Hadron Form Factors at Large Momentum Transfer from Lattice QCD

Huey-Wen Lin
University of Washington
Outline

§ Why Higher-$Q^2$ form factors?
Talks at this workshop: G. Huber, K. de Jager...

§ The tool = Lattice Gauge Theory

\[ \frac{1}{Z} \int [dA][d\bar{\psi}][d\psi] \, O(\bar{\psi}, \psi, A) e^{i \int d^4x \, L^{QCD}(\bar{\psi}, \psi, A)} \]

Ph. Hagler (Tue), B. Musch (Fri)

§ Lattice Form Factor Calculations

✦ What’s been done in the past
✦ What’s new in this talk
✦ Some results (nucleon and pion)

§ Summary and Outlook
§ Challenge for lattice-QCD calculations

≈ Typical $Q^2$ range for nucleon form factors is $< 3.0$ GeV$^2$
≈ Examples from 2+1f cases

Higher-$Q^2$ calculations suffer from poor noise-to-signal ratios

RBC/UKQCD

Dirac FF
§ Problem: traditional approach

☞ Study hadron properties by looking at 2-point function

\[ \langle J_N J_N \rangle = \sum_n \langle J|n\rangle \langle n|J \rangle e^{-E_n t} \]
§ Problem: traditional approach

❖ Study hadron properties by looking at 2-point function
\[ \langle J_N J_N \rangle = \sum_n \langle J|n\rangle \langle n|J\rangle e^{-E_n t} \]

❖ Simplify to a one-state problem
Nucleon “effective mass”

\[ -\log[C(t)/C(t+1)] \]
§ Problem: traditional approach

riter  One-state problem

3pt correlator

\[ \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta (\vec{p}, t) \mathcal{O}(\tau) \bar{J}_\alpha (\vec{p}, 0) \rangle \]

\[ q = p_f - p_i \]

\[ p_{i,f} = (2\pi/L) n_{i,f} \alpha^{-1} \]
§ Problem: traditional approach

臊 Simplify to one-state problem

Example of Σ @ 600 MeV pion

\[
\sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta (\vec{p}, t) \mathcal{O}(\tau) \bar{J}_\alpha (\vec{p}, 0) \rangle
\]

3pt correlator

\[
p_{i,f} = (2\pi/L) n_{i,f} a^{-1}
\]

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§ Problem: traditional approach fails at large $Q^2$

§ Solution: confront excited states directly and allow operators to couple to excited states
§ $N_f = 2+1$ anisotropic clover fermions

Renormalized anisotropy $a_s/a_t = 3.5$

Better resolution in temporal direction

Correlators have time-dependent form $A e^{-Et}$

$a_s \approx 0.1227(8)$ fm (using $m_\Omega$)

Dynamical Anisotropic Lattices

§ $N_f = 2+1$ anisotropic clover vacuum structure ($M_\pi \approx 380$ MeV)

http://www.phys.washington.edu/users/hwlin/visQCD.html
§ Consider multiple-state 3pt correlators...

\[
\Gamma_{\mu,AB}^{(3),T}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = a^3 \sum_n \sum_{n'} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{Z_j \cdot 4E_n(p_f) E_0(p_i)} e^{-(t_f-t)E_{n'}(p_f)} e^{-(t-t_i)E_n(p_i)} \]

\[
\times \sum_{n,n'} T_{\alpha\beta} u_{n'}(p_f, s') \langle N_{n'}(p_f, s') | j_{\mu}(0) | N_n(p_i, s) \rangle \bar{u}_n(p_i, s) \]

Wanted
§ Consider multiple-state 3pt correlators...

\[ \Gamma^{(3)}_{\mu,AB}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = a^3 \sum_n \sum_{n'} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{Z_j \cdot 4E'_n(p_f)E_n(p_i)} e^{-(t_f-t)E'_n(p_f)} e^{-(t-t_i)E_n(p_i)} \]

\[ \times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f, s') \beta \langle N_{n'}(\vec{p}_f, s') | j_\mu(0) | N_n(\vec{p}_i, s) \rangle \bar{u}_n(\vec{p}_i, s) \]

§ “Variational method” for better determined Z’s and E’s
Form Factors

§ Consider multiple-state 3pt correlators...

\[ \Gamma^{(3),T}_{\mu,AB}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = a^3 \sum_n \sum_{n'} \frac{1}{Z_n, B} \frac{Z_{n, A}(p_i)}{4E_n(p_f)E_n(p_i)} e^{-(t_f-t)E'_n(p_f)} e^{-(t-t_i)E_n(p_i)} \]

\[ \sum_{s,s'} T_{\alpha \beta} u_{n'}(\vec{p}_f, s') \langle N_{n'}(\vec{p}_f, s') | j_\mu(0) | N_n(\vec{p}_i, s) \rangle \bar{u}_n(p_i, s) \]

