Deeply virtual Compton scattering Theory

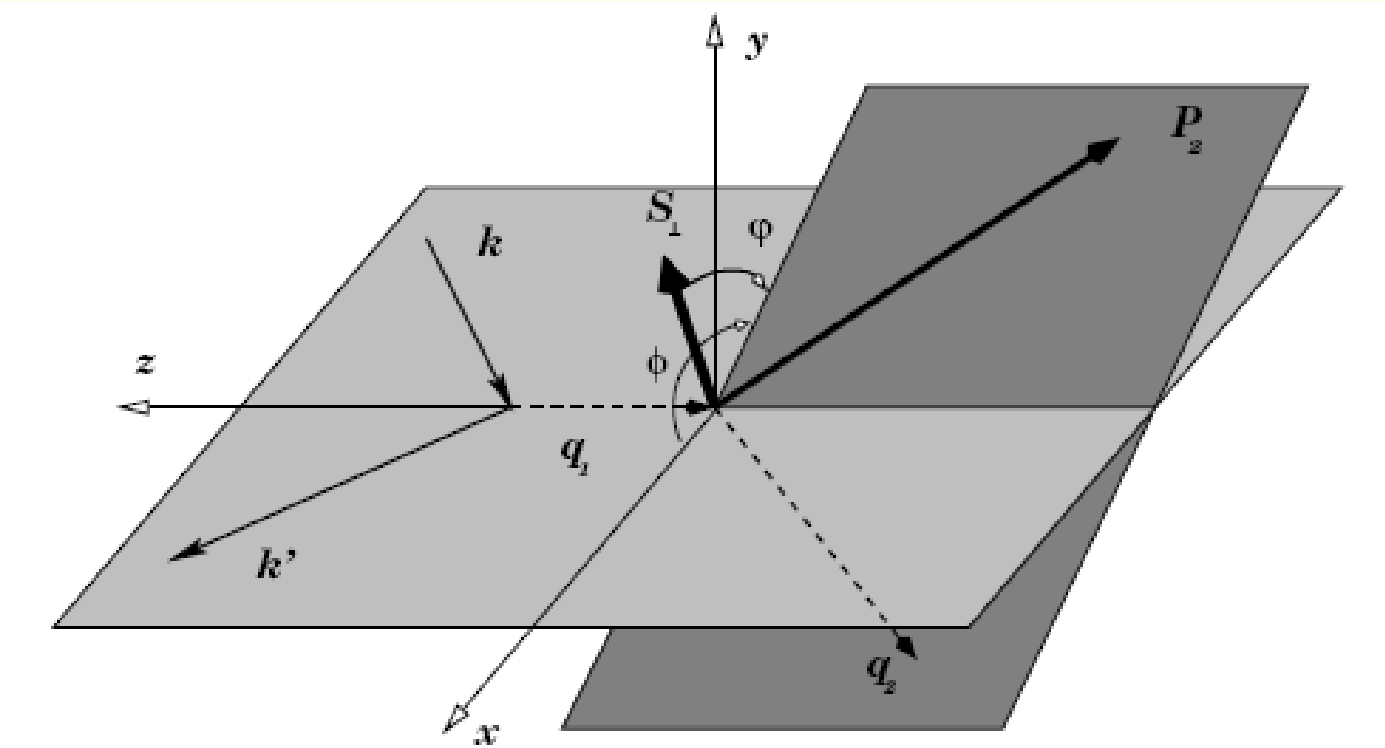
D. Müller, Ruhr Universität Bochum

- ***Observables in γ leptoproduction***
- ***GPD ansätze – how to improve them?***
- ***Perturbative corrections up to NNLO***
- ***Towards a fitting procedure***
- ***Conclusions***

Photon leptoproduction

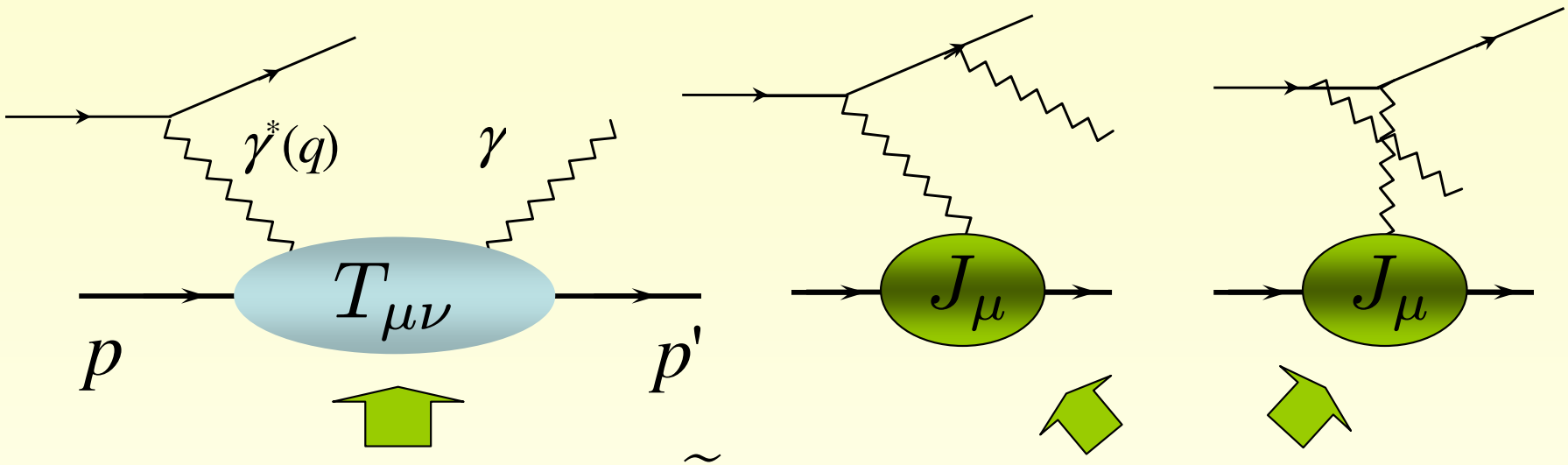
cross section for $e^{\pm}N \to e^{\pm}N\gamma$:

$$\frac{d\sigma}{dx_{\rm Bj}dyd|\Delta^2|d\phi_e d\phi_N} = \frac{\alpha^3 y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M_{Bj}^2{}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



$$\begin{aligned} x_{\rm Bj} &= \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi}, \\ y &= \frac{P_1\cdot q_1}{P_1\cdot k}, \\ \Delta^2 &= t \;\; (\text{fixed, small}), \\ \mathcal{Q}^2 &= -q_1^2 \;\; (>1\text{GeV}^2), \end{aligned}$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \cdots$ (helicity amplitudes)

elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

appr. known harmonics ⇕ **1:1** helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj}^y{}^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

appr. known harmonics ⇕ ~~**1:1**~~ helicity ampl.

CFFs are given in terms of generalized parton distributions

$$\mathcal{F}(\xi, \mathcal{Q}^2, \Delta^2) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, \Delta^2, \mu) + O(1/\mathcal{Q}^2)$$

Compton form factor

observable

hard scattering part

perturbation theory
(our conventions)

GPD

universal
(but conventional)

power
corrections

relations among CFFs and harmonics is based on $1/\mathcal{Q}$ expansion
(whole amplitude, including leptonic part at twist-3 level):

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{\mathcal{Q}} (\text{tw-2})^{\mathrm{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\mathrm{CS}} \propto (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathrm{CS}} \propto \frac{\Delta}{Q} (\text{tw-2})(\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathrm{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

How good is this approximation?

i. at least ***three different*** definitions of scaling variables are used:

physical definition: $\xi = \frac{-(q_1+q_2)^2}{2(P_1+P_2)\cdot(q_1+q_2)}, \quad \eta = \frac{(P_1-P_2)\cdot(q_1+q_2)}{(P_1+P_2)\cdot(q_1+q_2)},$

partonic definition: $\xi' = \frac{-n\cdot q_1}{n\cdot(P_1+P_2)}, \quad \xi = \frac{n\cdot(P_1-P_2)}{n\cdot(P_1+P_2)}$

$n^2=0$ definition of n is ambiguous !

