Real and Virtual Compton Scattering in Perturbative QCD

JLab, May 23, 2007
Motivation

• Forthcoming 12 GeV Upgrade and New JLab Data
  - Hall A Collab, PRL98, 152001(2007): \( s=5-11\text{GeV}^2, -t=2-7\text{GeV}^2 \)
  - Cornell Data, PRD19, 1921(1979): \( s=4.6-12.1\text{GeV}^2, -t=0.7-4.3\text{GeV}^2 \)

• Several Analyses of RCS but Different Results in the Past
  - M.Vanderhaeghen, P.Guichon and J.Van de Wiele, NPA622, 144(1997)

Recent Agreement with Brooks and Dixon’s Result

• Extension to Virtual Compton Scattering
  - Currently much interest in DVCS where GPD predictions are applicable
  - Only one previous PQCD calculation (Farrar and Zhang) and their RCS result is in disagreement with Brooks and Dixon’s and ours.
  - GPD predictions can be compared with the PQCD results for DVCS.
  This can shed light on the applicability of both GPD and PQCD methods.
  \( (Q^2 \gg -t \gg \Lambda^2) \)
Outline

• PQCD in Light-Front Dynamics (LFD)
  - Hard Scattering Amplitude
  - Distribution Amplitude
  - Remarks on Computational Methods

• Real Compton Scattering Results
  - Comparison with Previous Computations
  - Comparison with JLab Data

• Extension to Virtual Compton Scattering
  - Comparison with Previous Computations
  - Link to GPD and Handbag Dominance

• Conclusions
LFD in Exclusive Processes

\[ \sum = \text{Absent in } q^+ = 0 + \text{Absent} \]
LFD in Exclusive Processes

\[ T_H = \sum \left( \frac{\alpha_s^2}{Q^4} f(x_i, y_i) \right) \]

\[ |q^2| >> \Lambda_{QCD}^2 \]

\[ q^+ = 0 \quad \text{Absent} \]

\[ \text{Absent} \]
Classification of Diagrams

A, C, E \leftrightarrow B, D, F \text{ under } 1 \leftrightarrow 3;

Color factor for all type: \( C^{(d)} = 4/9; \)

Triple gluon contribution is absent due to the color factor.

For RCS,

\[
M_{hh'}^{\lambda\lambda'}(s,t) = M_{\bar{h}\bar{h}'}^{\lambda\lambda'}(s,t) \quad \text{due to Parity Inv.}
\]

\[
M_{hh'}^{\lambda\lambda'}(s,t) = M_{h'h}^{\lambda'\lambda}(s,t) \quad \text{Time-reversal Inv.}
\]
Number of Contributing Diagrams

Single Photon (e.g. $F_1^P$)
Attachment of a photon: $(6/2) \times 7 = 21$
Nonzero Diagrams: $A1, A4, A7, C2, C7, E1, E5$

Two Photons: $(6/2) \times 7 \times 8 = 168$
RCS needs only $A$ and $C$ types due to $T$-inv.
52 Nonzero Diagrams: $A11, A_{11}, \ldots, C_{12}, \ldots, C_{77}$.
Use $h=h'=1$ for proton helicities and
$|\gamma_{\text{in}}\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$, $|\gamma_{\text{out}}\rangle = \gamma |\uparrow\rangle + \delta |\downarrow\rangle$.

Virtual Compton Scattering can’t take advantage of $T$-inv.
96 Nonzero Diagrams: $A_{11}, \ldots, C_{77}; A_{77}, A_{77}, \ldots, E_{12}, \ldots, E_{55}$.
Virtual photon has also the longitudinal polarization.
tdiag:=[[1,p1],[p3,2],[p1,1,2,p2],[p2,p3,3,3]];
plotsett:=DrawDiag(3,2,tdiag);

A.Pang and C.Ji, J.Comp.Phys.115,267(94)
COMPUTE;FeynComp;FeynGen;FeynDraw

R. Thomson’s hard work!
Summary of Theory

Leading twist PQCD approximation for proton Compton scattering gives helicity amplitude

\[ M_{hh'}^{\lambda\lambda'} = \sum_{i,d} \int dx_1 dx_2 dx_3 dy_1 dy_2 dy_3 \phi_i(x) T_i^{(d)} \phi_i^*(y) \]

subject to constraints \( x_1 + x_2 + x_3 = 1 \) and \( y_1 + y_2 + y_3 = 1 \)

\( h, \lambda(h', \lambda') \) helicities of i/c (o/g) proton, photon

\( x(y) \) longitudinal momentum fractions of i/c (o/g) quarks

\( i \) labels independent Fock states of the proton with distribution amplitudes \( \phi_i(x) \)

