Inclusive Neutrino-Nucleus Scattering:

- Motivation
- Ingredients: Interactions and Currents
- Correlations
- Simplified Models of Response
- Sum Rules
- Euclidean Response (concept)

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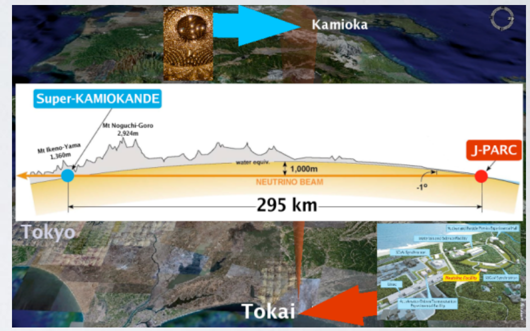
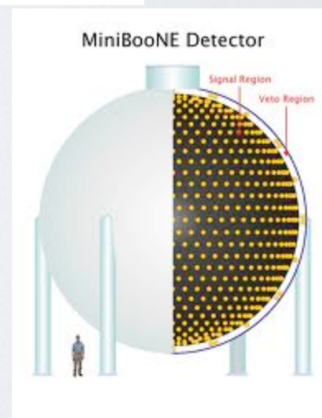
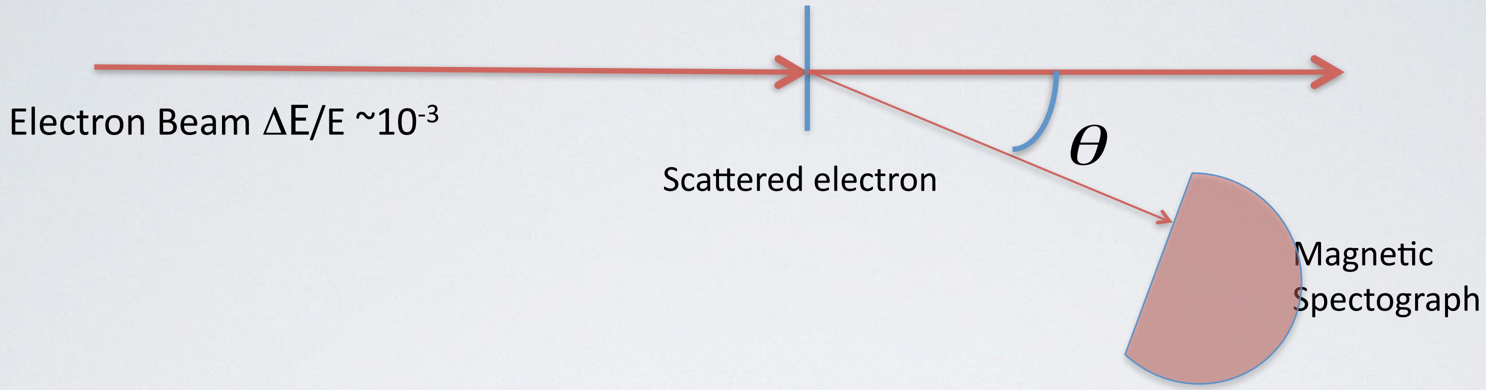
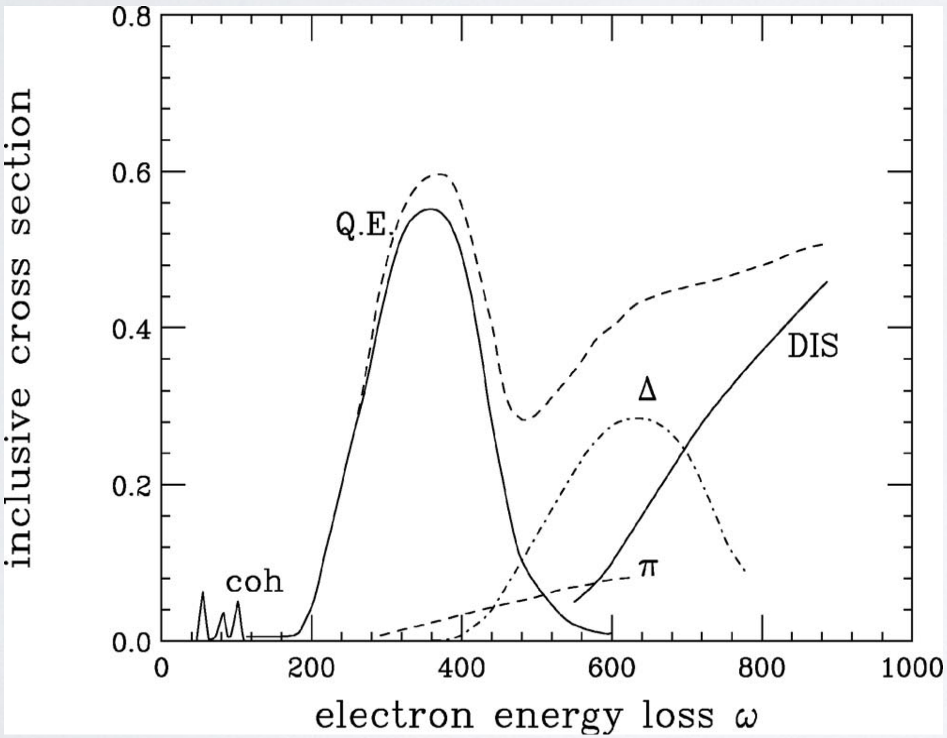


T2K



Inclusive Electron Scattering





$$(E,0,0,p),\ \ (E',p'\sin\theta,0,p'\cos\theta)$$

$$\omega \equiv E - E'$$

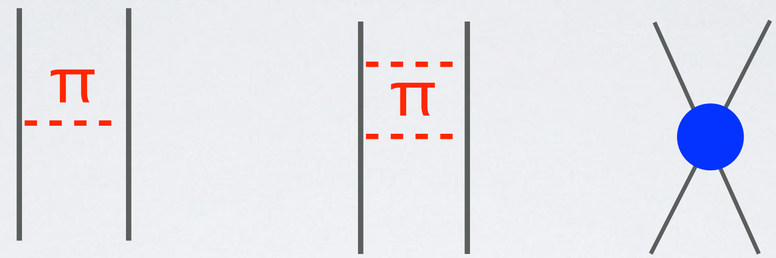
$$\vec{q} = \vec{p} - \vec{p}\,'$$

Thus q and ω are precisely known <u>without any reference to the nuclear final state</u>

