs, $D$-term, kinematical corrections at low $Q^2$, low $W$, large $\xi$ and new data from COMPASS, JLAB12 and EIC