§ “Variational method” for better determined Z’s and E’s

§ \( n = n' = 0 \) gives us nucleon Matrix Element \( \langle N | V_\mu | N \rangle(q) \)

and solve linear equations for form factors
\[ (r^2_{1,2}) = (-6) \frac{d}{dQ^2} \left( \frac{F_{1,2}^\nu(Q^2)}{F_{1,2}^\nu(0)} \right) \bigg|_{Q^2=0} \]

The form factors are buried in the amplitudes \( F^2(0) \)

Consistency Checks

\[ \langle r^2 \rangle (\text{fm}^2) \]

\[ m_{\pi}^2 (\text{GeV}^2) \]

\[ F_2(0) \]

\[ \kappa'(\mu_N) \]

\[ f_{d,\text{bare}}^d \]
Our Results

$N_f = 0$ anisotropic lattices, $M_\pi \approx 480, 720, 1080$ MeV

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Our Results

$N_f=2+1$ anisotropic lattices, $M_\pi \approx 450, 580, 875$ MeV

$G_E^p$, $G_M^p$, $G_E^n$, $G_M^n$
\[ F_1 = \frac{a_0 + \sum_{i=1}^{k-2} a_i \tau^i}{1 + \sum_{i=1}^{k} b_i \tau^i} \]

\[ \tau = \frac{Q^2}{4 m_N^2} \]

§ Phenomenological choice with dimensionless parameter

\( N_f = 2+1 \) anisotropic lattices, \( M_\pi \approx 450, 580, 875 \) MeV

HWL et al., arXiv: 1005.0799
Transverse Charge Density

§ Infinite-momentum frame

\[ \rho(b) \equiv \int \frac{d^2q}{(2\pi)^2} F_1(q^2) e^{iq \cdot b} = \int_0^\infty \frac{Q dQ}{2\pi} J_0(bQ) F_1(Q^2) \]

§ How does high-\(Q^2\) affect charge density?

- **Red** band uses lattice data \(\leq 2.0 \text{ GeV}^2\)
- **Blue** band uses lattice data \(\leq 4.0 \text{ GeV}^2\)

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G. A. Miller, arXiv: 1002.0355

H. W. Lin et al., arXiv: 1005.0799
Transverse charge density in infinite-momentum frame

\[
\rho(b) = \int_0^\infty \frac{Q dQ}{2\pi} J_0(bQ) F_1(Q^2)
\]

Proton

G. A. Miller, arXiv: 1002.0355

HWL et al., arXiv: 1005.0799
Transverse Charge Density

§ Transverse charge density in infinite-momentum frame

\[ \rho(b) = \int_0^\infty \frac{Q dQ}{2\pi} J_0(bQ) F_1(Q^2) \]

Neutron

HWL et al., arXiv: 1005.0799

G. A. Miller, arXiv: 1002.0355
§ Magnetization density in infinite-momentum frame

\[ b \sin^2 \phi \int_0^\infty \frac{Q^2 dQ}{2\pi} J_1(bQ)F_2(Q^2) \]

§ \( \varphi = \pi/2 \)

Proton

HWL et al., arXiv: 1005.0799

G. A. Miller, arXiv: 1002.0355
§ Magnetization density in infinite-momentum frame

\[ b \sin^2 \phi \int_0^\infty \frac{Q^2 dQ}{2\pi} J_1(bQ) F_2(Q^2) \]

§ \( \phi = \pi/2 \)

\[ \tilde{\rho}_M^n \text{ (fm}^{-2}) \]

HWL et al., arXiv: 1005.0799

G. A. Miller, arXiv: 1002.0355
Nucleon Axial Form Factors

§ \( N_f = 2+1 \) anisotropic lattices, \( M_\pi \approx 450, 580, 875 \) MeV

\[
\bar{u}_B(p') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B} \right] u_B(p)
\]

\( G_A^{u-d}(Q^2) \)

\( \rho^{\uparrow}, u-d \) (fm\(^{-2}\))

\( \rho^{\downarrow}, u-d \) (fm\(^{-2}\))

Preliminary
§ $N_f=2+1$ anisotropic lattices, $M_\pi \approx 875, \ 1350$ MeV

Preliminary

§ Disconnected contribution $O(10^{-2})$ for EM form factor
- Small for most of the form factors but could be significant for neutron electric form factor

§ To get larger momentum, we use $O(ap) \approx 1$
- Rome was not built in a day…
- Methodology for improving a traditional lattice calculation

§ Possible future improvement
- Step-scaling through multiple lattice spacings and volumes
- Higher momentum transfer
Summary

Exploratory study of large momentum transfer on the lattice

§ A Novel Strategy

▷ Include operators that couple to high-momentum and excited states
▷ Explicitly analyze excited states to get better ground-state signal

§ What We Show

▷ Demonstrated results for heavier pions
▷ Transverse densities

§ Future Work

▷ Smaller src-snk separation for better signal
▷ Extend to other hadrons or isotropic lattices
▷ Multiple lattice spacings to study/reduce systematic error