$$\xi = \xi' + O(\Delta^2/\mathcal{Q}^2) + O(x_{\rm Bj}^2 M^2/\mathcal{Q}^2) \quad \overset{\textbf{Bjorken limit}}{\Rightarrow} \quad \frac{x_{\rm Bj}}{2-x_{\rm Bj}}$$

different kinematical conventions yield different GPDs (different corrections)

for fixed target kinematics there is a ***5% - 10% uncertainty*** (scaling variable)

for the time being one perhaps can agree on (? DDVS)

$$\xi = \eta, \; x_{\rm Bj} = \frac{2\xi}{1+\xi}, \; \mathcal{Q}^2 = -q_1^2$$

imaginary part of a CFF reads then at LO:

$$\mathcal{F} = \pi \left(F(\xi = \frac{x_{\rm Bj}}{2-x_{\rm Bj}}, \eta = \frac{x_{\rm Bj}}{2-x_{\rm Bj}}, \Delta^2, \mathcal{Q}^2) \pm \{\xi \to -\xi\} \right)$$

ii. ***various*** definitions of DVCS-tensor/amplitude

parameterization should lead to `convenient' relations of CFFs and helicity amplitudes

iii. treatment of squared amplitude

numerically squared [VGG code] leads to the question: What can we extract?

square first and expand to get analytic expressions [A.V. Belitsky, DM., A. Kirchner]

numerical differences between both treatments for fixed target kinematics

e.g., real part of the interference term for scalar target,
unpolarized beam JLAB kinematics: $x_{\rm Bj} = 0.3, t = -0.3{\rm GeV}^2, Q^2 = 2{\rm GeV}^2$

$$I \propto -2.3\Re\mathcal{H} - 12.9\Re\mathcal{H}\cos(\phi) + 1.1\Re\mathcal{H}_3\cos(2\phi) + \cdots\cos(3\phi)$$

$$\propto -2.43\Re\left[\mathcal{H} - 0.06\mathcal{H}_3\right] - 7.54\Re\left[H + 0.02\mathcal{H}_3\right]\cos(\phi) + 1.2\Re\left[\mathcal{H}_3 - 0.64\mathcal{H}_T\right]\cos(2\phi) + \cdots$$

take exact leptonic part and differences should ***diminish***, e.g., in c_1

$$\sqrt{1-y}\left(2-2y+y^2\right) \Rightarrow \sqrt{1-y\left(1+\frac{4x_{\rm Bj}^2M^2}{Q^2}y/4\right)} \times \left[\tfrac{1}{2}(2-y)^2\cdots+\cdots\right]$$

not a too small number

A popular GPD ansatz

spectral representation is **not unique** and reads for $F = \{H, E, \widetilde{H}, \widetilde{E}, \cdots\}$

$$F(x,\eta,\Delta^2) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz x^p \delta(x-y-z\eta) f(y,z,\Delta^2) + \theta(|\eta|-|x|)|\eta|^{P-1} S\left(\frac{x}{\eta},\Delta^2\right)$$

Radyushkin's ansatz for spectral function is within *p=0*:

$$f(y,z,\Delta^2) = F(\Delta^2) q(y,\Delta^2) \frac{\Pi(z/(1-y))}{(1-y)}, \quad \int_{-1}^{1} dz\, \Pi(z) = 1, \quad \Pi(z) \geq 0;$$

Polyakov, Weiss suggestions $E: \quad p = 0, P = 1, \quad S(x,\Delta^2) = D(x,\Delta^2)$

$\widetilde{E}: \quad P = 0, \quad S(x,\Delta^2)$ pion pole contribution

! *D*-term and pion pole *contribute* only to the *real part* of the amplitude

constraints:

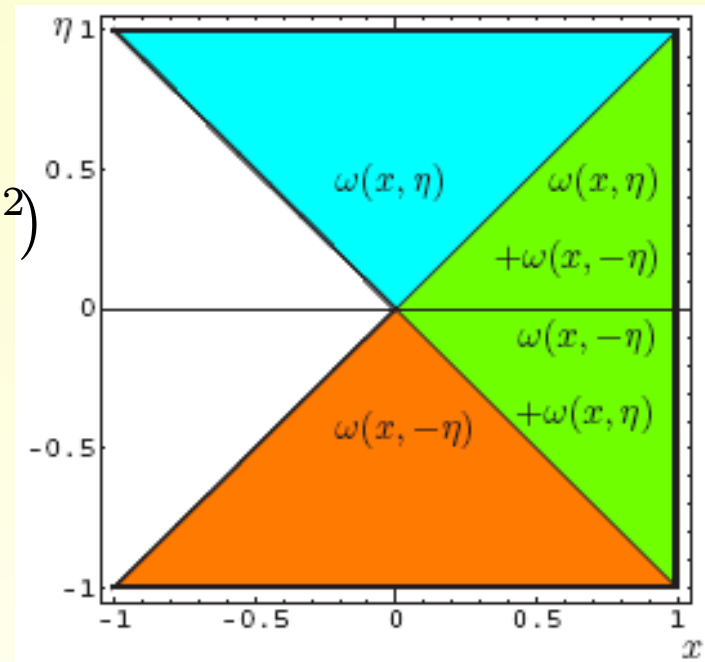
- ✓ polynomiality is satisfied
- ✓ $\int_0^1 dy\, q(\Delta^2) f(y,\Delta^2)$ partonic form factor – related to observables
- ✓ $q(y,\Delta^2 = 0)$ parton densities
- ! positivity constraints [P.Pobylitsa] (requirement on GPDs and scheme)

Support of GPDs – a hint for duality

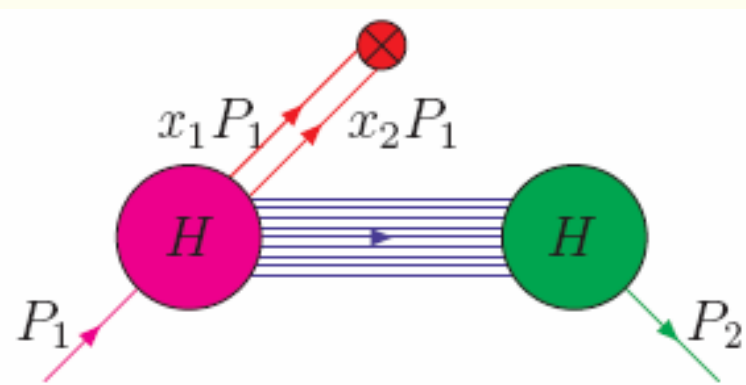
consider a quark GPD (anti-quark $x \to -x$)

$$F = \theta(-\eta \le x \le 1)\,\omega\left(x,\eta,\Delta^2\right) + \theta(\eta \le x \le 1)\,\omega\left(x,-\eta,\Delta^2\right)$$

$$\omega\left(x,\eta,\Delta^2\right) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y,(x-y)/\eta,\Delta^2)$$

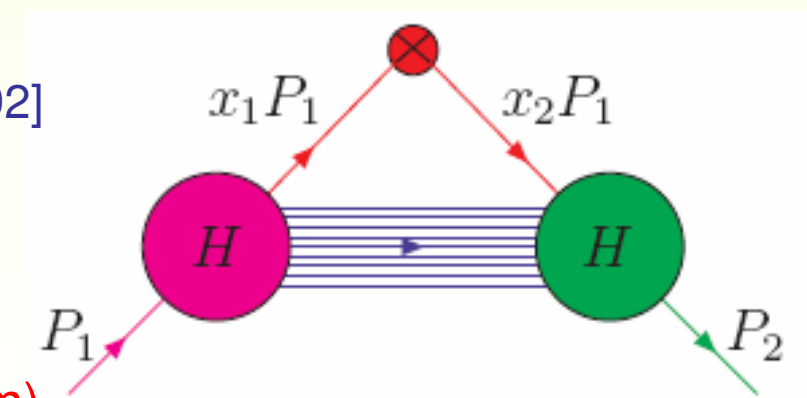


a naive ***dual*** interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term) [DM, A. Schäfer (05)]



central region - $\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

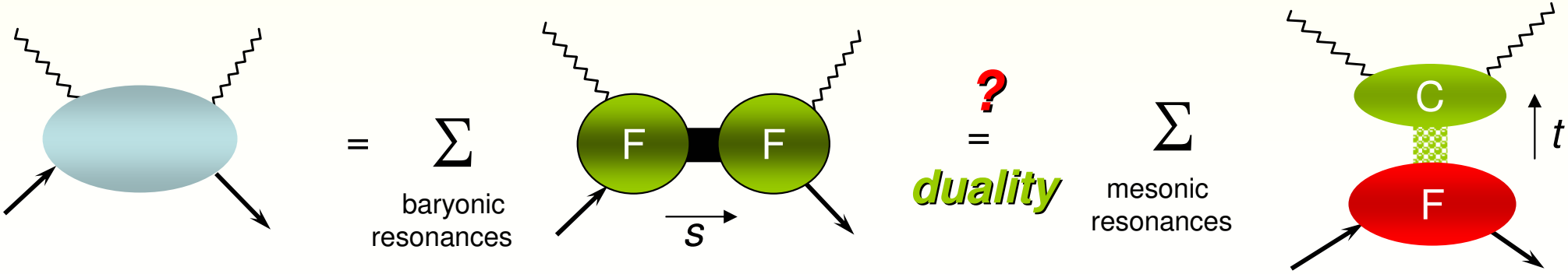
partonic exchange in *s*-channel

How to get more insight into GPDs?

