Paving to Way for $\gamma p \to K^+ K^- p$: KN Scattering

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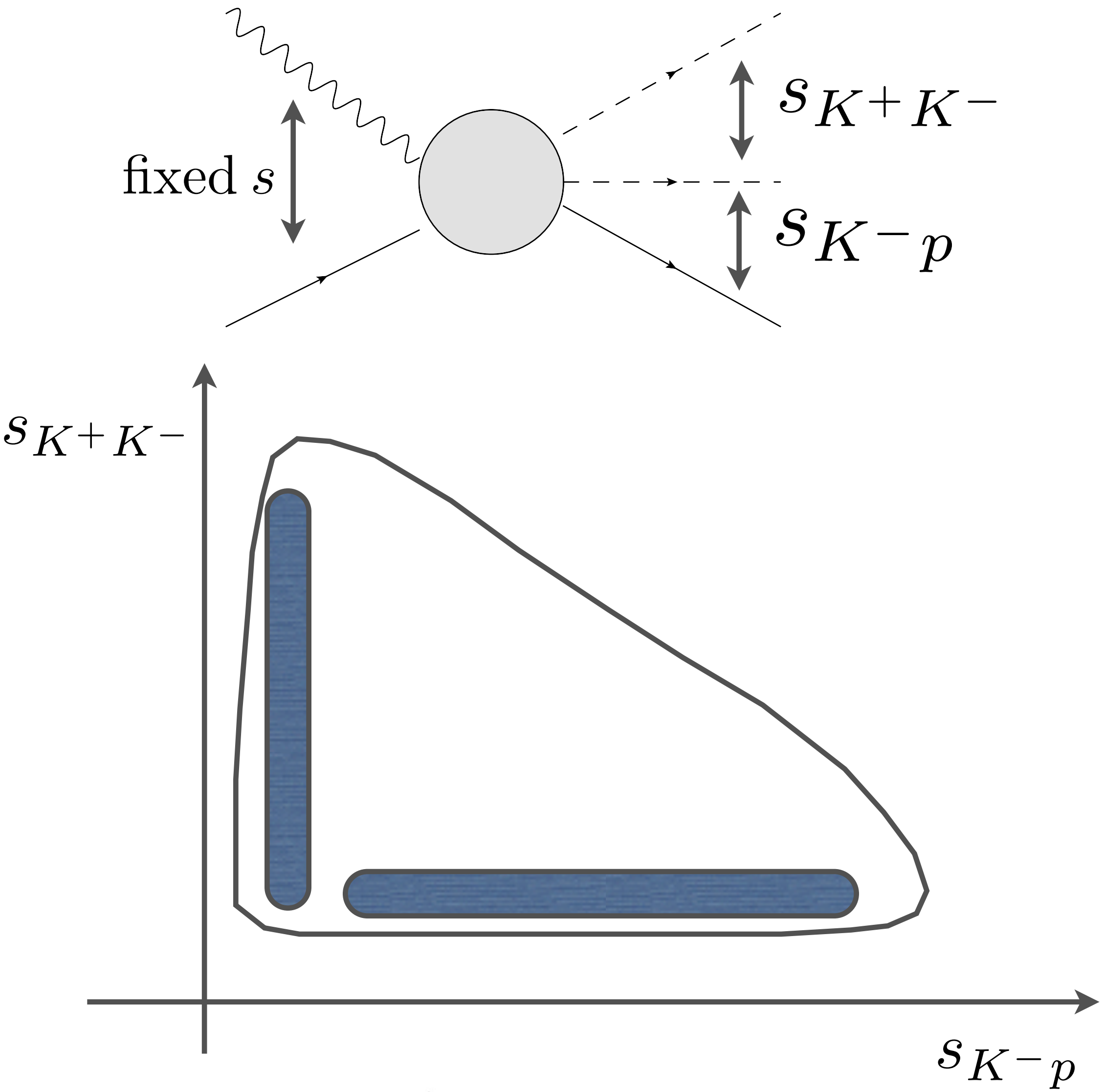
Motivation

- For two kaon photo production
  - Map *strangeonia* spectrum
  - Hybrids/exotics search
- For KN scattering
  - Next slides

Dalitz Plot



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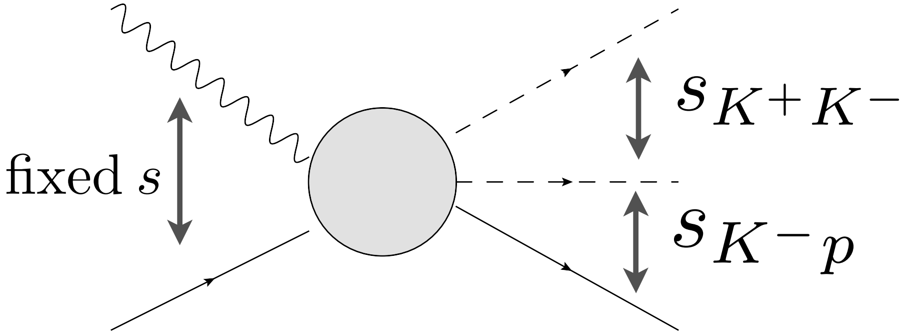