from Benhar, Day, Sick, RMP 2008

Input Ingredients

Hamiltonian: two-nucleon (+ 3 nucleon) interactions



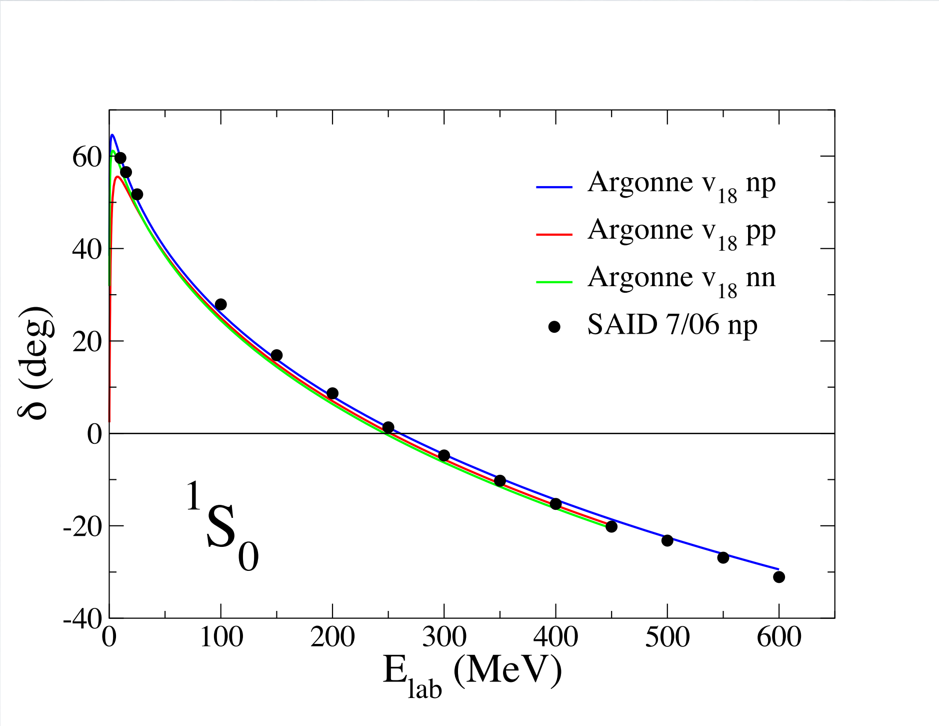
Currents: 1 + 2-nucleon currents + …

virtual pions, deltas, …

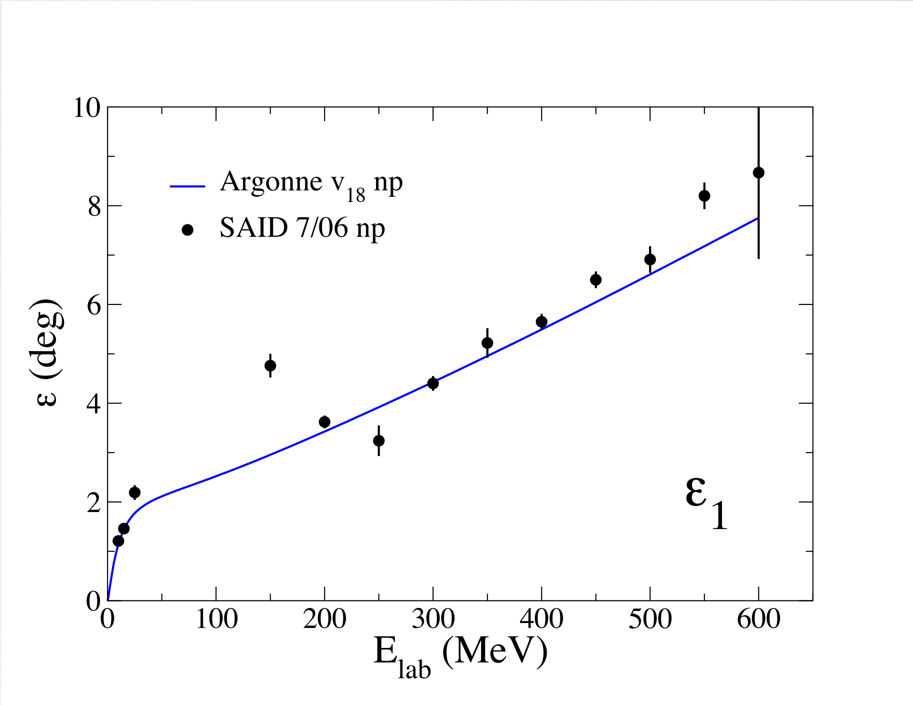
yields ground state, current, FSI, …

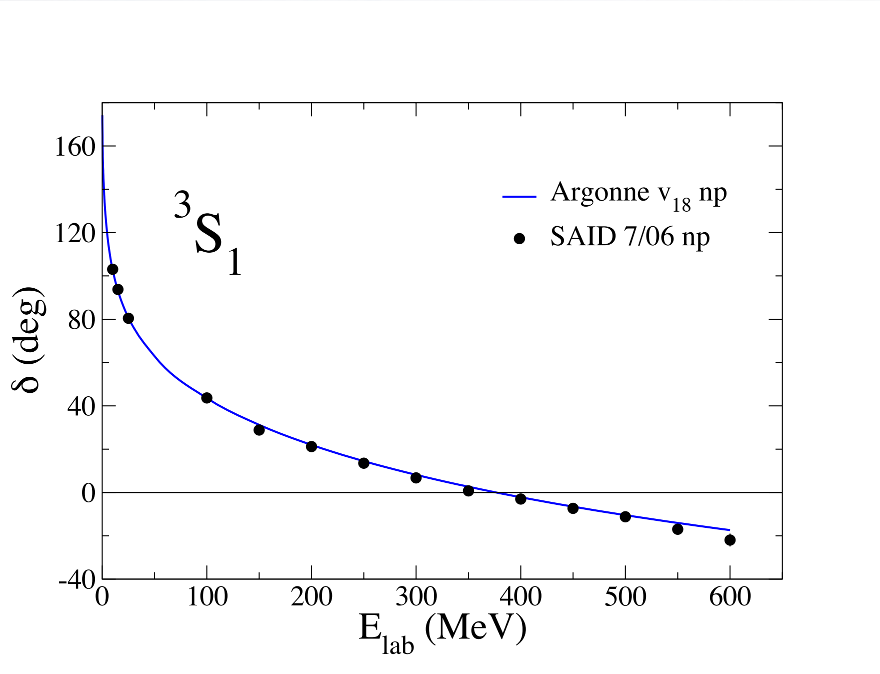
Same model for beta-decay, astrophysical neutrinos, double-beta decay, accelerator neutrinos