\( d \) labels the Feynman diagrams that contribute to hard scattering amplitude \( T_i^{(d)} \)
Distribution Amplitude

\[ | p^\uparrow \rangle = \frac{f_N}{8\sqrt{6}} \int [dx] \sum_i \phi_i(x_1, x_2, x_3) | i, x_1, x_2, x_3 \rangle; \]

\[ | 1; x_1, x_2, x_3 \rangle = | u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3) \rangle, \]
\[ | 2; x_1, x_2, x_3 \rangle = | u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3) \rangle, \]
\[ | 3; x_1, x_2, x_3 \rangle = | d^\uparrow(x_1)u^\downarrow(x_2)u^\uparrow(x_3) \rangle; \]

\[ \phi_2(x_1, x_2, x_3) = -[\phi_1(x_1, x_2, x_3) + \phi_1(x_3, x_2, x_1)], \]
\[ \phi_3(x_1, x_2, x_3) = \phi_1(x_3, x_2, x_1); \]
\[ [dx] = dx_1dx_2dx_3\delta(1 - x_1 - x_2 - x_3); \]
\[ f_N = (5.2 \pm 0.3) \times 10^{-3} (GeV / c)^2. \]
V. Chernyak and I. Zhitnitsky, Nucl. Phys. B246, 52 (84)
\[
\phi_{CZ} = 120 x_1 x_2 x_3 (1.69 - 9.26 x_1 - 10.94 x_3 + 22.70 x_1^2 + 13.45 x_3^2 + 9.26 x_1 x_3)
\]

\[
\phi_{COZ} = 120 x_1 x_2 x_3 (5.880 - 25.956 x_1 - 20.076 x_3 + 36.792 x_1^2 + 19.152 x_3^2 + 25.956 x_1 x_3)
\]

I. King and C. Sachrajda, Nucl. Phys. B279, 785 (87)
\[
\phi_{KS} = 120 x_1 x_2 x_3 (8.40 - 26.88 x_1 - 35.28 x_3 + 35.28 x_1^2 + 37.80 x_3^2 + 30.24 x_1 x_3)
\]

M. Gari and N. G. Stefanis, Phys. Lett. B175, 462 (86); PRD35, 1074 (87)
\[
\phi_{GS} = 120 x_1 x_2 x_3 (6.040 - 16.775 x_1 - 34.985 x_3 - 1.027 x_1^2 + 12.307 x_3^2 + 111.320 x_1 x_3)
\]

Asymptotic DA
\[
\phi_{ASY} = 120 x_1 x_2 x_3
\]

AdS/CFT?
It is possible to write the helicity amplitude in the form

\[ M_{hh'}^{\lambda\lambda'} = \frac{4}{9}(4\pi\alpha_{em})(4\pi\alpha_s)^2 \left( \frac{120f_N}{8\sqrt{6}} \right)^2 \sum_d \sum_{m,n} C^{(d)}(m_1,m_3,n_1,n_3)I^{(d)}(m_1,m_3,n_1,n_3) \]

where

\[ I^{(d)}(m_1,m_3,n_1,n_3) = \int_0^1 dx_1dx_2dx_3dy_1dy_2dy_3 \tilde{T}^{(d)}(x_1^{m_1+1}x_2x_3^{m_3+1}y_1^{n_1+1}y_2y_3^{n_3+1}) \]

are powers of momentum fractions (from \( \phi_i \))

is a coefficient that sums contributions for 3 Fock states.

It is dependent on \( Z_i^{(d)} \) (product of charges of struck quarks) and the coefficients appearing in \( \phi_i \).

\( f_N \) is a normalization constant (taken as 0.0052 GeV\(^2\))

\( \tilde{T}^{(d)} \) is the color/flavor independent part of \( T_i^{(d)} \)
Kinematics

Proton o/g $p'$

Photon i/c $k$

Proton i/c $p$

Photon o/g $k'$

$p = P(1,0,0,1)$

$k = P(\sqrt{1 - Q^2 / P^2}, 0, 0, -1)$

$p' = E(1, \sin \theta, 0, \cos \theta)$

$k' = E(1, -\sin \theta, 0, -\cos \theta)$

- Define $S = (2E)^2$ (Mandelstam invariant)
- $R = 1 + Q^2 / S$ (Virtuality parameter)

- For real photon: $R=1$, for DVCS: $R=2$
- $\tilde{T}^{(d)}$ calculated as a function of $S$ and $R$: e.g.

$$\tilde{T}^{↑↑}_{A16} = \frac{8R^{3/2}S^4}{S^2c} \frac{1 - Rx_3}{\langle \vec{y}_1, x_3 \rangle <\vec{y}_1, \vec{x}_1 > \langle y_3, x_3 > (1 - R\vec{x} + i\epsilon)}$$

$$< y, x >= y(1 - Rs^2x) - Rc^2x + i\epsilon$$

$s = \sin(\theta / 2), c = \cos(\theta / 2), \bar{y} = 1 - y$
• See details of calculation for A51 in Appendix A of Thomson, Pang and Ji, PRD73, 054023 (2006).