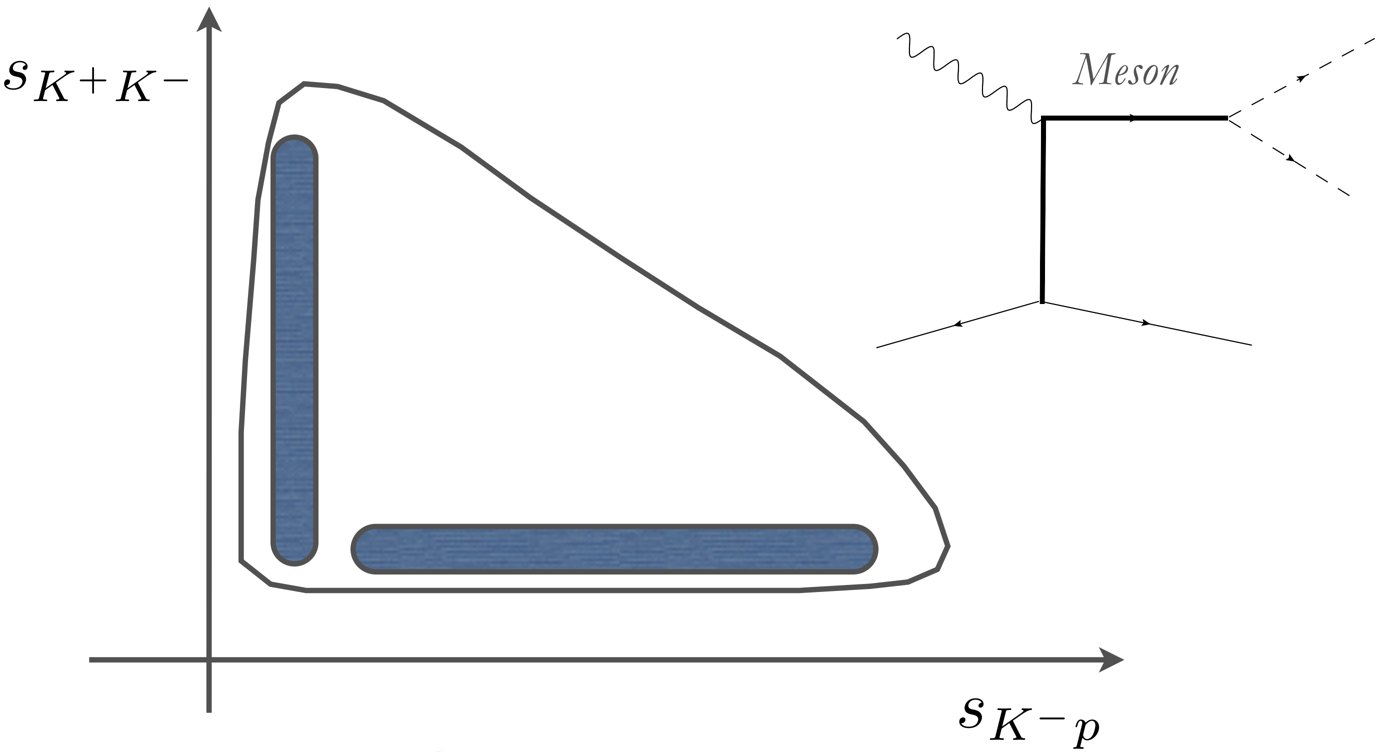

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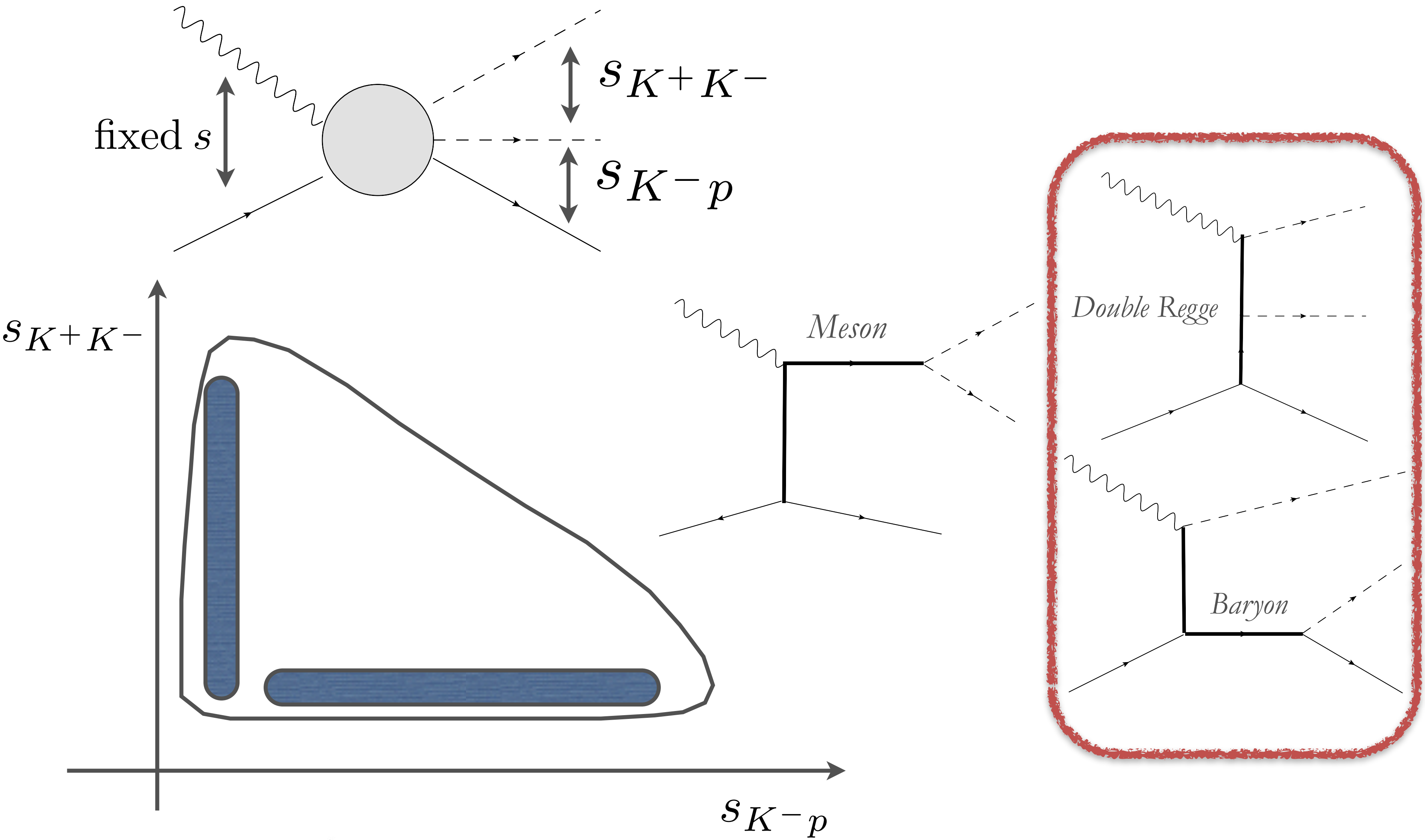