Phase Shifts



AV18 NN interaction

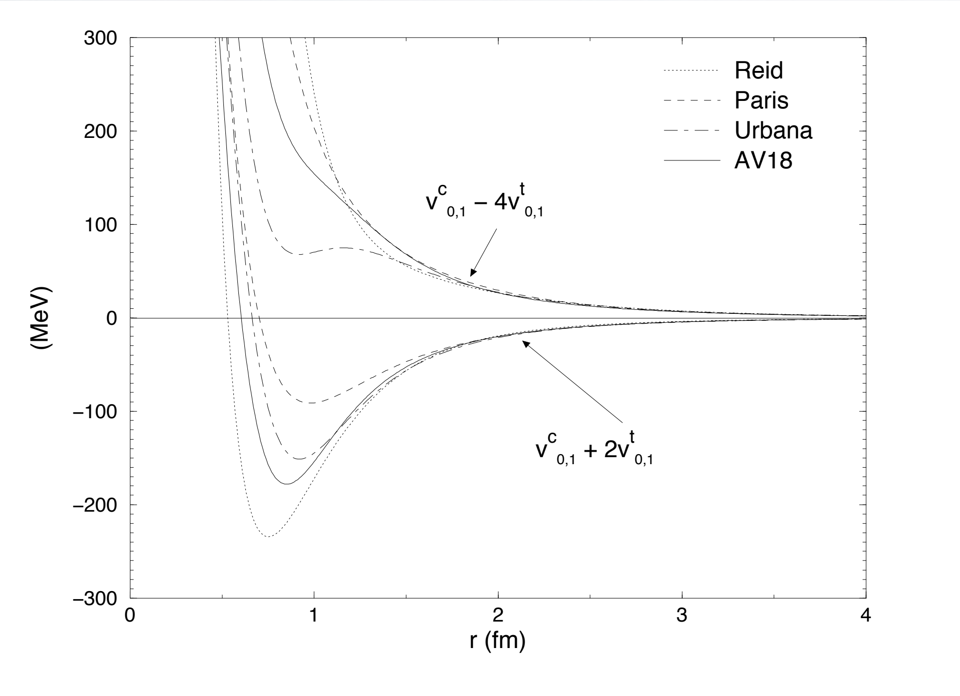


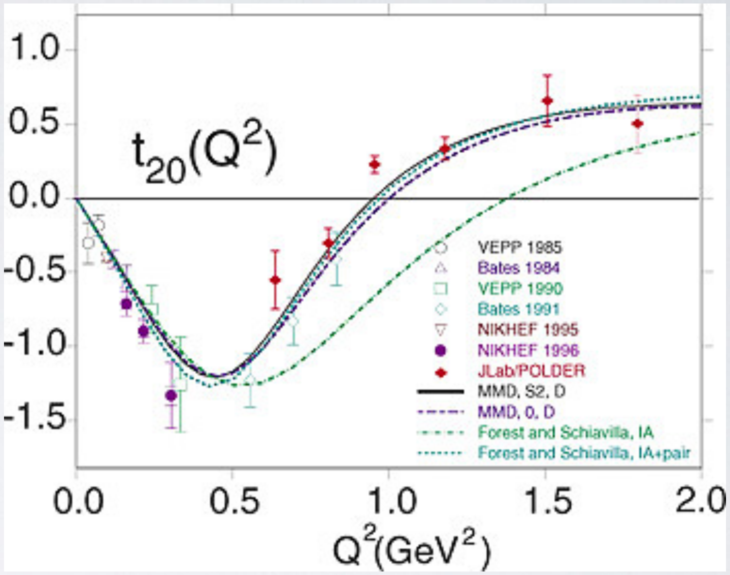


Phase shifts fit to 350 MeV lab, typically good to ~600 MeV lab

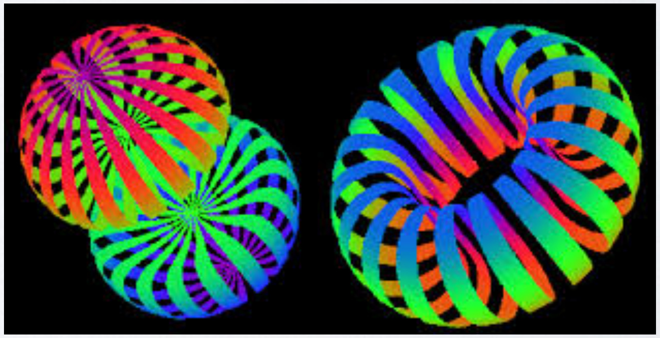
Nucleon-Nucleon Interactions

Deuteron Potential Models with Different Spin Orientations





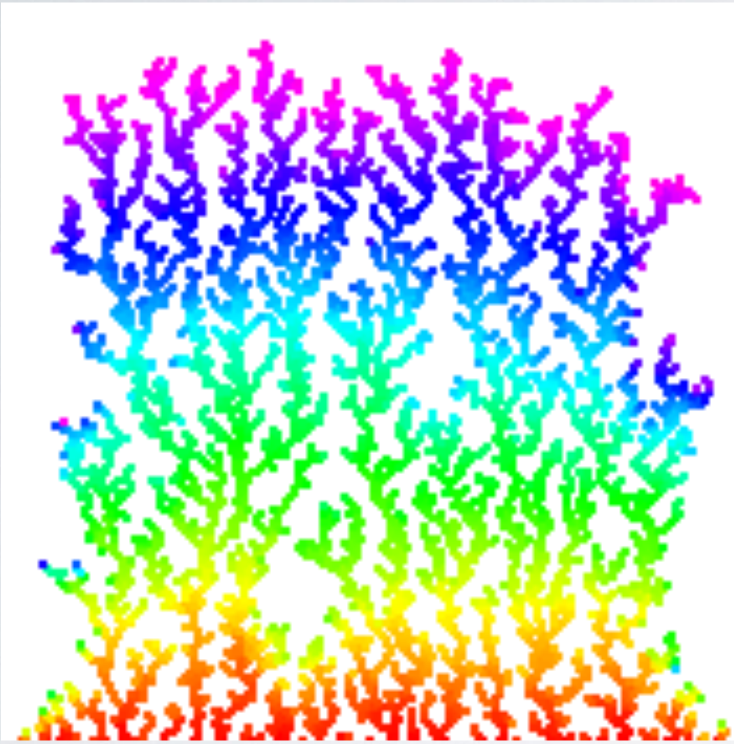
t20 experiment Jlab R. Holt



Forrest, et al, PRC 1996

Quantum Monte Carlo

Brownian motion

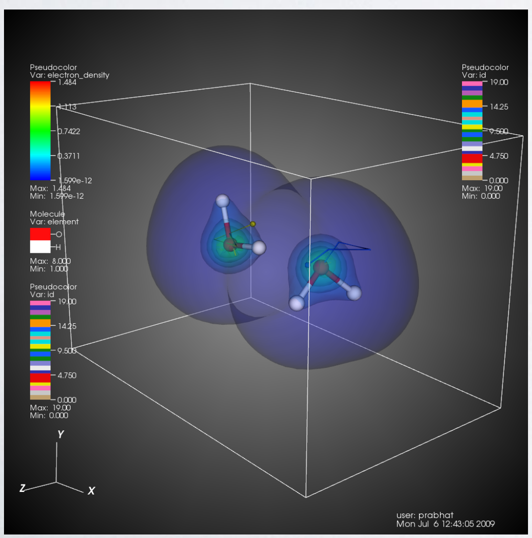


Zero Temperature

$$\Psi_0 = \exp\left[-H\tau\right]\,\Psi_T$$

