Nucleon Electromagnetic Form Factors

- Introduction
- Experimental Status of EMFF
- Analysis and Interpretation
- Outlook
- Summary

Kees de Jager
Exclusive Workshop
Jefferson Lab
May 18 - 21, 2010
Nucleon Electro-Magnetic Form Factors

- Fundamental properties of the nucleon
  - A testing ground for theories constructing nucleons from quarks and gluons
  - Provides insight in spatial distribution of charge and magnetization
  - Wavelength of probe can be tuned by selecting momentum transfer $Q$:
    - $< 0.1\, \text{GeV}^2$ integral quantities (charge radius, …)
    - $0.1-10\, \text{GeV}^2$ internal structure of nucleon
    - $> 20\, \text{GeV}^2$ pQCD scaling

  **Caveat:** If $Q$ is several times the particle that the virtual photon is interacting with (~Compton wavelength), dynamical (relativistic) effects make a physical interpretation more difficult

- Over the last decade there has been a dramatic improvement in precision and $Q^2$-coverage thanks to the development of polarized beam (> 100 $\mu$A, > 85 %), polarized targets and polarimeters with large analyzing powers

- The observation of a linear decrease with $Q^2$ of $G_E^p/G_M^p$ established the role of quark Orbital Angular Momentum (OAM) in the nucleon, confirmed by nucleon spin structure studies (“spin crisis”, $\Delta G$, …)
Formalism

Dirac (non-spin-flip) $F_1$ and Pauli (spin-flip) $F_2$ Form Factors

\[
\frac{d\sigma}{d\Omega}(E,\theta) = \frac{\alpha^2 E' \cos^2\left(\frac{\theta}{2}\right)}{4E^3 \sin^4\left(\frac{\theta}{2}\right)} \left[ (F_1^2 + \kappa^2 \tau F_2^2) + 2\tau(F_1 + \kappa F_2)^2 \tan^2\left(\frac{\theta}{2}\right) \right]
\]

with $E (E')$ incoming (outgoing) energy, $\theta$ scattering angle, $\mu$ anomalous magnetic moment and $\kappa = Q^2/4M^2$

Alternatively, Sachs Form Factors $G_E$ and $G_M$ can be used

\[
F_1 = G_E + \tau G_M \quad F_2 = \frac{G_M - G_E}{\kappa(1 + \tau)} \quad \tau = \frac{Q^2}{4M^2}
\]

\[
\frac{d\sigma}{d\Omega}(E,\theta) = \sigma_M \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right) \right]
\]

Separate the two Sachs FFs by measuring the cross section at one $Q^2$-value for various $\theta$-values (Rosenbluth separation).

In the Breit (centre-of-mass) frame the Sachs FF can be written as the Fourier transforms of the charge and magnetization radial density distributions
World Data Set on $G_{E^p}$ by mid 1990s

Relied on Rosenbluth separation

General assumption that $G_{E^p}/G_{M^p} \approx 1$

Although data showed large scatter

$$Q^2 = 4EE'\sin^2\left(\frac{\theta}{2}\right)$$

$$\varepsilon = \frac{1}{1 + 2(1 + \tau)\tan^2(\theta / 2)}$$

$$\sigma_R(Q^2, \varepsilon) = \varepsilon \left(1 + \frac{1}{\tau}\right) \frac{E}{E'} \frac{\sigma(E, \theta)}{\sigma_{Mott}} = (G_{M^p})^2(Q^2) + \frac{\varepsilon}{\tau} (G_{E^p})^2(Q^2)$$
Alternative: Spin Transfer Reaction $^1\text{H}(e,e'p)$