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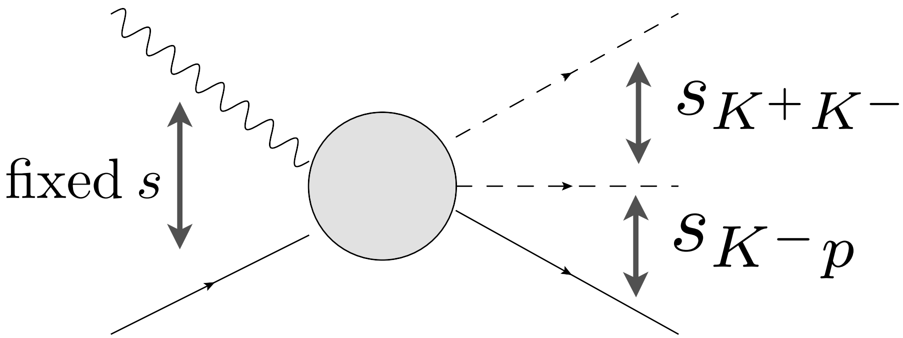
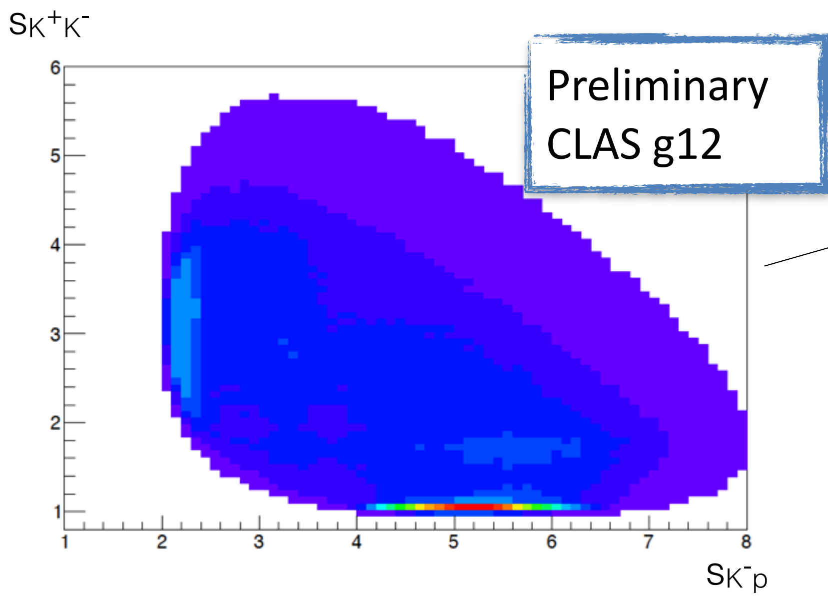
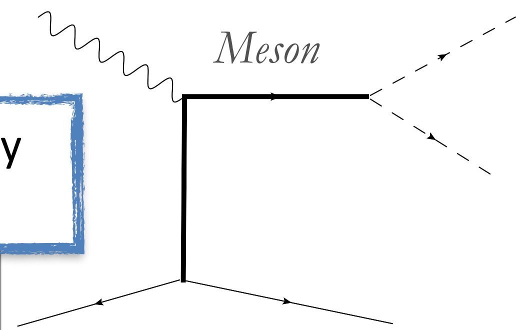
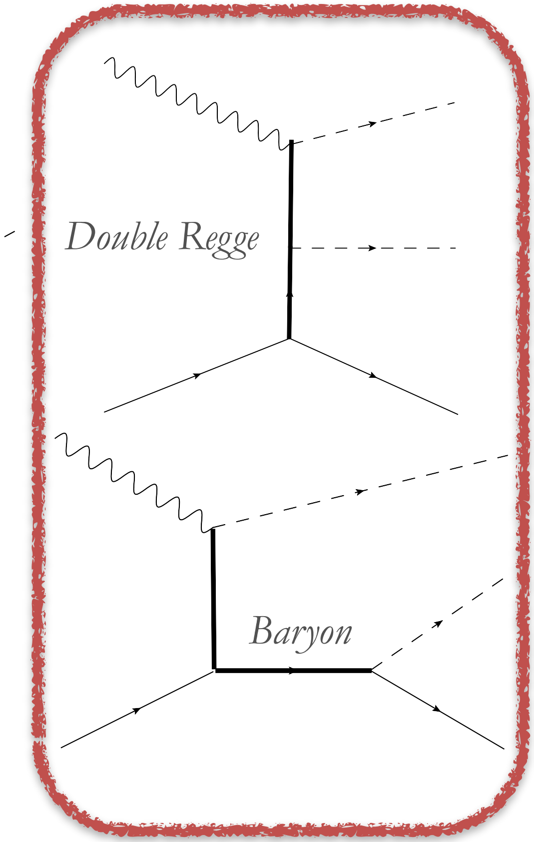


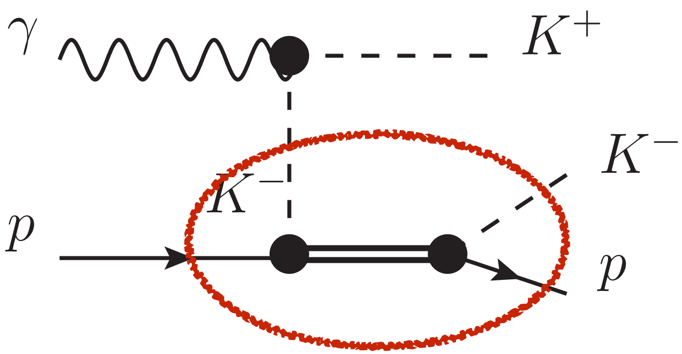


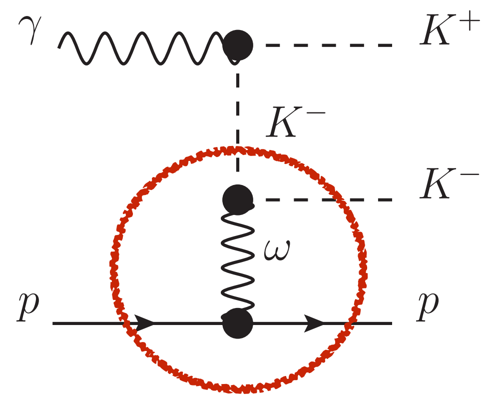
C. Salgado, D. Schott





A Model for $\gamma p \rightarrow K^+ K^- p$





$$K^- p \rightarrow K^- p$$

Low energy fit

FR et al.,

(in preparation)

$$K^- p \rightarrow K^- p$$

High energy fit

Mathieu et al.,

(in preparation)

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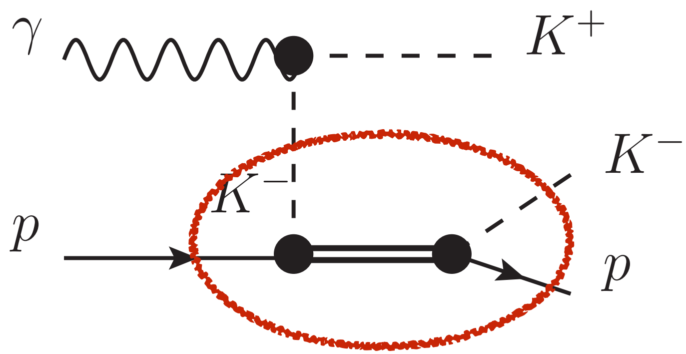