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_0\,\delta(\mathbf{r_{ij}})$$

Diffusion Branching



In nuclear physics, we have a set of amplitudes for each spin and isospin

$$\Psi = \sum_{\chi(\sigma)} \sum_{\chi(\tau)} a(\chi(\sigma), \chi(\tau))\, |\chi_\sigma\rangle\, |\chi_\tau\rangle$$

Ground States

- Non-relativistic nucleons only model
- AV18 + 3-nucleon interactions
- Includes pion exchange and fits phase shifts to fairly high energies (elastic threshhold)
- Also fits low energy properties of nuclei

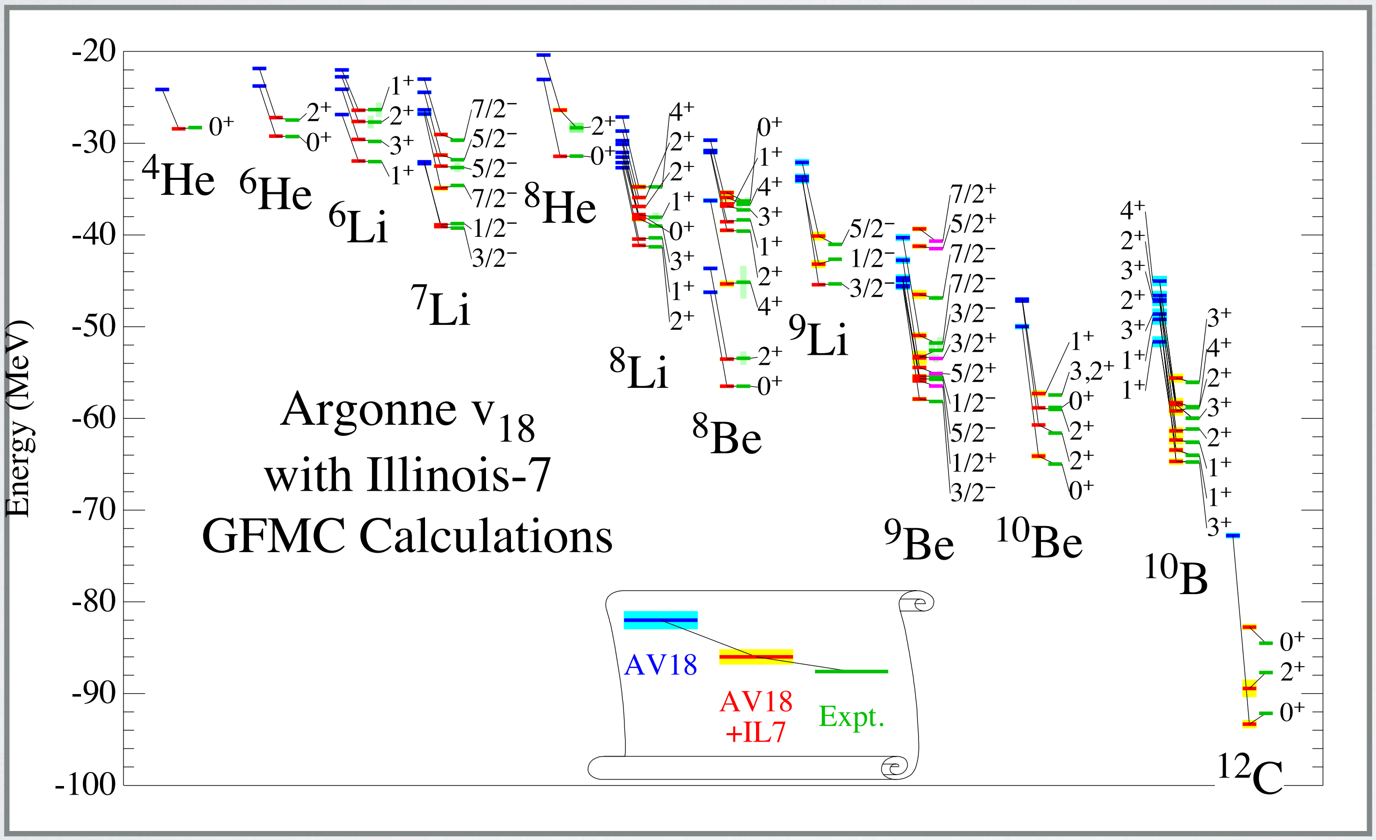
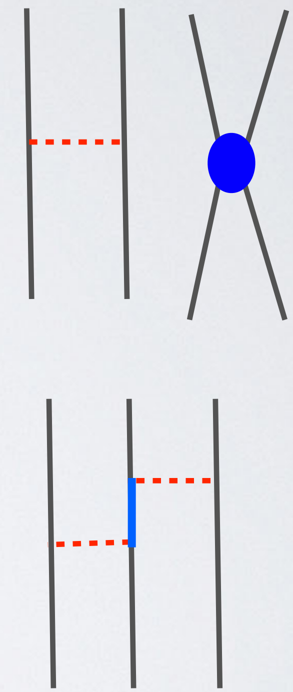