\[ P_n = 0 \]
\[ \pm hP_t = \mp h\frac{2\sqrt{\tau(1+\tau)}G_E^p G_M^p \tan \left( \frac{\theta_e}{2} \right)}{I_0} \]
\[ \pm hP_l = \pm h(E_e + E_{e'}) \left( G_M^p \right)^2 \sqrt{\tau(1+\tau)} \tan^2 \left( \frac{\theta_e}{2} \right) \frac{1 + 2(1+\tau)\tan^2 \left( \frac{\theta_e}{2} \right)}{M I_0} \]
\[ I_0 = \left[ G_E^p (Q^2) \right]^2 + \tau \left[ G_M^p (Q^2) \right]^2 \left[ 1 + 2(1+\tau)\tan^2 \left( \frac{\theta_e}{2} \right) \right] \]

\[ \frac{G_E^p}{G_M^p} = - \frac{P_t}{P_l} \frac{E_e + E_{e'}}{2M} \tan \left( \frac{\theta_e}{2} \right) \]

No error contributions from
- analyzing power
- beam polarimetry
Detailed reanalysis of SLAC data resulted in acceptable scatter of data
JLab Rosenbluth data in agreement with SLAC data
No reason to doubt quality of either Rosenbluth or polarization transfer data
Investigate possible theoretical sources for discrepancy
Speculation: missing radiative corrections

Speculation: there are radiative corrections to Rosenbluth experiments that are important and are not included in the analysis.

Missing correction: linear in $\varepsilon$, but with no strong $Q^2$-dependence.

$G_E$ term is proportionally smaller at large $Q^2$.

Effect more visible at large $Q^2$.

$$Q^2 = 6 \text{ GeV}^2$$

$$\frac{G_E^2}{\tau G_M^2} = \frac{4 M^2}{Q^2 \mu_P^2} = 7.5\%$$

if both FF scale in same way.

John Arrington
Calculations of TPE effects

\[ d\sigma = d\sigma_0 (1 + \delta) \]

\[ \delta = 2f(Q^2, \epsilon) + \frac{2\Re \left\{ \overline{M}_0 M_1 \right\}}{|M_0|^2} \]

\[ \delta_{2\gamma} = \frac{2\Re \left\{ M_\gamma \overline{M}_{2\gamma} \right\}}{|M_\gamma|^2} \]

\( f(Q^2, \epsilon) \) is the standard Mo & Tsai correction (soft photon exchange), which has some \( \epsilon \)-dependence and is IR divergent. IR divergent terms are canceled by soft-photon emission terms.

Two methods of calculating \( \delta_{2\gamma} \):

**Hadronic**
Use nucleon-pole diagrams with on-shell form factors in photon-nucleon vertices
Blunden, Melnitchouk, Tjon (BMT), PRC 72, 034612 (2005)

**Partonic**
Factorize TPE amplitude into hard process of e-q scattering and a soft process described by GPDs
Effect on L-T Extractions

Arrington, Melnitchouk, Tjon
PRC 76, 035205 (2007)

full reanalysis of data, incorporating
BMT calculations, but adding
extra (small) phenomenological
correction above $Q^2 = 1 \text{ GeV}^2$

\[ \delta_{2\gamma}^* = 0.01 (\varepsilon - 1) \frac{\ln Q^2}{\ln 2.2} \]

~1\% at 2 GeV$^2$, 2\% at 5 GeV$^2$

• Apply 100\% of the extra
correction as an uncertainty
(affects $G_M^p$ uncertainty)

• Corrections hardly
visible in $e^+/e^-$ ratio

New $e^+/e^-$ data expected soon: BINP (data), DESY (2012), CLAS (2012)
VEPP-3 positron-electron comparison

- Use VEPP-3 ring (50 mA)
- $e^+/e^-$ energy 1.6 GeV
- $\theta \approx 25^\circ$ and $65^\circ$
- $Q^2 \approx 0.3$ and $1.5$ GeV$^2$
- Internal gas target
- Luminosity $\approx 5 \cdot 10^{31} (\text{cm}^2\text{s})^{-1}$

J. Arrington, D. Nikolenko, spokespersons, nucl-ex/04-08020
Projected results for OLYMPUS@DESY

1000 hours each for $e^+$ and $e^-$
Lumi = $6 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$