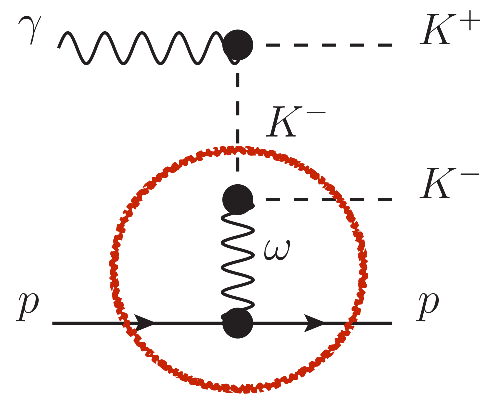

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Analytical continuation between the two regions via dispersion relations (FESR)

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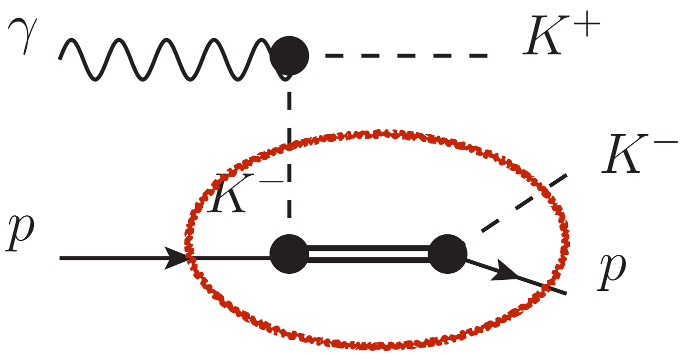

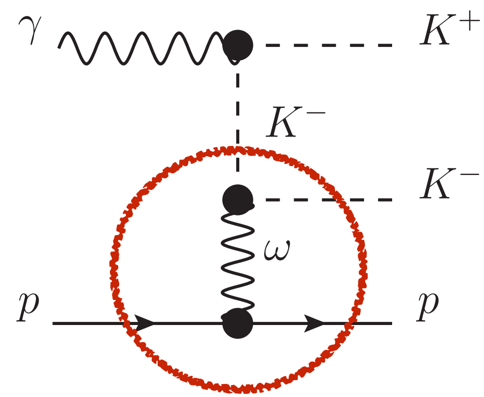

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Analytical continuation between the two regions via dispersion relations (FESR)

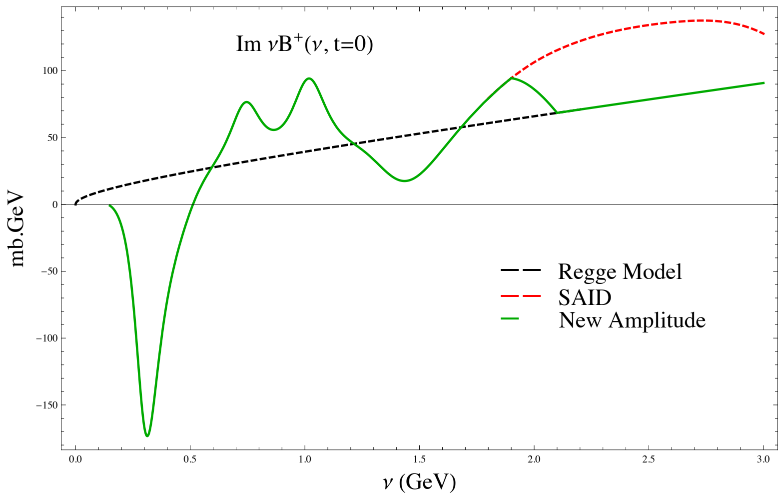
We feed amplitudes to experimentalists and they isolate mesons through PWA

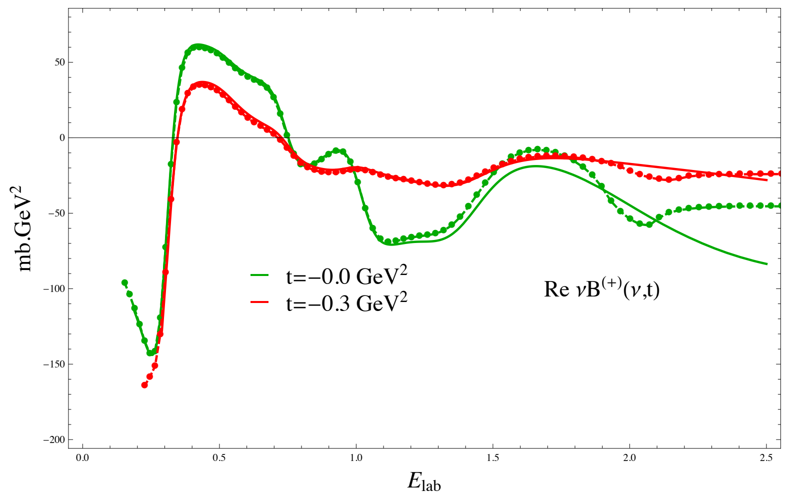
Finite Energy Sum Rules πN (playground)

- Discuss resonances together with Reggeons
- Construct Im(amplitude) from 0 to infinity via FESR
- Reconstruct Re(amplitude) from a dispersion relation

$$\pi N \to \pi N$$

$$\text{Re } \nu B^{(+)}(\nu, t) = \frac{g_r^2}{2m} \frac{2\nu^2}{\nu_m^2 - \nu^2} + \frac{2\nu^2}{\pi} \text{P} \int_{\nu_0}^{\infty} \frac{\text{Im } B^{(+)}(\nu', t)}{\nu'^2 - \nu^2} d\nu'$$





$\bar{K}N \to \bar{K}N, \pi\Sigma, \pi\Lambda$ in the Resonance Region

- Coupled channels, analytical and unitary
- We can use partial waves
- Resonances are incorporated employing relativistic Breit-Wigner
- Variation of Zhang et al., PRC 88 (2013) 035205 incorporating analyticity to the amplitudes and adapting for extension to two kaon photo production
- Single-energy partial waves from KSU analysis (Manley et al.) are fitted independently
- A lot of parameters but also a lot of data points
- We can go to the unphysical sheets and get the poles

Scattering Matrix (p.w.)

$$S = I + 2i\, T$$

$$T = T_B + B^T\, T_R\, B$$

$$S_R = I + 2i\, T_R$$

$$S_B = B^T\, B = I + 2i\, T_B$$

$\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, \pi\Sigma(1385), \pi\Lambda(1520), \bar{K}\Delta(1232), \bar{K}^* N, \sigma\Lambda, \sigma\Sigma$