- lattice simulations of GPD moments (first few, heavy pion world) [QCDSF,LHPC,…]
- bag model [Ji et al.], quark soliton model [Göke et al,…], BS-equation [Miller,…], ….
- overlap representation of LC wave functions [Brodsky, Diehl, Feldman, Kroll, …]
- approaches to describe hard amplitudes (better understanding as for GPDs)
  - resumming *s*-channel resonances [Close, Zhao]
  - vector dominance & Regge inspired description [Guidal et al., M. Capua et al., …]

s-channel contributions
(resonance region, large *x*)

t-channel contributions
(Regge phenomenology, small *x*)



take models (`knowledge') for the amplitude and extract GPDs

SO(2,1) (conformal) expansion of GPDs

GPD support is a consequence of Poincaré invariance (polynomiality)

$$F_n(\eta,\Delta^2,\mu^2) = \int_{-1}^{1} dx\, c_n(x,\eta) F(x,\eta,\Delta^2,\mu^2)\,, \qquad c_n(x,\eta) = \eta^n C_n^{3/2}(x/\eta)$$

conformal moments evolve *autonomously* (LO & NLO in a special scheme)

$$\mu\frac{d}{d\mu}F_n(\eta,\Delta^2,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_n^{(0)} F_n(\eta,\Delta^2,\mu^2)$$

inverse relation is given by a Mellin-Barnes integral:

$$F(x,\eta,\Delta^2,\mathcal{Q}^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\eta)}{\sin(\pi j)} \exp\left\{-\frac{\gamma_j^{(0)}}{2}\int_{\mathcal{Q}_0^2}^{\mathcal{Q}^2}\frac{d\sigma}{\sigma}\frac{\alpha_s(\sigma)}{2\pi}\right\} F_j(\eta,\Delta^2,\mathcal{Q}_0^2)$$

SO(2,1) partial waves (Legendre functions)

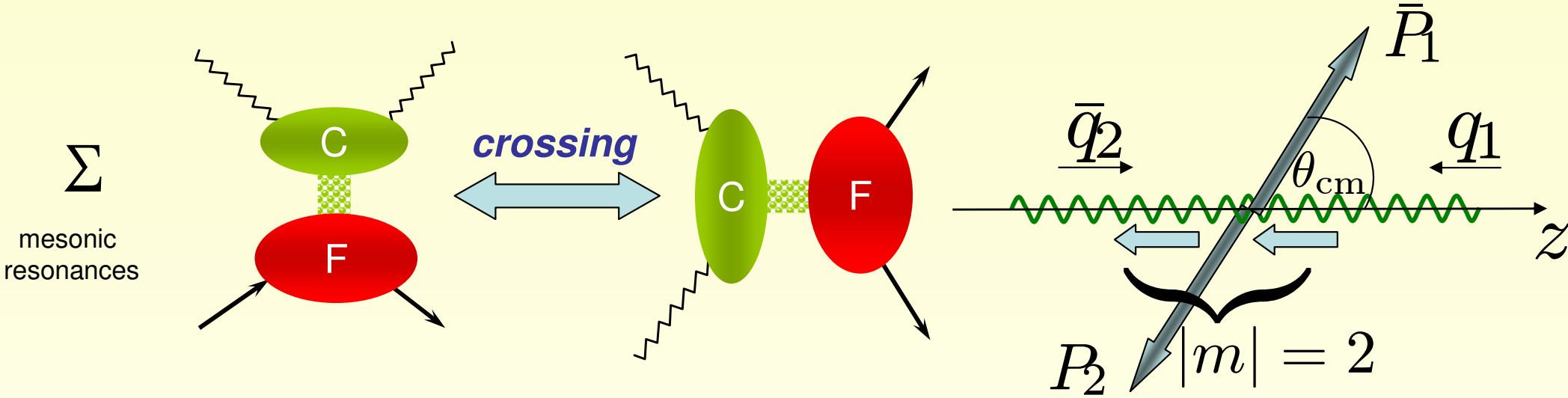
evolution (trivial)

analytic continuation of conformal GPD moments (polynomials)

reletated representations were proposed:

- smearing method [B. Geyer, A. Belitsky, D.M., L. Niedermeier, A. Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization (a mixture of both) [M. Polyakov, A. Shuvaev (02)]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- Mellin-Barnes integral [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

A closer look to the t-channel process



SO(3) partial wave expansion of helicity amplitudes in cm frame:

$$A^{(t)}_{\bar{b}d;a\bar{c}} = \sum_{J=J_{\rm min}}^{\infty} (2J+1)\, d^J_{\bar{b}-d,a-\bar{c}}(\theta)\, A^J_{\bar{b}d;a\bar{c}}(\bar{s},\bar{u})\,, \qquad d^J_{\mu,\nu}(\theta)$$

- reduced Wigner matrices

for twist-two CFFs $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$ we have $m = \mu = \pm 2$, e.g.,

$$\mathcal{H} + \mathcal{E} = \sum_{J=2}^{\infty} (2J+1) \cot(\theta) A^J_{\uparrow\uparrow} \left[d^J_{2,1}(\theta) + (-1)^J d^J_{-2,1}(\theta) \right]$$

$$\mathcal{E} = \sum_{\substack{J=2 \\ \rm even}}^{\infty} (2J+1) d^J_{2,0}(\theta) A^J_{\uparrow\downarrow} + \frac{4M^2}{\bar{s} - 4M^2} (\mathcal{H} + \mathcal{E})$$

for large virtualities we can employ (conformal) OPE

(Taylor expansion with respect to $\vartheta = (q_1^2 - q_2^2)/Q^2 = \eta/\xi$)

$$\mathcal{F}^{(t)} \simeq \sum_{j=0}^{\infty} [1 \mp (-1)^j]\, \vartheta^{j+1}\, C_j(\vartheta, Q^2/\mu^2, \alpha_s(\mu))\, F_j^{(t)}(\cos\theta, s, \mu^2)$$

angular momentum J partial wave expansion of GPD moments:

$$F_j^{(t)}(\cos\theta, s, \mu^2) = \sum_{\substack{J=J_{\min} \\ \text{even/odd}}}^{j+1} (2J+1) F_j^J(s, \mu^2) \omega_{\mathcal{F}}(\theta) d_{\mathcal{F}}^J(\theta)$$

crossing $F_j(\eta) = \eta^{j+1} F_j^{(t)}(\theta),\ \cos\theta \sim -1/\eta,$ yields

$$F_j(\eta, t) \sim \sum_\alpha \frac{F_j^{j+1,\alpha}(t)}{j+1-\alpha(t)} + \sum_\alpha \gamma_{j+1-\alpha(t)}(t) \eta^{j+1-\alpha(t)} \qquad \alpha(t) = \alpha(0) + \alpha' t$$

leading SO(3) partial wave *remaining ones* *Regge trajectories*

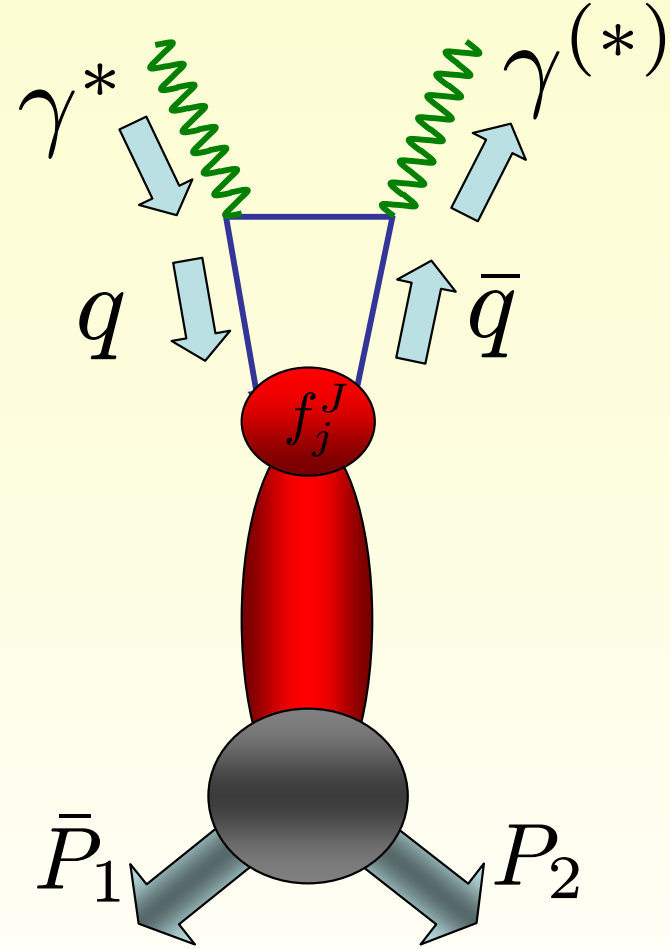
! matching scale should be rather low $Q^2 \sim 1\mathrm{GeV}^2, \quad \eta \lesssim 0.05$