2008 – Full proposal submitted and approved
2009/10 – Transfer of BLAST
2012 – OLYMPUS Running
SSA in elastic eN scattering

spin of beam OR target OR recoil proton NORMAL to scattering plane

on-shell intermediate state ($M_X = W$)

involves the imaginary part of two-photon exchange amplitudes

Target: general formula of order $e^2$

• GPD model allows connection of real and imaginary amplitudes
• Hadronic models sensitive to intermediate state contributions, no reliable theoretical calculations at present
• First data taken late 2008 in QE scattering off polarized $^3$He (E05-015)

Beam: general formula of order $m_e e^2$ (few ppm)

• Measured in PV experiments (longitudinally polarized electrons) at SAMPLE, A4 (Mainz), $G^0$ and HAPPEX (JLab)
• Only non-zero result so far for TPEX

Peter Blunden
Some available data on SSA

More recent calculations by Borisyuk & Kobushkin (arXiv:0812.0469)
• Bernauer et al. collected a large data set using all three A1 spectrometers
• The projected ≤1% accuracy will allow an L/T separation in a $Q^2$-range of 0.02 to 0.5 GeV$^2$
• Data analysis has been completed and will be available shortly
Polarization Transfer at low $Q^2$-values

Detailed understanding of Hall A HRS spectrometer optics and availability of BigBite spectrometer has made possible polarization transfer measurements with a ~1% accuracy in a $Q^2$-range from 0.3 – 0.7 GeV$^2$

Results are in disagreement (at a better precision) with earlier data

New data analyzed together with new cross-section data set measured at MAMI will allow to set sensitive limits to TPE effects at low $Q^2$

Further data at $Q^2$-values down to 0.1 GeV$^2$ are scheduled for late 2011 with a DNP target

Xiaohui Zhan
Ph.D. thesis
Measuring $G_M^n$

Old method: quasi-elastic scattering from $^2$H
large systematic errors due to subtraction of proton contribution

- Measure $(en)/(ep)$ ratio
  Luminosities cancel
  Determine neutron detector efficiency
  - On-line through $e+p \rightarrow e'+\pi^+(+n)$ reaction (CLAS)
  - Off-line with neutron beam (Mainz)

- Measure inclusive quasi-elastic scattering off polarized $^3$He

\[
R_D = \frac{\frac{d^3\sigma(eD \Rightarrow e'n(p))}{dE'd\Omega_{e'}d\Omega_{n}}}{\frac{d^3\sigma(eD \Rightarrow e'p(n))}{dE'd\Omega_{e'}d\Omega_{p}}}
\]

\[
A = \frac{-(\cos\theta^* v_T R_T + 2\sin\theta^* \cos\varphi^* v_{TL'} R_{TL'})}{v_L R_L + v_T R_T}
\]

$R_T$ directly sensitive to $(G_M^n)^2$
Overview of results for $G_M^n$

→ A systematic difference of several % between results from JLab and MAMI in $Q^2$-range 0.4 – 1.0 GeV^2
→ Reminder that at least two independent experiments are always needed

High-quality data set now available up to ~4.5 GeV^2
$G_E^n$ from polarized $^3$He target: $^3$He(e,e' n)

- New data more than double the $Q^2$-range of the world data set
- Roberts' dressed quark-diquark model using the Dyson-Schwinger and Faddeev equations in good agreement, better than Miller's CQM prediction
- Belitsky/Ji logarithmic scaling does not hold for the neutron in the $Q^2$-region where it was validated by the proton data
- New data will add significant constraints to GPD modeling

Seamus Riordan
(Logarithmic) Scaling

→ Basic pQCD scaling predicts $F_1 \propto 1/Q^4$; $F_2 \propto 1/Q^6 \rightarrow F_2/F_1 \propto 1/Q^2$

→ Data clearly do not follow this trend (yet?)