• The pole expressions have been expanded into real and imaginary parts using

\[ \frac{1}{\langle y, x \rangle + i\varepsilon} = P\left(\frac{1}{\langle y, x \rangle}\right) - i\pi\delta(\langle y, x \rangle) \]

where \( P \) means principal value.

• Delta function integrals have been evaluated explicitly.

• The remaining principal value integrals have been transformed using the ‘folding method’ of Kronfeld & Nizic which renders the integrand finite over the range of integration.

• Brooks and Dixon used a different method to do the integrations (contour deformation)

• The integration has then been completed numerically using a Monte Carlo algorithm in Fortran.

Real and imaginary parts of all helicity amplitudes were tabulated in the basis of Appell polynomial expansion up to \( A_3 \):

Thomson, Pang and Ji, hep-ph/0602164v2

for the computation of cross sections and phase calculations performed in the range of \( R \) values (1.0, 1.25, 1.5, 1.75, 2.0) and angles (20° to 160°).
Comparison with previous work for real photon

Initial-State Helicity Correlation

\[
A_{LL} (or \ K_{LL}) = \frac{\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt}} {\frac{d\sigma^+}{dt} + \frac{d\sigma^-}{dt}} = \frac{\mid M_{\uparrow\uparrow\uparrow\uparrow} \mid^2 + \mid M_{\uparrow\uparrow\downarrow\downarrow} \mid^2 - \mid M_{\uparrow\downarrow\uparrow\downarrow} \mid^2 - \mid M_{\uparrow\downarrow\downarrow\uparrow} \mid^2} {\mid M_{\uparrow\uparrow\uparrow\downarrow} \mid^2 + \mid M_{\uparrow\downarrow\uparrow\downarrow} \mid^2 + \mid M_{\uparrow\downarrow\downarrow\uparrow} \mid^2 + \mid M_{\downarrow\uparrow\downarrow\uparrow} \mid^2}
\]


--- M. Vanderhaeghen, P. Guichon and J. Van de Wiele, NPA622, 144 (1997)


JLab Data: $0.678 \pm 0.083 \pm 0.04$ at $120^0$
$s=6.9 \text{ GeV}^2$, $t=-4.0 \text{ GeV}^2$, $u=-1.1 \text{ GeV}^2$
PRL94,242001(2005)


Handbag approach to wide-angle RCS using GPD

• It is argued, at currently accessible kinematics, the Compton scattering amplitude is dominated by soft overlap contributions which can be described in light front gauge via the handbag diagram.

• In this approach, the helicity amplitude is given by

\[ M^{\lambda\lambda'}_{++}(s,t) = 2\pi\alpha_{em} \left( H^{\lambda\lambda'}_{++}(s,t)(R_{V}(t) + R_{A}(t)) + H^{\lambda\lambda'}_{--}(s,t)(R_{V}(t) - R_{A}(t)) \right) \]

where \( H^{\lambda\lambda'}_{++}(s,t) \) denotes the amplitude for the subprocess \( \gamma q \rightarrow \gamma q \) and \( R_{A}(t), R_{V}(t) \) are soft form factors which can be expressed in terms of GPDs. For example, summing over flavors \( a \)

\[ R_{V}(t) = \sum_{a} e_{a}^{2} \int_{-1}^{1} \frac{d\xi}{\xi} H^{a}_{\xi}(x, \xi = 0, t) \]
Scaled Unpolarized RCS Cross Section

Hall A Collab, PRL98, 152001(2007)

Power $n=6$ scaling is not yet reached.
12 GeV upgrade is anticipated.

Suppress the Uncertainty in Normalization:

$0.19 < \alpha_s < 0.39$, $0.75 < \phi_{\text{evol}} < 1.33$, $f_N = (5.2 \pm 0.3) \times 10^{-3}$ GeV$^2$. 
Virtual photon comparison: longitudinal polarization
(l to d)

The results have been compared with Farrar and Zhang, PRD41,3348(1990).
Virtual photon comparison: longitudinal polarization
(l to u)
Virtual photon comparison: up to up
Virtual photon comparison: up to down
Virtual photon comparison: down to up
Virtual photon comparison: down to down
Checking Handbag Dominance in PQCD

- FeynComp converted to work in Light Front gauge.
- Hand calculation of a few sample diagrams has been used to check the output of FeynComp.
- Explicit calculation using FeynComp shows the gauge invariance:

\[ \sum_{\text{LightFront}} \text{amplitudes} = \sum_{\text{Feynman}} \text{amplitudes} \]

where the sum is over diagrams with photons attached to the same quarks.

Work in progress …
Conclusions

• Agreement with Brooks & Dixon in RCS results gives more confidence on our VCS calculation over the previous one by Farrar & Zhang.

• JLab 12 GeV upgrade is highly desirable to shed some light on the validity of PQCD in exclusive processes.

• GPDs can be expressed in terms of DAs using PQCD at large momentum transfer and the previous results are currently under investigation.

• Handbag dominance check in PQCD analysis is also in progress.