Photoproduction of a $\pi \rho_T$ pair with a large invariant mass and Transversity Generalized Parton Distribution

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Transversity of the nucleon using hard processes

What is transversity?

Transverse spin content of the proton:

$| \uparrow \rangle (x) \sim | \rightarrow \rangle + | \leftarrow \rangle$

$| \downarrow \rangle (x) \sim | \rightarrow \rangle - | \leftarrow \rangle$

spin along $x$ helicity states

Observable sensible to helicity flip thus give access to transversity $\Delta T q(x)$. Very poorly known

Transversity GPDs are completely unknown

For massless (anti)particles, chirality = (-)helicity

Transversity is thus a chiral-odd quantity

Since QCD and QED are chiral even, the chiral odd quantities which one want to measure should appear in pairs
Transversity of the nucleon using hard processes: using a two body final state process?

How to get access to transversity?

- the dominant DA of $\rho_T$ is of twist 2 and chiral odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- unfortunately $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$
  - this is true at any order, because this would require a transfer of helicity of 2 from photon: impossible!

lowest order diagrammatic argument:

$$\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$$
Transversity of the nucleon using hard processes: using a two body final state process?

Can one circumvent this vanishing?

- this vanishing is true only a twist 2
- at twist 3 this process does not vanish
- however processes involving twist 3 DAs may face problems with factorization (end-point singularities)
- the problem of classification of twist 3 chiral-odd GPDs is still open

Our process: $\gamma N \rightarrow \pi^+ \rho_T^0 N'$

$\gamma N \rightarrow \pi^+ \rho_T^0 N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \rightarrow \pi + \rho$ at large $s$ and fixed angle (i.e. fixed ratio $t'/s$, $u'/s$)
  $\implies$ factorization of the amplitude for $\gamma + N \rightarrow \pi + \rho + N'$ at large $M_{\pi\rho}^2$

- a typical non-vanishing diagram:

- these processes with 3 body final state can give access to all GPDs. $M_{\pi\rho}^2$ plays the role of $\gamma^*$ in usual DVCS, and can be scanned.
Master formula based on leading twist 2 factorization

\[ \mathcal{A} = \frac{1}{\sqrt{2}} \int_{-1}^{1} dx \int_{0}^{1} dv \int_{0}^{1} dz \left( T_u^u(x, v, z) - T_d^d(x, v, z) \right) \Phi_{\pi^+}(z) \Phi_{\rho^0}(v) + \cdots \]
Non-perturbative matrix elements

One needs to encode the matrix elements of two kinds of chiral-odd operator:

- **transversity GPDs** (twist-2 level):

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) i\sigma^{+i} \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle \\
= \frac{1}{2P^+} \tilde{u}(p_2, \lambda_2) \left[ H_T^q(x, \xi, t) i\sigma^{+i} + \tilde{H}_T^q(x, \xi, t) \frac{P^+ \Delta^i - \Delta^+ P^i}{M_N^2} \right] \gamma^+ \frac{\Delta^i - \Delta^+}{2M_N} u(p_1, \lambda_1) + \tilde{E}_T^q(x, \xi, t) \frac{\gamma^+ P^i - P^+ \gamma^i}{M_N} \gamma^+ \frac{\Delta^i - \Delta^+}{2M_N} u(p_1, \lambda_1)
\]

for \( \Delta_\perp = 0 \) each above factors vanishes except for \( H_T^q \) which thus dominates in the small \( t \) domain

- **transversity DAs** (twist-2 level):

\[
\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p, s) \rangle = \frac{i}{\sqrt{2}} (\sigma_\rho^{\mu} p^\nu - \sigma_\rho^\nu p^\mu) f_{\rho} \int_0^1 du \ e^{-iu p \cdot x} \phi_\perp(u)
\]
Kinematics