→ The introduction of a quark orbital angular momentum component results in
  $F_2/F_1 \propto 1/Q$

→ Belitsky et al. have included logarithmic corrections in pQCD limit

→ Proton data appear to follow this scaling behaviour, but new neutron data do not
Comparison with Theory
Significant progress in LQCD, but still limited to $m_π \geq 300$ MeV and neglect of disconnected diagrams, resulting in large underestimates of e.g. isovector charge radius

Bratt et al., arXiv: 1001.3620
Nucleon densities and relativity

\[ \rho(r) = \frac{2}{\pi} \int_{0}^{\infty} dk \, k^2 \, j_0(k \, r) \, \tilde{\rho}(k) \]

rest frame density

\[ \tilde{\rho}(k) = G(Q^2) \]

intrinsic FF

Q\(^2\)-evolution of quark mass (nucl-th/9812063)

rest frame

non-relativistic limit: \[ \tilde{\rho}(k) = G(Q^2) \]

importance of relativity (with increasing Q\(^2\)):
Lorentz contraction of spatial distributions in Breit frame

\[ k^2 = Q^2/(1 + \tau) \quad \tau = Q^2/(4M^2) \]

\[ \tilde{\rho}_{E,M}(k) = G_{E,M}(Q^2)(1 + \tau)^2 \]

limit: \( k = 2 \, M \) (Compton wavelength)
Thus, Fourier transform remains valid for \( r > r_{\text{min}} \approx 0.3 \, \text{fm} \)

At \( Q \approx 0.6 \, \text{GeV} \) (\( r \approx 0.3 \, \text{fm} \)) \( m_{u/d} \approx 0.3 \, \text{GeV} \)
Kelly and Friedrich and Walcher have performed simultaneous fit to all four EMFF in coordinate space. Both observe a structure in the proton and neutron densities at ~0.9 fm which they assign to a pion cloud.

Crawford et al. performed a global fit to all four EMFF within the framework of Lomon’s VMD parametrization, including an estimate of the unmeasured high-$Q^2$ region with either a Fourier-Bessel or a Laguerre expansion. A straightforward transformation to coordinate space is shown below.

C. Crawford et al., arXiv:1003.0903
$F_{1,2}$ form-factor decomposition

\[
F_{1,2}^p = \frac{2}{3} F_{1,2}^u - \frac{1}{3} F_{1,2}^d; \quad F_{1,2}^n = \frac{2}{3} F_{1,2}^d - \frac{1}{3} F_{1,2}^u
\]

assuming isospin symmetry: $F_{1,2}^{u,p} = F_{1,2}^{d,n}$ and $F_{1,2}^{d,p} = F_{1,2}^{u,n}$

- New data have been used to flavor separate the Pauli and Dirac FFs
- Assuming that the $s$-quark contribution is negligible (based on the HAPPEEx plus G0 results)
- Clearly, the $F_2$ ratio provides a sensitive test for theoretical predictions
Nucleon transverse charge density

Light Front Kinematics

Photon only couples to forward moving quarks

Transverse (Dirac) charge density of unpolarized nucleon

\[ \rho_0^N (\vec{b}) = \frac{1}{2P^+} \left\langle P^+, \frac{q_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{q_\perp}{2}, \lambda \right\rangle \]

\[ = \int_0^\infty \frac{dQ}{2\pi} Q J_0 (bQ) F_1 (Q^2) \]

\[ \rho_0^p, \rho_T^p \text{ [1/fm]} \]

Unrelated to rest-frame charge density

J. Miller; C. Carlson & M. Vanderhaegen
Mapping of nucleon constituents (in the proton)

impact parameter $b$

is defined relative to the transverse center of the quark’s longitudinal momentum fractions

$$R = \sum x_i r_i$$

$$\rho_{\text{Dirac}}(b) = \int_0^\infty \frac{QdQ}{2\pi} J_0(bQ) F_1(Q^2)$$

$$\rho_{\text{Pauli}}(b) = \int_0^\infty \frac{Q^2dQ}{4\pi M} J_1(bQ) F_2(Q^2)$$

- The flavor-separated $F_1$ and $F_2$ ratios were then used to extract the transverse densities for the $u$- and $d$-quark (in the proton).