! the forward limit is `nontrivial' $F_j(\eta \to 0, t) \sim \frac{\beta(t)}{j+1-\alpha(t)} + \gamma(t)\eta^{j+1-\alpha(t)}$

contrary to some claims about small *x* GPD behavior

Ansatz for partonic partial wave amplitudes

- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2* (helicity conservation)
- they form an intermediate mesonic state with total angular momentum ***J***
  strength of ***coupling*** is $f_j^J,\ J \leq j+1$
- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$
- decaying into a nucleon anti-nucleon pair with given spin *S* and angular momentum *L*, described by an ***impact form factor***



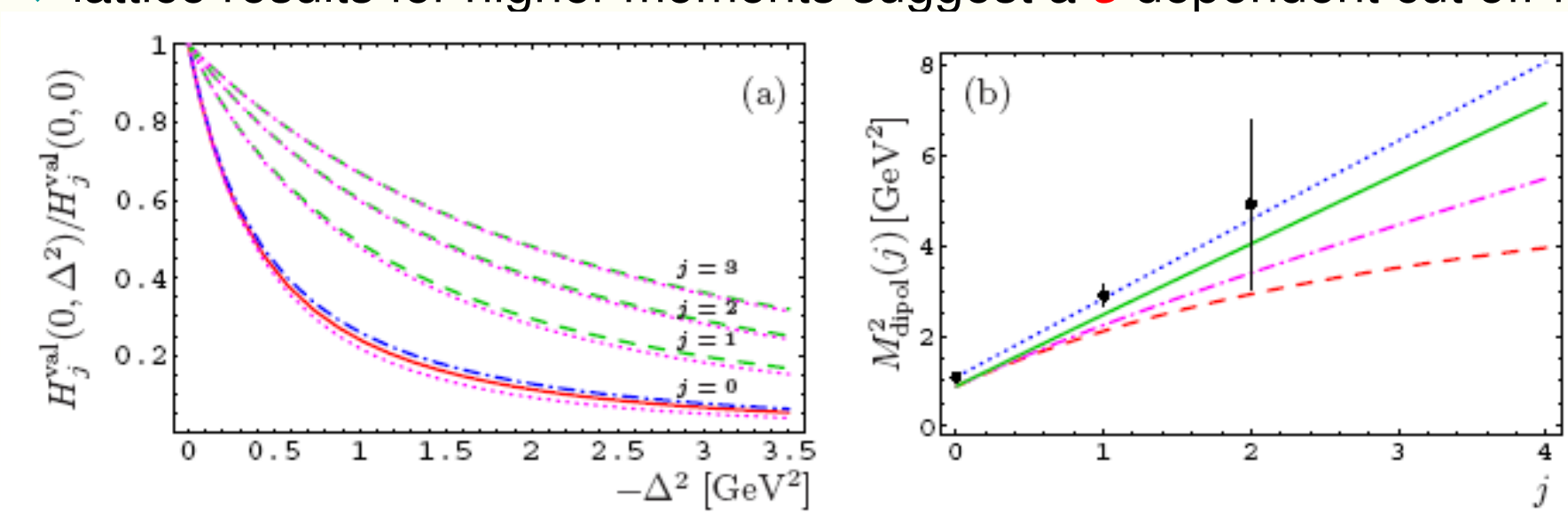
$$F_j^J(\Delta^2) = \frac{f_j^J}{J-\alpha(\Delta^2)} \frac{1}{(1-\frac{\Delta^2}{M^2(J)})^p}$$

! GPD *E* is zero if chiral symmetry holds (partial waves are Gegenbauer polynomials with index 3/2)

D-term arises from the SO(3) partial wave *J=j+1* *(j → -1)*

Constraints for partonic partial wave amplitudes

- moments of parton distributions $$F_j^{J=j+1}(\Delta^2=0) = \frac{f_j^{j+1}}{j+1-\alpha(\Delta^2=0)}$$
- partonic form factors $$F_{j=0}^{J=1}(\Delta^2) = \frac{f_0^1}{1-\alpha(\Delta^2)}\frac{1}{(1-\frac{\Delta^2}{M^2(0)})^p}$$
- angular momentum sum rule $$(H+E)_{j=1}^{J=2}(\Delta^2) = \frac{(h+e)_1^2}{2-\alpha(\Delta^2)}\frac{1}{(1-\frac{\Delta^2}{M^2(1)})^p}$$
- positivity constraints? $$\sum_{\substack{j=0\\ \text{even}}}^{\infty} C_{j+J}(\vartheta, Q^2/\mu^2, \alpha_s(\mu^2))\, F_{j+J}^J(\Delta^2,\mu^2) \geq 0$$
- lattice results for higher moments suggest a ***J*** dependent cut off-mass $M^2(J)$



Perturbative and higher twist corrections

- ***perturbative next--to--leading order corrections*** [conformal approach D.M. (94)]
  - ✓ hard scattering part for photon/meson electroproduction [A. Belitsky, D.M. (00,01)
  - ✓ flavor singlet part for meson electroproduction [D. Ivanov, L. Szymanowski (04)]
  - ✓ for all then flavor singlet twist--two anomalous dimensions [A. Belitsky, D.M. (98)]
  - ✓ and flavor singlet twist--two evolution kernels [A. Belitsky, D.M., A. Freund (99,00)]
- ***evaluation of higher twist contributions***
  - ✓ completing the twist-three sector [A. Belitsky, D.M. (00)]
  - ❖ target mass corrections (twist-4) to photon electroproduction [A.Belitsky,D.M.(01)
  - ❖ WW-approximation to helicity flip DVCS contribution [N. Kivel, L. Mankiewicz (01)]
  - o higher twist corrections are not well understood
- ***perturbative next--to--next--to--leading order corrections to DVCS***
  [D.M. (05); K.Kumerićki, K.Passek-Kumerićki, D.M., A. Schäfer (06/07)]

Dispersion relation & conformal OPE

i. use analyticity with respect to $\nu = \frac{s-u}{4M} = \frac{Q^2}{2M}\xi^{-1}$

$$\mathcal{F}(\xi,\vartheta,\Delta^2,Q^2) = \frac{1}{\pi}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right) \Im\mathrm{m}\mathcal{F}(\xi'-i0,\vartheta,\Delta^2,Q^2) + \mathcal{C}(\vartheta,\Delta^2,Q^2)$$

photon asymmetry $\vartheta=\eta/\xi$ is *0* for DIS *1* for DVCS

subtraction needed for $\mathcal{H}$ and $\mathcal{E}$

ii. Taylor expansion in the unphysical region, i.e., $\omega=1/\xi$ in the vicinity $\omega=0$

$$\mathcal{F}(\xi,\vartheta,\Delta^2,Q^2) = \sum_{j=0}^{\infty}\left[1\mp(-1)^j\right]\omega^{j+1}\,\mathcal{F}_j(\vartheta,\Delta^2,Q^2) + \mathcal{C}(\vartheta,\Delta^2,Q^2)\,,$$

$$\mathcal{F}_j(\cdots) = \frac{1\mp(-1)^j}{2\pi}\int_0^1 d\xi\,\xi^j\,\Im\mathrm{m}\mathcal{F}(\xi-i0,\cdots)$$

iii. (conformal) operator product expansion gives the Taylor coefficients

$$\mathcal{F}_j \simeq \sum_{\substack{n=0\\ \text{even}}}^{\infty} C_{j+n}(\vartheta,Q^2/\mu^2,\alpha_s(\mu))\,\vartheta^n\,F^{(n)}_{j+n}(\Delta^2,\mu^2)\,, \qquad F^{(l)}_j = \frac{1}{l!}\frac{d^l}{d\eta^l}F_j(\eta,\Delta^2,\mu^2)\Big|_{\eta=0}$$