- use a Sudakov basis: light-cone vectors $p$, $n$ with $2p \cdot n = s$
- assume the following kinematics:
  - $\Delta_\perp^\mu$ small
  - $M^2, m^2, m^2_\rho \ll M^2_{\pi\rho}$
- initial state particle momenta:
  $$q^\mu = n^\mu, \quad p^\mu_1 = (1 + \xi)p^\mu + \frac{M^2}{s(1+\xi)} n^\mu$$
- final state particle momenta:
  $$p^\mu_2 = (1 - \xi)p^\mu + \frac{M^2 + \vec{\Delta}_t^2}{s(1 - \xi)} n^\mu + \Delta_\perp^\mu$$
  $$p^\mu_\pi = \alpha n^\mu + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2 + m^2_\pi}{\alpha s} p^\mu + p^\mu_\perp - \frac{\Delta_\perp^\mu}{2}$$
  $$p^\mu_\rho = \alpha_\rho n^\mu + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m^2_\rho}{\alpha_\rho s} p^\mu - p^\mu_\perp - \frac{\Delta_\perp^\mu}{2}$$
Total center-of-mass energy squared of the $\gamma$-N system

$$S_{\gamma N} = (q + p_1)^2$$

**Hard scale:** invariant squared mass of the $(\pi^+, \rho^0)$ system

$$M_{\pi\rho}^2 = (p_\pi + p_\rho)^2 \simeq -u' = -(p_\rho - q)^2 \simeq -p_{\perp}^2$$

**Transferred squared momentum:**

$$t = (p_2 - p_1)^2 \quad \text{small} \quad t$$

**Skewedness:**

$$\xi = \frac{\tau}{2-\tau}$$

with $\tau = \frac{M_{\pi\rho}^2}{S_{\gamma N} - M_{\pi\rho}^2}$ (generalized Bjorken variable for Drell Yan)
Computation of the hard part

Typical Feynman diagrams (62 in total)

Diagram with photon $u$-quark coupling

Diagram with photon $d$-quark coupling
Access to GPD through a 3 body final state

Transversity GPD and Double Distribution

Unpolarized Cross Section

Conclusion

representative diagram with a 3 gluon vertex
Tensorial structure of the amplitude

\[ A_{H_{T}}^{q} = (N_{\lambda_{1}\lambda_{2}}^{\perp} \cdot \epsilon_{\rho} \pm) (p_{\perp} \cdot \epsilon_{\gamma}) A + (N_{\lambda_{1}\lambda_{2}}^{\perp} \cdot \epsilon_{\gamma} \perp) (p_{\perp} \cdot \epsilon_{\rho} \pm) B \]

\[ + \ (N_{\lambda_{1}\lambda_{2}}^{\perp} \cdot p_{\perp}) (\epsilon_{\gamma} \perp \cdot \epsilon_{\rho} \pm) C \]

\[ - \ (N_{\lambda_{1}\lambda_{2}}^{\perp} \cdot p_{\perp}) (p_{\perp} \cdot \epsilon_{\gamma} \perp) (p_{\perp} \cdot \epsilon_{\rho} \pm) D \]

with

- $A, B, C, D$ scalar functions of $S_{\gamma N}$, $-u'$ and $M_{\pi \rho}^{2}$
- $\epsilon_{\gamma} \perp$ the transverse polarization of the on-shell photon
- $N_{\lambda_{1}\lambda_{2}}^{\perp} \mu = \frac{2i}{p \cdot n} g_{\mu \nu}^{\perp} \bar{u}(p_{2}, \lambda_{2}) \gamma_{\nu} \gamma_{5} u(p_{1}, \lambda_{1})$

Rich spin structure of $A_{H_{T}}^{q}$: access to the spin density matrix of $\rho_{T}^{0}$, polarization asymmetries, ...
A model based on Double Distribution

Realistic Parametrization of $H_T^q$

- GPDs can be represented in terms of **Double Distribution** (Radyushkin)
  based on **Schwinger** representation of a toy model for GPDs which has the structure of a triangle diagram in scalar $\phi^3$ theory

\[ H_T^q(x, \xi, t = 0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(\beta + \xi\alpha - x) \, f_T^q(\beta, \alpha) \]

- ansatz for these Double Distribution (Radyushkin):
  \[ f_T^q(\beta, \alpha) = \Pi(\beta, \alpha) \, \Delta_T q(\beta) \]
  \[ \Delta_T q(x) : \text{chiral-odd PDF (Anselmino et al.)} \]
  \[ \Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3} : \text{profile function (} f_T^q(\beta, 0) = \Delta_T q(\beta) \text{)} \]

- ansatz for the $t$-dependence:

\[ H_T^q(x, \xi, t) = H_T^q(x, \xi, t = 0) \times F_H(t) \]

with \[ F_H(t) = \frac{C^2}{(t-C)^2} \text{ a standard dipole form factor (} C = .71 \text{ GeV)} \]
Plots of our model for transversity GPD