Resonant Part: Single Resonance

$$[K_a(s)]_{jk} = \tan\delta_a(s)\,\frac{\phi_{\ell,j}(s)\phi_{\ell,k}(s)}{\Gamma_a(s)}\,x_j^a x_k^a$$

$$T_a(s) = K_a(s)\,\left[I - iK_a(s)\right]^{-1}$$

$$[T_a(s)]_{jk} = \frac{M_a}{M_a^2 - s - iM_a\Gamma_a(s)}\,\phi_{\ell,j}(s)\phi_{\ell,k}(s)\,x_j^a x_k^a$$

$\phi_{\ell,j}(s)$ has the angular momentum barrier

$\Gamma_a(s) = \sum_j^{n_c} \phi_{\ell,j}^2(s)\,x_j^a$ is the total width

Resonant Part: Two Resonances

$$[K_{ab}]_{jk} = \tan\delta_a(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^a x_k^a + \tan\delta_b(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^b x_k^b$$

$$\begin{aligned}[T_{ab}]_{jk} &= c_{aa}(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^a x_k^a + c_{ab}(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^a x_k^b \\ &+ c_{ba}(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^b x_k^a + c_{bb}(s)\,\phi_{\ell,j}(s)\,\phi_{\ell,k}(s)\,x_j^b x_k^b\end{aligned}$$

$$c_{aa}(s) = \frac{1}{\mathcal{C}(s)}\,\frac{M_a}{M_a^2 - s - iM\Gamma_a(s)}$$

$$c_{bb}(s) = \frac{1}{\mathcal{C}(s)}\,\frac{M_b}{M_b^2 - s - iM\Gamma_b(s)}$$

$$c_{ab}(s) = c_{ba}(s) = \frac{i\varepsilon_{ab}(s)}{\mathcal{C}(s)}\,\frac{M_a}{M_a^2 - s - iM\Gamma_a(s)}\,\frac{M_b}{M_b^2 - s - iM\Gamma_b(s)}$$

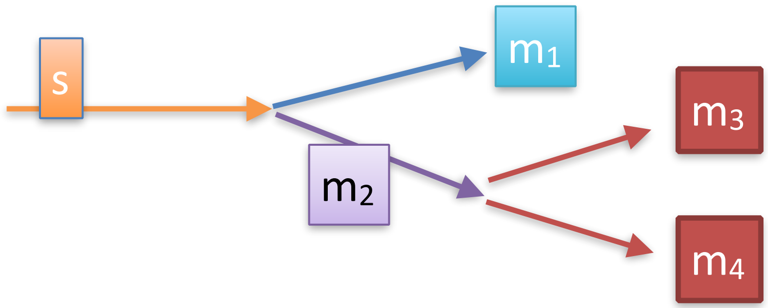
$$\mathcal{C}(s) = 1 + [\varepsilon_{ab}(s)]^2\,\frac{M_a}{M_a^2 - s - iM\Gamma_a(s)}\,\frac{M_b}{M_b^2 - s - iM\Gamma_b(s)}$$

$$\varepsilon_{ab}(s) = \sum_j^{n_c} \phi_{\ell,j}^2(s)\,x_j^a x_j^b$$

Energy Dependence

$$\phi^2_{\ell,j}(s) = \frac{\bar{q}_k(s,m_1,m_2)}{w_0}\mathcal{B}^2_\ell\left[\bar{q}^2_k(s,m_1,m_2)r^2\right]$$

$$\bar{q}_k(s,m_1,m_2) = \frac{m_1 m_2}{(m_1+m_2)^2}\left[s-(m_1+m_2)^2\right]$$



$$\phi^2_{\ell,j}(s) = \frac{\Gamma_2}{2\pi w_0}\int_{m_3+m_4}^{\Lambda} dx\, \frac{\bar{q}_k(s,m_1,x)\mathcal{B}^2_\ell\left[\bar{q}^2_k(s,m_1,x)r^2\right]}{(x-m_2)^2+(\Gamma_2/2)^2}$$

Background

$$\left[T_p^B\right]_{jk} = \frac{\epsilon_p\, M_p}{M_p^2 + s - i\epsilon M_p \Gamma_p(s)} \phi_{\ell,j}(s)\, \phi_{\ell,k}(s)\, y_j^p y_k^p$$

$$T_p^B = (B_p^T B_p - I)/2i$$

$$B = \prod_j B_p$$

$$T_p^B = (B_p^T B_p - I)/2i$$

Summary of fits (current version)

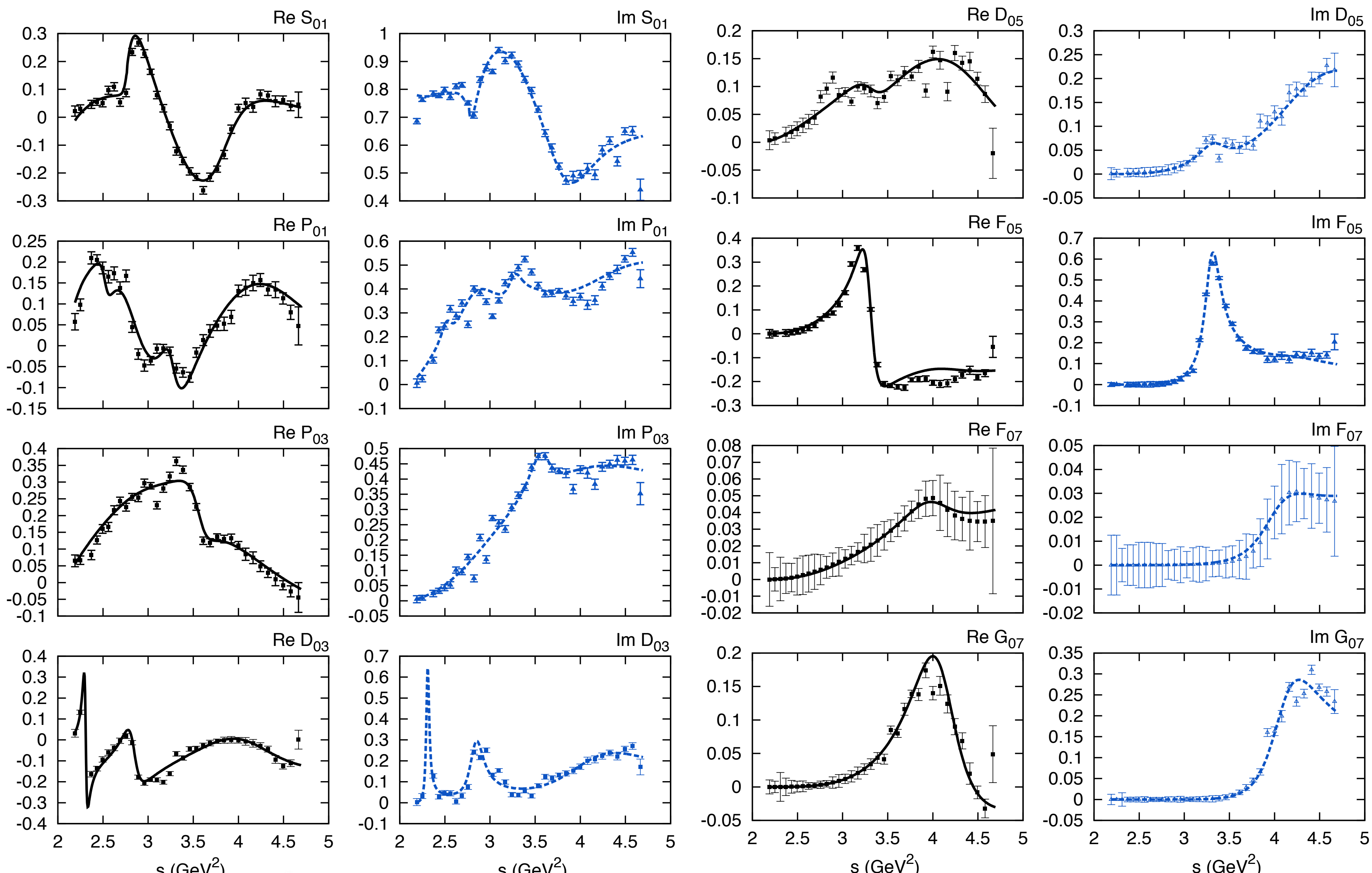
Fit single energy partial waves from Kent State University analysis of:

- ~8000 exp. data for $\bar{K}N \to \bar{K}N$
- ~4500 exp. data for $\bar{K}N \to \pi\Lambda$
- ~5000 exp. data for $\bar{K}N \to \pi\Sigma$

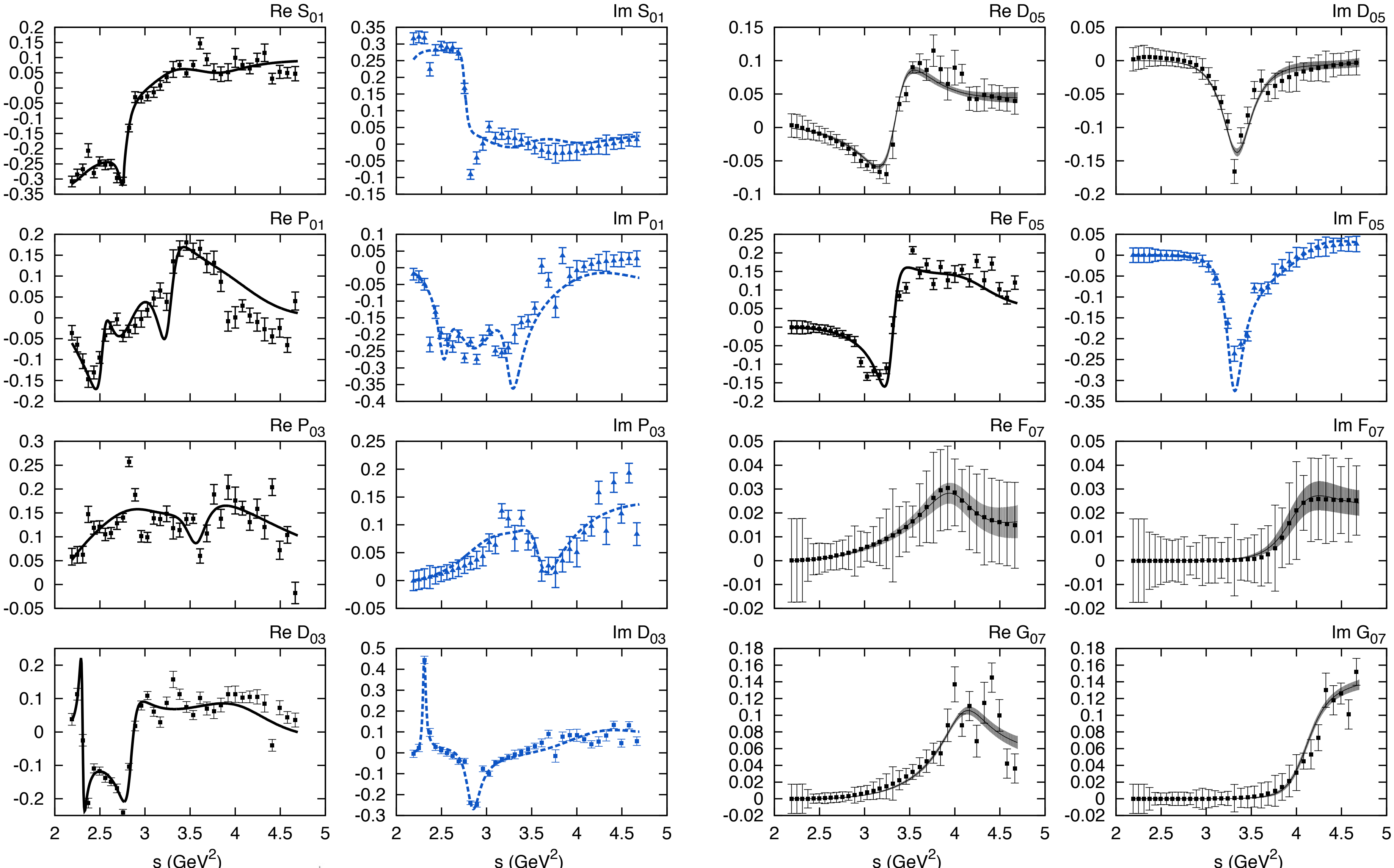
TABLE I. Summary of the fitted single-energy partial waves. Notation: n_R: number of resonances; n_B: number of backgrounds; n_C: number of channels; N: number of fitted single-energy points; $dof = N - n_p$: degrees of freedom.