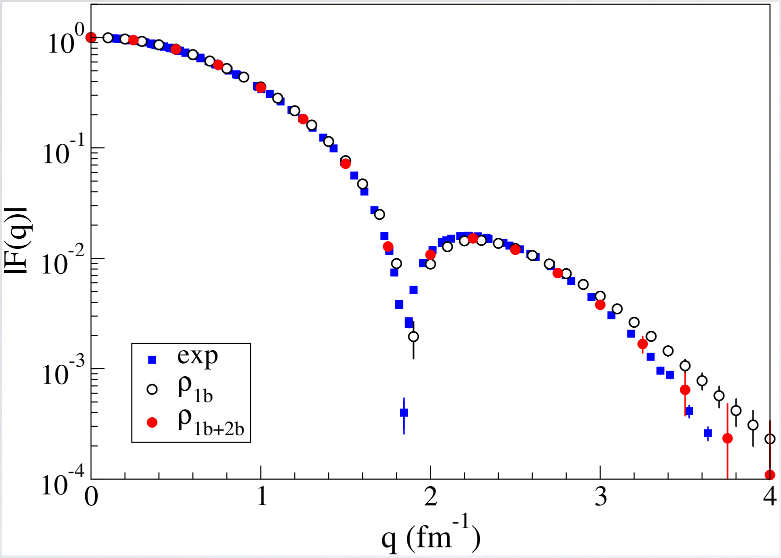
FIG. 2 GFMC energies of light nuclear ground and excited states for the AV18 and AV18+IL7 Hamiltonians compared to experiment.



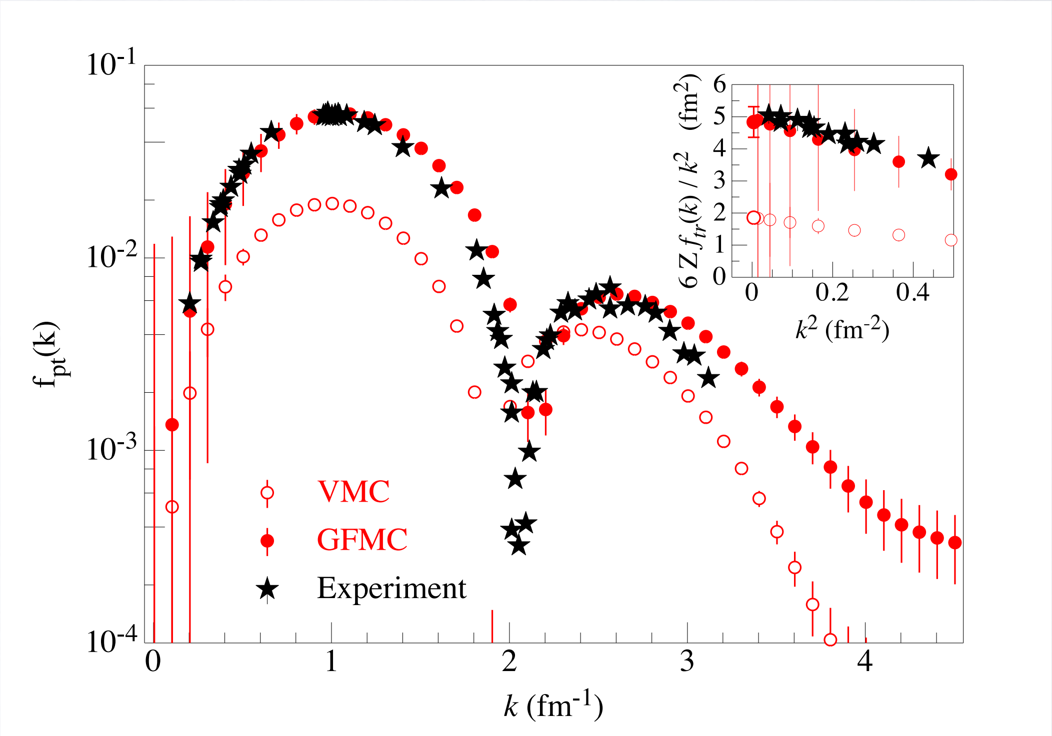
Carlson, et al, arXiv:1412.3081
to appear in Rev. Mod. Phys. (2015)

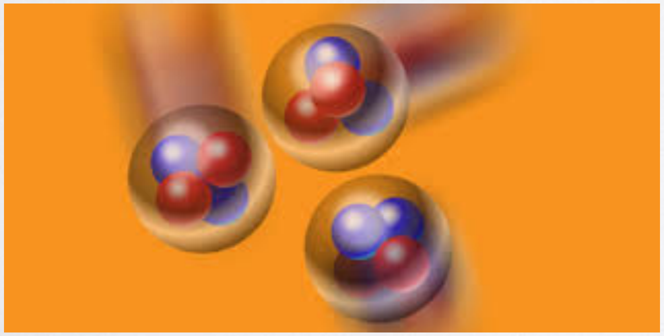
Currents / Form Factors

^{12}C elastic form factor



Hoyle state transition form factor



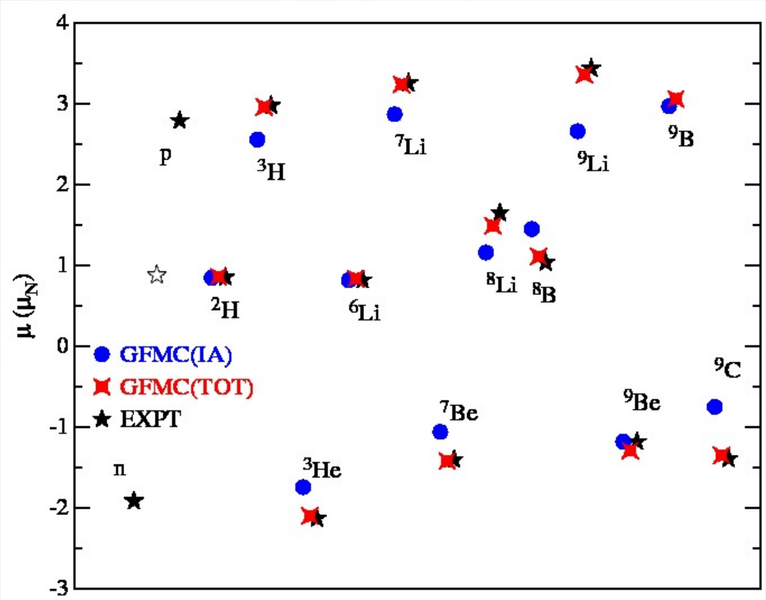


http://physics.aps.org/articles/v4/94

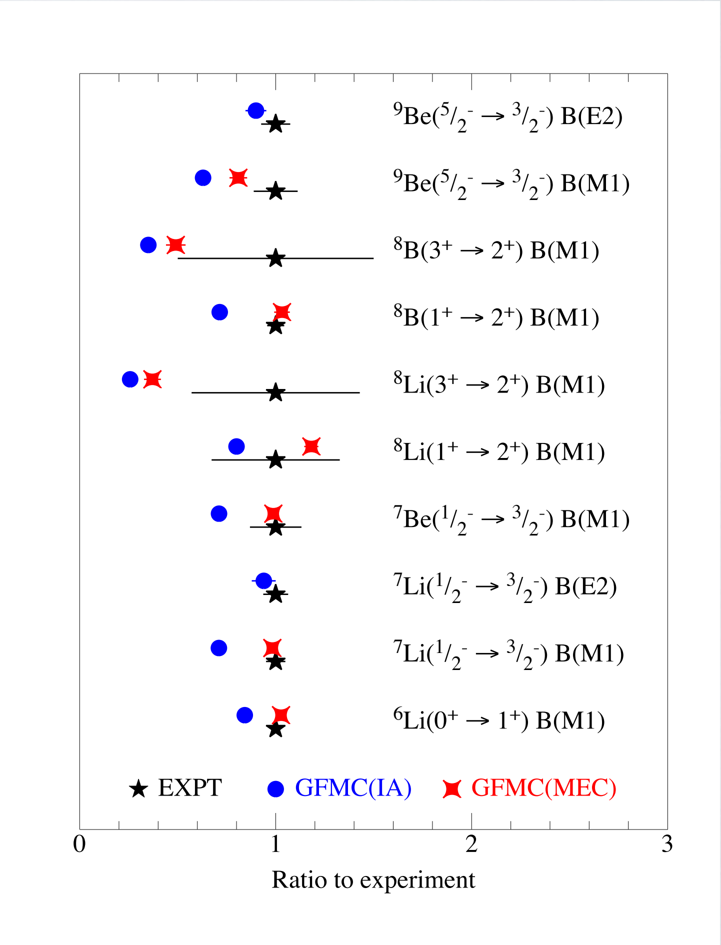
2-Nucleon Currents essential even at low q, E

Low Energy Transitions

Magnetic Moments



EM transitions and 2 nucleon currents



Pastore, Pieper, Schiavilla, Wiringa

Inclusive Scattering

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}|,\omega) \right]$$

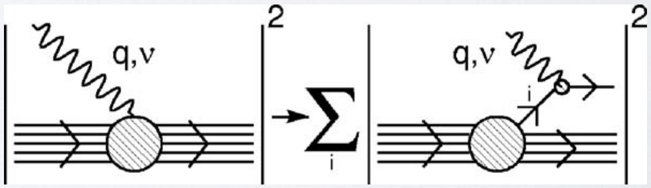
electron scattering

$$R(q,\omega) = \sum_f \langle 0|\, \mathbf{j}^\dagger(q)\, |f\rangle\langle f|\, \mathbf{j}(q)\, |0\rangle\, \delta(w - (E_f - E_0))$$

$$R(q,\omega) = \int dt\, \langle 0|\, \mathbf{j}^\dagger(q)\, \exp[i(H-\omega)t]\, \mathbf{j}(q)\, |0\rangle$$

Full Response: Ground State (Hamiltonian)
Currents
Propagation for final states

Impulse Approximation for quasi-elastic
incoherent sum over single nucleons
requires momentum distributions and spectral functions

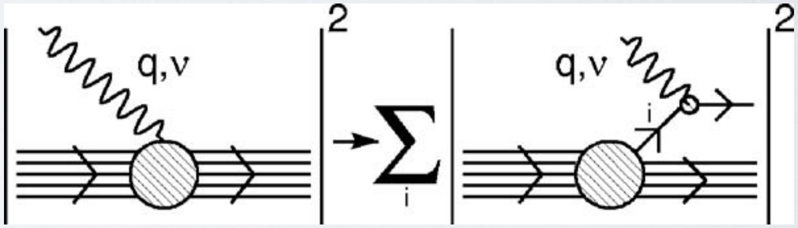


Response in PWIA

$$R\ (q,\omega)\ =\ \sum_i \langle\ 0\ |\rho_i^\dagger(q;r')\ \rho_i(q;r)|0\rangle\ \delta(E_F - E_I - \omega)$$

Requires one-body off-diagonal density matrix:
momentum distribution

$E_F = q^2/(2m) + \Delta$ including a mean-field shift



Spectral function:

includes energy of A-1 particles not interacting with the probe

$$R\ (q,\omega)\ =\ \sum_i \sum_f \langle 0\ |a_i^\dagger(q;r')|f_{A-1}\rangle\ \langle f_{A-1}|\ a_i(q;r)|0\rangle\ \delta(E_F - E_I - \omega)$$

$E_F = q^2/(2m) + \Delta + E_{A-1}$

Why are 'local' properties enough?
Simple view of Nuclei: inclusive scattering

Charge distributions of different Nuclei:

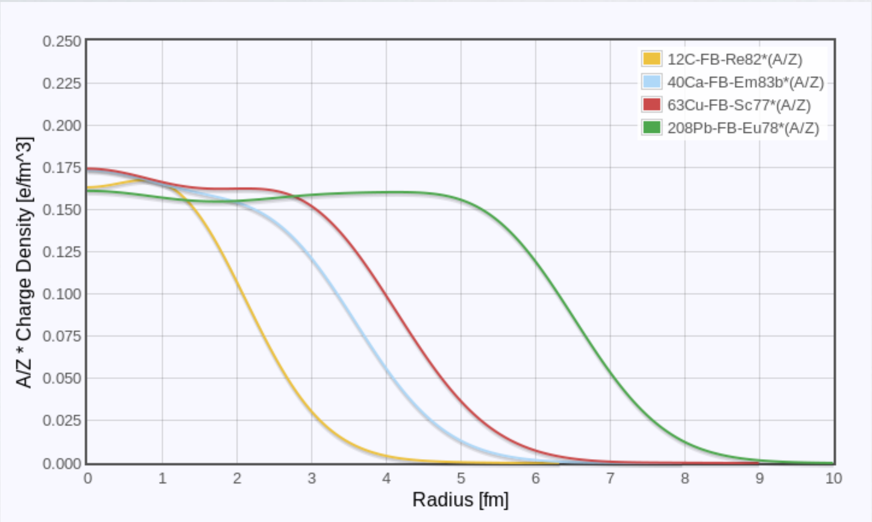
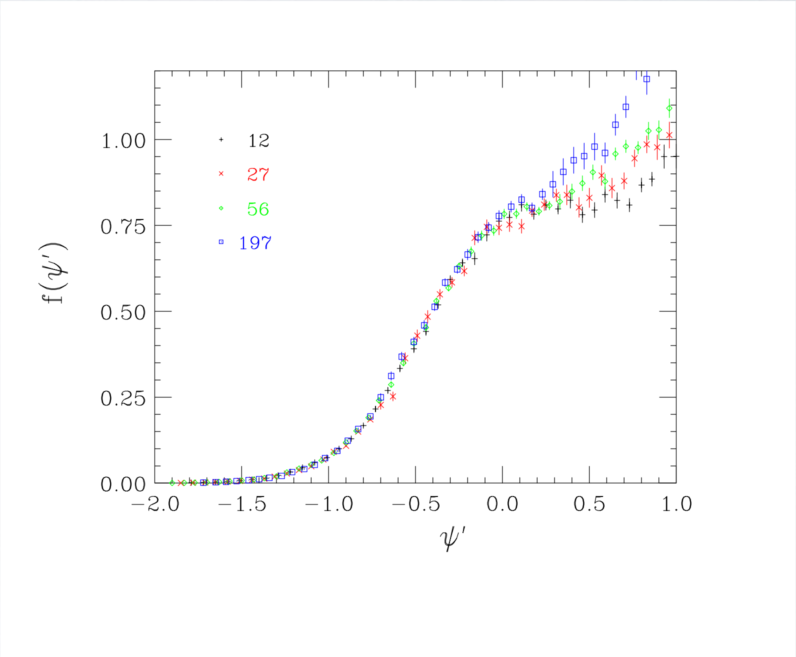


figure from faculty.virginia.edu/ncd
based on work of Hofstadter, et al.: Nobel Prize 1961

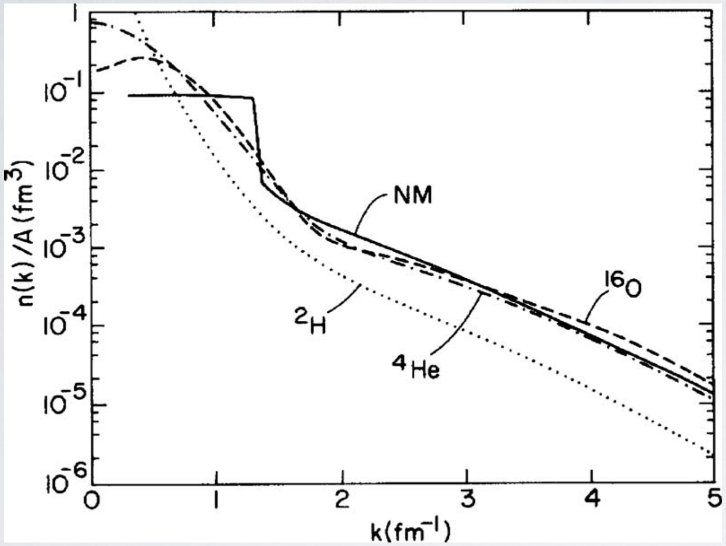
Scaling (2nd kind) different nuclei



Donnelly and Sick, 1999

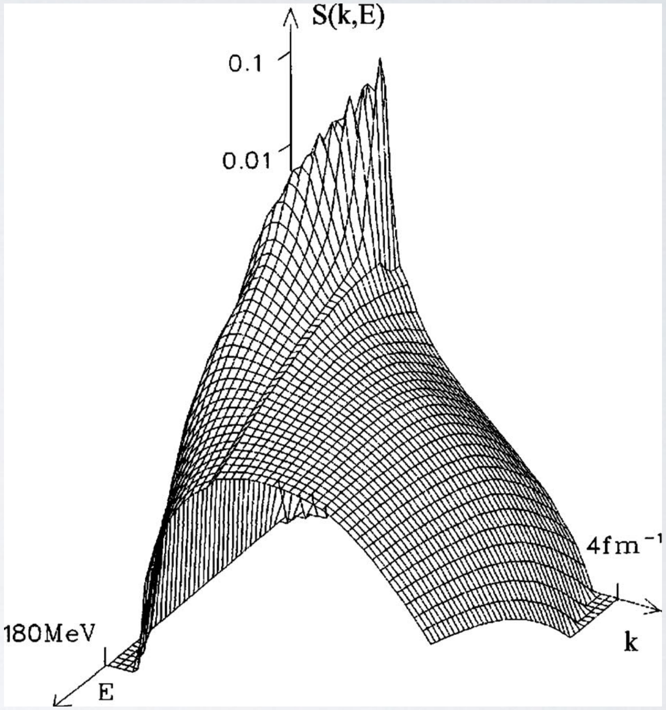
Inclusive scattering measures properties at distances ~ π / q ≲ 1 fm