$x$ and $\xi$-dependence of $H_T^q(x, \xi, t = 0)$
Plots of our model for transversity GPD

$x$-dependence of $H_T^q(x, \xi, t = 0)$ for fixed values of $\xi$

Same order of magnitude but significant differences with other parametrizations (Pincetti et al.) and lattice calculations (Göckeler et al.)

$$A_{T10}^u(t \sim 0) \approx 0.4(0.9)$$

$$A_{T10}^d(t \sim 0) \approx -0.1(-0.2)$$
Unpolarized differential cross section

\[ \frac{d\sigma}{dt \, du' \, dM_{\pi\rho}^2} \bigg|_{t=t_{min}} = \frac{|M|^2}{32 S_{\gamma N}^2 M_{\pi\rho}^2 (2\pi)^3} \]

- **Validity of the factorization of the partonic amplitude:**
  \(-t', -u' > \Lambda^2 \Rightarrow \Lambda_{QCD}^2 \) with \( \Lambda \sim 1 \text{ GeV} \)

- **Suppress final states interactions (to justify factorization):**
  \( M_{\pi N'}^2, M_{\rho N'}^2 > M_R^2 \) with \( M_R^2 = 2 \text{ GeV}^2 \)

  \[-u'_{min}(res.) (t, S_{\gamma N}, M_{\pi\rho}^2) \]

- **Cuts over \(-t'\) and \(M_{\rho N'}^2\):**
  \(-u'_{max} (t, S_{\gamma N}, M_{\pi\rho}^2) \)

\[
\begin{align*}
S_{\gamma N} &= 20 \text{ GeV}^2, M_{\pi\rho}^2 = 3 \text{ GeV}^2 \\
M_{\pi N'}^2 &= 3 \text{ GeV}^2, M_{\rho N'}^2 > M_R^2
\end{align*}
\]
Differential cross section for $M_{\pi\rho}^2 = 6 \text{ GeV}^2$

- $S_{\gamma N} = 20 \text{ GeV}^2$
  \[
  \frac{d\sigma}{dt\,du'\,dM_{\pi\rho}^2} \bigg|_{t=t_{\text{min}}} \propto 10 \text{ nb. GeV}^{-6}
  \]

- $S_{\gamma N} = 200 \text{ GeV}^2$
  \[
  \frac{d\sigma}{dt\,du'\,dM_{\pi\rho}^2} \bigg|_{t=t_{\text{min}}} \propto 0.01 \text{ nb. GeV}^{-6}
  \]
Predictions

$M_{\pi\rho}^2$-dependence of the differential cross section $\frac{d\sigma}{dM_{\pi\rho}^2}$

\[
\frac{d\sigma}{dM_{\pi\rho}^2} = \int_{-0.5}^{t_{\text{min}}} dt \int_{-u'_{\text{max}}}^{-u'_{\text{min}}} d(-u') \frac{d\sigma}{dt\,du'dM_{\pi\rho}^2} \bigg|_{t=t_{\text{min}}}
\]

\[
\frac{d\sigma}{dM_{\pi\rho}^2} \quad \text{(nb.GeV}^{-2})
\]

Total cross sections for photoproduction:

$\sigma(S_{\gamma N} = 20 \text{ GeV}^2) \simeq 33 \text{ nb}$  $\sigma(S_{\gamma N} = 200 \text{ GeV}^2) \simeq 0.1 \text{ nb}$
Predictions

$S_{\gamma N}$-dependence of the differential cross section $\sigma$

With our cuts

- $-t',-u' > 1 \text{ GeV}^2$
- $M_{\pi N'}^2, M_{\rho N'}^2 > 2 \text{ GeV}^2$
Muoproduction at Compass (CERN)

Very sizable rates

- denote $\Gamma_T^{\mu}(Q^2, \nu)$ the quasi real (transverse) photon flux ($E_\mu = 160$ GeV).