Wilson coefficients

Taylor expansion of GPD moments

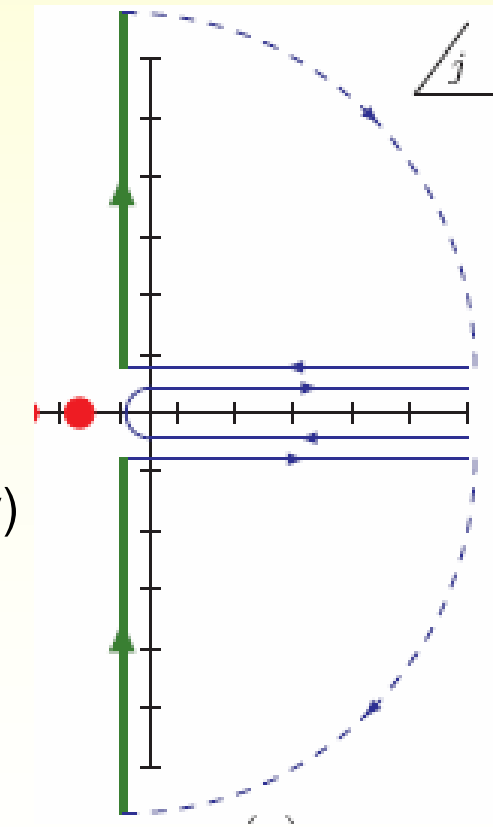
iv. invert Mellin transform yields ***conformal partial wave expansion***:

$$\mathcal{F} = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i + \left\{\begin{matrix}\tan \\ -\cot\end{matrix}\right\}\left(\frac{\pi j}{2}\right)\right] C_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) F_j(\xi, \Delta^2, \mu^2)$$

signature

Wilson coefficients (hard scattering)

GPD moments contain subtraction constant



- integration path is parallel to the imaginary axis, where singularities (Regge poles and cuts) are on the l.h.s.
- for a scheme that respects conformal symmetry:
- ✓ C_j and γ_j are ***known*** to **NNLO** (from DIS)
- Conformal symmetry breaking (arising from the trace anomaly) is shifted to the NNLO evolution and remains unknown
- ✓ standard MS scheme can be also implemented to NLO

various representations of that kind are well known

- smearing method [B. Geyer, A. Belitsky, D.M., L. Niedermeier, A. Schäfer (97/99)]
- generating function, resummation of SO(3) partial waves [M. Polyakov, A. Shuvaev (02)]
- Mellin-Barnes representation [DM, A. Schäfer (05); also A. Manashov, M. Kirch, A. Schäfer (05)]

Remarks on the subtraction constant

subtraction constant is ***predicted*** by the OPE:

$$\mathcal{C}(\vartheta, \Delta^2, Q^2) \simeq 2 \sum_{\substack{n=2 \\ \text{even}}}^{\infty} C_{n-1}(\vartheta, Q^2/\mu^2, \alpha_s(\mu)) \vartheta^n \, E_{n-1}^{(n)}(\Delta^2, \mu^2) \,,$$

it is expressed by the coefficients of the highest possible power in η appearing in the conformal GPD moments (*D*-term contribution)

predicted to be zero for $\mathcal{H} + \mathcal{E}$

and in forward kinematics $\mathcal{C}(\vartheta = 0, \Delta^2, Q^2) \simeq 0$

Is the subtraction constant related to the imaginary part?

yes, for the case of oversubtraction $\mathcal{C}(\vartheta, \ldots) = \frac{2}{\pi} \int_0^\infty d\xi \, \xi^{-1} \, \Im\mathrm{m} \mathcal{E}(\xi, \vartheta, \ldots)$

likely, in reality $\mathcal{C}(\vartheta, \ldots) = \frac{2}{\pi} \lim_{j \to -1} \left\{ \int_0^\infty d\xi \, \xi^j \, \Im\mathrm{m} \left[\mathcal{E}(\xi, \vartheta, \ldots) - \mathcal{E}(\xi, \vartheta = 0, \ldots) \right] \right\}_{\mathrm{AC}}$

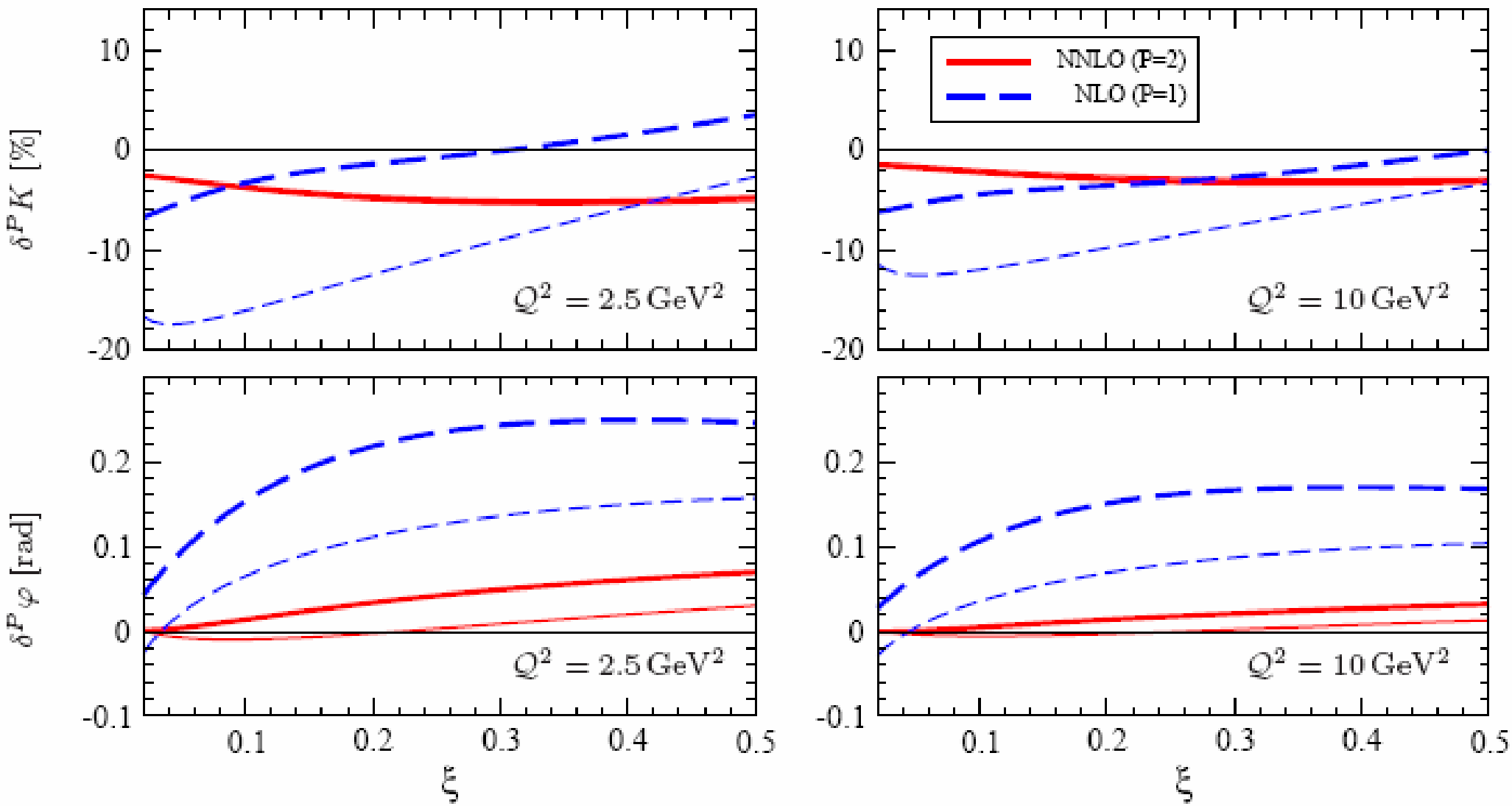
fixed pole

Brodsky, Close, Gunion (72)

only a Kronecker-δ singularity in the *j* plane justifies an independent *D*-term