L_{IJ}	n_R	n_B	n_C	N	n_P	dof	χ^2/N	χ^2/dof
S_{01}	4	2	7	360	41	319	2.09	2.36
P_{01}	4	2	6	358	42	316	4.70	5.33
P_{03}	2	2	8	508	36	472	1.45	1.56
D_{03}	3	1	6	372	28	344	1.83	1.98
D_{05}	2	1	5	302	18	284	0.64	0.68
F_{05}	2	1	8	460	27	433	2.14	2.27
F_{07}	1	1	4	208	10	198	0.08	0.09
G_{07}	1	1	6	350	14	336	0.65	0.68
S_{11}	4	3	10	546	77	469	3.08	3.59
P_{11}	2	3	9	546	50	496	1.48	1.63
P_{13}	2	2	11	722	60	662	0.55	0.60
D_{13}	1	2	13	814	42	772	0.77	0.81
D_{15}	2	1	11	714	36	678	1.52	1.60
F_{15}	2	2	12	782	52	730	0.11	0.12
F_{17}	1	1	11	704	24	680	0.37	0.38
G_{17}	1	1	10	580	22	558	0.09	0.09

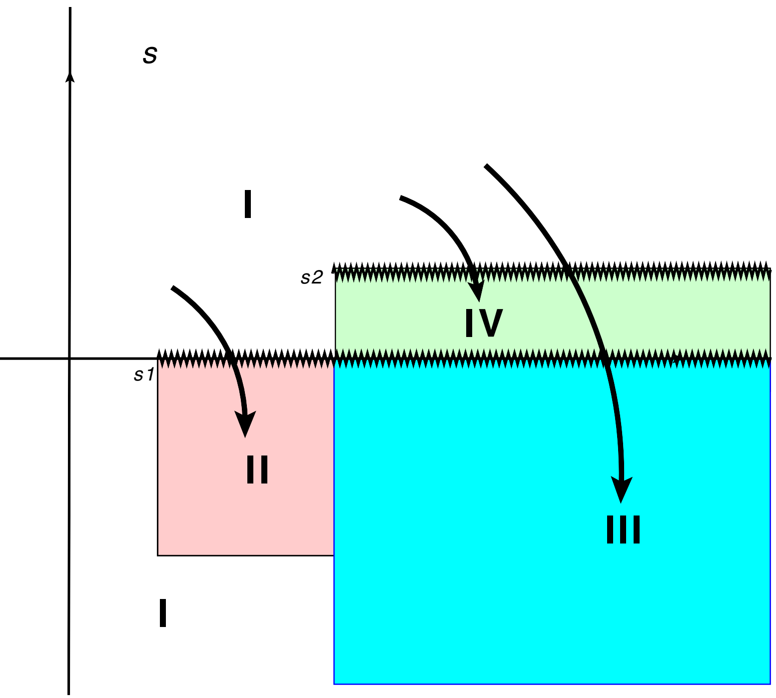
Some Fits: $\bar{K}N \rightarrow \bar{K}N$

