Two-Photon Exchange Contribution to Elastic ep Scattering in a Non-local Field Formalism

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Experimental Disagreement

The observed trend in polarization transfer experiment is different from Rosenbluth separation.
Two-photon Exchange

Two photon exchange diagrams have been proposed as a possible solution to this problem.
Two-photon Exchange

Since one of the proton line is off-shell, one needs a more general electromagnetic vertex and proton propagator. **Else the Ward identity is not satisfied**

\[
q^\mu \Gamma_\mu (p', p) = S_F^{-1}(p') - S_F^{-1}(p)
\]
We construct a gauge invariant non-local model for the electromagnetic interaction of proton.

The scale of non-locality is equal to the scale of compositeness of hadrons.

The theory is appropriate for dealing with reactions below this momentum scale.

We include all operators up to dimension 5
Effective Non-local Action

\[ L = \bar{\psi} (i \tilde{D} - M_p) \psi + \frac{a'}{2 M_p} \bar{\psi} \left( \sigma_{\mu\nu} f_2 \left[ \frac{\partial^2}{\Lambda^2} \right] F^{\mu\nu} \right) \psi + \frac{b'}{2 M_p} \bar{\psi} (i \tilde{D} - M_p)^2 \psi \]

where

\[ i\tilde{D} = i\partial - ef_1 \left[ \frac{\partial^2}{\Lambda^2} \right] A. \]

\( L \) is invariant under the non-local gauge transformations:

\[ \delta A_\mu = - \partial_\mu \alpha(x); \psi \rightarrow e^{i ef_1 \left[ \frac{\partial^2}{\Lambda^2} \right] a(x)} \psi; \bar{\psi} \rightarrow \bar{\psi} e^{-i ef_1 \left[ \frac{\partial^2}{\Lambda^2} \right] a(x)} \]
Effective Non-local Action

Higher dimensional operators will contribute at higher order in

\[(P^2 \cdot M^2)/\Lambda^2\]

and hence are negligible if the proton is not too far off-shell.
Effective Non-local Action

The model so far produces a spurious pole in the proton propagator. We therefore modify it further

\[ L = \overline{\psi} \left( i\mathcal{D} - M_p \right) \exp \left[ \frac{b'}{2M_p} \left( i\mathcal{D} - M_p \right) \right] \psi + \frac{\alpha'}{2M_p} \overline{\psi} \left( \sigma_{\mu\nu} f'_2 \left[ \frac{\partial^2}{\Lambda^2} \right] F_{\mu\nu} \right) \psi \]

After expanding the exponential, keeping terms upto \((b')^2\) and field transformation, we find only one extra term if we restrict to operators of \(\text{dim} \leq 5\)
Feynman Diagrams
Feynman Diagrams

Order \((b')^2\)
We need $G_E$ and $G_M$ both in the space-like region and time-like region.

In the space-like region $G_M$ is known reasonably well. In the time-like region experimental data exists for $4M_P^2 < q^2 < 14 \text{ GeV}^2$.

In the unphysical region $G_M$ has been extracted by using the dispersion relations.

$G_E$ is not known very well in the time-like region. We use two different models for $G_E$
It shows two resonances at masses $M \sim 770$ MeV and $M \sim 1600$ MeV. The phase shows a large variation in the unphysical region.

Baldini et al 1999
Model $G_M$ and $G_E$

\[
G_M(q^2) = \mu_p \sum_{a=1}^{4} \frac{A_a}{q^2 - m_a^2 + i m_a \Gamma_a}
\]

\[
G_E(q^2) = \sum_{a=1}^{6} \frac{B_a}{q^2 - m_a^2 + i m_a \Gamma_a}
\]
$G_M, G_E$ (model)
$G_M, G_E$ (model)
The calculation is performed using Feynman parametrization.

We are unable to do all the integrals analytically since the form factors are complex in the unphysical region.

We insert a small photon mass $\mu$ to regulate the infrared divergence. The infrared divergent $\mu$ dependence has to be removed.
Numerically we are not able to go below $\mu \approx 50 \text{ MeV}$. At this value of $\mu$ there are corrections to the log-$\mu$ dependence of the infrared divergence

$$\sigma_R = (a_0 + a_1 \mu^2) + (b_{IR} + b_1 \mu^2) \ln(\mu^2)$$

where $b_{IR}$ is the analytic result for IR divergence

$a_0$ can be extracted from this fit, which gives the required two photon contribution
Results:
Reduced cross section
(Box +Cross Box)

\[
\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{\varepsilon(1 + \tau)} \sigma^R_R
\]

\(\varepsilon = \text{photon long. polarization}\)
Results: Reduced cross section \(((b')^2 \text{ diagram})\)

set \(b' = 1\)

This contribution turns out to be very small and in the direction opposite to that required for explaining the data
$F_2-F_2$ contribution

$Q^2 = 4.10 \text{ GeV}^2$
Results

\[ \mu_p G_E/G_M \]

Graph showing the relationship between \( Q^2 \) and \( \mu_p G_E/G_M \). The graph includes data points with error bars, indicating variability in the measurements.
Results (with $\varepsilon \leq 0.5$)
Conclusions

We have used a non-local model to compute the two photon exchange contributions.

The model preserves gauge invariance and can be used reliably as long as the proton is not too far off-shell.

It involves only one unknown parameter $b'$ whose contribution is found to be small.
Conclusion

The $\varepsilon$ dependence of $\sigma_R$ is slightly non-linear. This trend is not seen in data but is still within the error bars.

The resulting two photon contribution explains about 70% of the discrepancy at $Q^2 = 2.64 \text{ GeV}^2$, but only about 20% at $Q^2 = 4.10 \text{ GeV}^2$.

If we use only the results at $\varepsilon \leq 0.5$, then we are able to explain 50% of the difference at $Q^2 = 4.10 \text{ GeV}^2$. 
Thank you