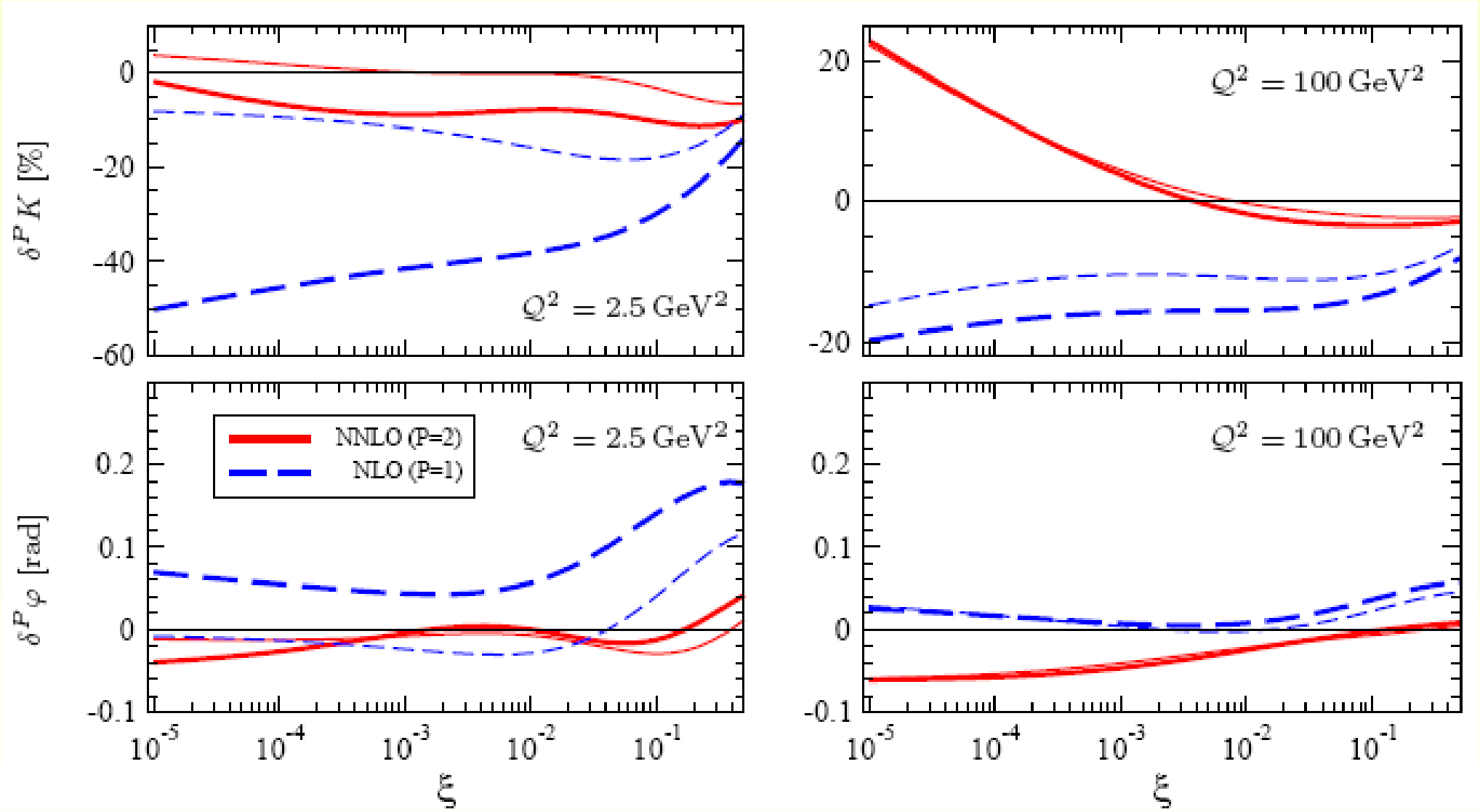
NLO & NNLO corrections – non-singlet

K. Kumerički, D.M., A. Schäfer
K. Passek-Kumerički (06/07)



- ✓ NLO corrections are moderate & factorization scale dependence is strongly reduced
- ✓ at NNLO perturbation theory is stabilizing for hard-scattering part and evolution
- ✓ both factorization and renormalization scale dependencies are reduced to 2% and 3%

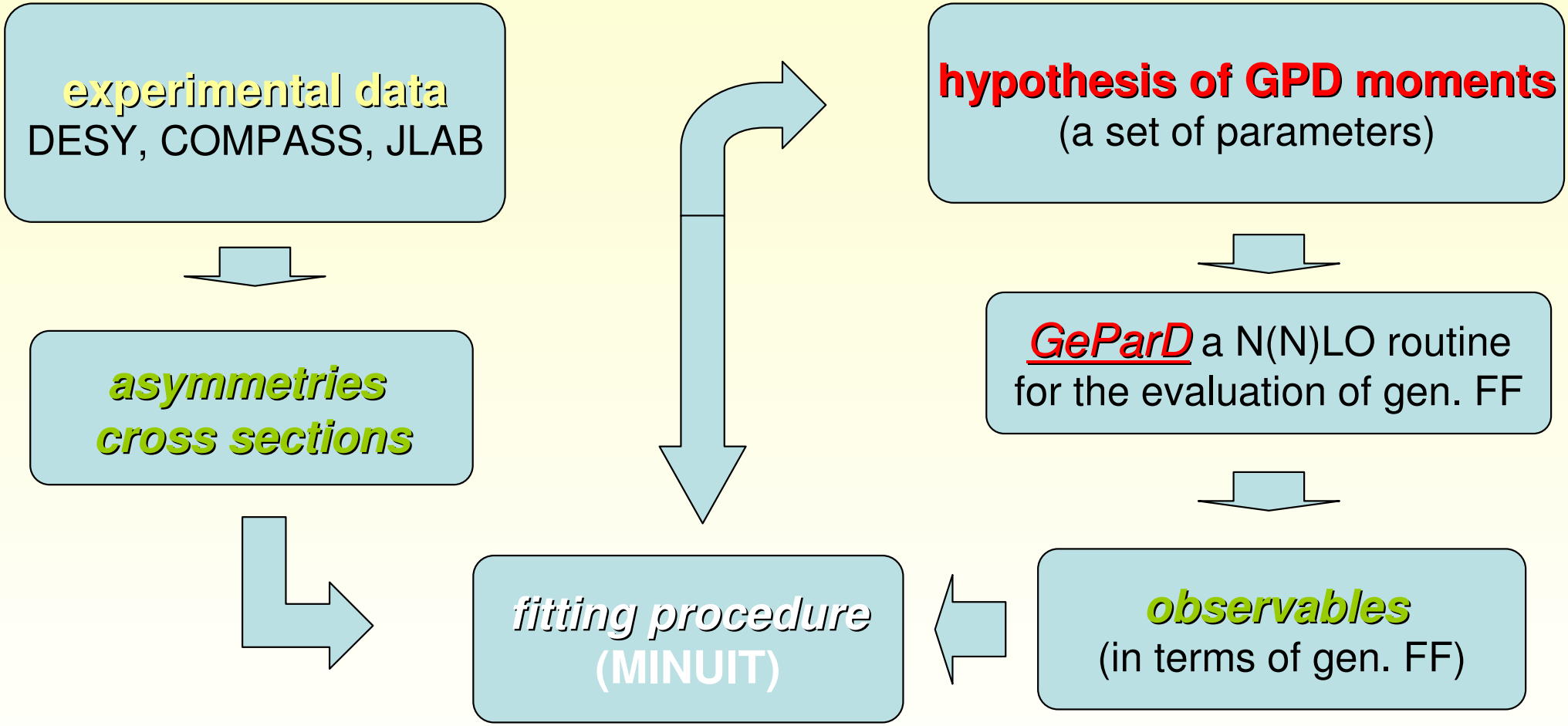
NLO & NNLO corrections– singlet sector



- NLO corrections strongly depend on the gluon entry
- at NNLO drastically reduction of perturbative corrections to the hard scattering part & reduction of renormalization scale dependence
- but perturbative predictions for the evolution is unstable
  no improvement of factorization scale dependence

Ready for a GPD fitting procedure?