→ Why is the $d$-quark so much wider?
The SuperBigbite project

- The Super Bigbite project:
  - large dipole magnet
  - GEM trackers (~100,000 channels)
  - hadron and EM calorimeter
  - Trigger and DAQ

- operating in open geometry at a luminosity of $10^{38} \text{ cm}^{-2}\text{s}^{-1}$
- will extend measurements of EMFFs to double the existing $Q^2$-range
- Included in the JLab long-term capital funding request to DOE
Projected EMFF data with SBS @ 12 GeV
Impact of EMFF on GPDs

1. Allows for a unified description of form factors and parton distributions
2. Describes correlations of quarks/gluons
3. Allows for Transverse Imaging

Fourier transform in momentum transfer gives transverse spatial distribution of quark (parton) with momentum fraction $x$ and related to EMFFs through first moments

$$\sum_{q} e_{q} \int dx H^q(x, \xi = 0, Q^2) = F_1(Q^2); \quad \sum_{q} \kappa_{q} \int dx E'^q(x, \xi = 0, Q^2) = F_2(Q^2)$$

4. Allows access to quark angular momentum (in model-dependent way)
Summary and Outlook

• Very active experimental program on nucleonelectro-magnetic form factors thanks to development of polarized beam (> 100 µA, > 85 %), polarized targets and polarimeters with large analyzing powers
  → $G_E^p$ discrepancy between Rosenbluth and polarization transfer not an experimental problem, but probably caused by TPE effects.
  → Broad ongoing program to obtain quantitative information on TPE.
  → Observation of linear decrease with $Q^2$ of $G_E^p/G_M^p$ established role of quark Orbital Angular Momentum (OAM) in nucleon
  → $G_M^n$ precise data up to $Q^2 = 4.5 \text{ GeV}^2$, but inconsistency at $\sim 1 \text{ GeV}^2$
  → $G_E^n$ precise data up to $Q^2 = 3.5 \text{ GeV}^2$ provides strong indication that OAM has different effect on neutron than on proton
  → The SuperBigBite project, to be implemented once the JLab 12 GeV upgrade has been completed, will extend the present knowledge of the nucleon EMFF $G_E^p$, $G_E^n$ and $G_M^p$ to double or triple the $Q^2$-range covered by existing data
  → It is imperative that this experimental program is accompanied by a similar progress in our theoretical understanding of the nucleon
THANK YOU!
Flavor separation

- Especially $F_2$ ratio sensitive test of predictions (RCQM, GPDs)
- Transverse charge densities shown for proton, $d$-quark much more spread out than $u$-quark (large $x$-behaviour?)

$$F_{1,2}^p = \frac{2}{3} F_{1,2}^u - \frac{1}{3} F_{1,2}^d, \quad F_{1,2}^n = \frac{2}{3} F_{1,2}^d - \frac{1}{3} F_{1,2}^u$$

assuming isospin symmetry: $F_{1,2}^{u,p} = F_{1,2}^{d,n}$ and $F_{1,2}^{d,p} = F_{1,2}^{u,n}$

Unpolarized Transverse Charge Density

- Red: proton
- Blue: neutron (x -4)
- Dashed: $u$ quark (x 0.5)
- Dotted: $d$ quark (x 2)
Hall A: Two High Resolution ($10^{-4}$) Spectrometers
Maximum luminosity $10^{38}$ cm$^{-2}$s$^{-1}$

Hall B
CLAS

Hall C
HMS+BigCal
Add new hall

Upgrade magnets and power supplies

Add 5 cryomodules

20 cryomodules

CHL-2

Add arc

Enhance equipment in existing halls

Add 5 cryomodules

20 cryomodules
Beyond form factors and quark distributions

Generalized Parton Distributions (GPDs)


Proton form factors, transverse charge & current densities

Correlated quark momentum and helicity distributions in transverse space - GPDs

Structure functions, quark longitudinal momentum & helicity distributions