- Total cross section for the muoproduction $\mu N \rightarrow \mu\pi^+ \rho_{T}^0 N'$

$$\sigma_\mu = \int_{0.02}^{1} dQ^2 \int_{16}^{144} d\nu \ \Gamma_T^{\mu}(Q^2, \nu) \ \sigma_{\gamma^* N \rightarrow \pi^+ \rho_{T}^0 N'}(Q^2, \nu) \approx 0.25 \text{ pb}$$

- Experimental rate: For a muon beam luminosity of $2.5 \times 10^{32} \text{ cm}^{-2}\text{.s}^{-1}$, $R \approx 6 \times 10^{-2} \text{ Hz}$

Very sizable
Rate estimates at JLab

**Very high rates**

- **CLAS12 Hall B:**
  
  with a photon (7 - 10.5 GeV) flux $N_\gamma \sim 5 \times 10^7$ photons/s

  **Experimental rate:** $R \sim \mathbf{0.1 \ Hz}$

- **Hall D (12 GeV)**

  - photon (8 - 9 GeV) flux $N_\gamma \sim 10^8$ photons/s
  - number of protons per surface unit $N_p \sim 1.27 \ \text{b}^{-1}$ (target: liquid hydrogen (30 cm))

  **Experimental rate:** $R = \sigma \times N_\gamma \times N_p \sim \mathbf{5 \ Hz}$
Photoproduction of a $\pi \rho_T^0$ pair with a large hard scale $M_{\pi\rho}^2$ sensitive to the transversity GPDs even for unpolarized target and at twist-2 level

Parametrization of the dominant chiral-odd GPD $H_T^q$ based on double distribution

Promising way to get informations on the generalized chiral-odd quark content of the nucleon:

- large enough rates to extract transversity GPDs, at COMPASS and JLab@12 GeV
- Possibility to access to:
  - spin density matrix of $\rho_T^0$
  - spin asymmetries
  - chiral-even GPDs $H$ and $\tilde{H}$ with $\rho_L^0$

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Such processes with 3 body final state are also promising for non transversity GPD measurement, on top of the now standard DVCS based studies
Introduction  Access to GPD through a 3 body final state  Transversity GPD and Double Distribution  Unpolarized Cross Section
Transverse polarization of $\rho_T^0$

$$
\epsilon^\mu_\pm(p_\rho) = \left( \frac{\vec{p}_\rho \cdot \vec{\epsilon}_\pm}{m_\rho}, \, \vec{\epsilon}_\pm + \frac{\vec{p}_\rho \cdot \vec{\epsilon}_\pm}{m_\rho(E_\rho + m_\rho)} \vec{p}_\rho \right)
$$

$$
\Rightarrow \quad 2\bar{\alpha} \frac{\hat{p}_t \cdot \vec{\epsilon}_\pm}{\bar{\alpha}^2 s + \vec{p}_t^2} (p^\mu + n^\mu) + (0, \vec{\epsilon}_\pm)
$$

$$
\Rightarrow \quad 2\bar{\alpha} \frac{\hat{p}_t \cdot \vec{\epsilon}_\pm}{\bar{\alpha}^2 s + \vec{p}_t^2} \left[ 1 - \frac{\vec{p}_t^2}{\bar{\alpha}^2 s} \right] p^\mu + 2\frac{\hat{p}_t \cdot \vec{\epsilon}_\pm}{\bar{\alpha}^2 s + \vec{p}_t^2} p^\mu_T + (0, \vec{\epsilon}_\pm)
$$
Transversity PDFs

\[ \Delta_T u(x) = 7.5 \times 0.5 \times (1 - x)^5 (x \times u(x) + x \times \Delta u(x)) \]
\[ \Delta_T \bar{u}(x) = 7.5 \times 0.5 \times (1 - x)^5 (x \times \bar{u}(x) + x \times \Delta \bar{u}(x)) \]
\[ \Delta_T d(x) = 7.5 \times (-0.6) \times (1 - x)^5 (x \times d(x) + x \times \Delta d(x)) \]
\[ \Delta_T \bar{d}(x) = 7.5 \times (-0.6) \times (1 - x)^5 (x \times \bar{d}(x) + x \times \Delta \bar{d}(x)) \]

Polarized PDFs

\[ \Delta u(x) = \sqrt{x} u(x) \]
\[ \Delta \bar{u}(x) = -0.3 \times x^{0.4} \bar{u}(x) \]
\[ \Delta d(x) = -0.7 \sqrt{x} d(x) \]
\[ \Delta \bar{d}(x) = -0.3 \times x^{0.4} \bar{d}(x) \]