[K. Kumerički, D.M., K. Passek-Kumerički, hep-ph/0703179]



partially ***YES*** but it is ***NOT*** completed yet:

- reasonable well motivated hypotheses of GPD moments must be implemented
- some technical, however, straightforward work is left
  (like a reevaluation of observables)

Lessons from DVCS fits for H1 and ZEUS data

DVCS cross section has been measured in the small $\xi = \mathcal{Q}^2/(2W^2 + \mathcal{Q}^2)$ region

$$40\mathrm{GeV} \lesssim W \lesssim 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\mathrm{GeV}^2, \quad |t| \lesssim 0.8\mathrm{GeV}^2$$

and it is predicted by

$$\frac{d\sigma}{d\Delta^2}(W, \Delta^2, \mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4} \frac{W^2\xi^2}{W^2 + \mathcal{Q}^2} \left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_\mathrm{p}^2} |\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2 \right] (\xi, \Delta^2, \mathcal{Q}^2) \Big|_{\xi = \frac{\mathcal{Q}^2}{2W^2 + \mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

LO [Belitsky, DM, Kirchner (01), Guzey, Teckentrup (06)]

data are described within ***questionable*** *t*-slope parameters

NLO [Freund, M. McDermott (02)]

results strongly depend on used parton density parameterization

⇨ ***do a simultaneous fit to DIS and DVCS***

Ansatz for conformal GPD moments

$$H_j^{\mathrm{G}}(\eta,\Delta^2,\mu_0^2) = N_{\mathrm{G}} \frac{B(1-\alpha_{\mathrm{G}}(0)+j,6)}{B(2-\alpha_{\mathrm{G}}(0),6)} \frac{1}{1-\frac{\Delta^2}{(m_j^{\mathrm{G}})^2}} \frac{1}{\left(1-\frac{\Delta^2}{(M_j^{\mathrm{G}})^2}\right)^2} + \mathcal{O}(\eta^2)$$

$$H_j^{\Sigma}(\eta,\Delta^2,\mu_0^2) = N_{\Sigma} \frac{B(1-\alpha_{\Sigma}(0)+j,8)}{B(2-\alpha_{\Sigma}(0),8)} \frac{1}{1-\frac{\Delta^2}{(m_j^{\Sigma})^2}} \frac{1}{\left(1-\frac{\Delta^2}{(M_j^{\Sigma})^2}\right)^3} + \mathcal{O}(\eta^2)$$

some simplifications in the ansatz:

- neglecting η dependence
- only designed for small x (no momentum sum rule)
- flavor non-singlet contribution is neglected (5% effect)
- fixed numbers of quarks *(n_f=4)*

parameters @ fixed input scale $Q^2 = 4\ GeV^2$

- 2x normalization N, 2x intercept α, 2x cut-off mass M_0
- little sensitivity of slope α' (=0.15/GeV^2)
- little sensitivity on j-dependence in M_j

order (scheme)	$\alpha_s(M_Z)$	N_Σ	$\alpha_\Sigma(0)$	M_Σ^2	N_G	$\alpha_G(0)$	M_G^2	χ^2	χ^2/d.o.f.	$\chi^2_{\Delta^2}$
LO	0.130	0.157	1.17	0.228	0.527	1.25	0.263	100	0.85	38.5
NLO ($\overline{\text{MS}}$)	0.116	0.172	1.14	1.93	0.472	1.08	4.45	109	0.92	4.2
NLO ($\overline{\text{CS}}$)	0.116	0.167	1.14	1.34	0.535	1.09	1.59	95	0.80	2.2
NNLO ($\overline{\text{CS}}$)	0.114	0.167	1.14	1.17	0.571	1.07	1.39	91	0.77	2.2

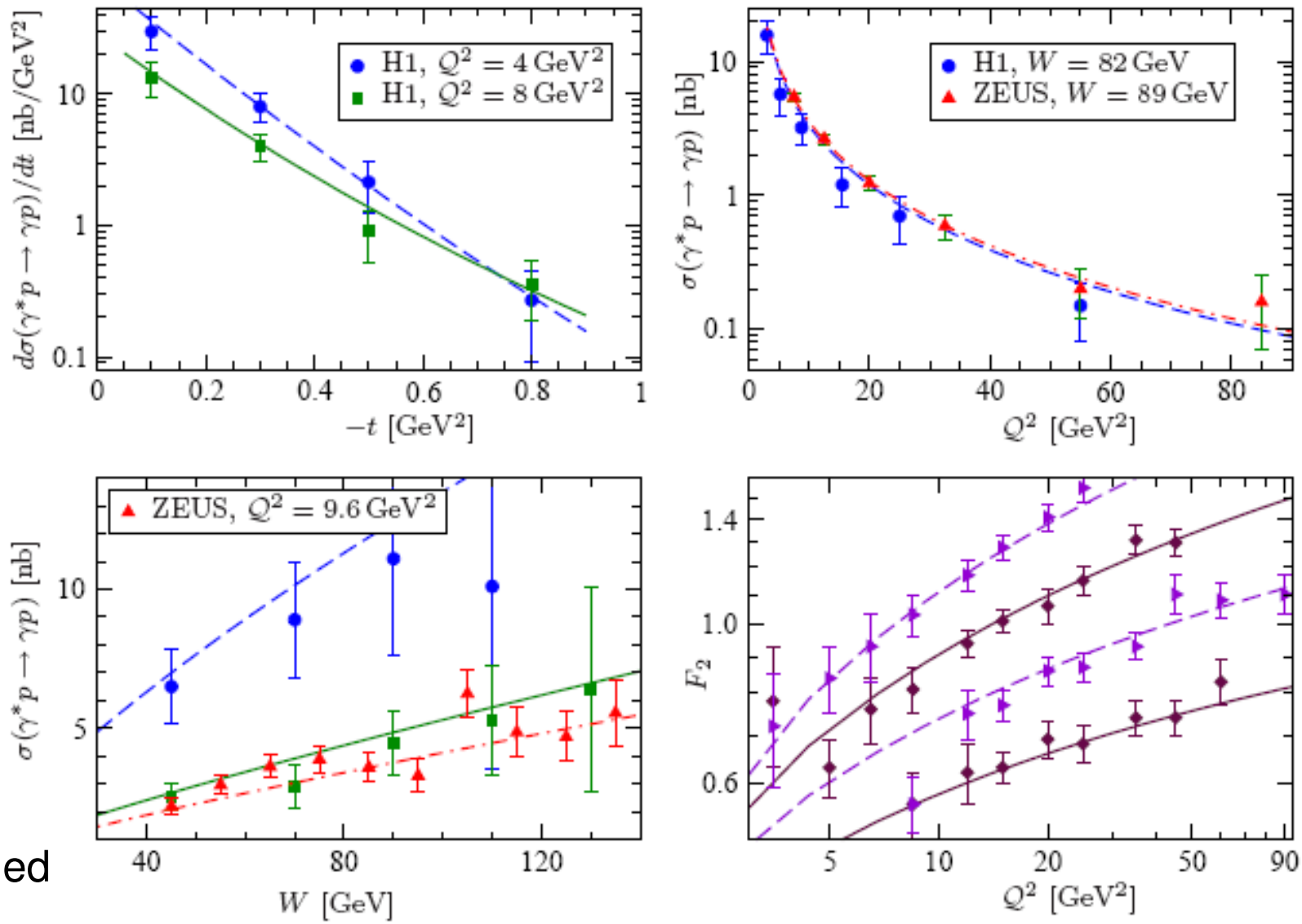
simultaneous NNLO fit to DVCS and DIS

LO & MS NLO fits are not optimal

missing parameter

CS beyond LO yields good fits

neglecting η is justified ? just luck



Can one do better?

Yes, introduce a distribution of SO(3) partial waves in conformal GPD moments

toy example: take two partial waves

η dependence can be safely neglected

$$F_j(\eta,\Delta^2) = \frac{f_j^{j+1}}{(1-\frac{\Delta^2}{M^2(j+1)})^p}\left(\frac{1}{j+1-\alpha(\Delta^2)}P^{\mathcal{F}}_{j+1}(\eta) + \frac{s\,\eta^2}{j-1-\alpha(\Delta^2)}P^{\mathcal{F}}_{j-1}(\eta)\right)$$

effective relative strength of remaining partial waves

now we get a very good LO fit:

- fixed $s_G = 0$, $M=M_G = M_\Sigma$
- X^2/d.o.f. = 0.52, $s_\Sigma = -0.75$,
- *other parameters are consistent with previous fits*

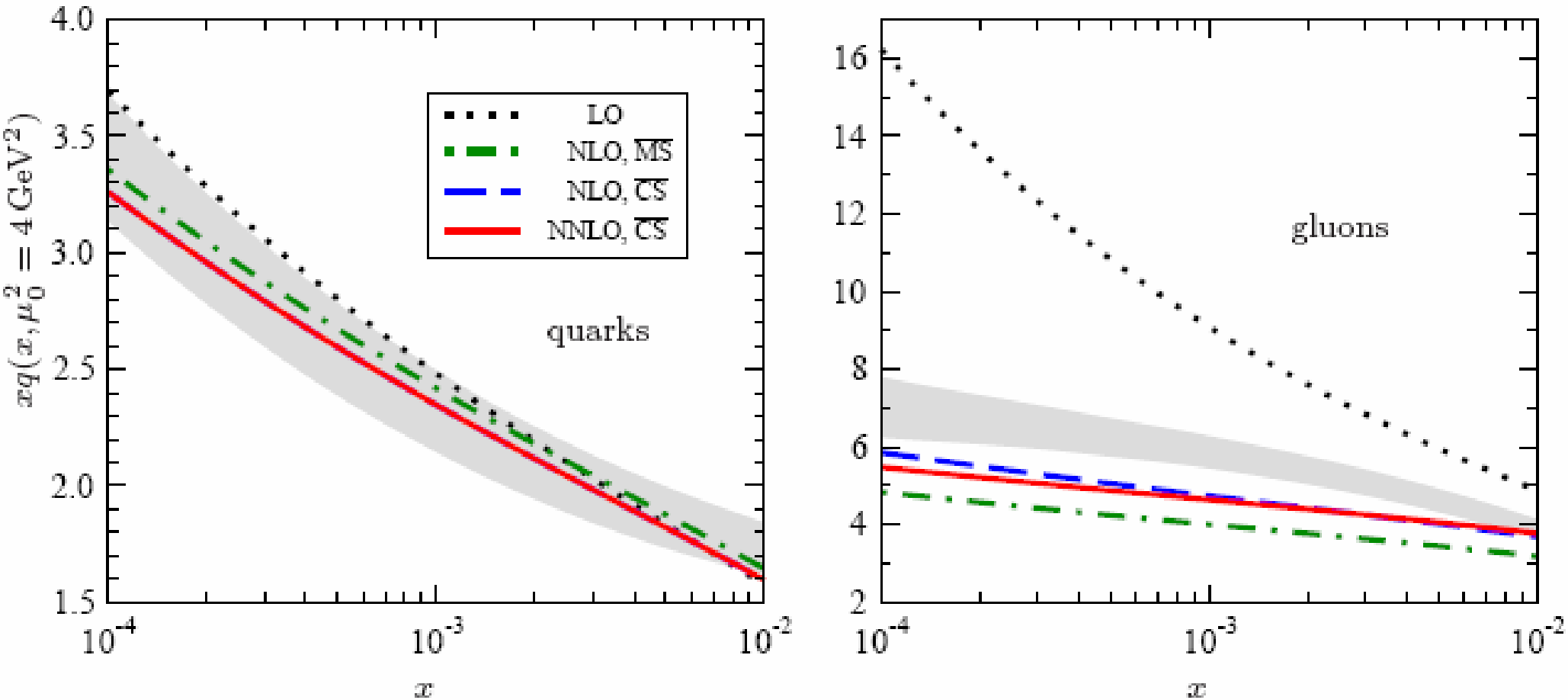
  $N_\Sigma = 0.14$, $\alpha_\Sigma = 1.20$, $N_G = 0.8$, $\alpha_G = 1.16$
- $X^2_t = 2.61$, $M^2 = 0.86$

⇨ ***`negative'*** skewness dependence is ***required*** at LO

Partonic picture: longitudinal degrees

our fits are ***compatible*** with Alekhin's NLO PDF parameterization:

- ✓ central value of our quark densities lies in Alekhin's error band
- ✓ gluons are less constrained by DIS fit
  (error bands would overlap)

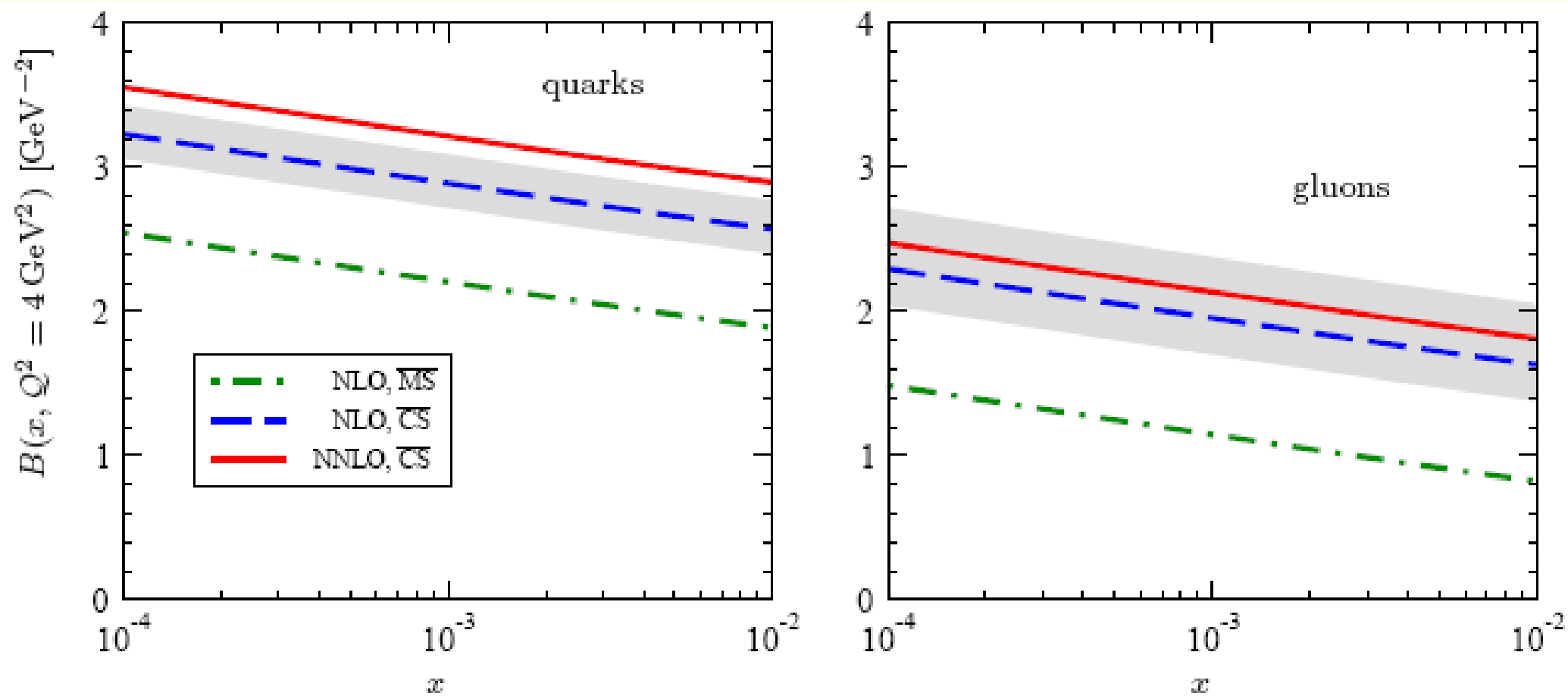


Partonic picture: transversal degrees

transversal distribution of partons in the infinite momentum frame:

$$H(x,\vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x,\eta=0,\Delta^2=-\vec{\Delta}^2)$$

the average distance of partons is: $\langle \vec{b}^2\rangle(x,\mathcal{Q}^2) = \dfrac{\int d\vec{b}\,\vec{b}^2 H(x,\vec{b},\mathcal{Q}^2)}{\int d\vec{b}\, H(x,\vec{b},\mathcal{Q}^2)} = 4B(x,\mathcal{Q}^2)$



Conclusions

✓ Mellin-Barnes representation offers a ***new view*** on GPDs
e.g., conformal GPD moments are simply related to:

i. Lattice simulations (non-negative integer j)
ii. Regge phenomenology (complex valued j, behavior at small η)
iii. s-channel physics, i.e., behavior at large j

✓ observables are sensitive to ***SO(3) partial waves***

☹ their decomposition in terms of ***conformal partial waves*** is ***not accessible***
(except in double DVCS, or photon fusion of two virtual photons)

✓ numeric is ***fast*** and ***reliable*** [even at NLO for MS scheme]

✓ perturbative expansion in DVCS ***works*** – except for ***evolution*** at ***small x***

✓ fitting procedure (better than comparing model A, B, ..., with data) ***can be set up***

✓ a `global' analysis of GPD related data ***requires NLO***

To do list:

❖ improving twist expansion for DVCS observables

❖ How conjectured duality can be implemented in GPD moments?

❖ extend formalism to hard meson electroproduction (straightforward @ NLO)