Analyticity Constraints in Hadron Spectroscopy:

Example on Pion-Nucleon Scattering and Prospects on Beam Fragmentation

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Future Directions in Spectroscopy Analysis
JLab November 2014

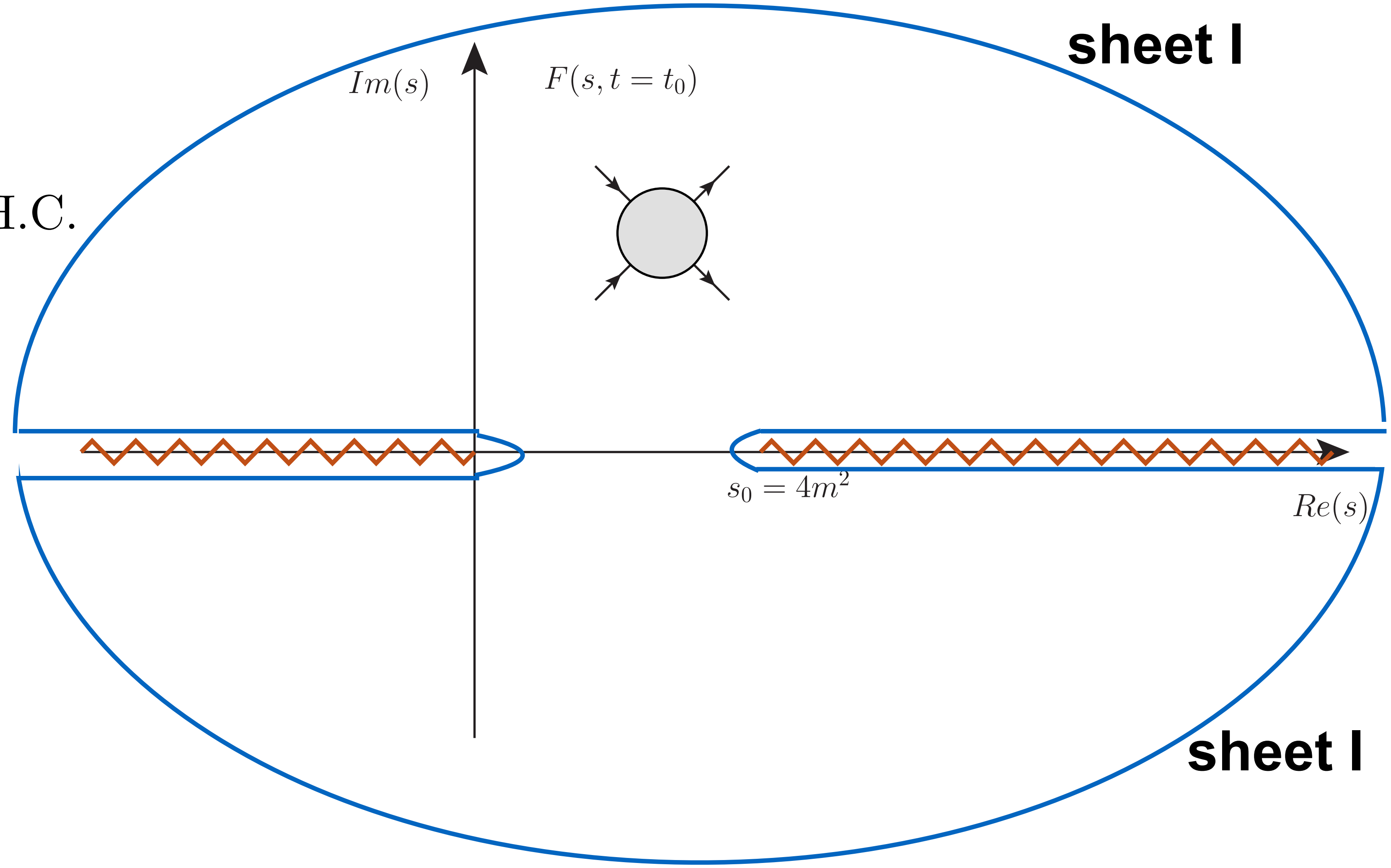
INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Analytic Structure of Amplitudes

Analyticity:

$$A(s,t) = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\text{Im }A(s',t)}{s'-s} ds + \text{L.H.C.}$$



Unitarity:

$$\text{Im } A_\ell(s) = \rho(s)|A_\ell(s)|^2$$

$$\rho(s) = \sqrt{1 - 4m^2/s}$$

Continuation on sheet II:

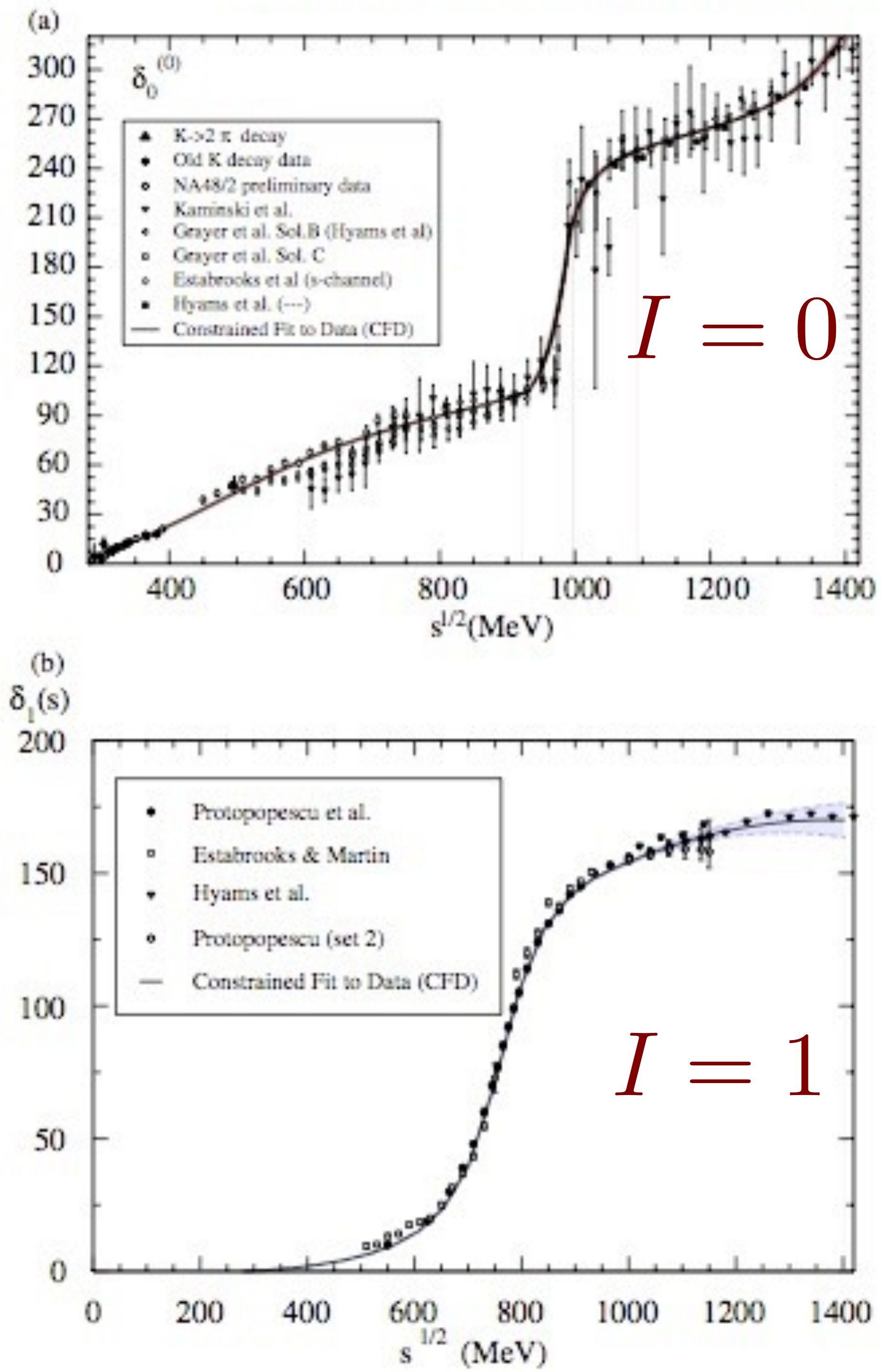
$$A_\ell^{II}(s) = \frac{A^I(s)}{1 - 2i\rho(s)A^I(s)}$$

Extraction of poles

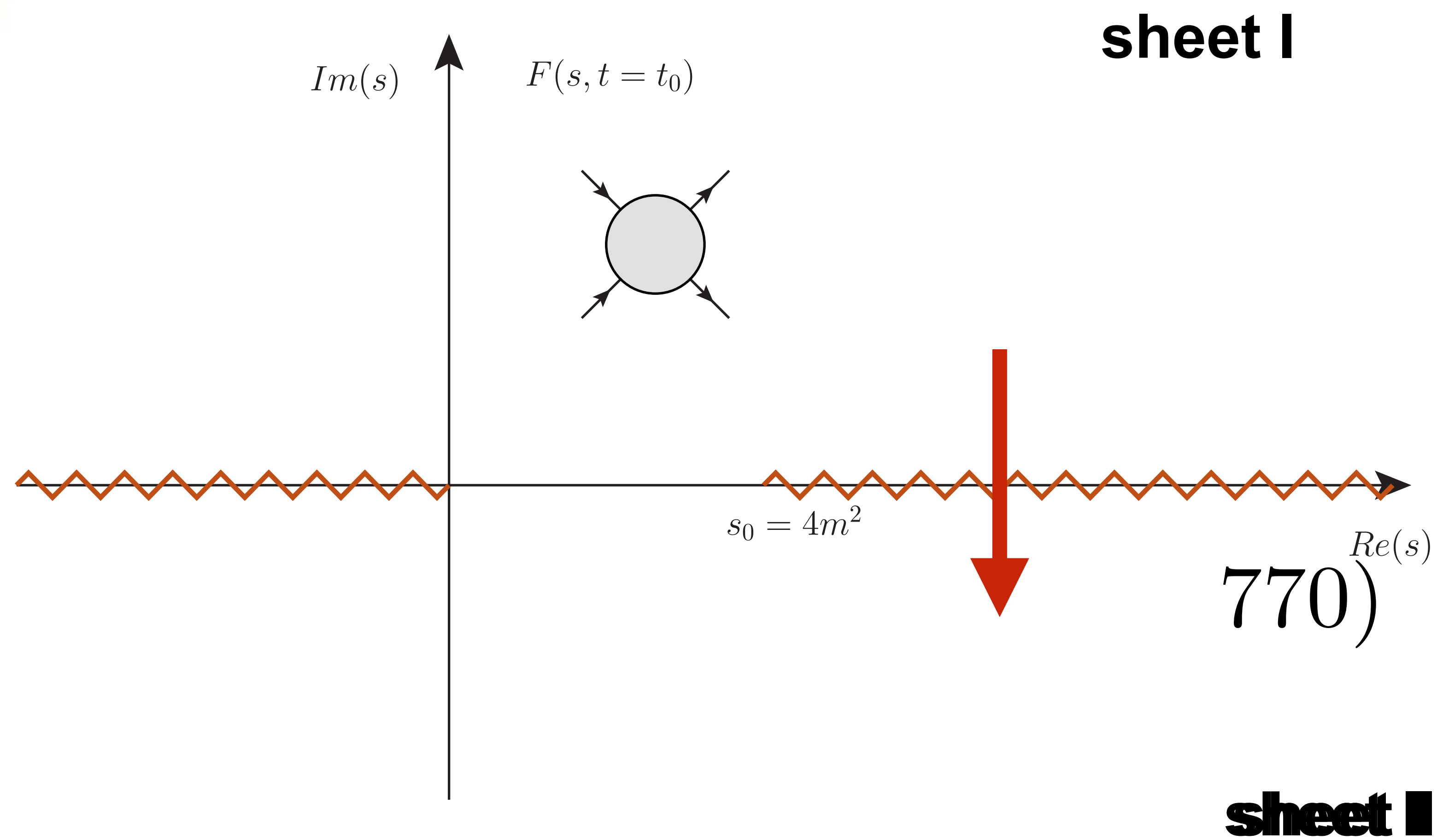
Procedure:
Amplitudes are
1. **fitted on data**
2. **checked against constraints**
3. **continued on sheet II**

Analytic Structure of Amplitudes

Pelaez PHYSICAL REVIEW D 77, 054015 (2008)



$$\tan\delta(s) = \frac{m\Gamma}{m^2 - s}$$



Unitarity:

$$\text{Im } A_\ell(s) = \rho(s)|A_\ell(s)|^2$$

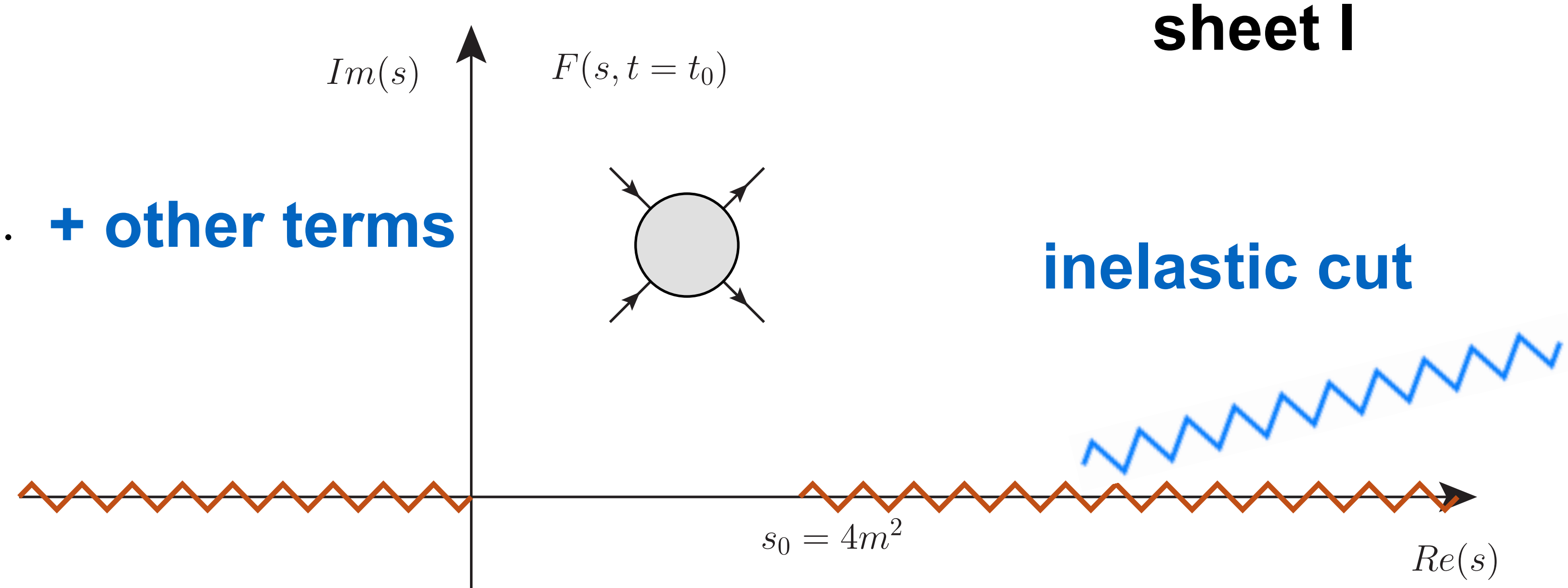
$$A_\ell(s) = \frac{m\Gamma/\rho}{m^2 - s - im\Gamma}$$

Analytic Structure of Amplitudes

sheet I

Analyticity: no data !

$$A(s,t) = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\text{Im } A(s',t)}{s'-s} ds + \text{L.H.C.}$$ + other terms



Unitarity:

$$\text{Im } A_\ell(s) = |A_\ell(s)|^2 \sqrt{1 - 4m^2/s}$$ + other terms

sheet I

Problems:

1. Inelasticities

2. large s data

3. spin

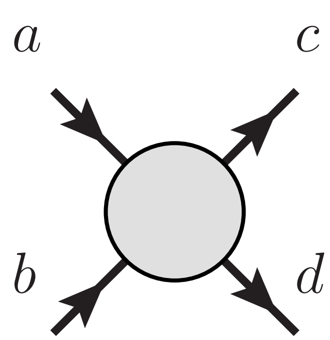
Partial Waves vs Regge

Two different representations of amplitudes

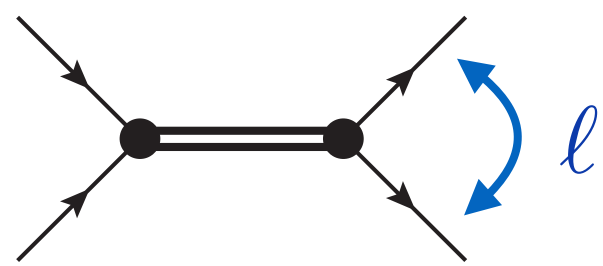
How to take advantages of both ?

Problems:

- **truncated sum**
- **Regge only at high energies**

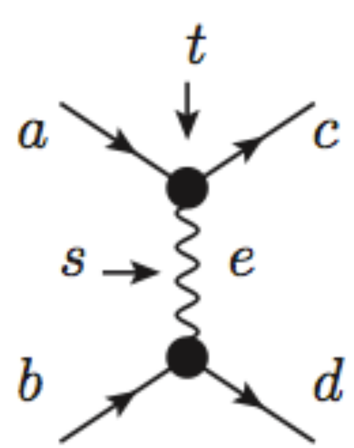


$$= \sum_{\ell=0}^{\infty \to L_{max}}$$



Partial wave series

$=$

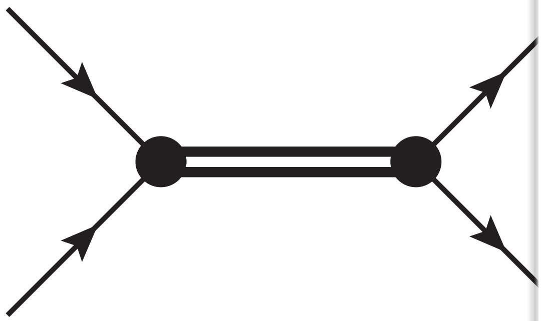


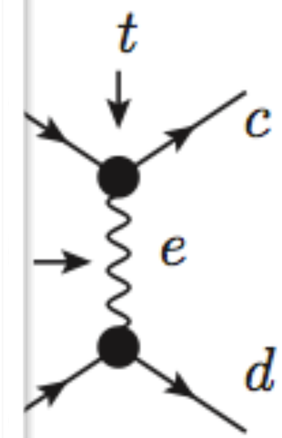
$$+\,\mathcal{O}\left(\frac{1}{\sqrt{s}}\right) \;?$$

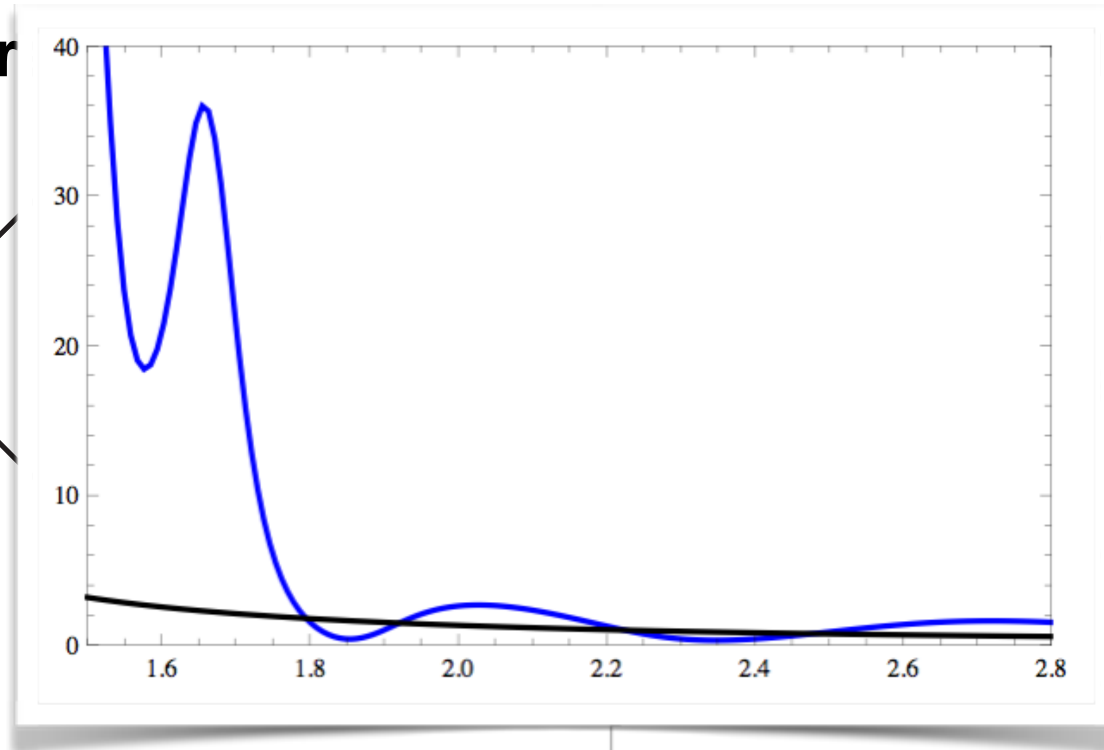
Sum over Regge poles
+ background integral

$\pi^- p \to \pi^0 n$

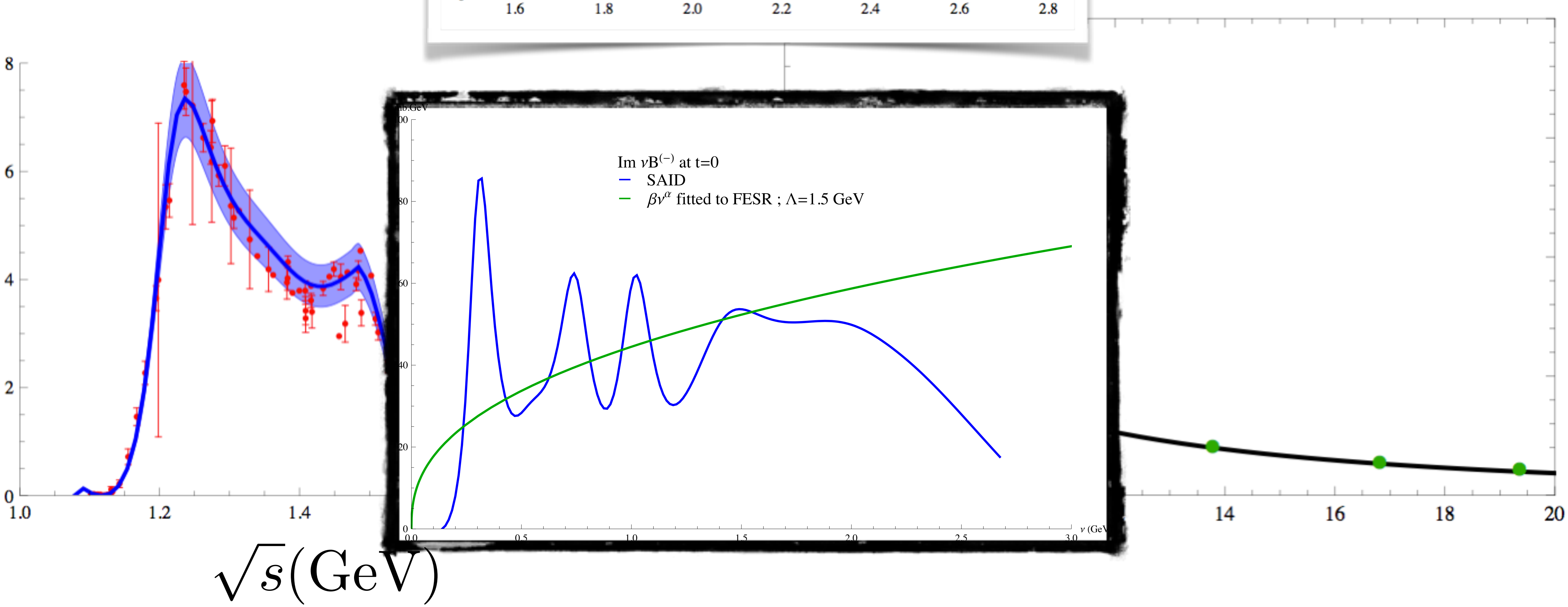
Low energy: **baryon r** … **Regge exchange**



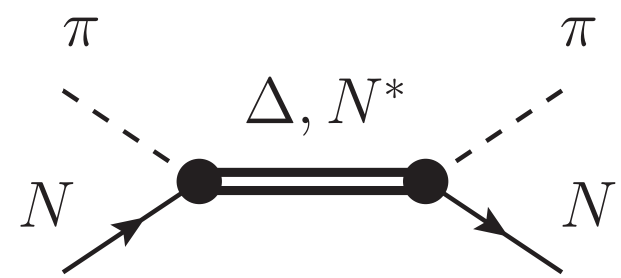




Total cross section



How to Deal with Spin ?



$$T = \bar{u}(p_4, \lambda_4) \left(A + \frac{1}{2} (\not{p}_1 + \not{p}_3)\, B \right) u(p_2, \lambda_2)$$

$$A \equiv A(\nu, t)$$
$$B \equiv B(\nu, t)$$

$$\nu = \frac{s-u}{4m}$$

Same analysis for each invariant amplitude

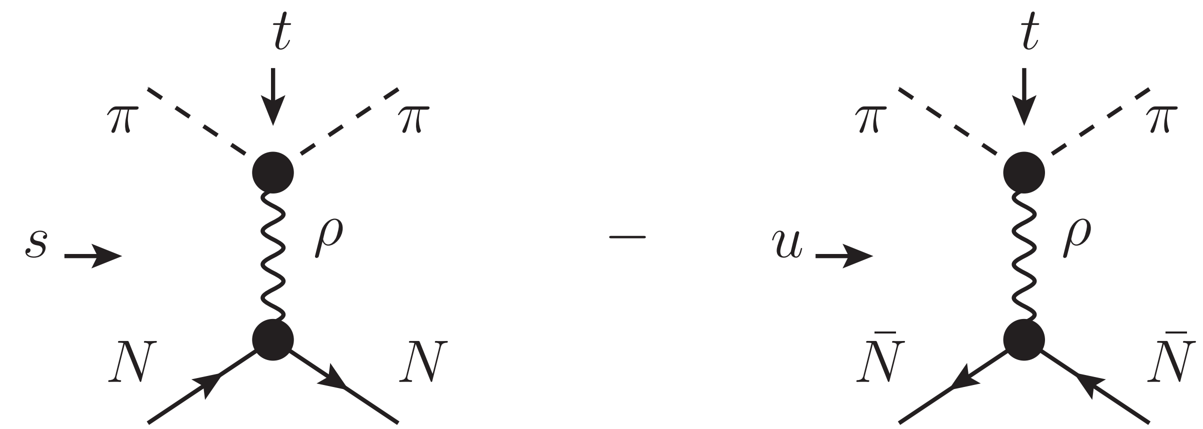
Example with the simplest case:

$$\pi^- p \to \pi^0 n \qquad \pi^\pm p \to \pi^\pm p$$

Regge Pole

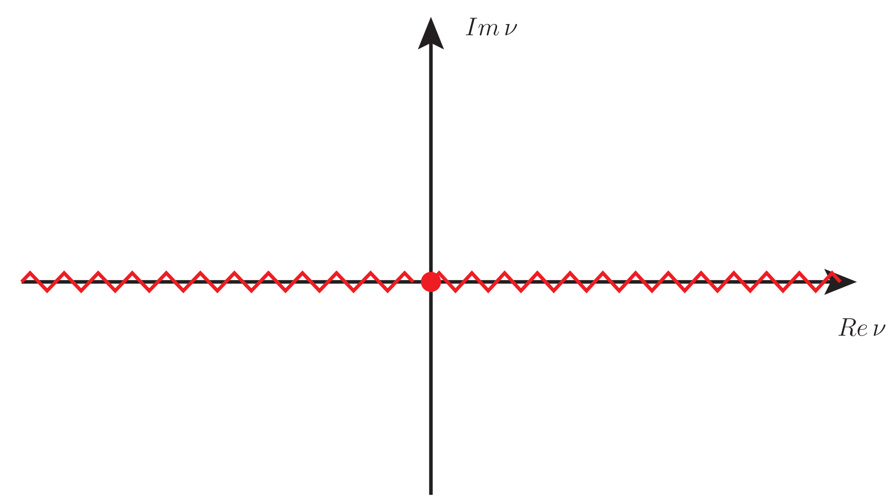
Good symmetry w.r.t crossing variable

$$\nu = \frac{s-u}{4m}$$



$$R_\rho(\nu,t) = \beta(t)\frac{\nu^{\alpha_\rho(t)} - (-\nu)^{\alpha_\rho(t)}}{\sin\pi\alpha_\rho(t)}$$

$$= \beta(t)\frac{1-e^{-i\pi\alpha_\rho(t)}}{\sin\pi\alpha_\rho(t)}\nu^{\alpha_\rho(t)}$$



Satisfy dispersion relations

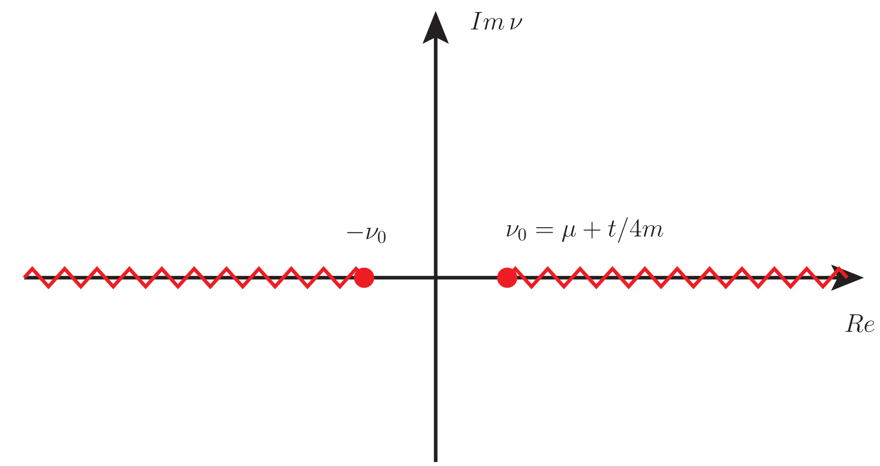
$$R_\rho(\nu,t) = \frac{2\nu}{\pi}\int_0^\infty \frac{\text{Im } R_\rho(\nu',t)}{\nu'^2-\nu^2}d\nu'$$

$$\text{Im } R_\rho(\nu,t) = \beta(t)\nu^{\alpha_\rho(t)}$$

Finite Energy Sum Rules

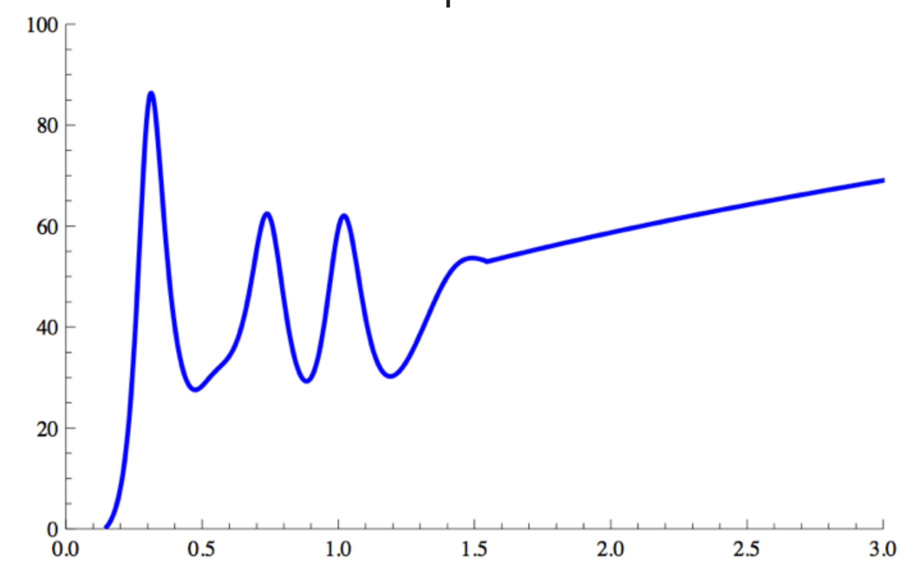
Satisfy dispersion relations

$$A^{(-)}(\nu,t) = \frac{2\nu}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{Im}\ A^{(-)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$



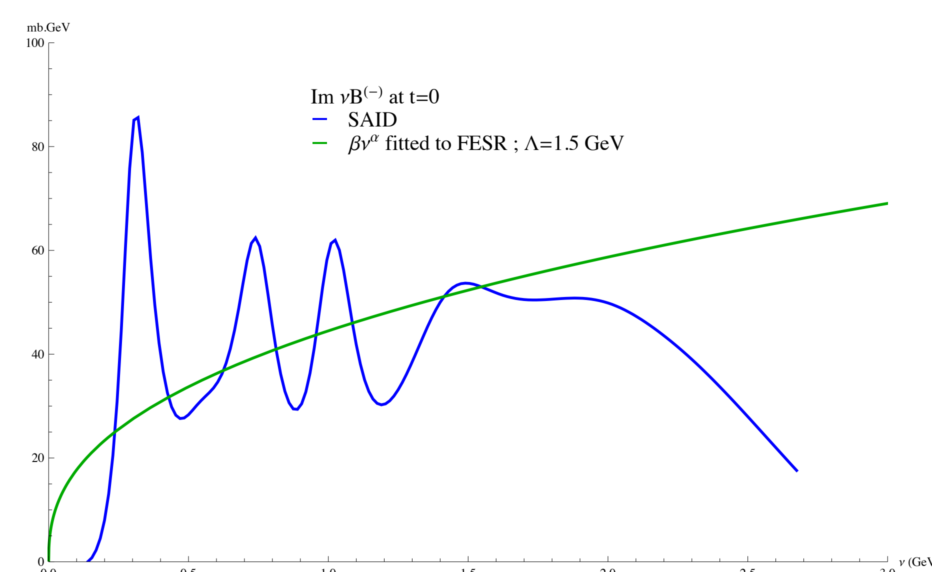
$\nu \to \Lambda$

$$\mathrm{Im}\ A^{(-)}(\nu,t) \longrightarrow \beta(t)\nu^{\alpha_\rho(t)}$$



Analyticity implies FESR

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A^{(-)}(\nu',t)\nu'^{2k}d\nu' = \beta(t)\frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t)+2k+1}$$



Finite Energy Sum Rules

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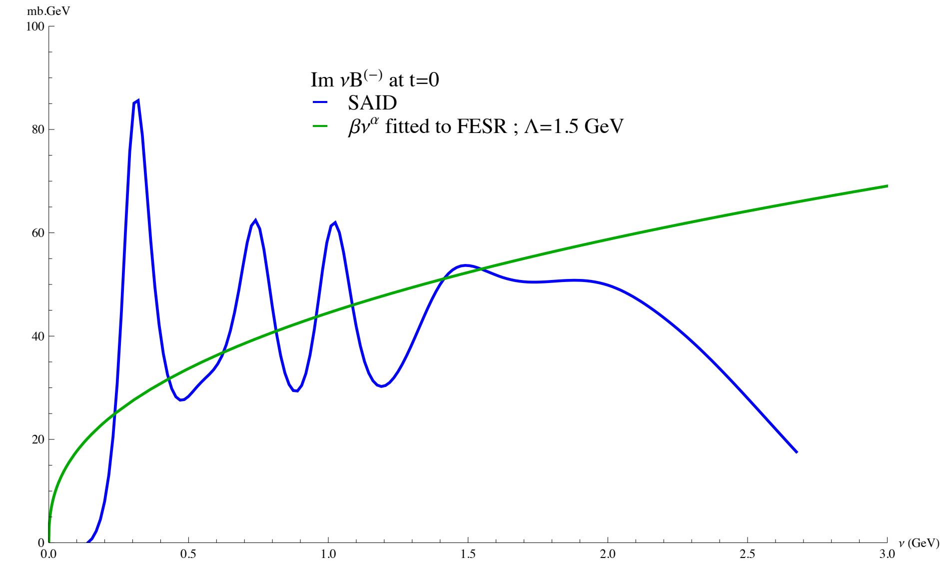


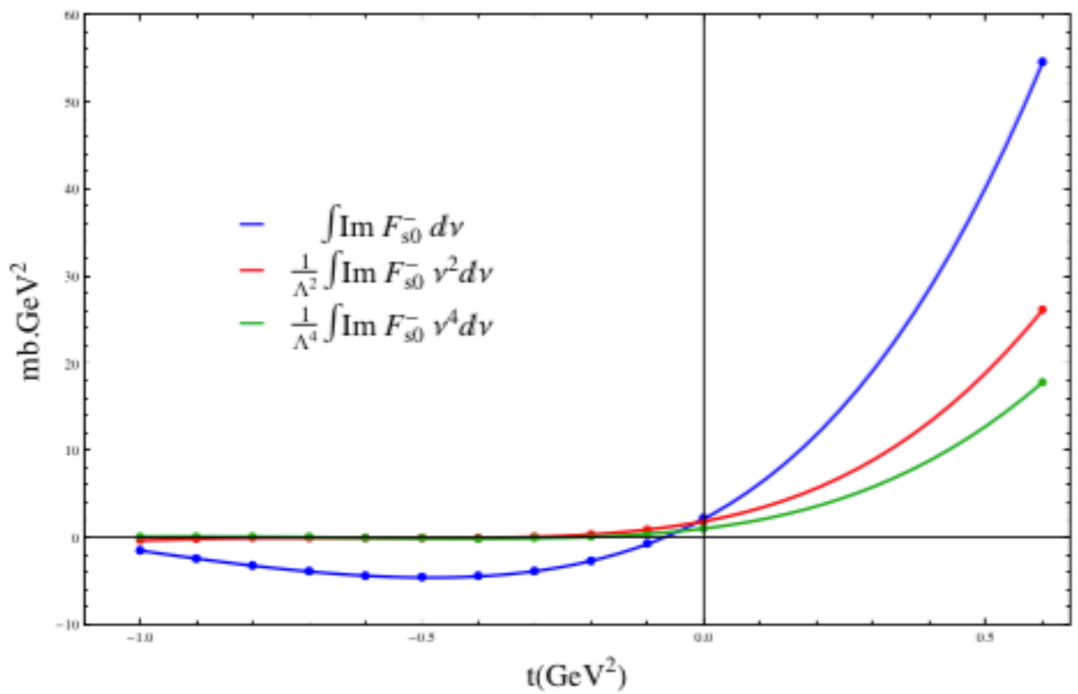
Resonances predict Regge parameters $\alpha(t), \beta(t)$

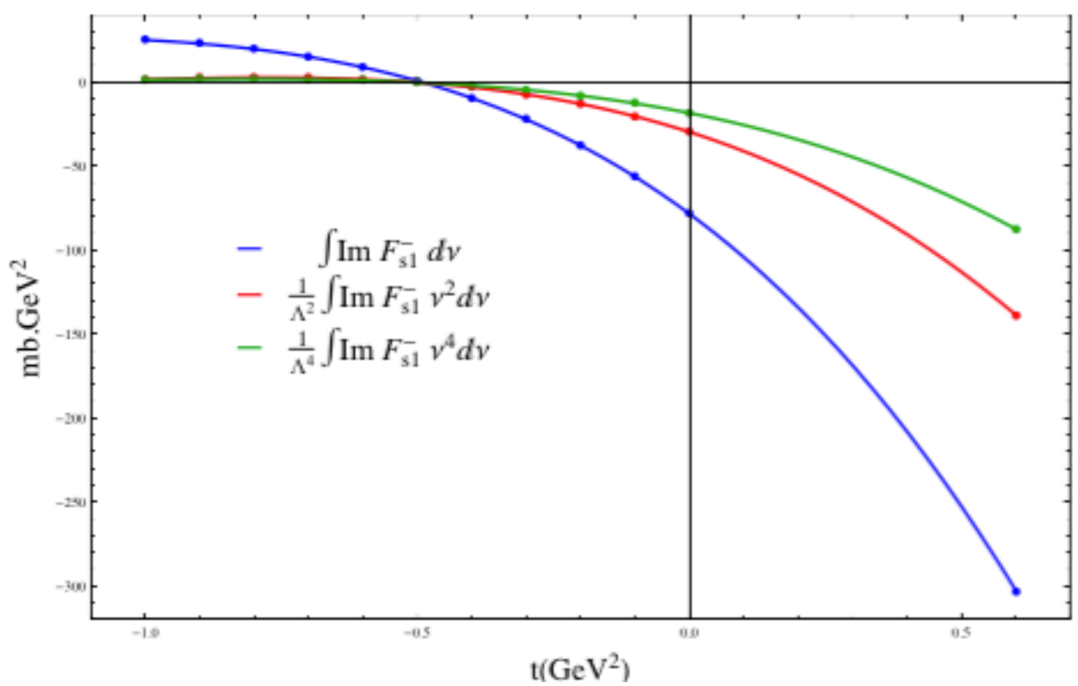
Amplitude is known above Λ

$$A(s,t) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im}\ A(s',t)}{s'-s} ds + \mathrm{L.H.C.}$$

Only Im A(s,t) for $s < \Lambda$ is needed

Isovector: ρ



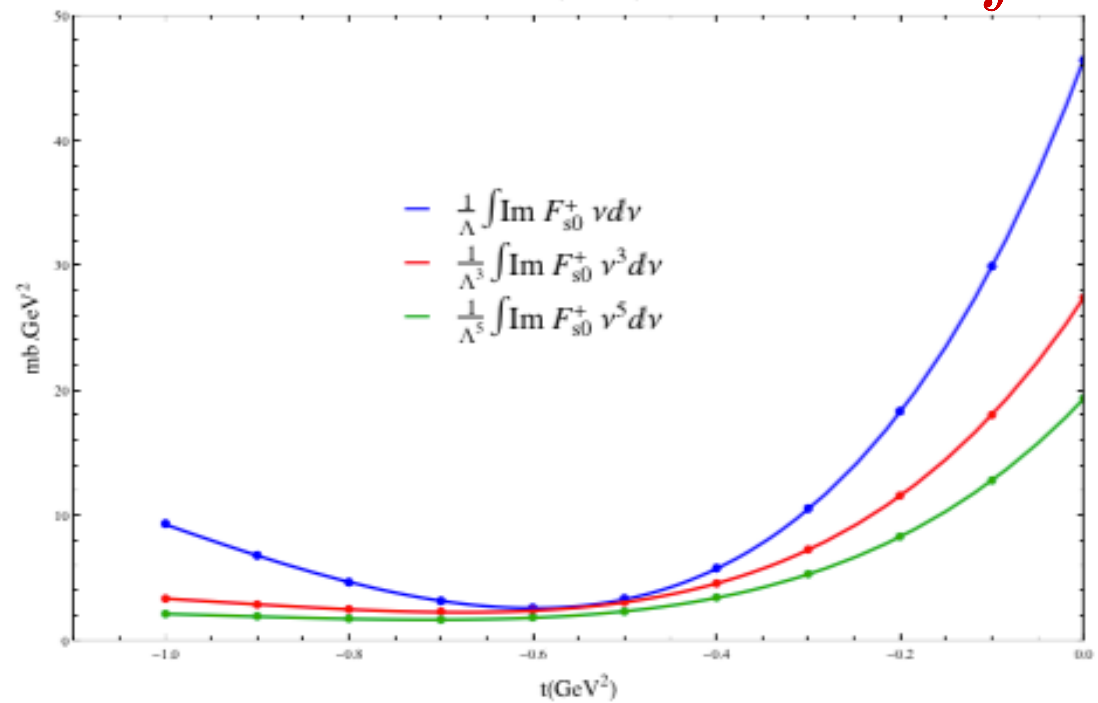


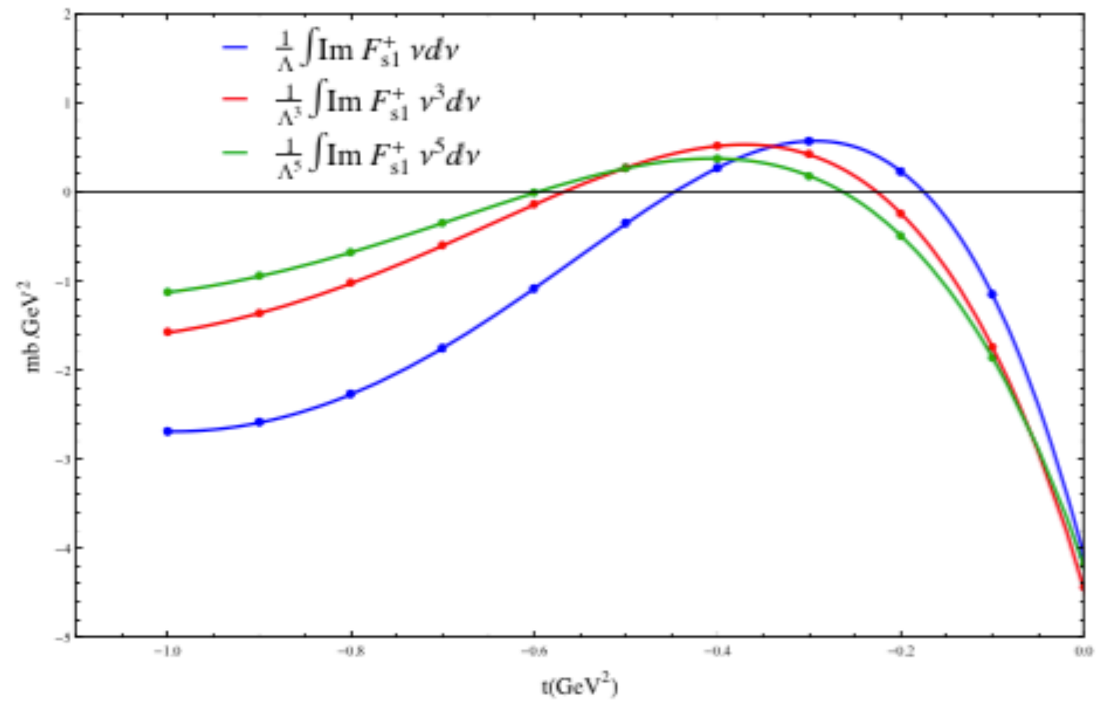
$$T^{\rho}_{++} = -\beta^{\rho}_{++}(t)\frac{\pi(1+\gamma t)}{\Gamma(\alpha_\rho+1)}\frac{e^{-i\pi\alpha_\rho}-1}{2\sin\pi\alpha_\rho}\nu^{\alpha_\rho}$$

$$T^{\rho}_{+-} = -\frac{\sqrt{-t}}{2M}\beta^{\rho}_{+-}(t)\frac{\pi}{\Gamma(\alpha_\rho)}\frac{e^{-i\pi\alpha_\rho}-1}{2\sin\pi\alpha_\rho}\nu^{\alpha_\rho}$$

SAID: WI08 R. Workman et al. 2008

Isoscalar: $\mathbb{P}+f$



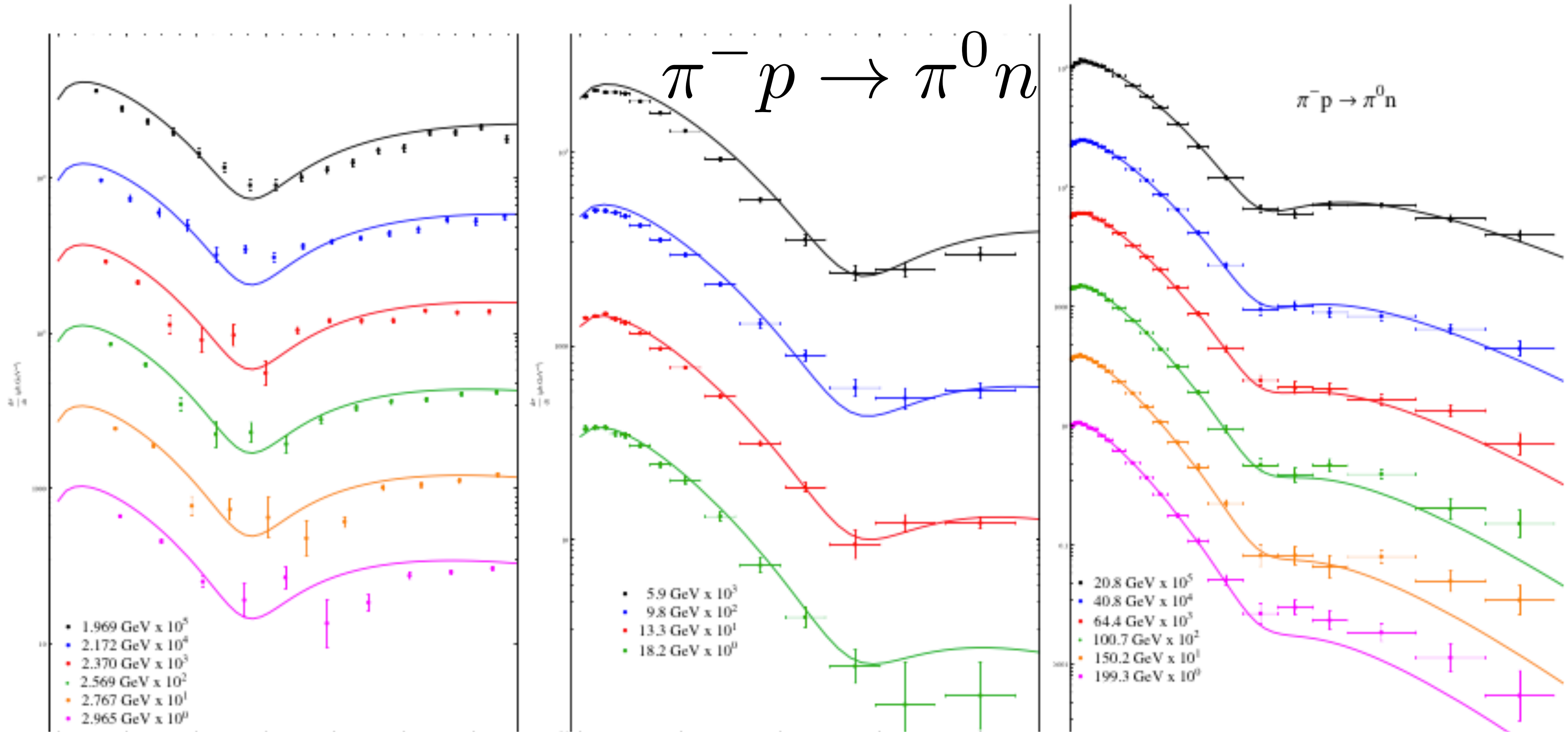


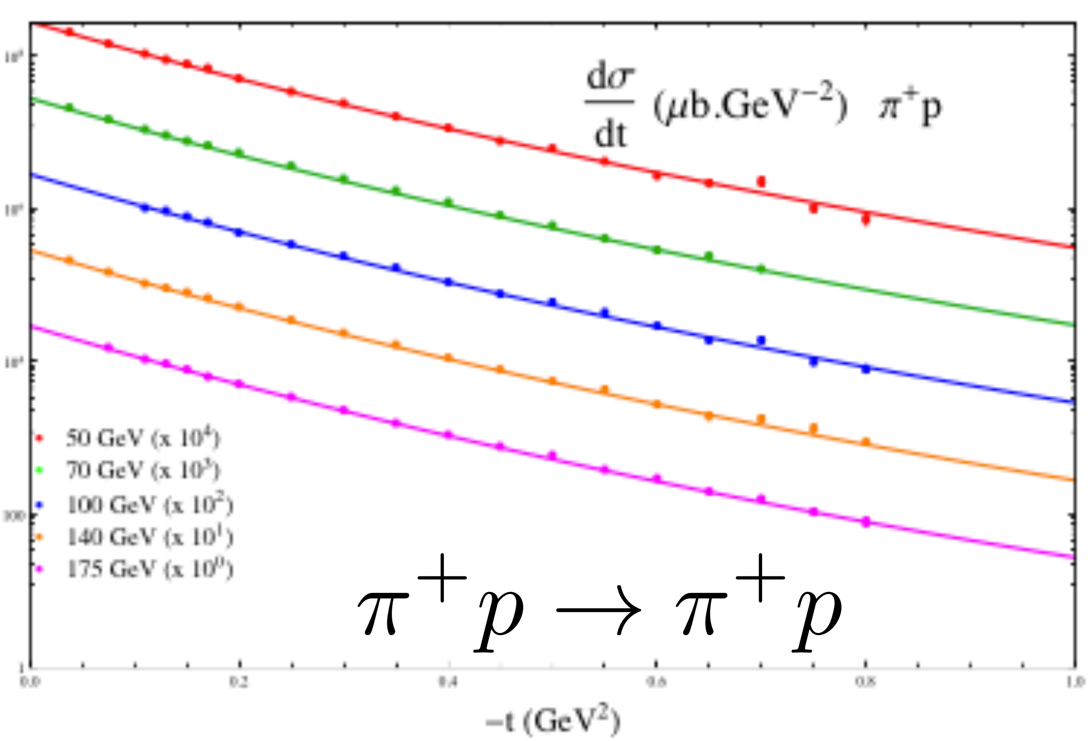
$$T^{f}_{++} = -\beta^{f}_{++}(t)\frac{\pi}{\Gamma(\alpha_f)}\frac{e^{-i\pi\alpha_f}+1}{2\sin\pi\alpha_f}\nu^{\alpha_f}$$

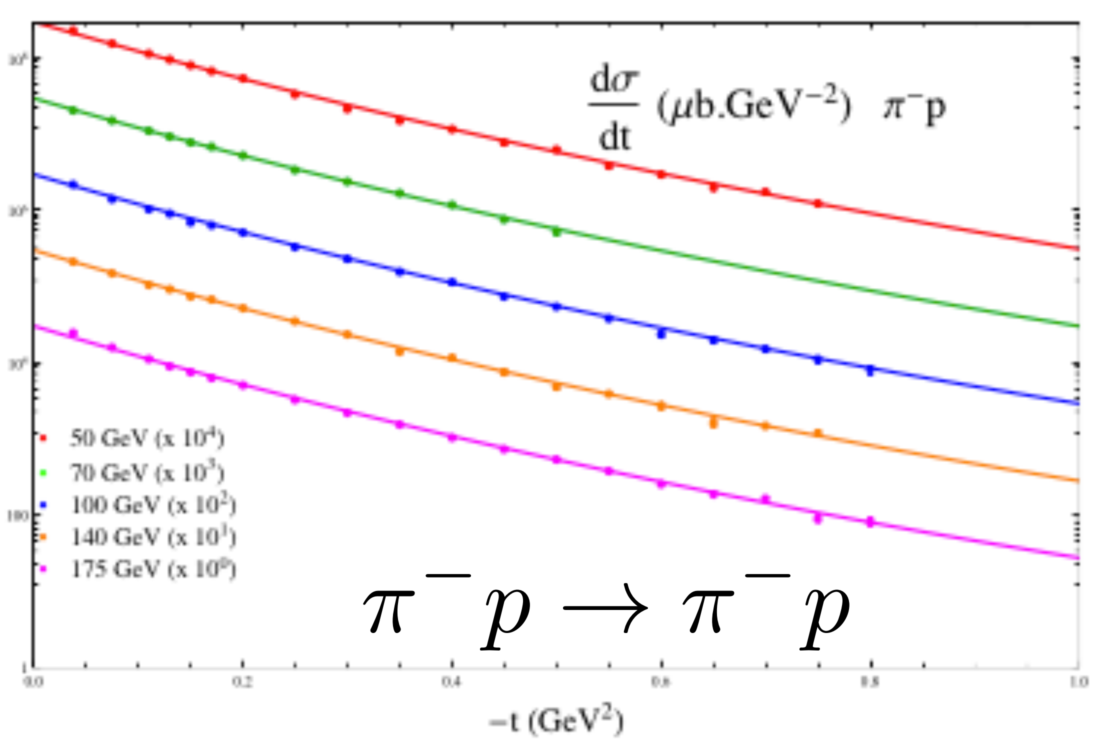
$$T^{\mathbb{P}}_{++} = -\beta^{\mathbb{P}}_{++}(t)\frac{\pi}{\Gamma(\alpha_{\mathbb{P}})}\frac{e^{-i\pi\alpha_{\mathbb{P}}}+1}{2\sin\pi\alpha_{\mathbb{P}}}\nu^{\alpha_{\mathbb{P}}}$$

$$T^{f}_{+-} = T^{\mathbb{P}}_{+-} = 0$$

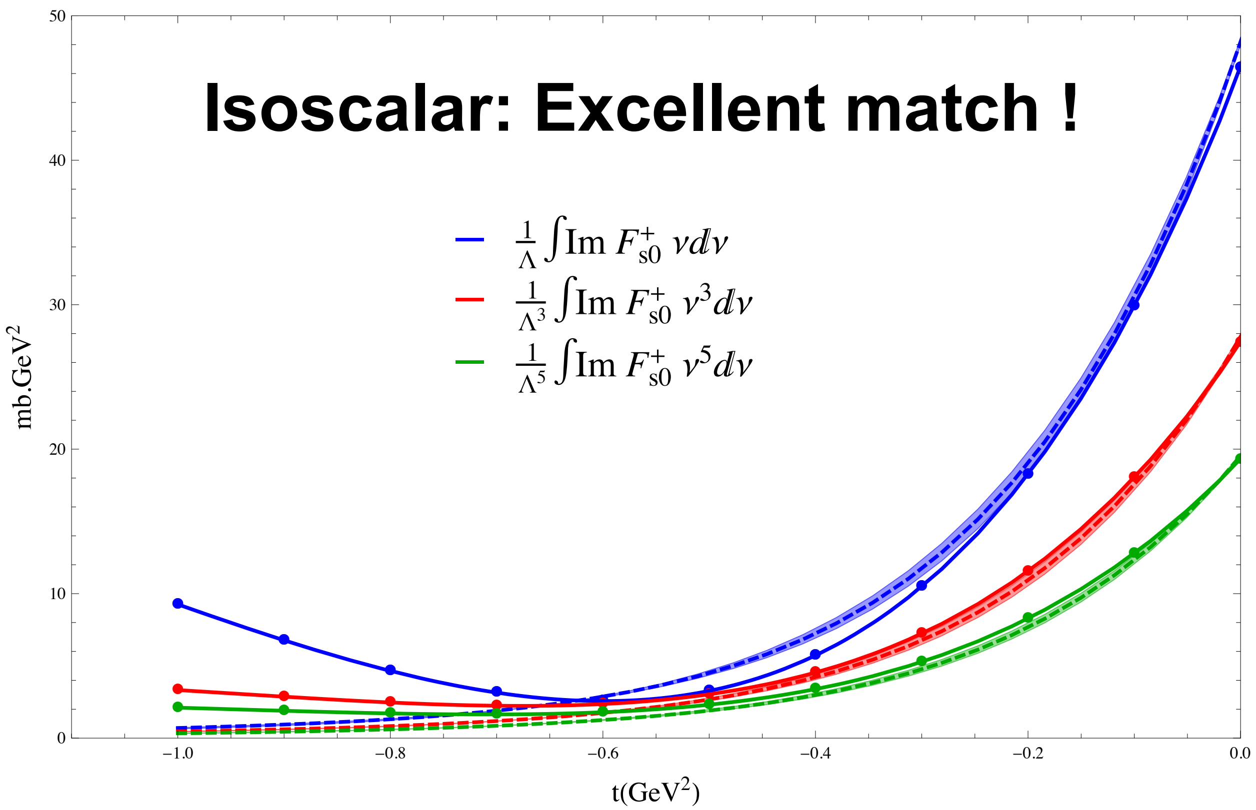
Parametrization of High Energy Data

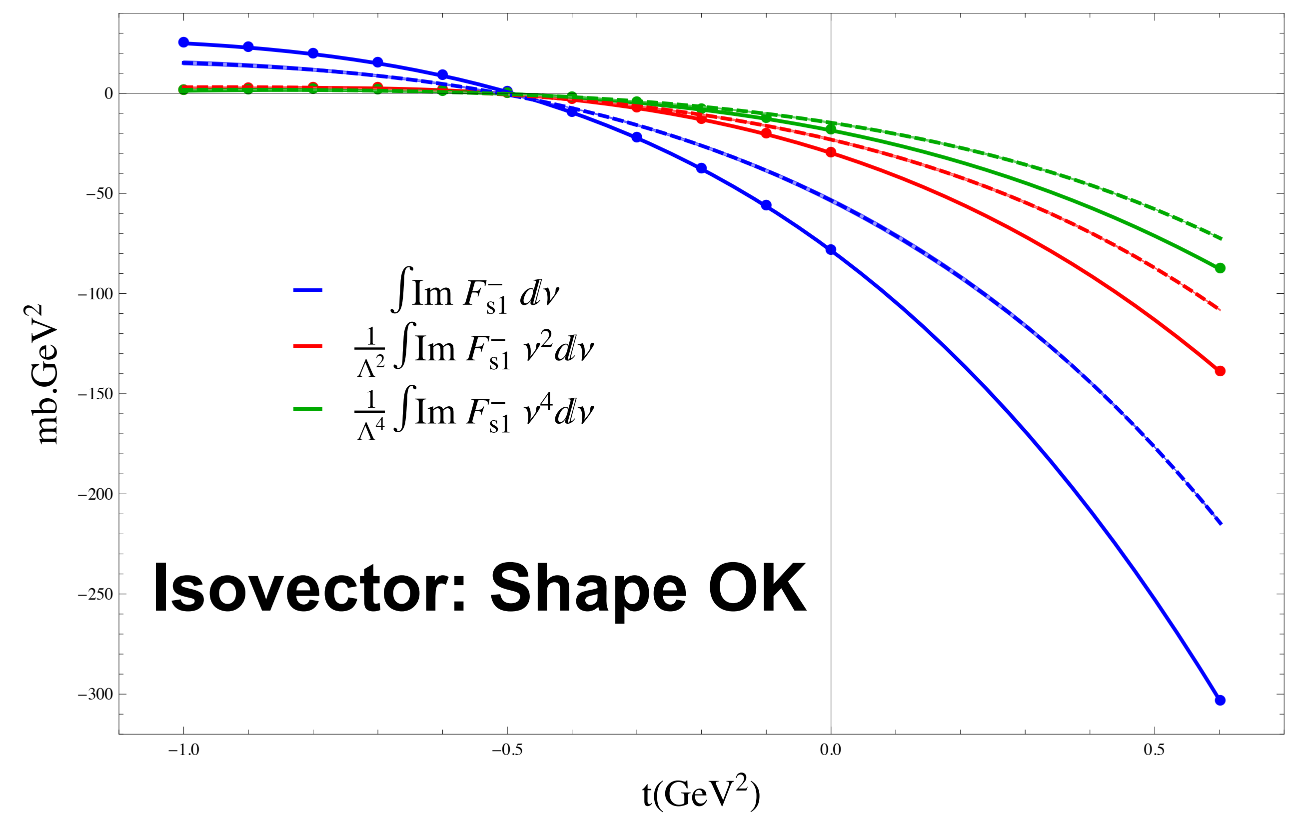


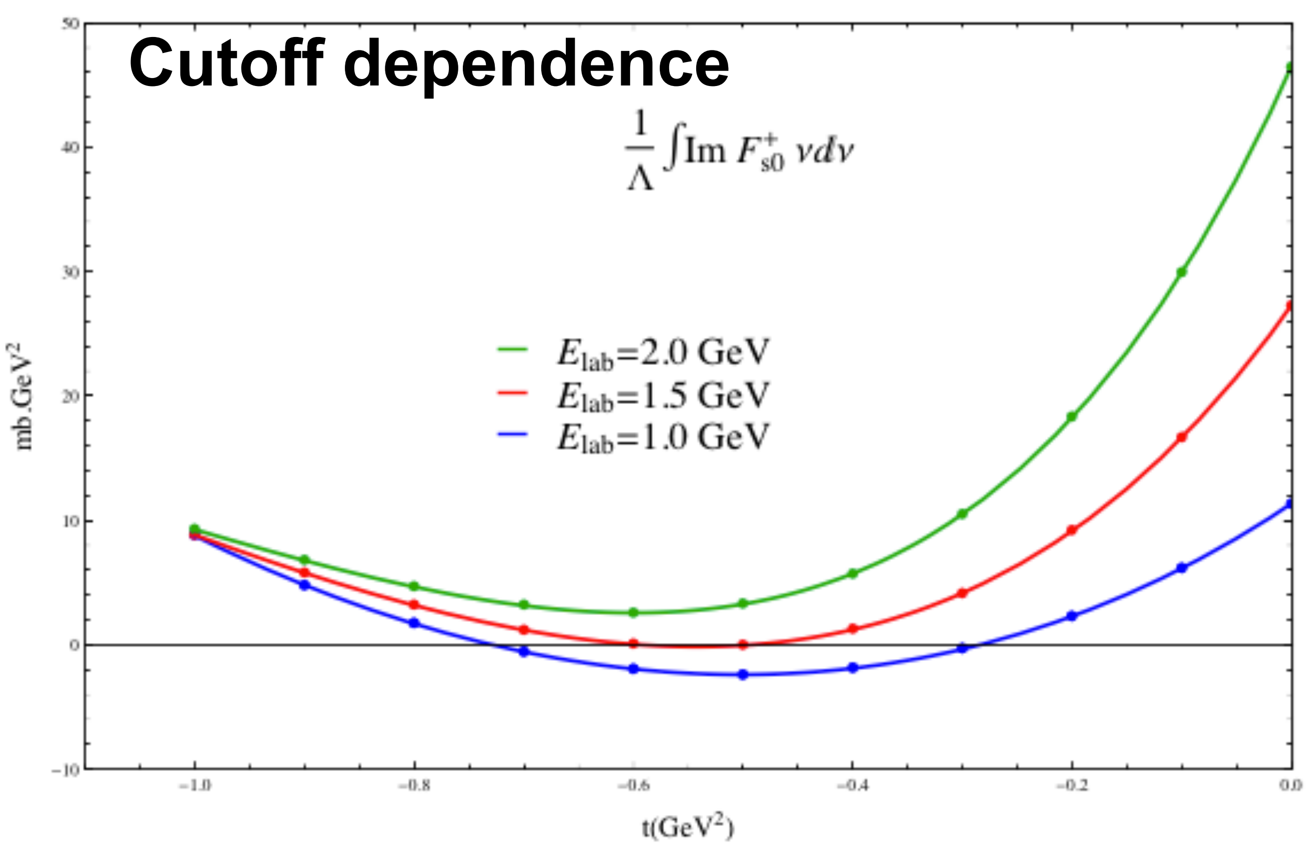


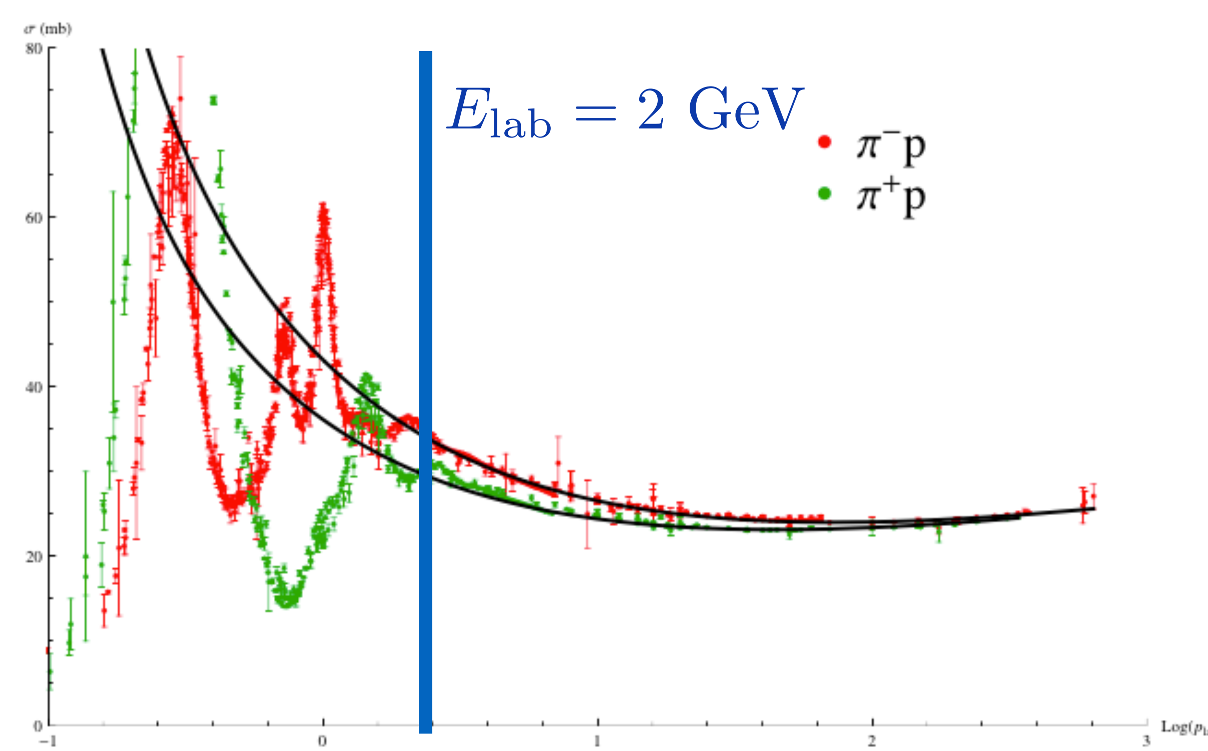


Comparison between Low and High Energy Amplitudes









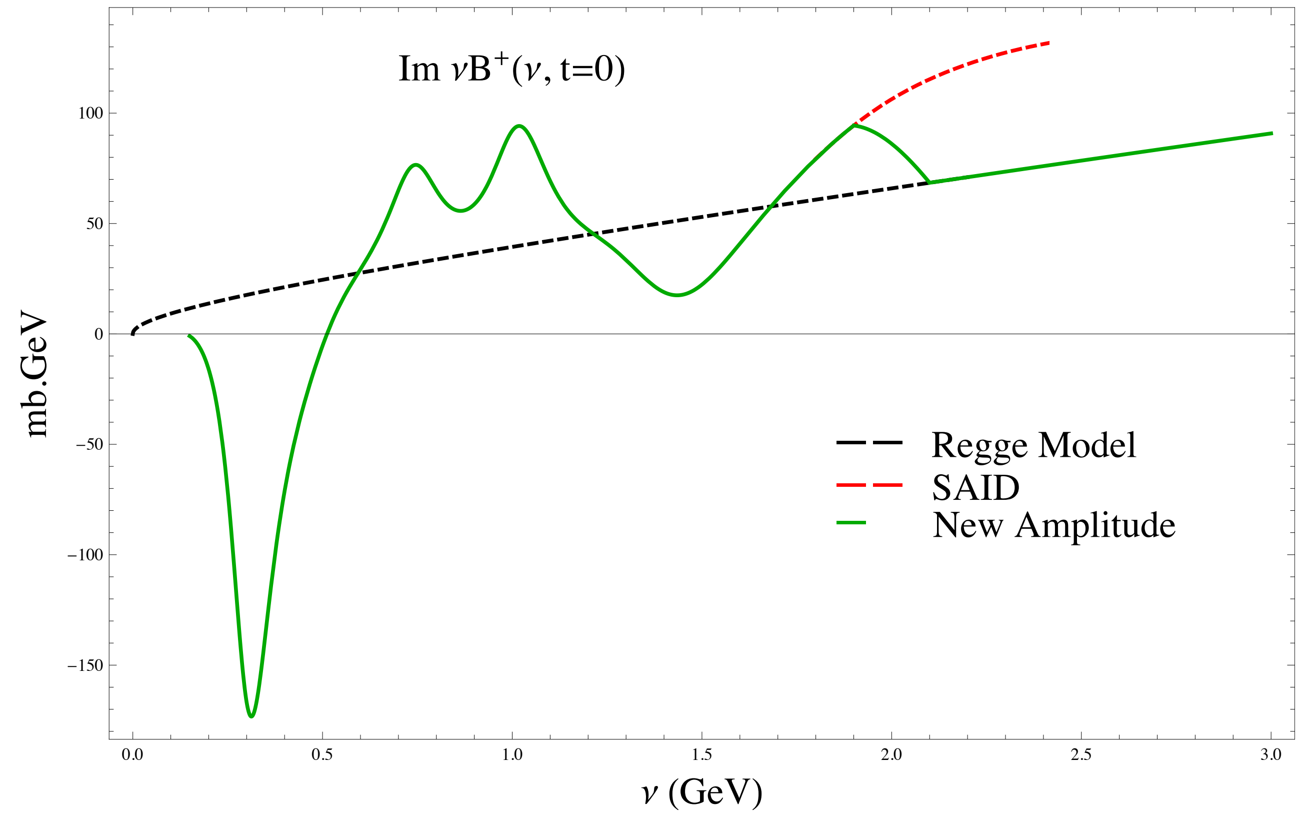
VM et al. (in preparation)

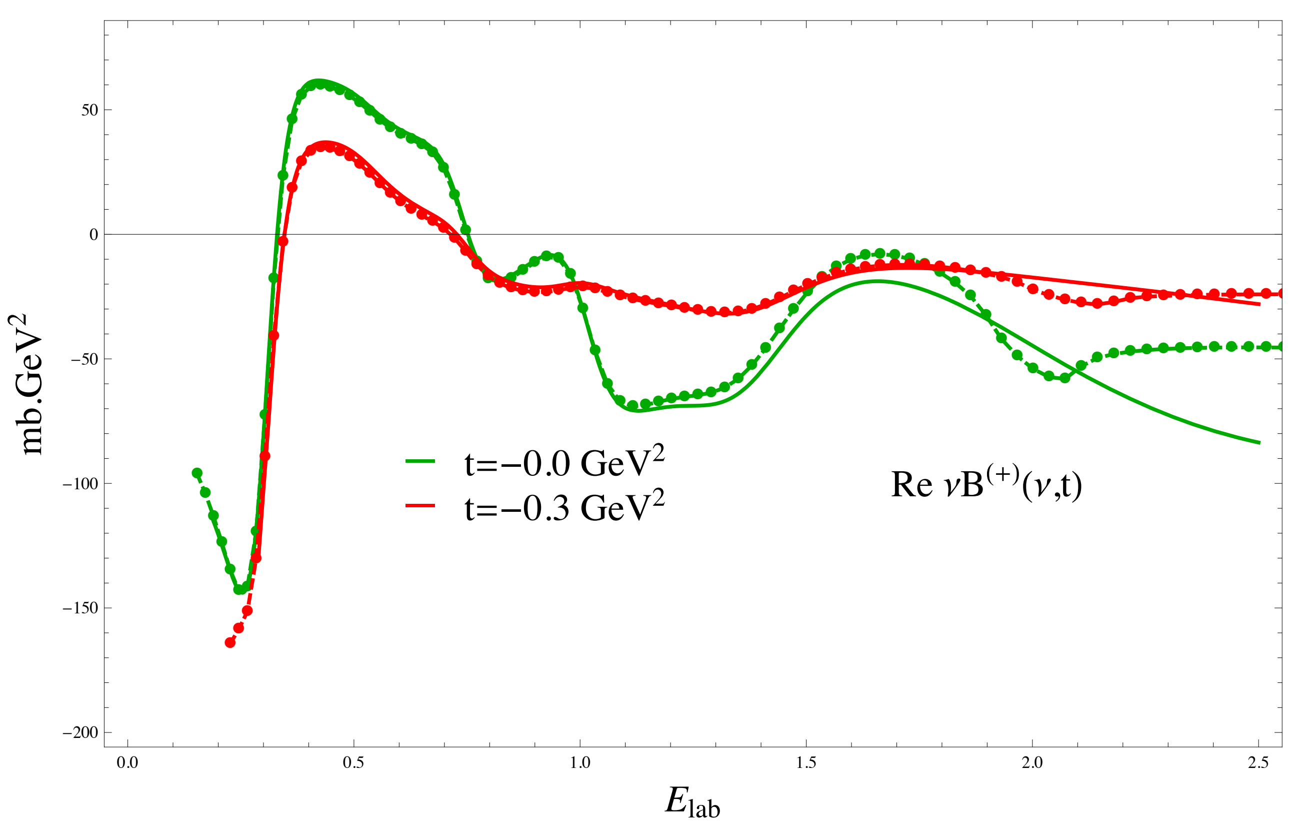
Finite Energy Sum Rules

$$\pi N \to \pi N$$

Construct Im(amplitude) from 0 to infinity via FESR
Reconstruct Re(amplitude) from dispersion relation

$$\text{Re } \nu B^{(+)}(\nu,t) = \frac{g_r^2}{2m}\frac{2\nu^2}{\nu_m^2-\nu^2} + \frac{2\nu^2}{\pi}\text{P}\int_{\nu_0}^{\infty}\frac{\text{Im } B^{(+)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$

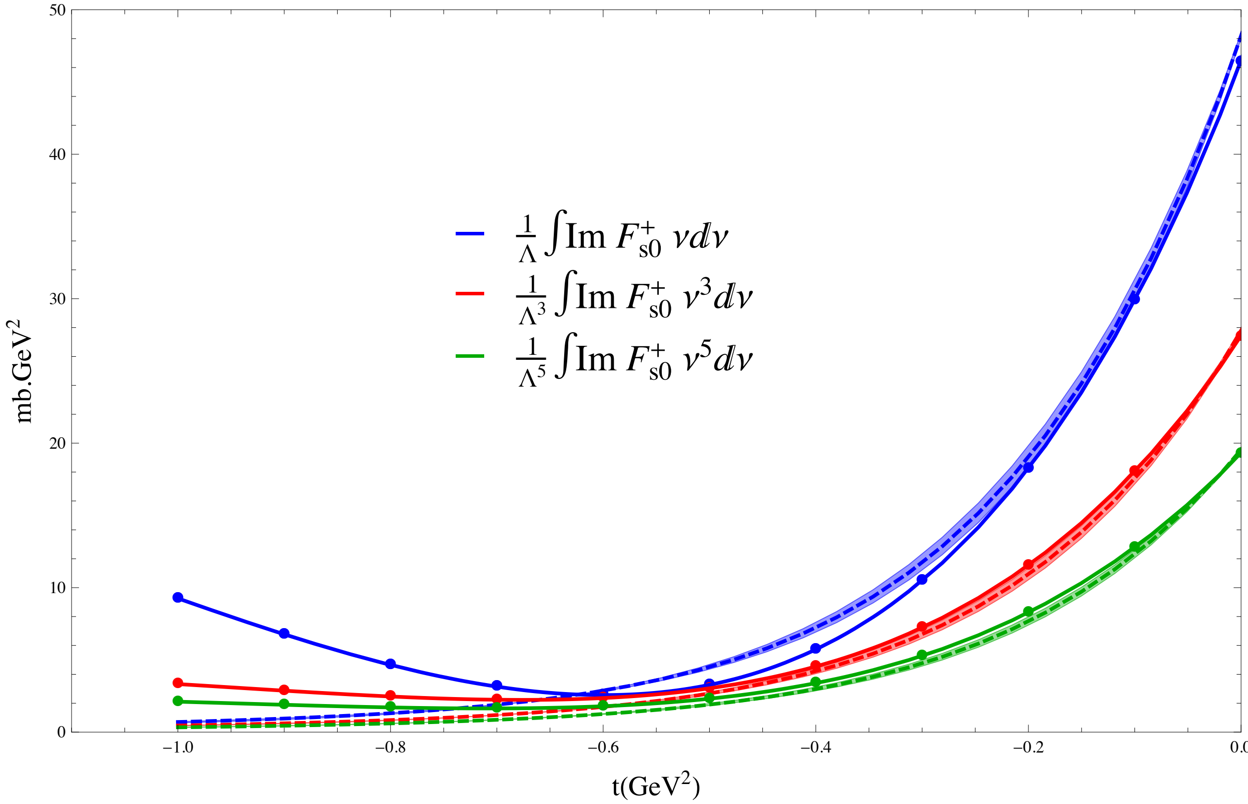


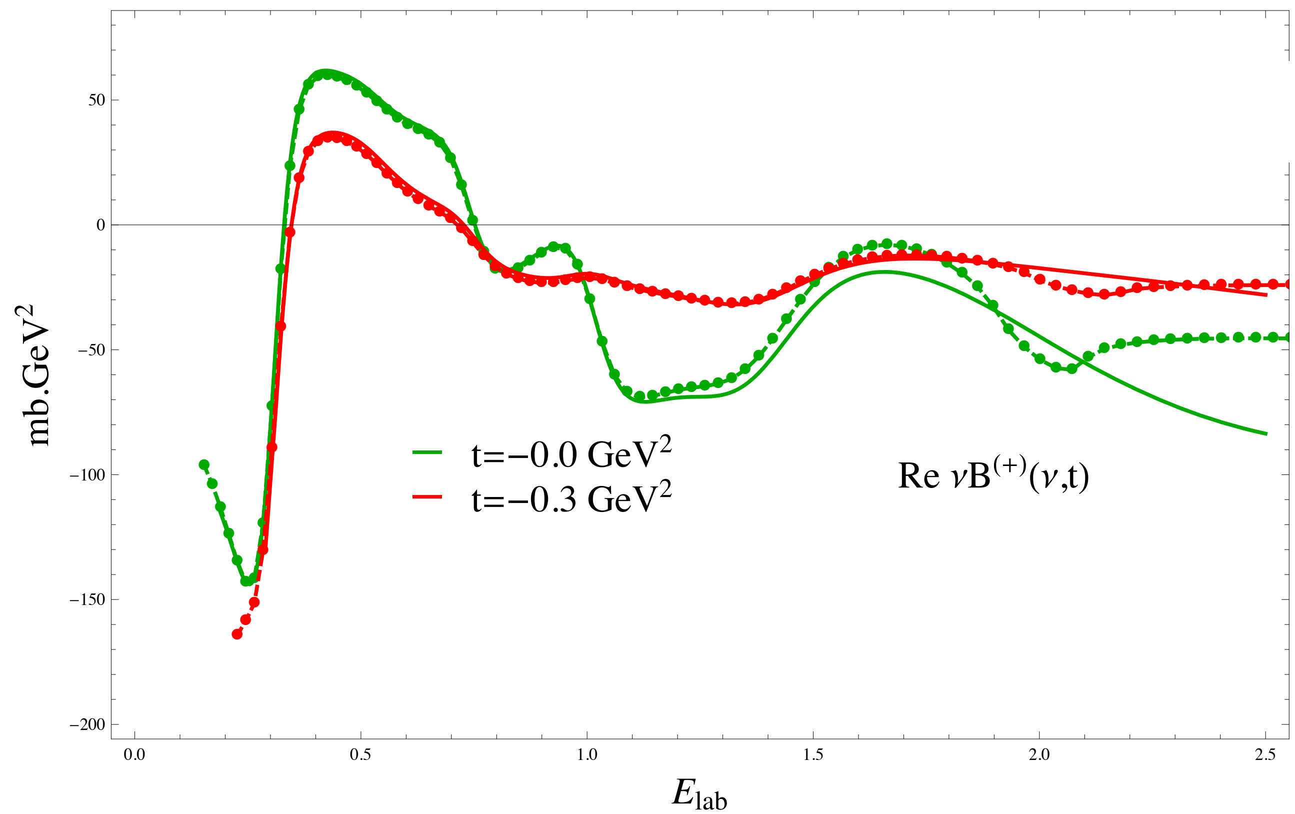


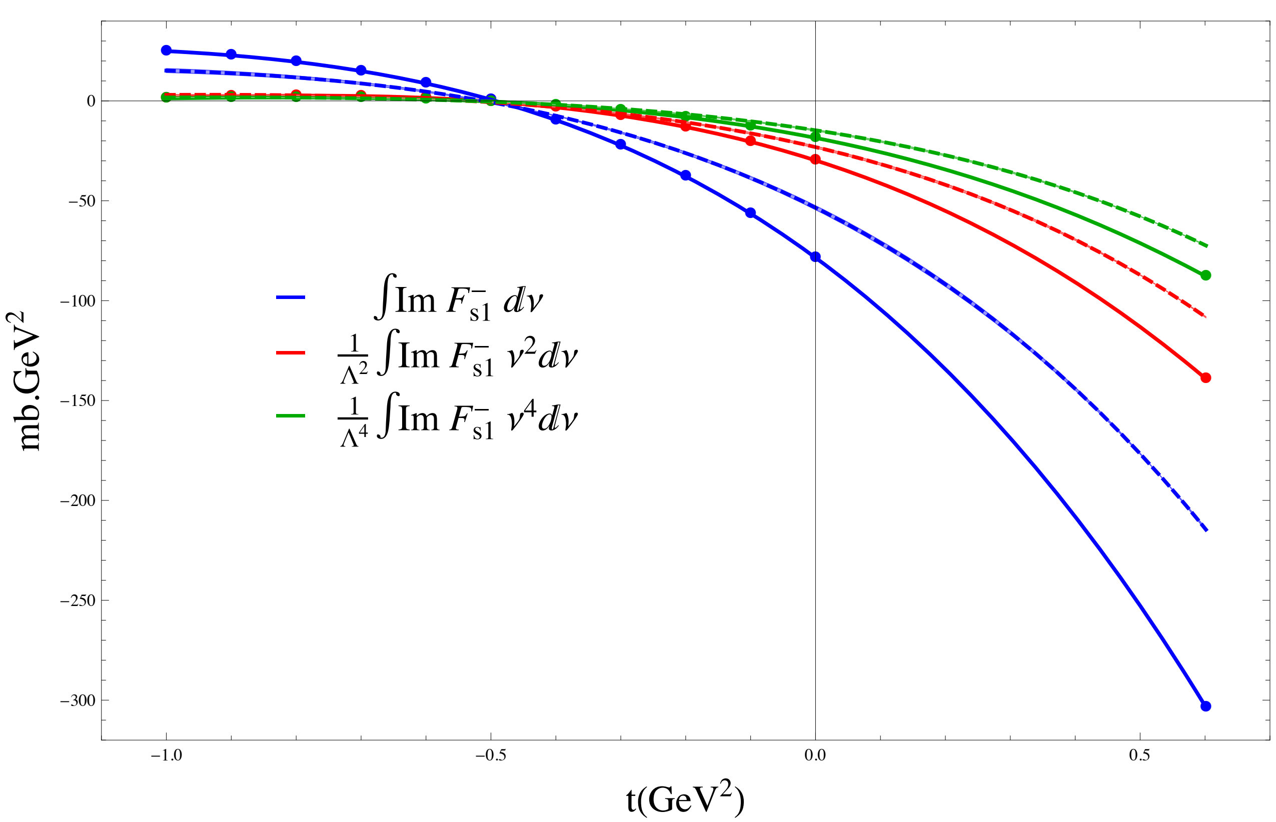
Excellent Match between Re(SAID) Solid lines and Re(Reconstructed) Dashed-Dotted line!

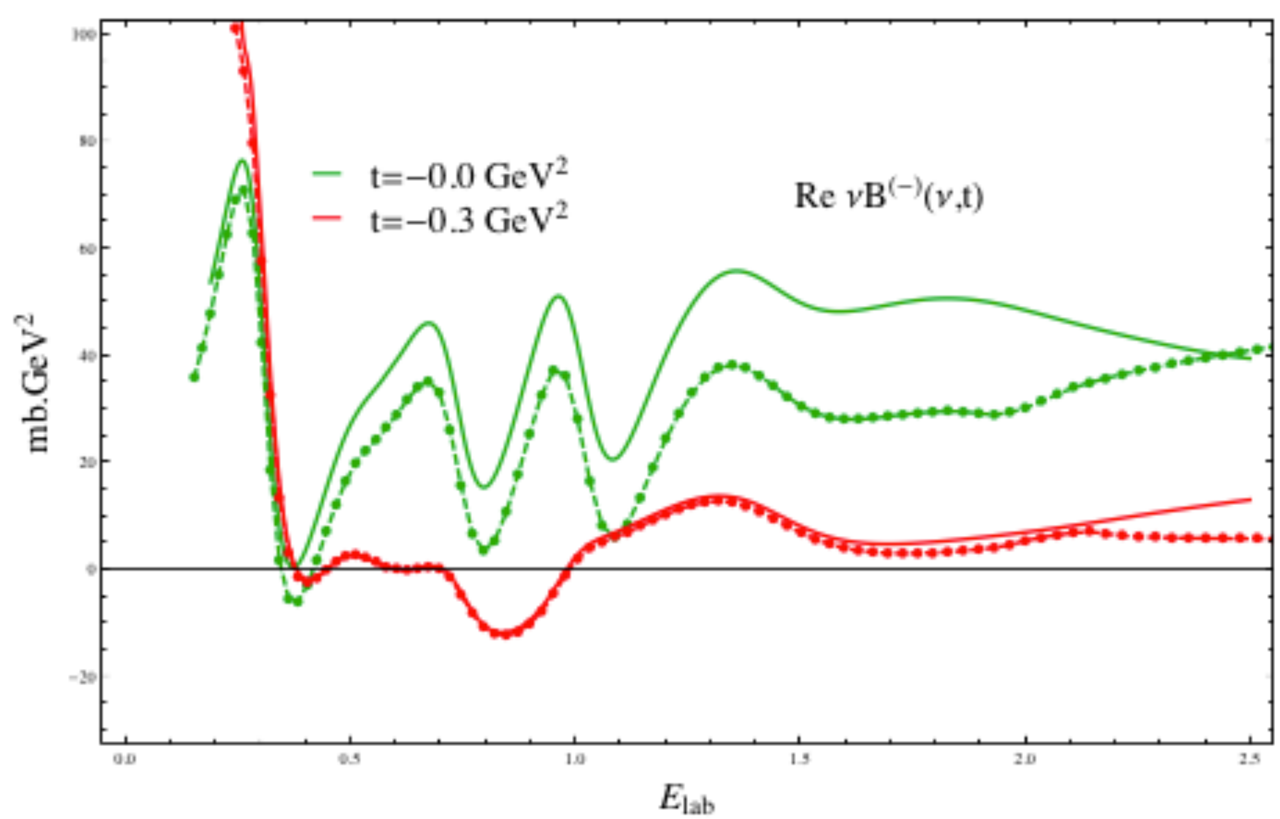
Finite Energy Sum Rules

$\pi N \to \pi N$

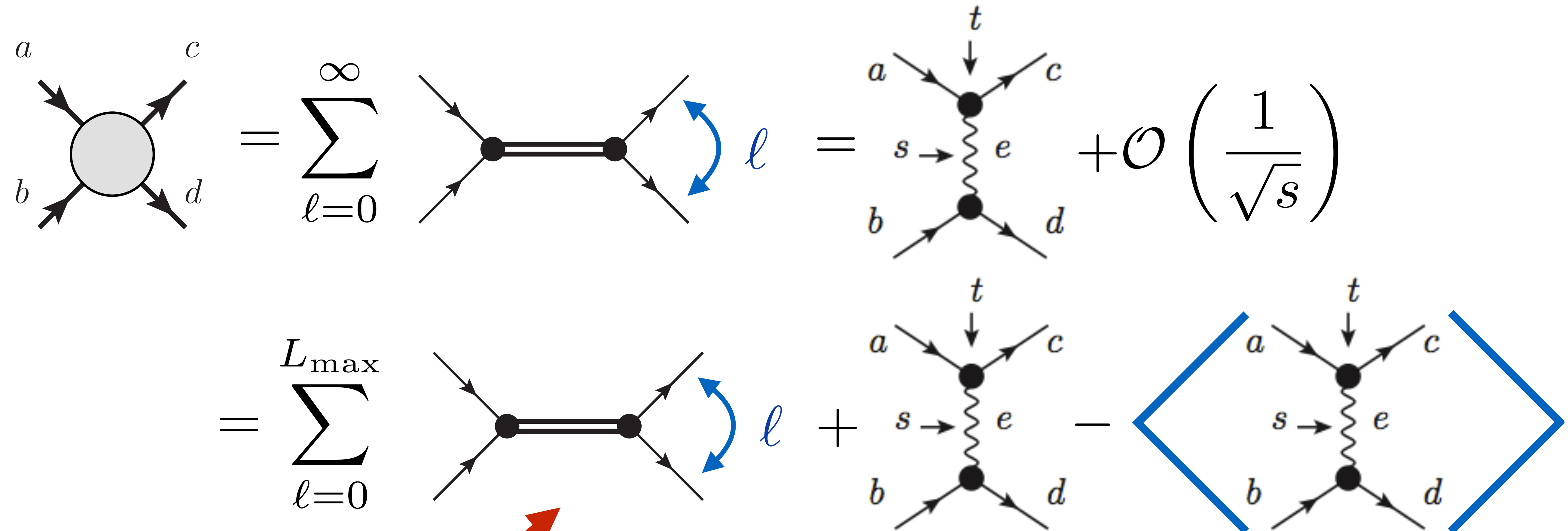


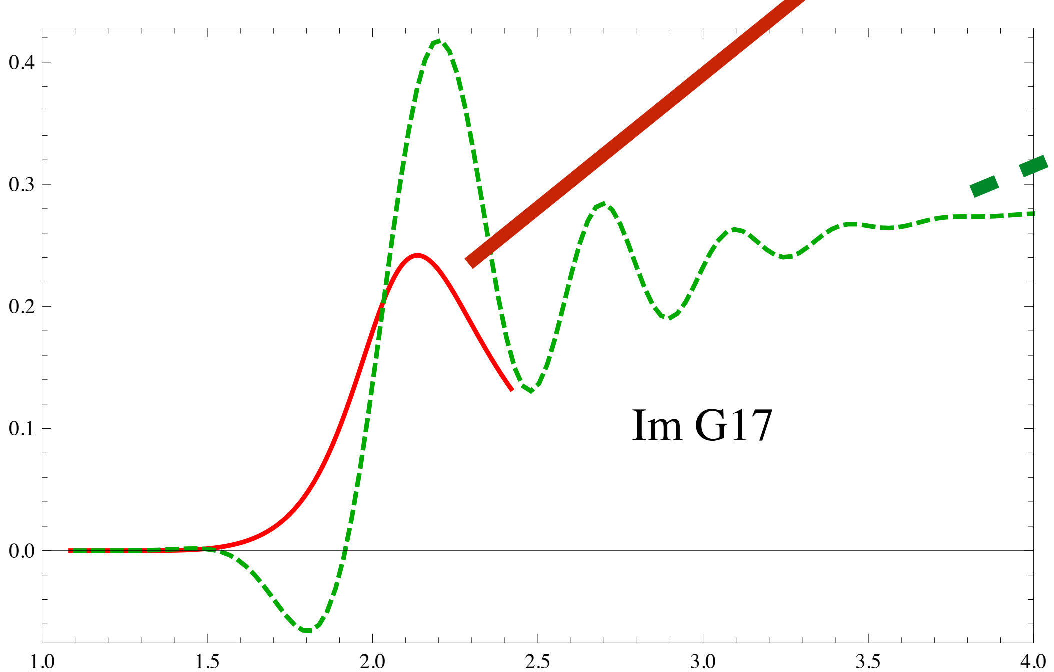


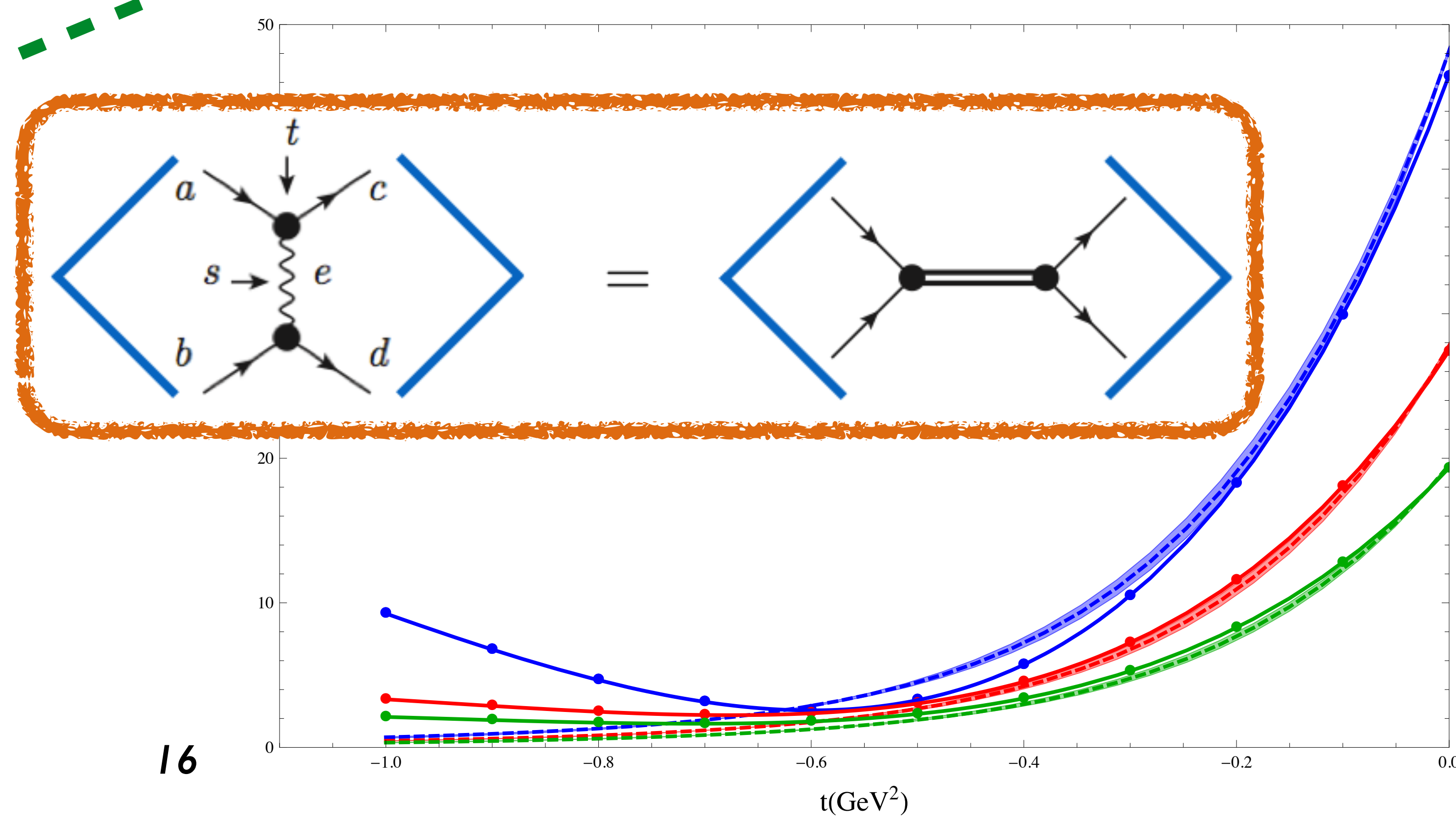




Partial Waves with Regge

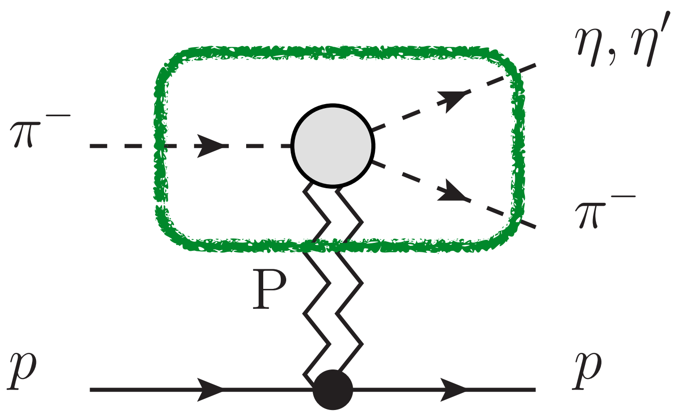


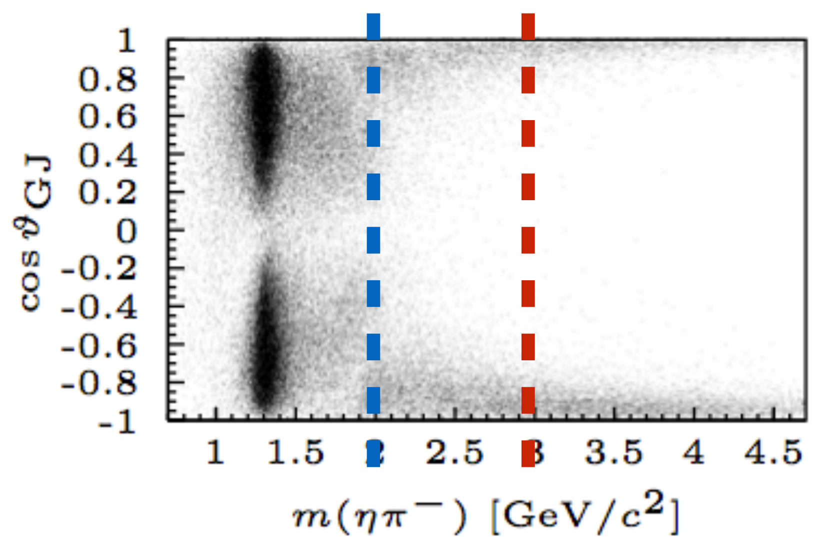


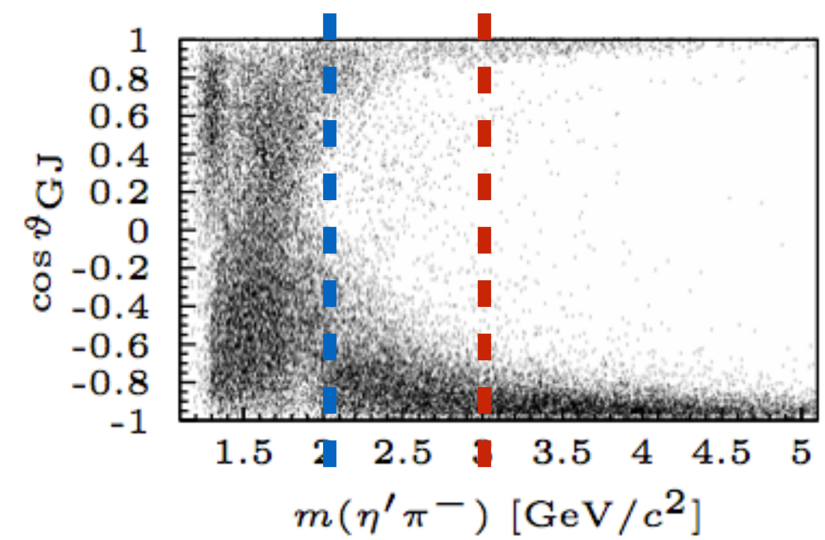


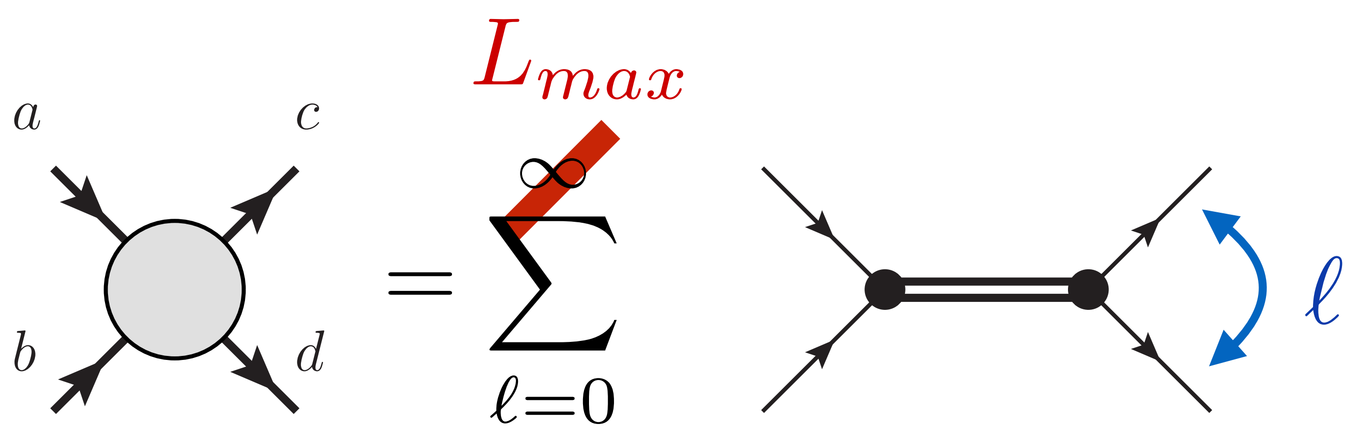
COMPASS: Eta-Pi

COMPASS 1408.4286

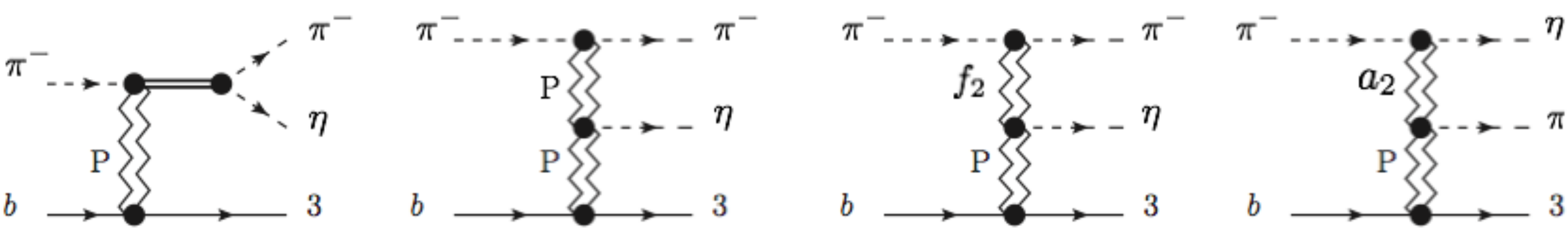








Lmax = 6



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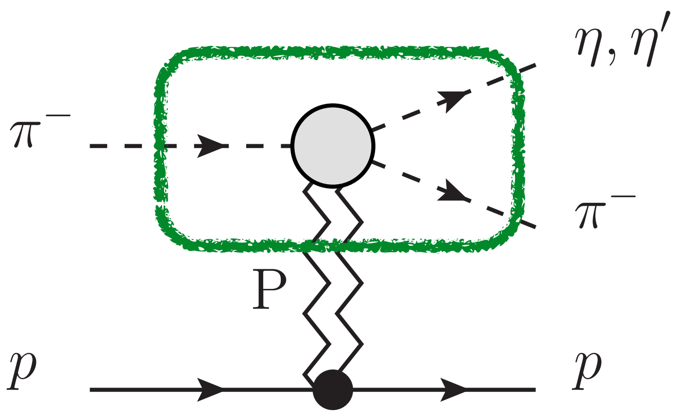
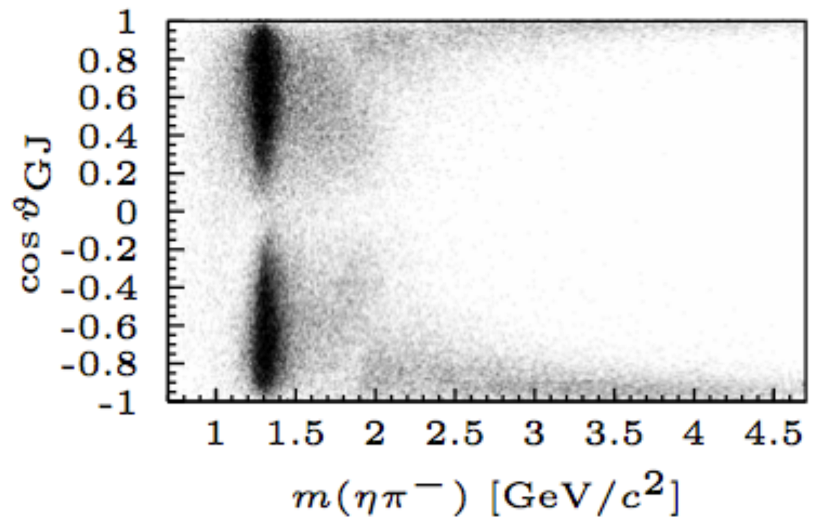
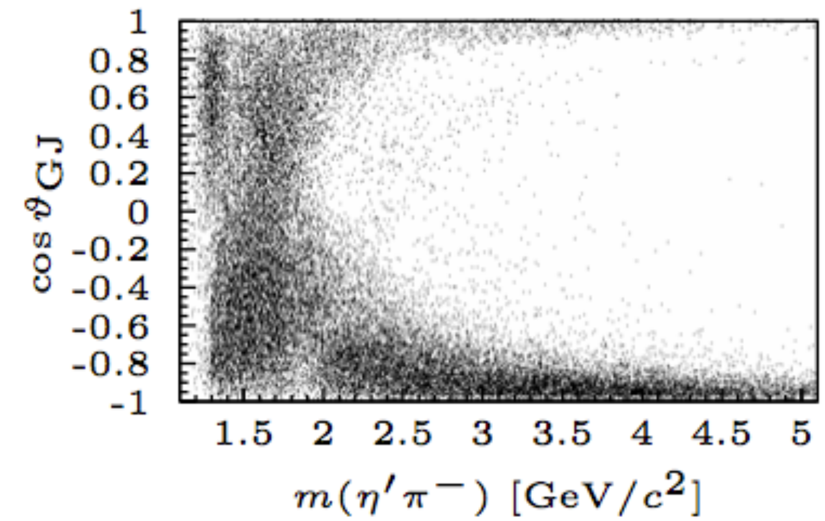
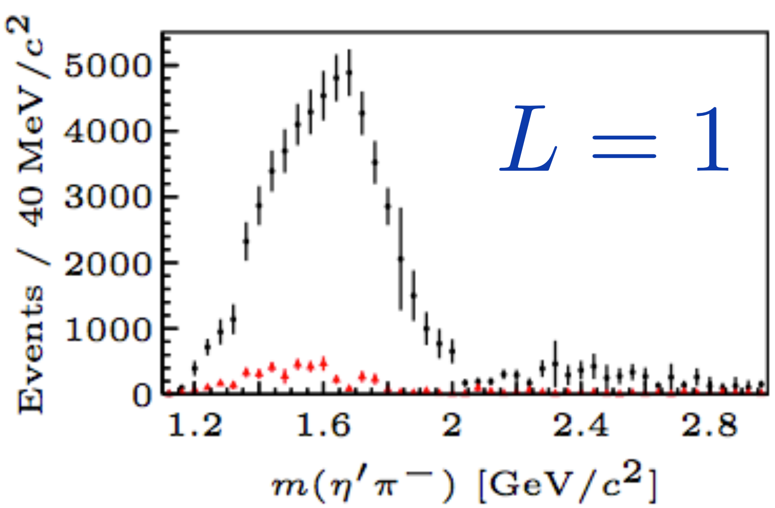
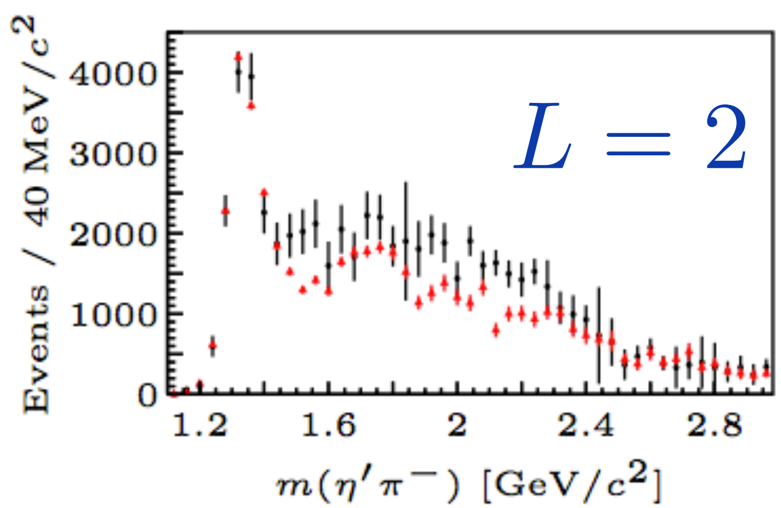
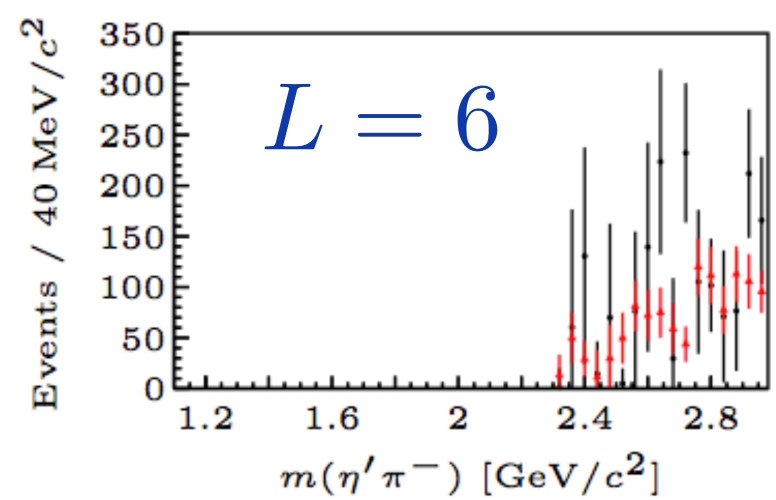


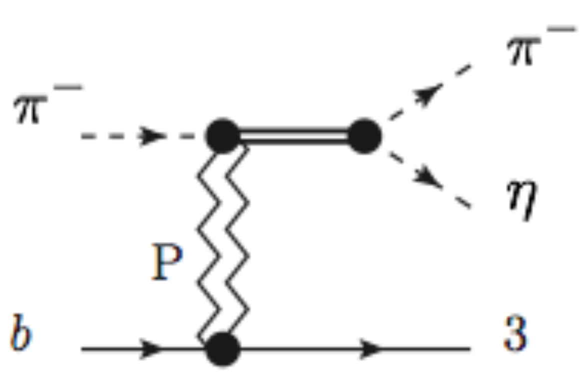


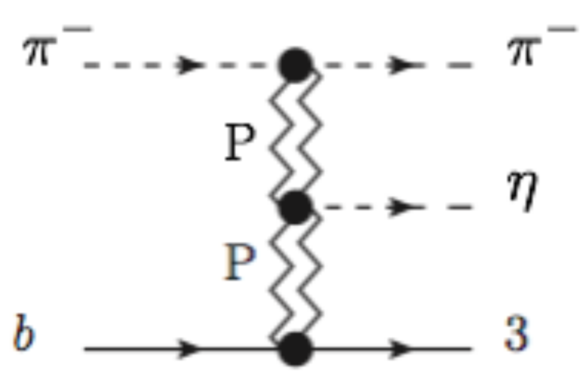


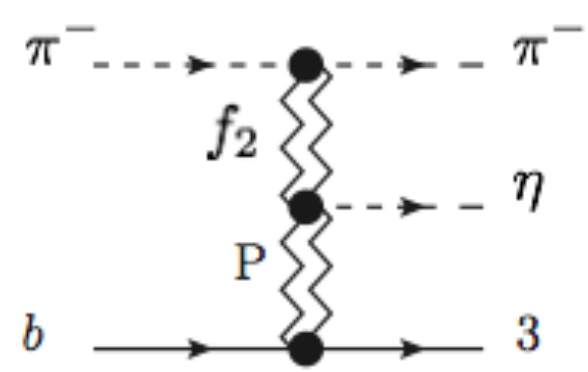


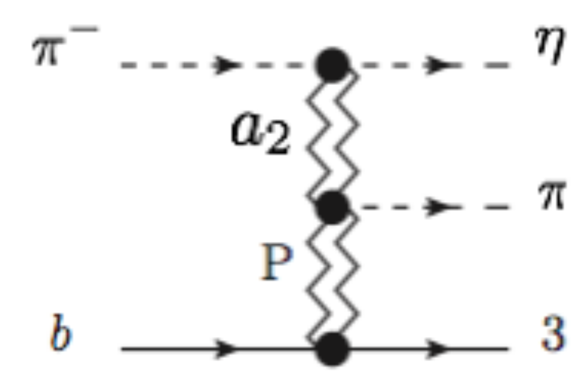




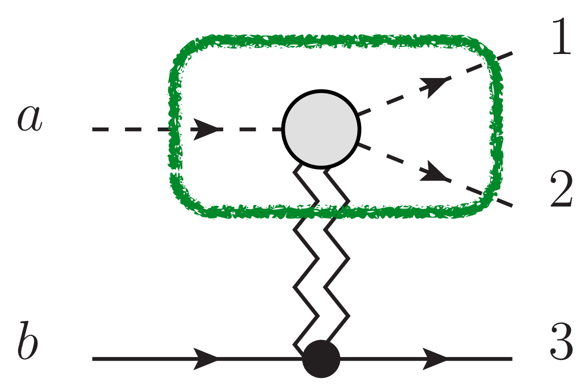


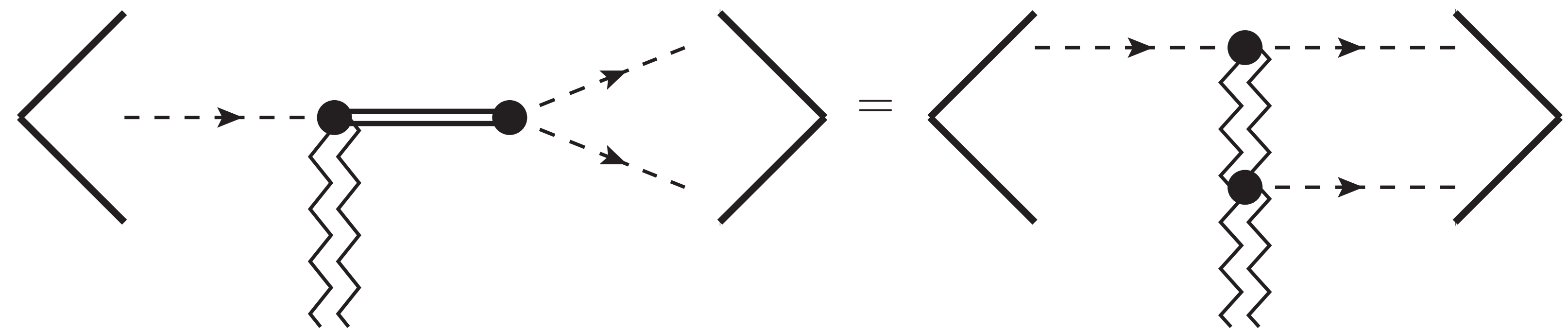






Finite Energy Sum Rules





Pion beam @COMPASS

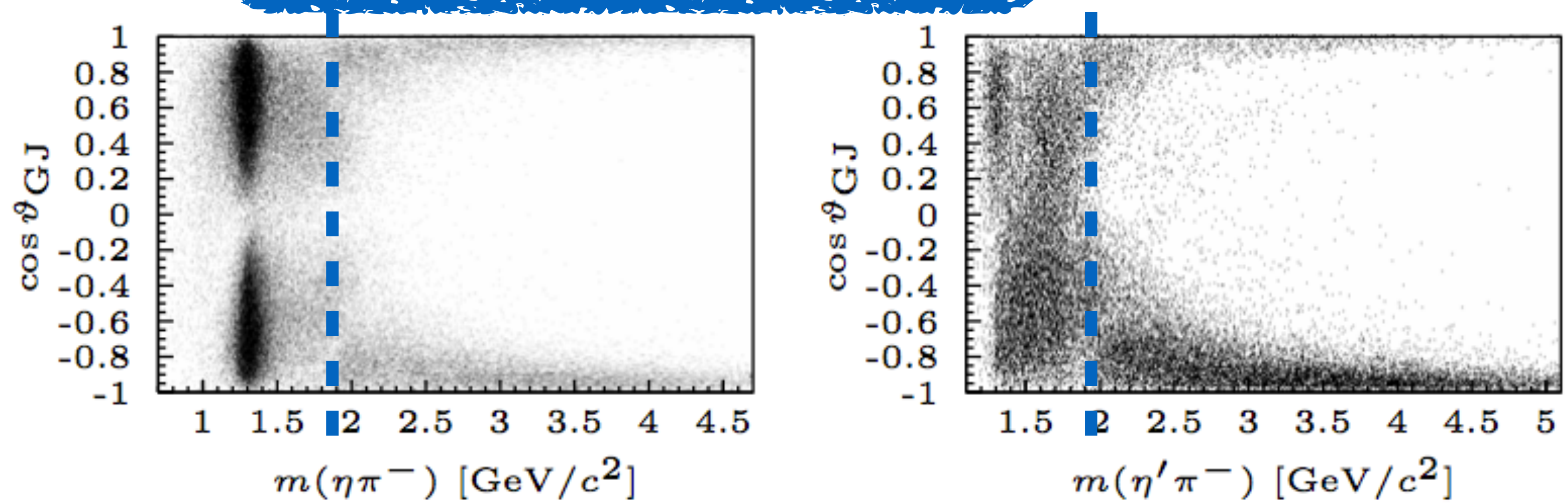
Exotic meson
$\pi_1(1400/1600)??$

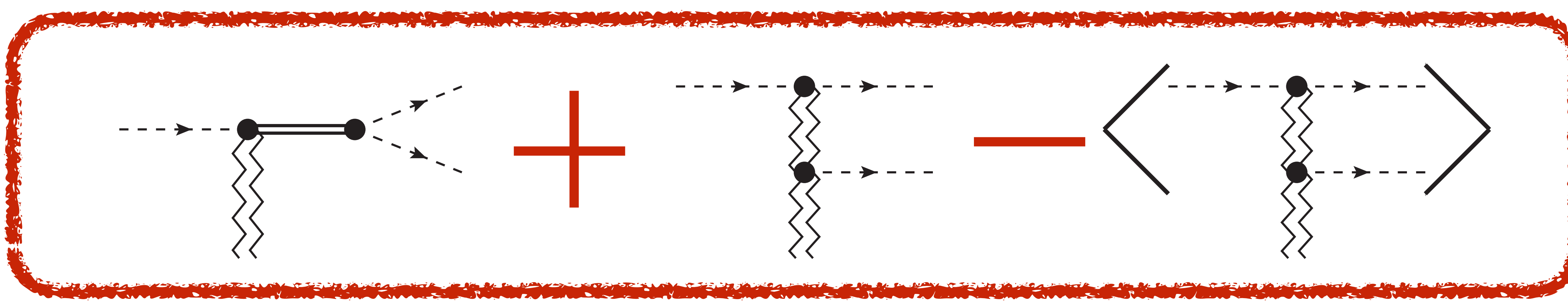
COMPASS 1408.4286

2-to-2 scattering

$$\pi^- p \to \pi^- \eta p$$

$$\pi^- p \to \pi^- \eta' p$$



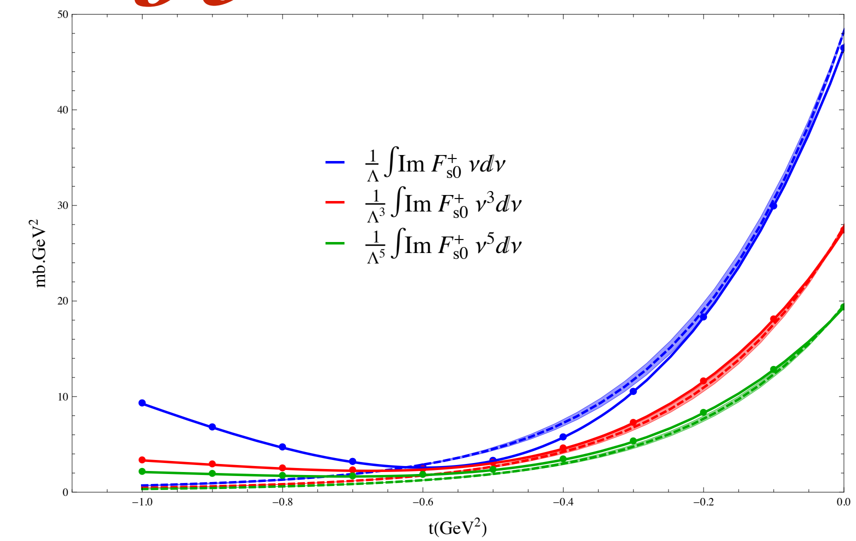


Summary: Methodology

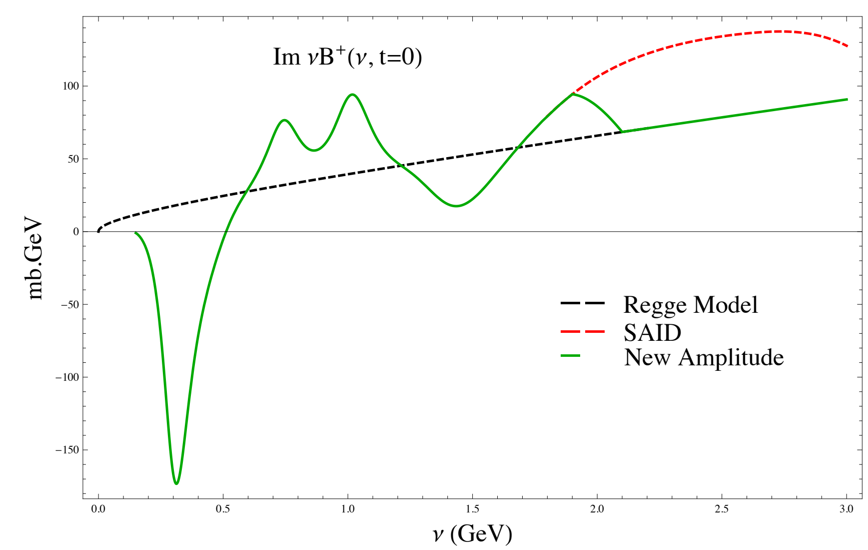
Going beyond partial waves truncation

Use FESR to extrapolate to high energy

$$\int_{\nu_0}^{\Lambda} \text{Im } A^{(-)}(\nu', t)\nu'^{2k} d\nu' = \beta(t) \frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

