Extraction of the Compton Form Factor $\mathcal{H}$ from recent DVCS measurements at JLab

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1. Preliminary analysis

2. Fitting strategies

3. Results
DVCS described by 4 Compton Form Factors.
Approximations: quark sector, leading twist and leading order.

- Example: GPD $H$

$$H = \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

- Integration yields real and imaginary parts to $H$:

$$\text{Re}H = \mathcal{P} \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right)$$

$$\text{Im}H = \pi \left( H(\xi, \xi, t) - H(-\xi, \xi, t) \right)$$

- Relation between $\text{Im}H$ and $\text{Re}H$ weakly constrained by dispersion relations. However see:

K. Kumericki and D. Müller, arXiv:0904.0458
G. Goldstein and S. Liuti, DIS2009
Selected JLab data: recent DVCS measurements. Fine kinematic binning and large kinematic coverage.

Hall A: helicity-dependent and independent cross sections

- 12 bins: 1 value of $x_B$, 3 values of $Q^2$ and 4 values of $t$.
- Each kinematic bin contains 24 $\phi$-bins.
- Statistical uncertainties:
  - helicity-dependent: at least 20%
  - helicity-independent: $\simeq$ 5%

Hall B: Beam Spin Asymmetries

- 62 bins: 5 value of $x_B$, 4 values of $Q^2$ and 5 values of $t$.
- Each kinematic bin contains (at most) 12 $\phi$-bins.
- Statistical uncertainties: $\simeq$ 25%
Analytic $ep \to ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example: DVCS helicity-dependent cross section at twist 2

- BKM formalism:
  \[ C_1 \sin \phi \text{Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left(1 + \frac{F_2}{F_1}\right) \tilde{\mathcal{H}} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right) \]

A.V. Belitsky, D. Mueller and A. Kirchner

- GV formalism:
  \[ C_2 \sin \phi \text{Im} \left( \mathcal{H} + c_\mathcal{E} \mathcal{E} + c_{\tilde{\mathcal{H}}} \tilde{\mathcal{H}} + c_{\tilde{\mathcal{E}}} \tilde{\mathcal{E}} \right) \]

P.A.M. Guichon and M. Vanderhaeghen, unpublished
Analytic $ep \rightarrow ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example: DVCS helicity-dependent cross section at twist 2

- BKM formalism: coefficients do not depend on $Q^2$

$$ C_1 \sin \phi \text{Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left( 1 + \frac{F_2}{F_1} \right) \mathcal{H} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right) $$

A.V. Belitsky, D. Mueller and A. Kirchner

- GV formalism: coefficients depend on $Q^2$

$$ C_2 \sin \phi \text{Im} \left( \mathcal{H} + \frac{c_{\mathcal{E}}}{20 \%} \mathcal{E} + \frac{c_{\tilde{\mathcal{H}}}}{20 \%} \tilde{\mathcal{H}} + \frac{c_{\tilde{\mathcal{E}}}}{30 \%} \tilde{\mathcal{E}} \right) $$

P.A.M. Guichon and M. Vanderhaeghen, unpublished
Main assumptions.
Expectation: extraction of $\mathcal{H}$ with $\geq 40\%$ total uncertainty.

- **Twist 2 accuracy**
  - Early $Q^2$-scaling was observed in Hall A.
  - Similar recent result concerning a subset of JLab data.
    - M. Guidal, arXiv:1003.0307
  - Small higher twist contribution in Hermes data.
    - D. Zeiler et al., DIS2008

- **$H$-dominance**
  - Dramatically decreases the number of degrees of freedom in the fits.
  - Expectations: *systematic error between 20 and 50 %.*
  - Systematic error $\lesssim 25\%$ from direct test of hypothesis with VGG model.
  - The most questionable assumption so far?
Local fits.
Fits on each kinematic bin to twist 2 expressions.

- Keep bins with $\frac{|t|}{Q^2} < \frac{1}{2}$.
- Low model dependence ($H$-dominance, twist 2).
- But fits may still be underconstrained.

- **Estimation** of systematic errors caused by $H$-dominance hypothesis by fitting data with subdominant GPDs set to 0 or to their VGG value.
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
\[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:
\[
2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{\frac{3}{2}} \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)
\]
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Legendre polynomial
Global fit.
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  \]
  Gegenbauer polynomial

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Gegenbauer polynomial
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  Support: Resummed
Global fit.
Fit to a parametrization from the dual model.

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Model
$t$–dep.

with $B_{nl}(t, Q^2) = \left(\ln \frac{Q_0^2}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2}\right)^{\frac{\gamma_p}{\beta_0}} B_{nl}(t, Q_0^2)$. 

Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  \[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:
  \[
  \sum_{n=0}^{N} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^2 \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right)
  \]
  Model
  $t$-dep.
  with $B_{nl}(t, Q^2) = \left( \ln \frac{Q_0^2}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2} \right)^{\gamma p/\beta_0} \frac{a_{nl}}{1 + b_{nl}(t-t_0)^2}$.

- Non-trivial correlation between $x$ and $t$.
- $a_{nl}$ and $b_{nl}$ are fitted. $t_0$ is chosen prior to the fits.
Global fit.
Iterative fitting procedure and systematic uncertainties.

- Keep bins with $\frac{|t|}{Q^2} < \frac{1}{2}$ (1001 $\phi$-bins fitted).

- $\frac{N(N+3)}{2}$ fitted coefficients for a given truncation $N$.
  - 10, 18 and 28-parameter fits for $N = 2, 3$ and 4.
  - **Estimation** of the truncation error by comparison of the results of these 3 fits.

- Iterative fitting procedure to handle large number of parameters.

- **Estimation** of systematic errors caused by $H$-dominance hypothesis by fitting data with subdominant GPDs set to 0 or to their VGG value.

- Purpose: smooth parametrization of data. **No extrapolation** outside the domain of the fit.
Effect of the truncation of the series.
Hall B data.

- 3 global fits qualitatively similar:
  \[ N \chi^2/d.o.f. \]
  
  \[
  \begin{array}{c|c}
  N & \chi^2/d.o.f. \\
  \hline
  2 & 1.73 \\
  3 & 1.61 \\
  4 & 1.78 \\
  \end{array}
  \]

- No differences on Hall A data (next slide).

- \( N=2 \) fails to reproduce BSAs at small \( \xi \).

- \( N=3 \) always good and close to local fits.

- \( N=4 \) is uncontrolled at large \( \xi \).
Effect of the truncation of the series.
Hall A data.

- Local fits
- Global fit

Effect of the truncation of the series.
Hall A data.

- Local fits
- Global fit

Results

- Im\( \mathcal{H} \) and Re\( \mathcal{H} \)

Discussion

Conclusions
$\Im H$ on Hall B kinematics.
$Q^2$-dependence.

- Compatible results of local and global fits: strong consistency check.
- Realistic estimation of systematic uncertainties:
  - Comparable accuracy from local and global fits.
  - Accuracy in agreement with expectations.
- Restricted kinematic region suitable for GPD-analysis.
**ReH on Hall B kinematics.**

\( Q^2 \)-dependence.

- **Large fluctuations in \( ReH \) from local fits. Global fit is smoother.**

- **Unreliable extraction of \( ImH \) or \( ReH \) at large \( \xi \).**

- **\( ReH \) weakly constrained.**
$\text{Im} \mathcal{H}$ on Hall A kinematics.

$t$-dependence.

- Good agreement between results of local and global fits but...
- Discrepancy seems to be larger at small $|t|$!
- Sizeable scaling deviation for $t = -0.17 \text{ GeV}^2$.
- Noticeable deviations if
  
  $$\xi = x_B \left( \frac{1 + \frac{t}{2Q^2}}{2 - x_B + \frac{x_Bt}{Q^2}} \right) \to \frac{x_B}{2 - x_B}$$

- Call for a \textbf{twist 3 analysis}!
Im\(\mathcal{H}\) and Re\(\mathcal{H}\) on Hall A kinematics.

\(t\)-dependence.
Comparison with other studies (Hall A data).
Several approaches : BKM, BKM + ”hot fix”, GV, VGG.

- First extraction : BKM formalism without ”hot fix”.
  
  C. Muñoz Camacho et al.

- Model-dependent prediction. Fit in progress.
  
  S. Ahmad et al., arXiv:0708.0268

- VGG fitter code.
  
  M. Guidal, EPJA 37, 319 (2008)
  M. Guidal, arXiv:1003.0307

- ”Hot fix” for power suppressed contributions in BKM.
  
  A. Belitsky and D. Müller, PRD79, 014017 (2009)

- Global fit for all unpolarized proton target with BKM + ”hot fix”.
  
  K. Kumericki and D. Müller, arXiv:0904.0458
Comparison with previous studies (Hall A data).
Where are we today?

![Graph comparing results with previous studies](image)
Comparison to the VGG model.
Similar $x_B$-dependence but loss of information during the extraction.
Conclusions.
JLab DVCS measurements are a challenge to phenomenology.

- $\text{Im}\mathcal{H}$ extracted with 20 to 50% accuracy on a wide kinematic range.
- Realistic first estimation of systematic errors.
- Plausible early $Q^2$-scaling but twist 3 study necessary.
- Working without $H$-dominance hypothesis? In progress.
- More generally, a global fitting strategy is still missing.
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References for this work: