Consistent treatment of one- and two-nucleon currents in the spectral function formalism

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Outline

- ★ Structure of the nuclear transition amplitude
  - ▹ Initial state
  - ▹ Current operator
  - ▹ Final state
- ★ Quasielastic (nucleon-only) processes
  - ▹ Accessible final states
  - ▹ Consistent treatment of one-and two-body currents: why worry?
  - ▹ Extension of the spectral function formalism
- ★ Summary & Outlook

Preamble: the lepton-nucleus x-section

★ Double differential cross section of the process $\ell + A \to \ell' + X$

$$\frac{d\sigma_A}{d\Omega_{k'} dk'_0} \propto L_{\mu\nu} W_A^{\mu\nu}$$

- $L_{\mu\nu}$ is fully specified by the lepton kinematical variables (warning: not all of them are known in the case of ν scattering)
- The determination of the target response tensor

$$W_A^{\mu\nu} = \sum_N \langle 0|J_A^{\mu\,\dagger}|N\rangle\langle N|J_A^{\nu}|0\rangle \delta^{(4)}(P_0 + k - P_N - k')$$

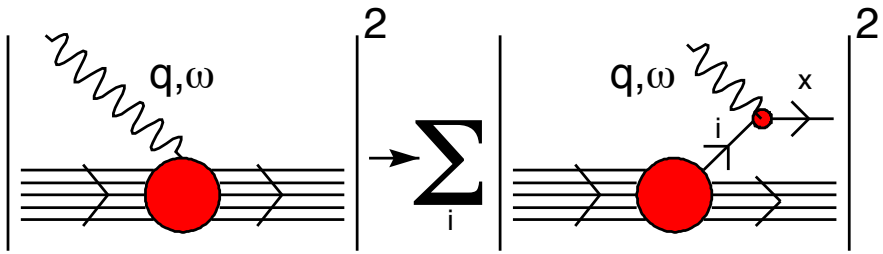
  requires a *consistent* description of the target initial and final states and the nuclear current. Accurate calculations are feasible in the non relativistic regime, corresponding to $|\mathbf{q}| \lesssim 500$ MeV
- In the kinematical regime in which relativistic effects become important, approximations are needed to describe the $|\mathbf{q}|$-dependent current operator and final state

Accessible final states

- ★ Within the independent particle model (IPM) , one-body operators induce transitions to one particle-one hole (1p1h) final states only, while two-body operators can excite both 1p1h and two particle-two hole (2p2h) states
- ★ While the results obtained within the IPM suggest that interference effects in the 1p1h sector are small, sizable effetcs have been predicted by calculations carried out within more realistic approaches, taking into account nucleon-nucleon correlations [PRC 55, 338 (1995)]
- ★ In correlated systems, 2p2h final states can be excited thorugh different mechanisms, involving both the one- and two-body currents
  - ▹ Ground state correlations
  - ▹ Meson-exchange current
  - ▹ Final state interactions
- ★ In view of the evidence of strong correlation effects provided by electron-nucleus scattering data, the role of interference in the 2p2h sector must be carefully analysed

The impulse approximation (IA)

★ At $\lambda = 2\pi/|\mathbf{q}| \gg d_{\rm NN}$, the average NN distance in the target nucleus



- neglect the contribution of the two-nuleon current

$$J^{\mu}_{A}(q) = \sum_{i} j^{\mu}_{i}(q) + \sum_{j>i} j^{\mu}_{ij}(q) \approx \sum_{i} j^{\mu}_{i}(q)$$

- write the final state in the factorized form

$$|N\rangle \rightarrow |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle \, .$$

- at zero-th order, neglect final state interactions (FSI) between the outgoing nucleon and the spectator particles

Spectral function

- within the IA scheme the nuclear matrix element of the one-nucleon current reduces to

$$\langle N|j_i^\mu|0\rangle = \int d^3k\, M_m(\mathbf{k})\, \langle \mathbf{p}|j_i^\mu|\mathbf{k}\rangle \,,$$

with

$$M_n(\mathbf{k}) = \{\langle n_{(A-1)}, \mathbf{p}_n| \otimes \langle \mathbf{k}|\}\, |0\rangle \,.$$

- The nuclear spectral function, yielding the probability of removing a nucleon of momentum $\mathbf{k}$ from the target nucleus, leaving the residual system with excitation energy E, is defined as

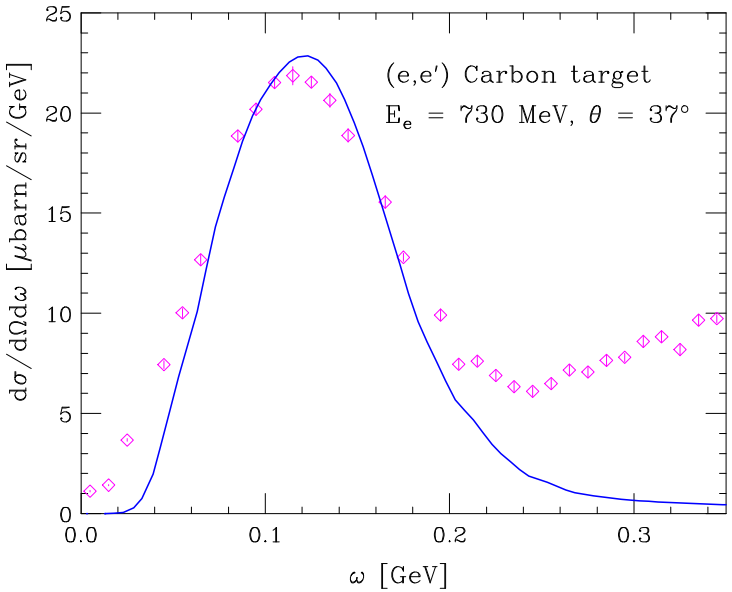
$$P(\mathbf{k}, E) \sum_n |M_n(\mathbf{k})|^2 \delta(E_0 + E - E_n)$$

- Being an intrinsic property of the target, the spectral function can be computed within non relativistic approaches

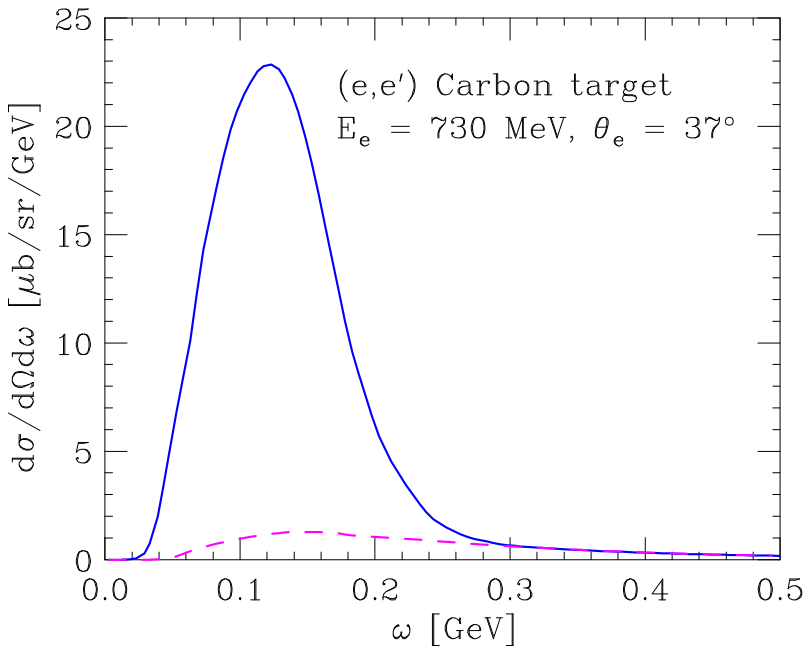
IA results compared to electron scattering data

★ Nuclear x-section $$d\sigma_A = \int d^3k dE\, d\sigma_N\, P(\mathbf{k}, E)$$

★ QE (nucleon-only final states) only



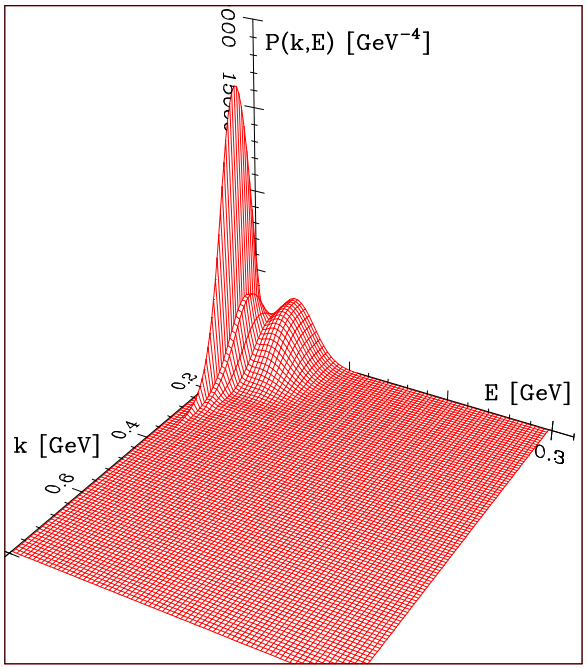
★ Correlation tail, arising from 2p2h final states, cleary visible

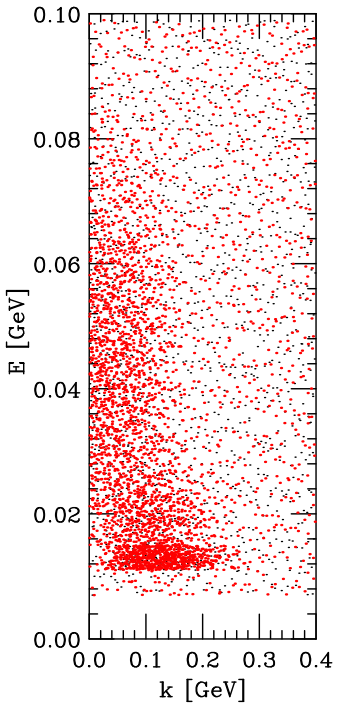


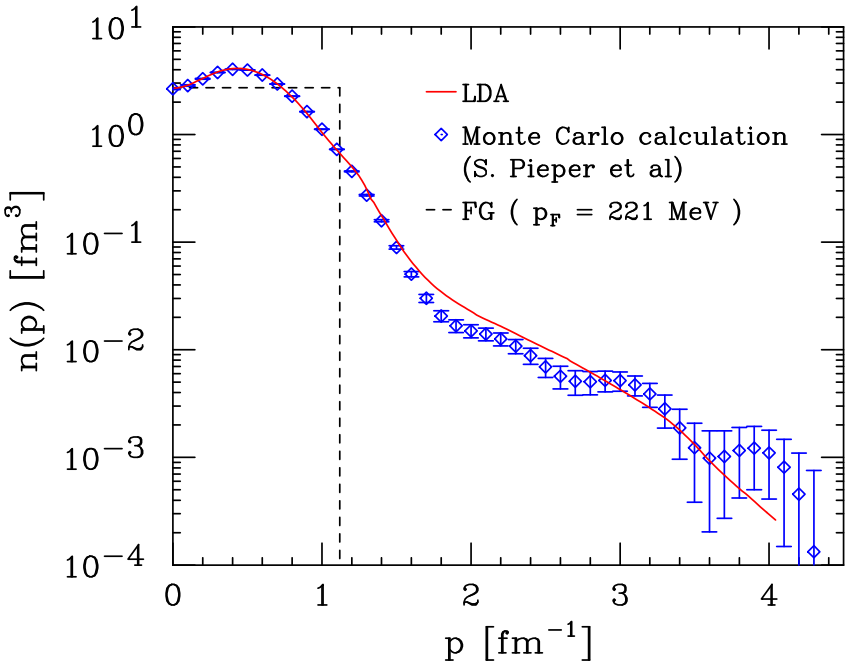
★ Position and width of the peak are reproduced

LDA spectral function of ^{16}O

- ★ The spectral function of medium-mass nuclei has obtained combining $(e, e'p)$ data with the results of theoretical nuclear matter calculations within the Local Density Approximation (LDA)



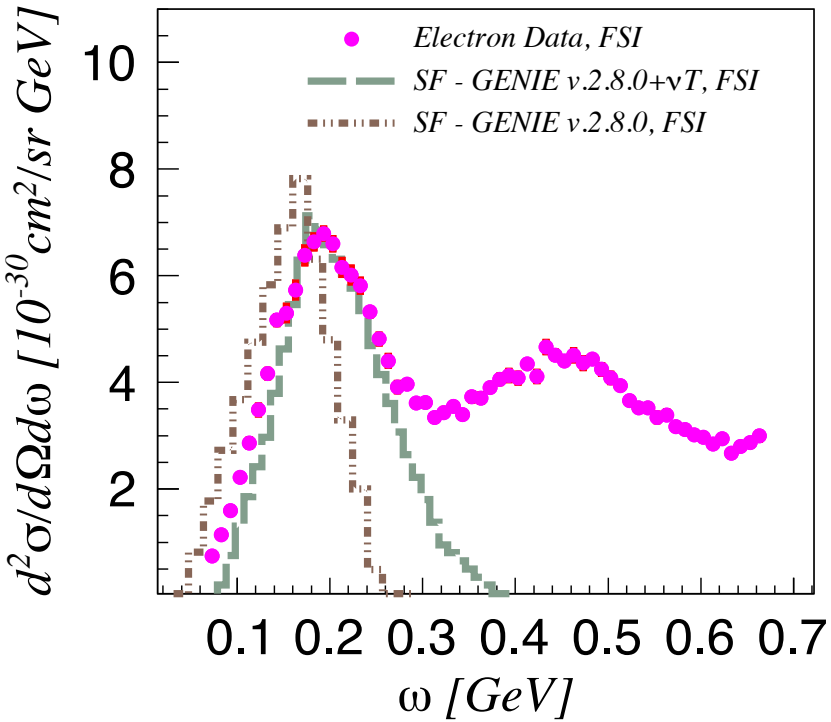




- ★ shell model states account for $\sim 80\%$ of the strenght
- ★ the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum *and* large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

Spectral functions for the analysis of neutrino experiments

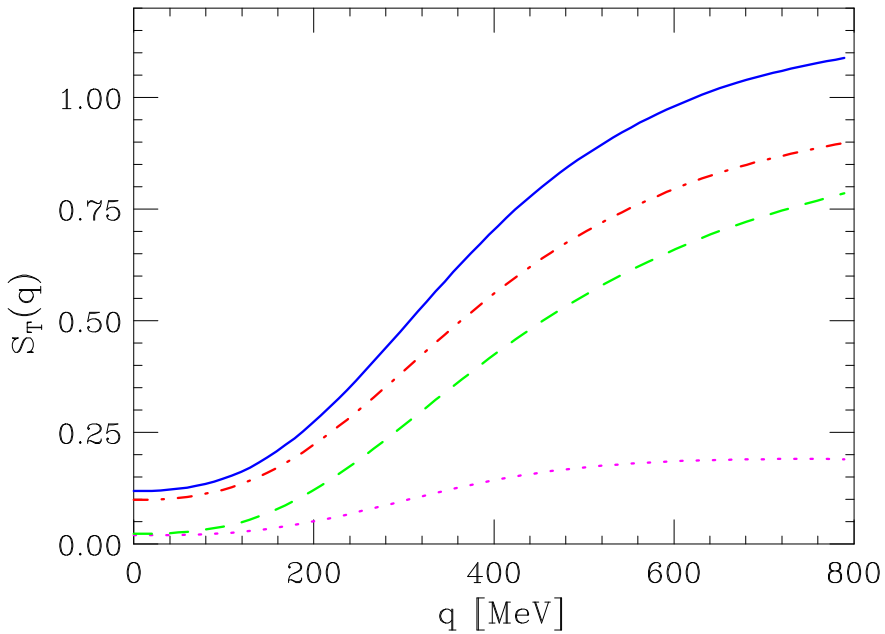
- ★ The LDA spectral functions of oxygen and carbon have been succesfully implemented into GENIE [Jen *et al*, PRD 92014)]
- ★ Carbon target, $E_e = 961$ MeV, $\theta_e = 37.5$ deg



- ★ The experimental input needed to extend the LDA approach to argon will be provided by a $(e, e'p)$ experiment, expected to run at JLab in 2016.

Enter the two-nucleon current

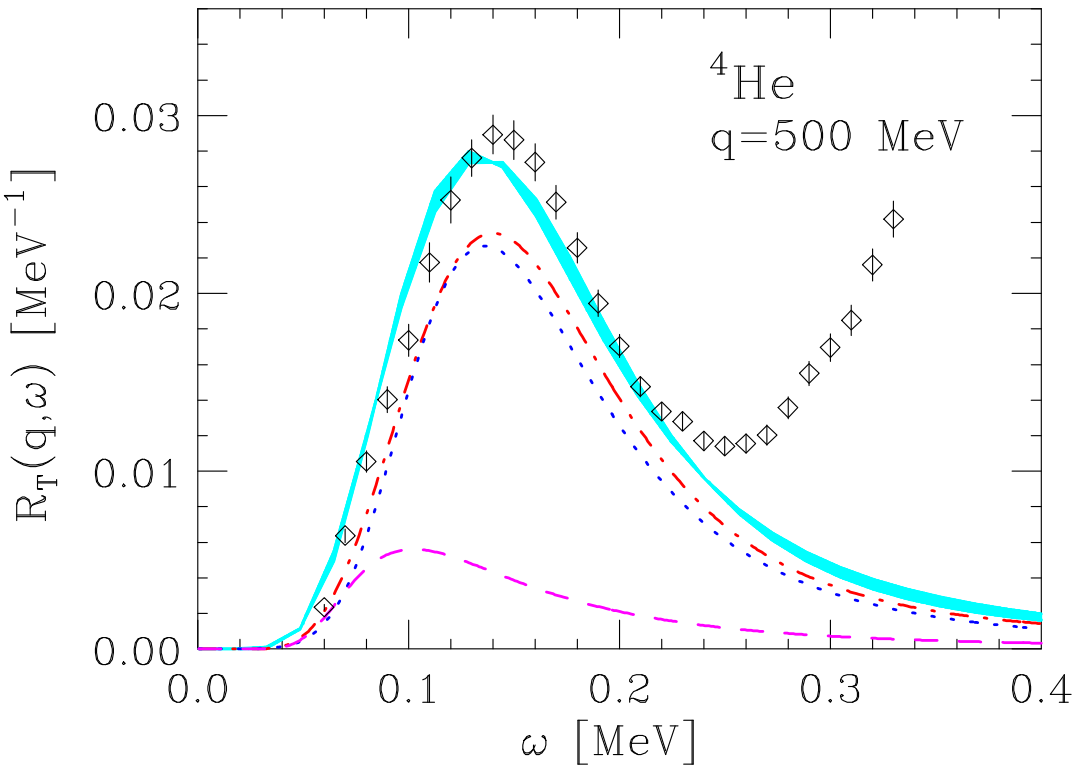
- ★ Two-nucleon current contributions can be accurately computed within the non relativistic approximation, using the GFMC approach
- ★ Energy-integrated electromagnetic response of carbon in the transverse channel (arXiv:1312.1210)



- ★ The effect of interference is large. Ground state correlations must be properly taken into account by the approximated approaches employed in the relativistic regime

Energy dependence of two-body current contributions

★ Transverse response of ^{4}He, obtained from inversion of the corresponding GFMC euclidean response (arXiv:1502.00887)



★ The large interference contribution is peaked at energy transfer $\omega < \omega_{QE}$

Two-body currents within the spectral function formalism

- ★ The generalisation of the factorisation scheme allows for a consistent treatment of ground state correlations and fully relativistic two-body currents
  - ▹ Rewrite the final state $|N\rangle$ in the factorized form

$$|N\rangle \rightarrow |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)},\mathbf{p}_n\rangle$$

$$\langle N|j_{ij}{}^{\mu}|0\rangle \rightarrow \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}')\,\langle \mathbf{p}\mathbf{p}'|j_{ij}{}^{\mu}|\mathbf{k}\mathbf{k}'\rangle$$

The amplitude

$$M_n(\mathbf{k},\mathbf{k}') = \{\langle n_{(A-2)}|\langle \mathbf{k},\mathbf{k}'|\} \otimes |0\rangle$$

is independent of $\mathbf{q}$, and can be obtained from non relativistic many-body theory

Two-nucleon spectral function

★ Calculations have been carried out for uniform isospin-symmetric nuclear matter using CBF perturbation theory [PRC 62, 034304 (2000)]

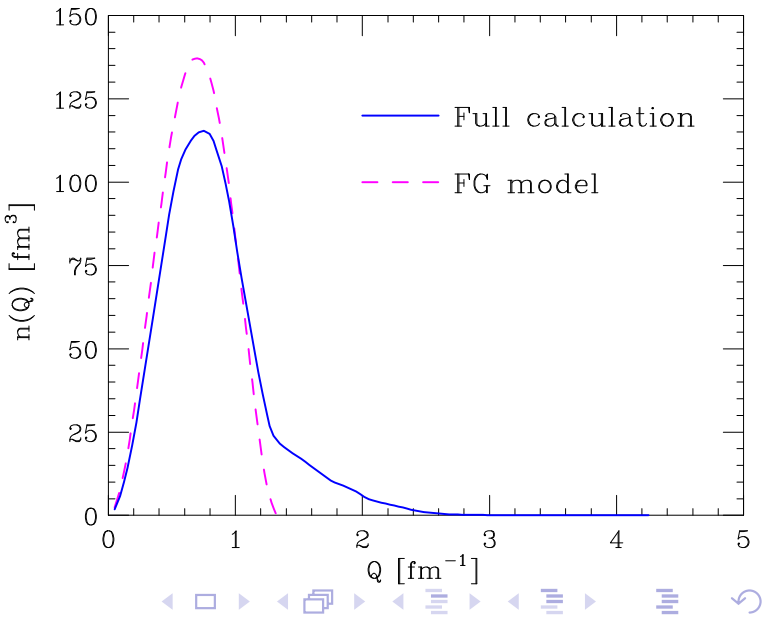
$$P(\mathbf{k},\mathbf{k}',E) = \sum_n |M_n(k,k')|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k},\mathbf{k}') = \int dE\, P(\mathbf{k},\mathbf{k}',E)$$

★ Relative momentum distribution

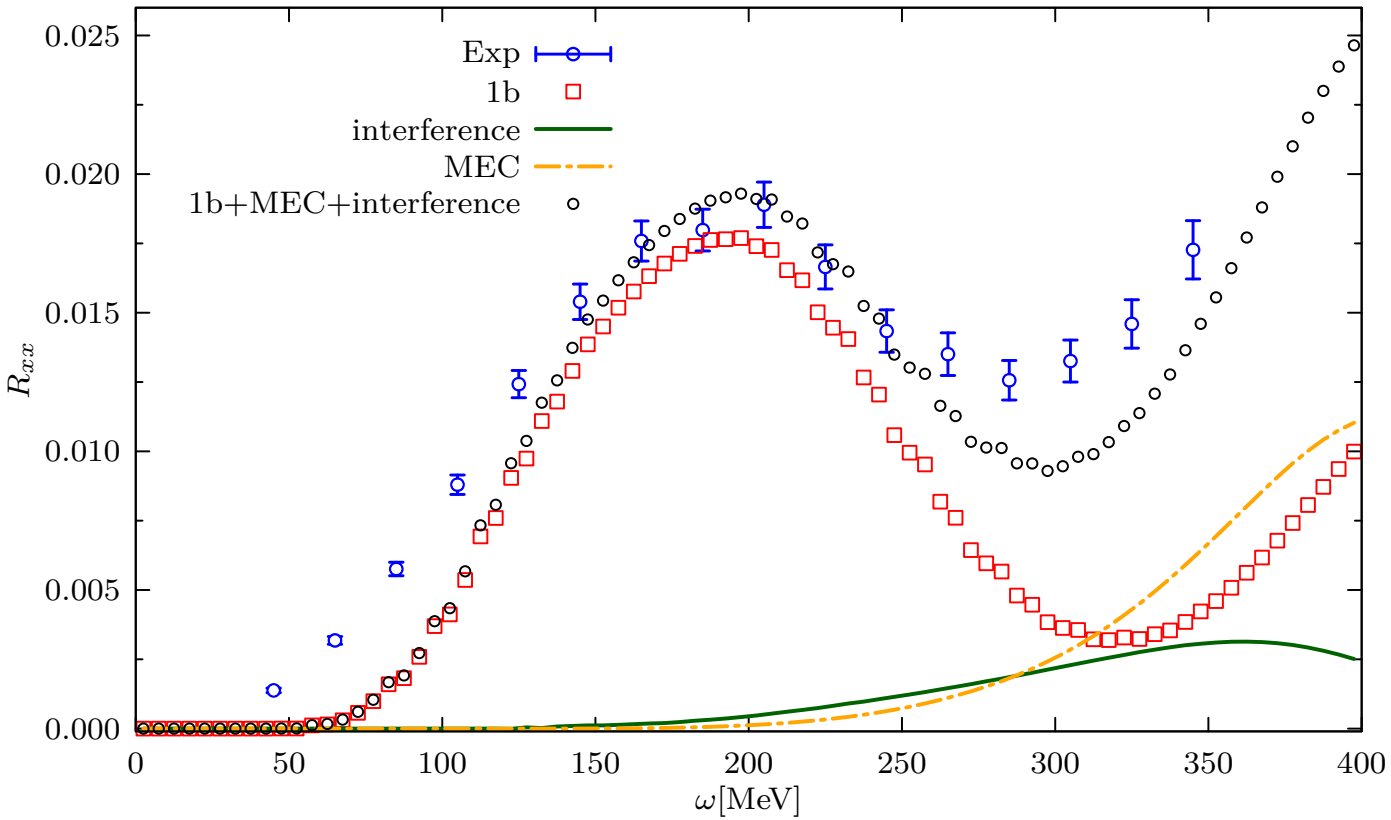
$$n(\mathbf{Q}) = 4\pi|\mathbf{Q}|^2 \int d^3q\, n\left(\frac{\mathbf{q}}{2} + \mathbf{Q}, \frac{\mathbf{q}}{2} - \mathbf{Q}\right)$$

$$\mathbf{q} = \mathbf{k} + \mathbf{k}' \ , \ \mathbf{Q} = \frac{\mathbf{k} - \mathbf{k}'}{2}$$



Results of the extended factorisation approach

★ Electromagnetic response of ^{12}C in the transverse channel (data from the global analysis of J. Jourdan)



★ Sizable interference contribution peaked at energy transfer $\omega > \omega_{\rm QE}$

Summary & Outlook

- ★ Results of exact non relativistic calculations suggest that interference between nuclear amplitudes involving one and two-nucleon currents is large
- ★ The spectral function formalism can be generalized to allow for a consistent treatment of processes involving one- and two- nucleon currents
- ★ Preliminary results of calculations of the electromagnetic response in the transverse channel are encouraging
- ★ The source of the different energy dependences of the interference contribution predicted by GFMC and the generalised factorisation scheme must be investigated
- ★ Corrections to the factorisation scheme, mainly arising from final state interactions can be systematically included