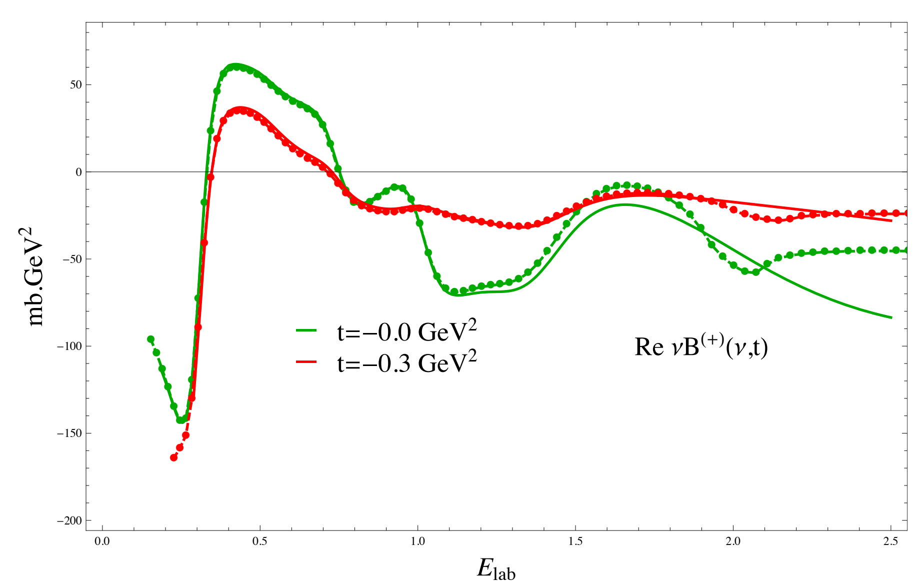


Reconstruct imaginary part
from threshold to infinity



Impose dispersion relation

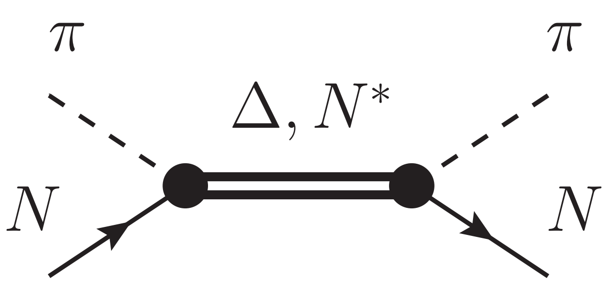
$$A^{(-)}(\nu, t) = \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} \frac{\text{Im } A^{(-)}(\nu', t)}{\nu'^2 - \nu^2} d\nu'$$



Analytically continue and extract poles

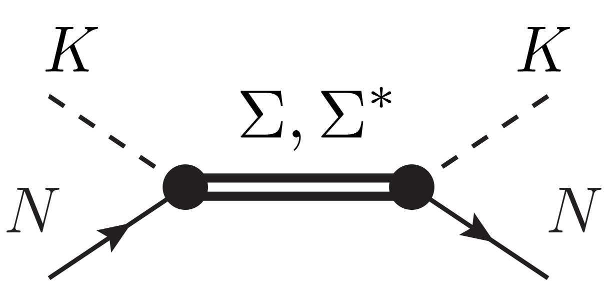
Same Procedure Applies for Other Reactions

$$\pi N \to \pi N$$



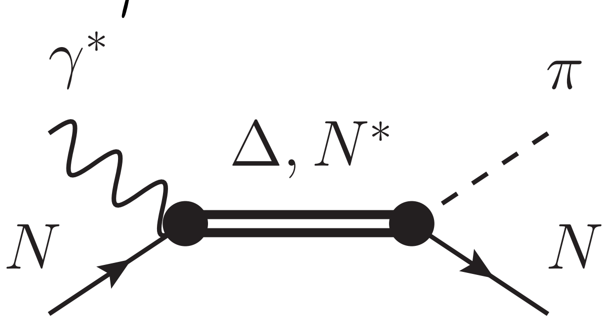
VM

$$K N \to K N$$

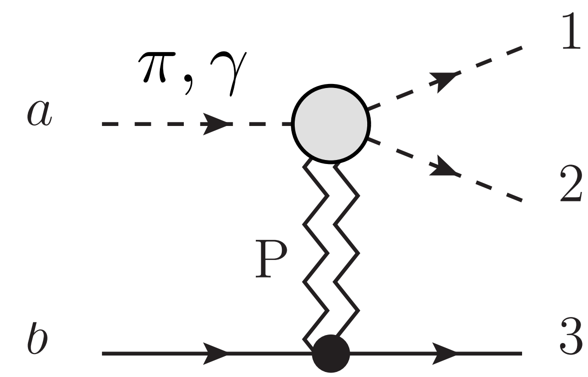


C. Fernandez-Ramirez

$$\gamma N \to \pi N$$



M. Doering
R. Workman



Pion beam @COMPASS

$$\pi^- p \to \pi^- \eta p$$
$$\pi^- p \to \pi^- \eta' p$$

Photoprod. @GlueX and CLAS12

$$\gamma p \to \pi^0 \eta p$$
$$\gamma p \to \pi^0 \eta' p$$

Backup Slides

Duality

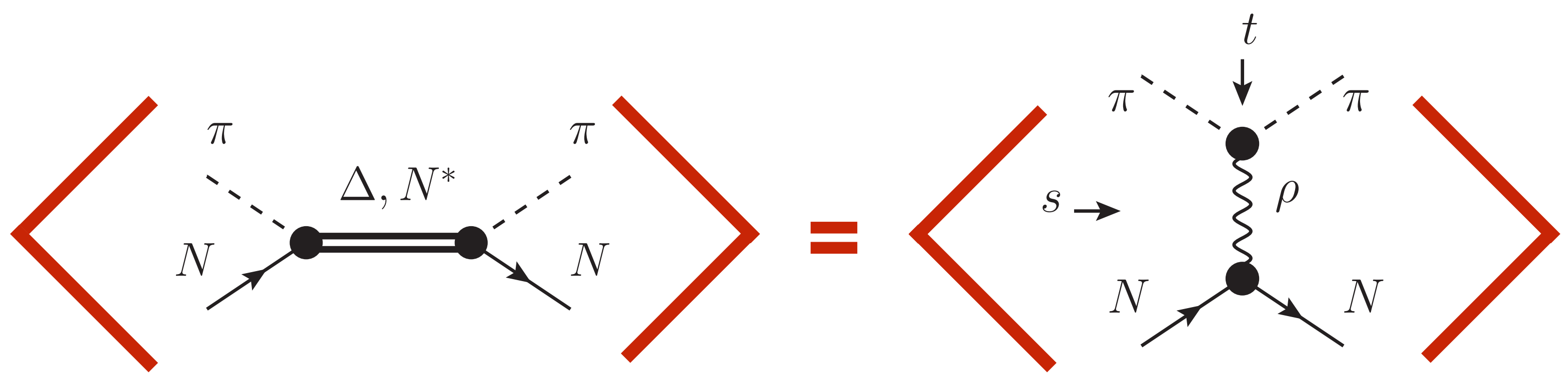
H. Harari, PRL 20 (1968) 1395

FESR are exact (from analyticity)

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A^{(-)}(\nu', t)\nu'^{2k} d\nu' = \beta(t)\frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

Can we separate background from resonances ?

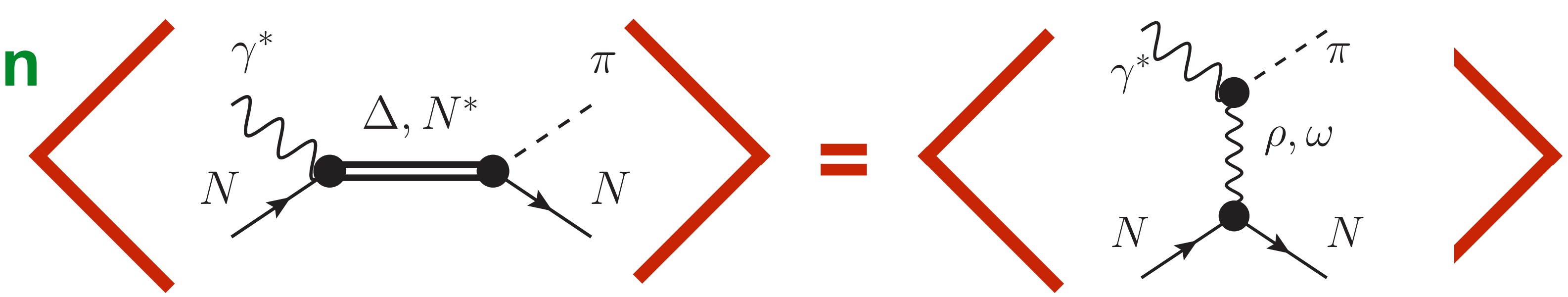
Duality hypothesis:
Resonance equal to exchange on average



Pomeron dual to background

Duality provides constraint on resonances

NO Pomeron in pseudoscalar photoproduction



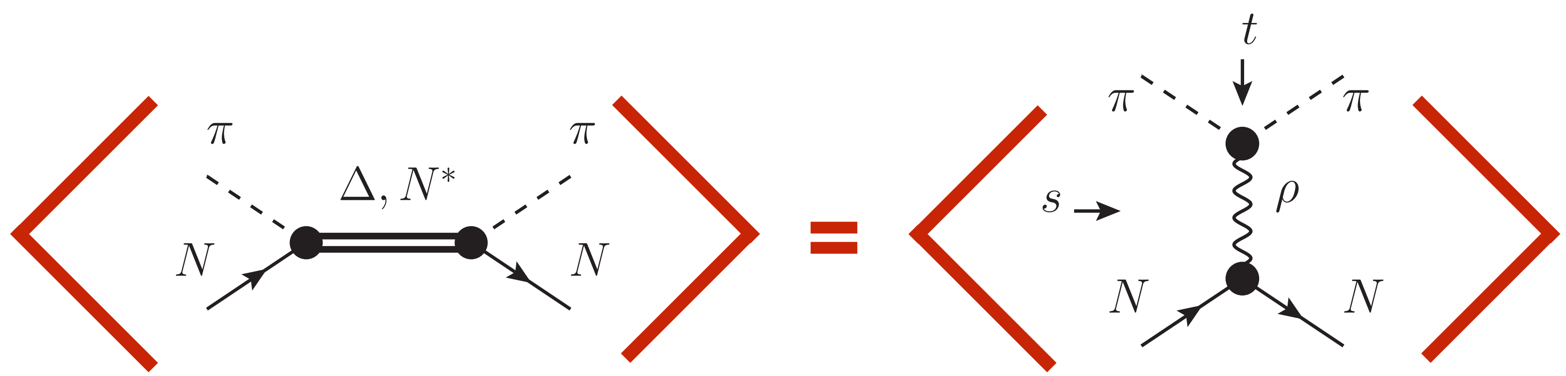
Duality

H. Harari, PRL 20 (1968) 1395

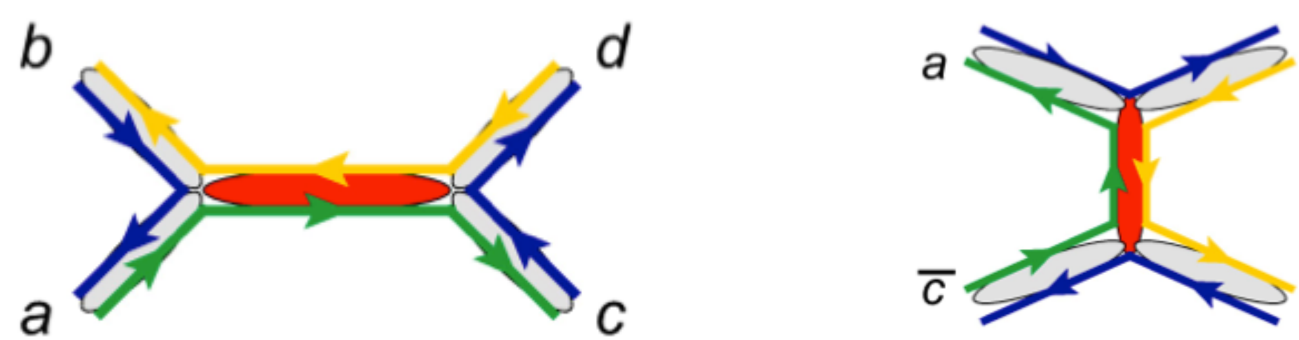
FESR are exact (from analyticity)

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A^{(-)}(\nu', t)\nu'^{2k} d\nu' = \beta(t) \frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

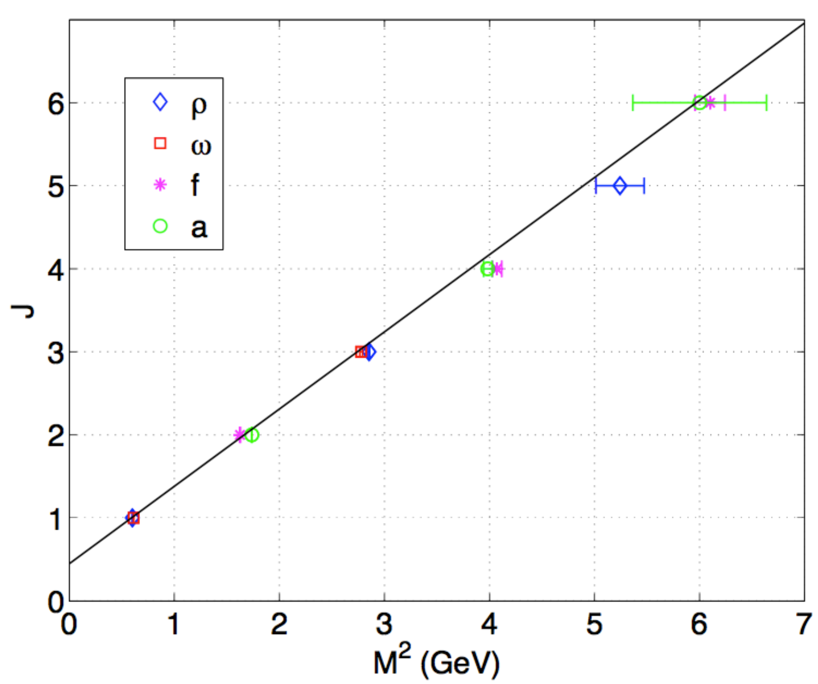
Duality hypothesis:
Resonance equal to exchange on average



Pomeron dual to background



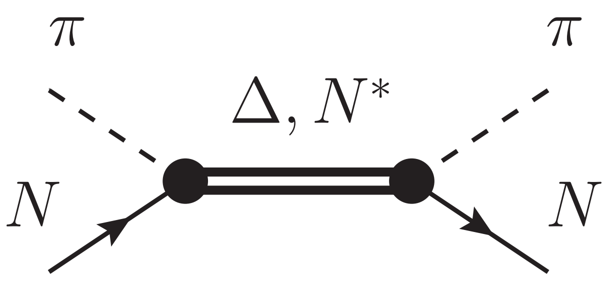
Supported by quark model



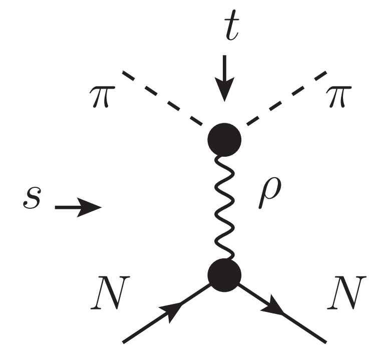
Predictions: degeneracy between exchanges

Duality provides constraint on resonances

Low energy: baryon resonances **High energy: Regge exchange**



$$(A, B) \to (f, g)$$

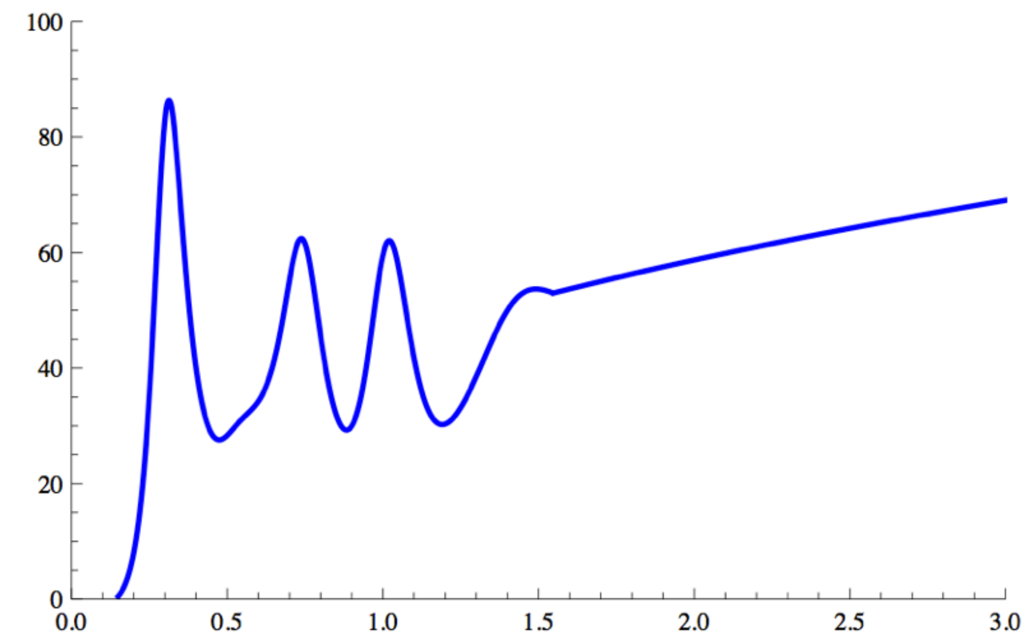


$$f(s,t) = \frac{1}{q}\sum_{\ell=0}^{\infty}\left[(\ell+1)f_{\ell+}(s) + \ell f_{\ell-}(s)\right] P_\ell(\cos\theta)$$

$$g(s,t) = \frac{1}{q}\sum_{\ell=1}^{\infty}\left[f_{\ell+}(s) - f_{\ell-}(s)\right] \sin\theta P'_\ell(\cos\theta)$$

Regge Pole

$$\nu B^{(-)}(\nu.t) = \frac{g_r^2}{2m}\frac{2\nu\nu_m}{\nu_m^2 - \nu^2} - \frac{2\nu}{\pi}\int_{\nu_0}^{\infty}\frac{\text{Im }\nu' B^{(-)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$



$\nu \to \infty$

$\nu > \Lambda$

$$\text{Im }\nu B^{(-)}(\nu,t) \longrightarrow \beta(t)\nu^{\alpha_\rho(t)}$$

$$\text{Im }R_\rho(\nu,t) = \beta(t)\nu^{\alpha_\rho(t)}$$

$$\int_0^\Lambda \text{Im }\nu B(\nu,t)d\nu = \beta_R\frac{\Lambda^\alpha}{\alpha+1}$$