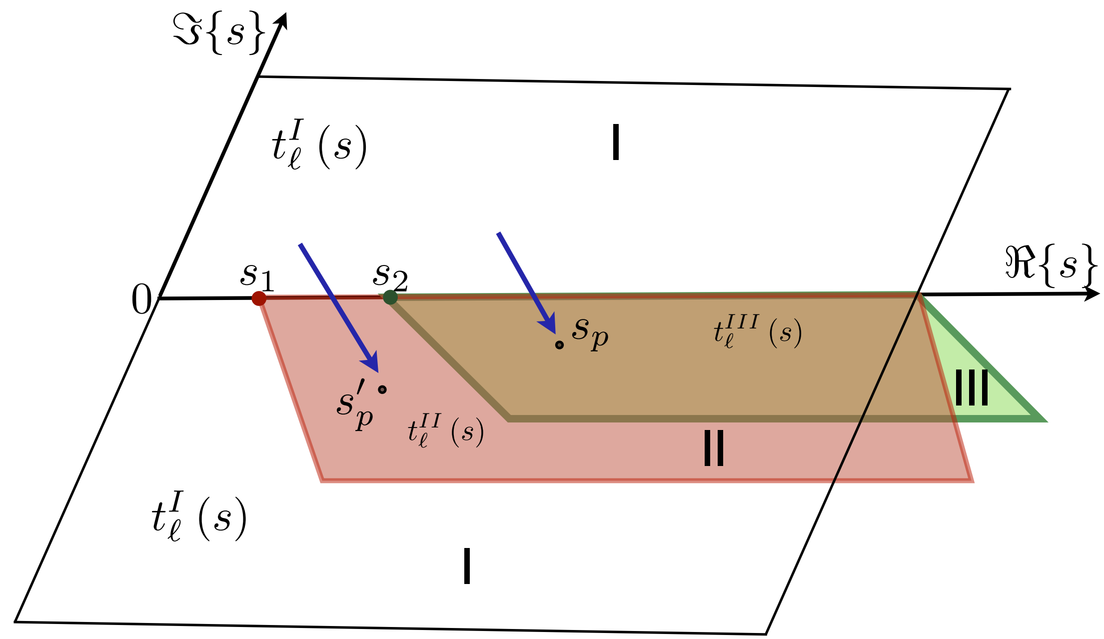


Some Fits: $\bar{K}N \to \pi\Sigma$



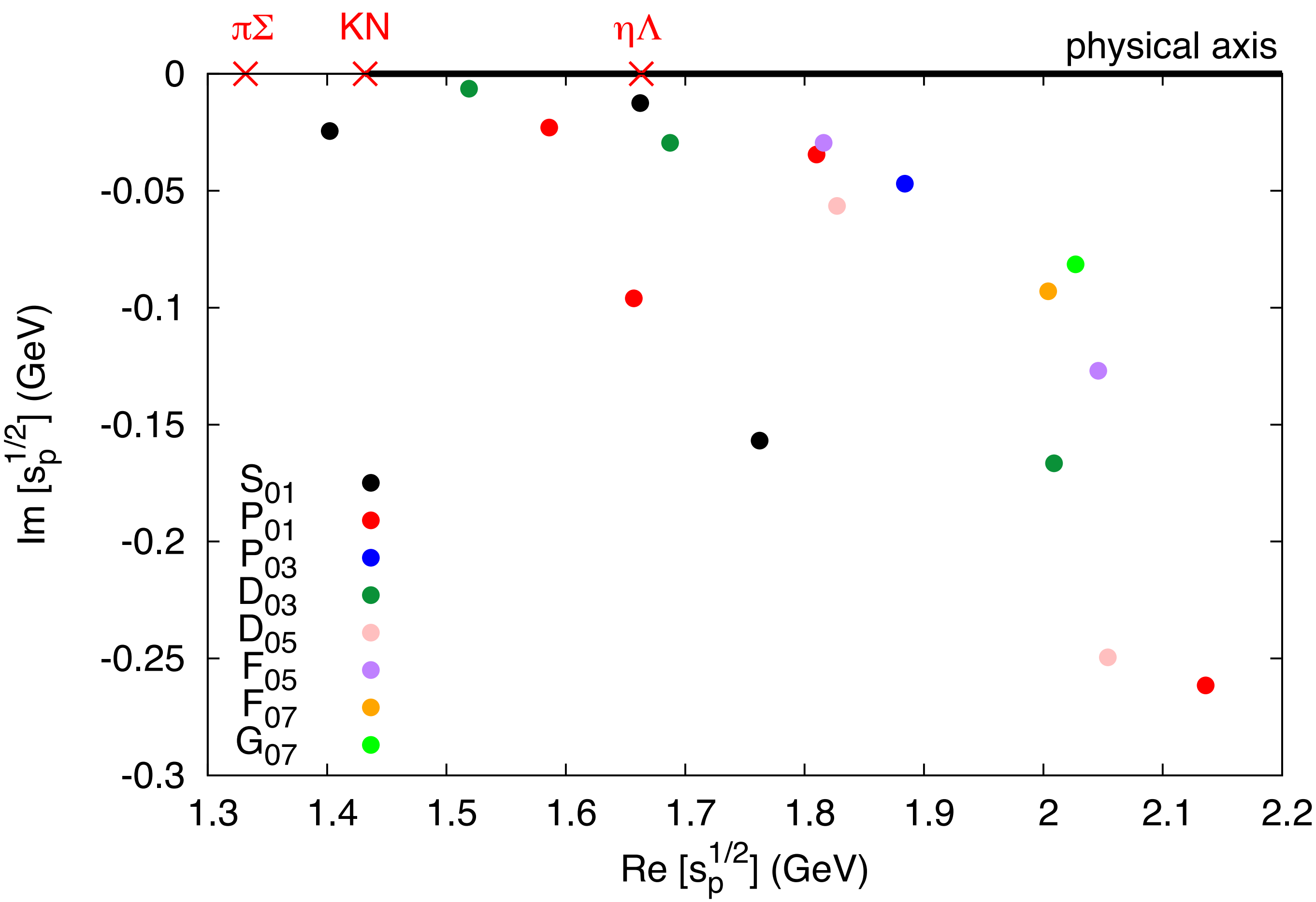
Unphysical Sheets



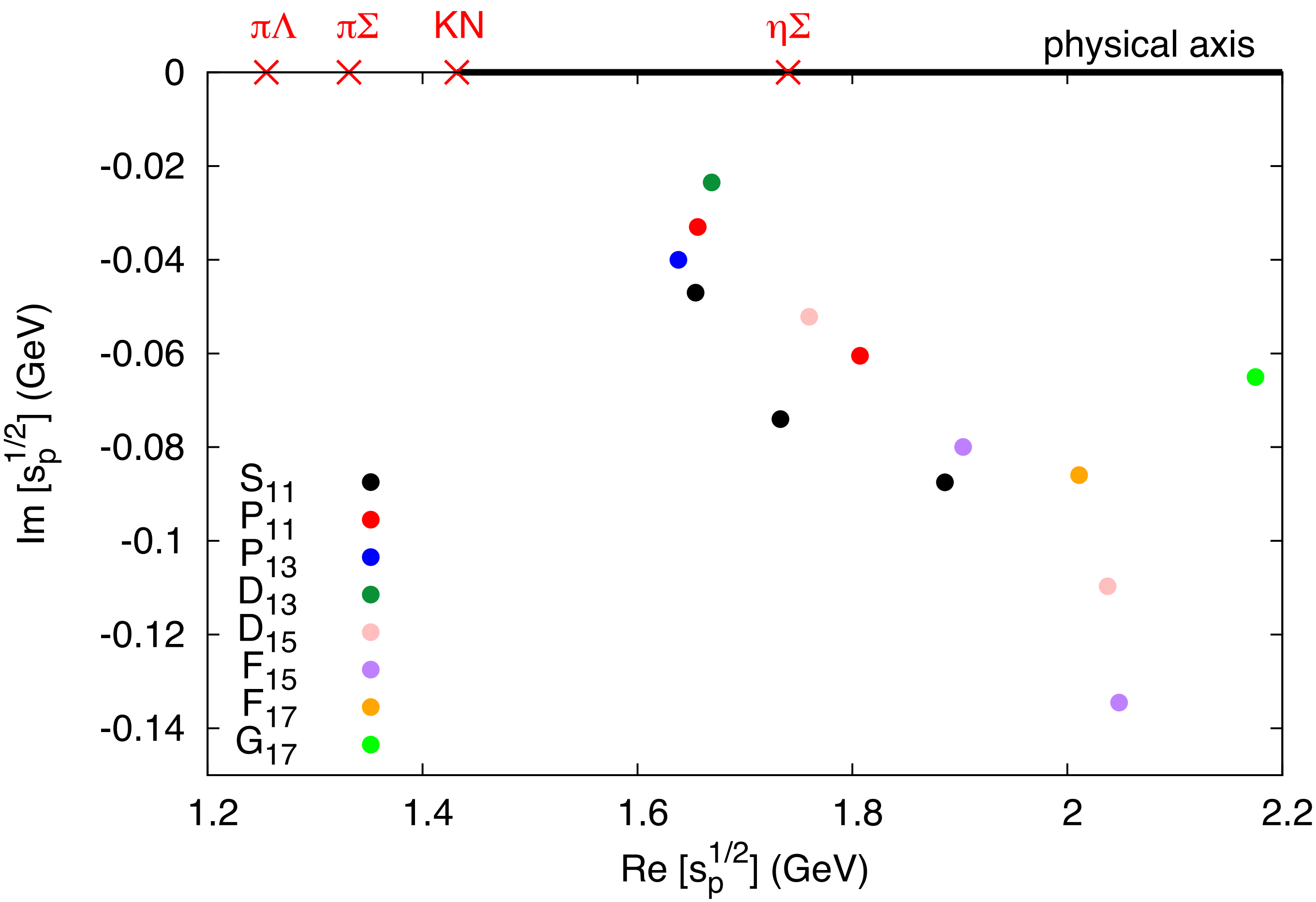


2^N Riemann Sheets

Poles, isospin 0



Poles, isospin 1



Error-bars: Bootstrap

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1. Randomize the experimental data

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3. Get a set of parameters

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4. Repeat 1→3 until I have enough statistics

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5. Compute observables, poles, …

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2. Fit them
3. Get a set of parameters
4. Repeat 1→3 until I have enough statistics
5. Compute observables, poles, …
6. Apply standard statistical methods

Summary and Future Directions

- Reasonable model for KN amplitude
- Poles obtained as a by-product (not main objective)

- Finalize analyticity implementation
- Refit single-energy partial waves
- Error calculations
- Observables
- Finite Energy Sum Rules

- Reggeization of incoming kaon
- Photon-Reggeon-Kaon vertex