Momentum Distributions and Spectral Functions



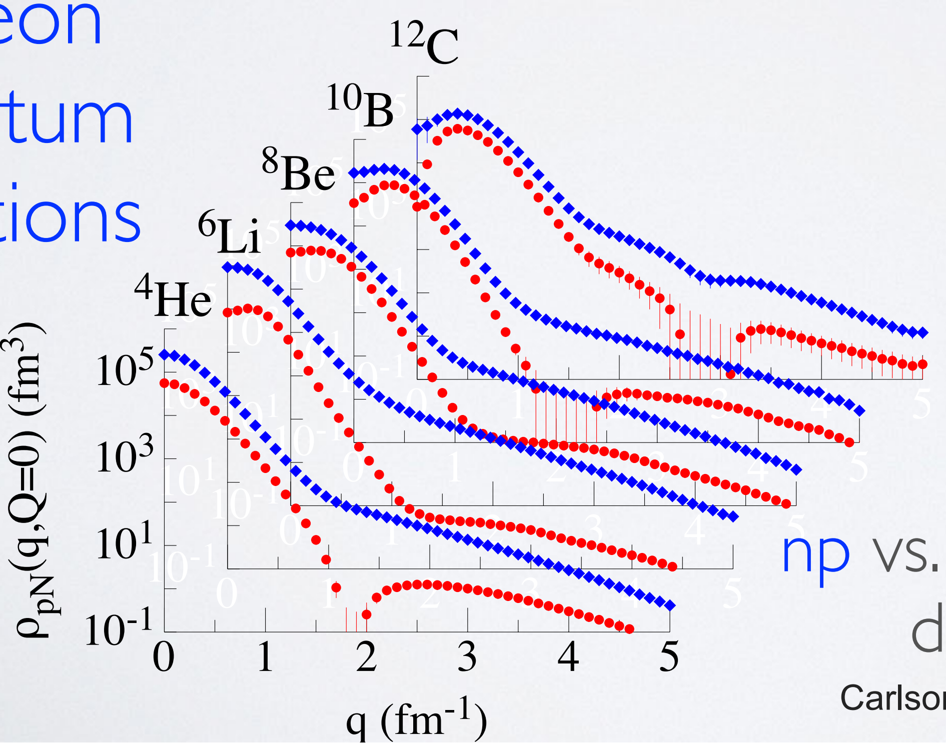
Schiavilla, et al 1986, Benhar, et al 1993

Spectral Function in NM



Benhar, 1989

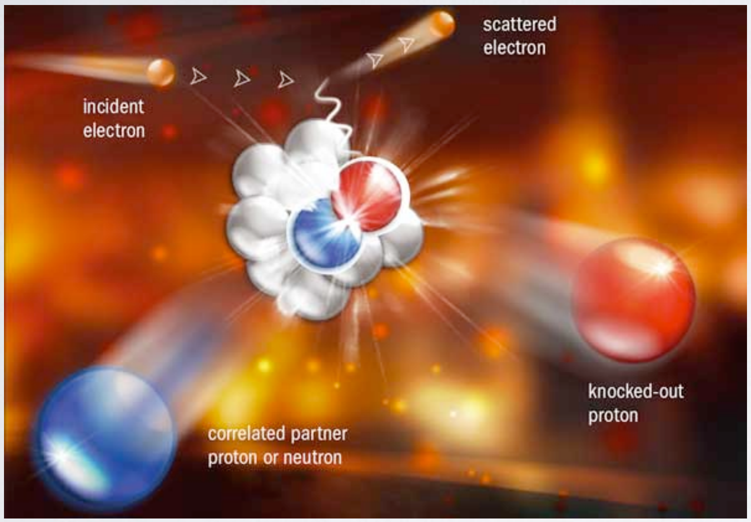
2-nucleon momentum distributions

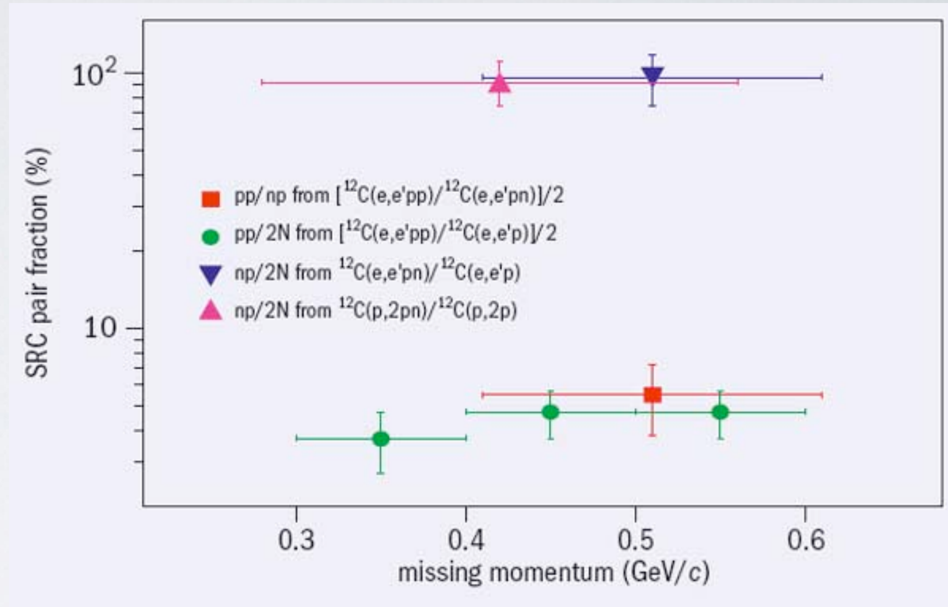


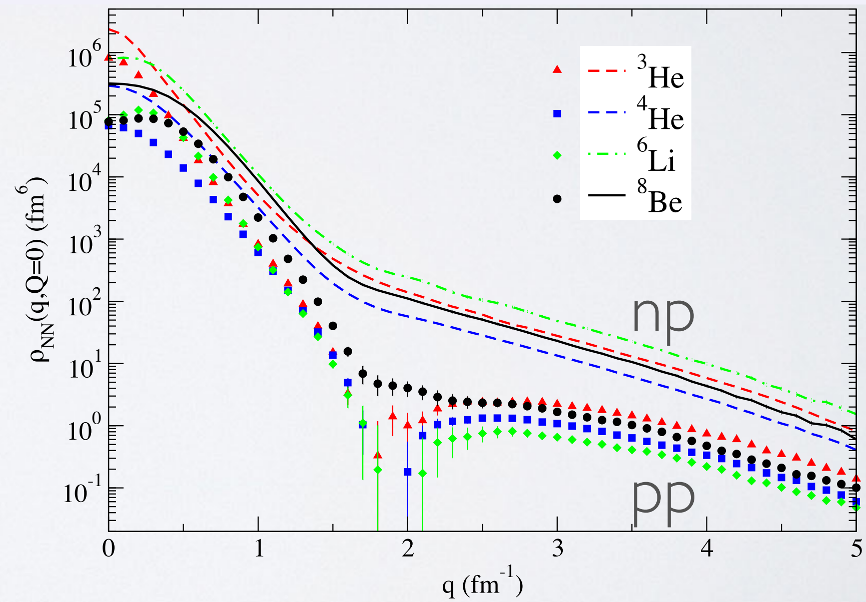
np vs. pp momentum distributions

Carlson, et al, arXiv:1412.3081

JLAB, BNL back-to-back pairs in 12C
np pairs dominate over nn and pp





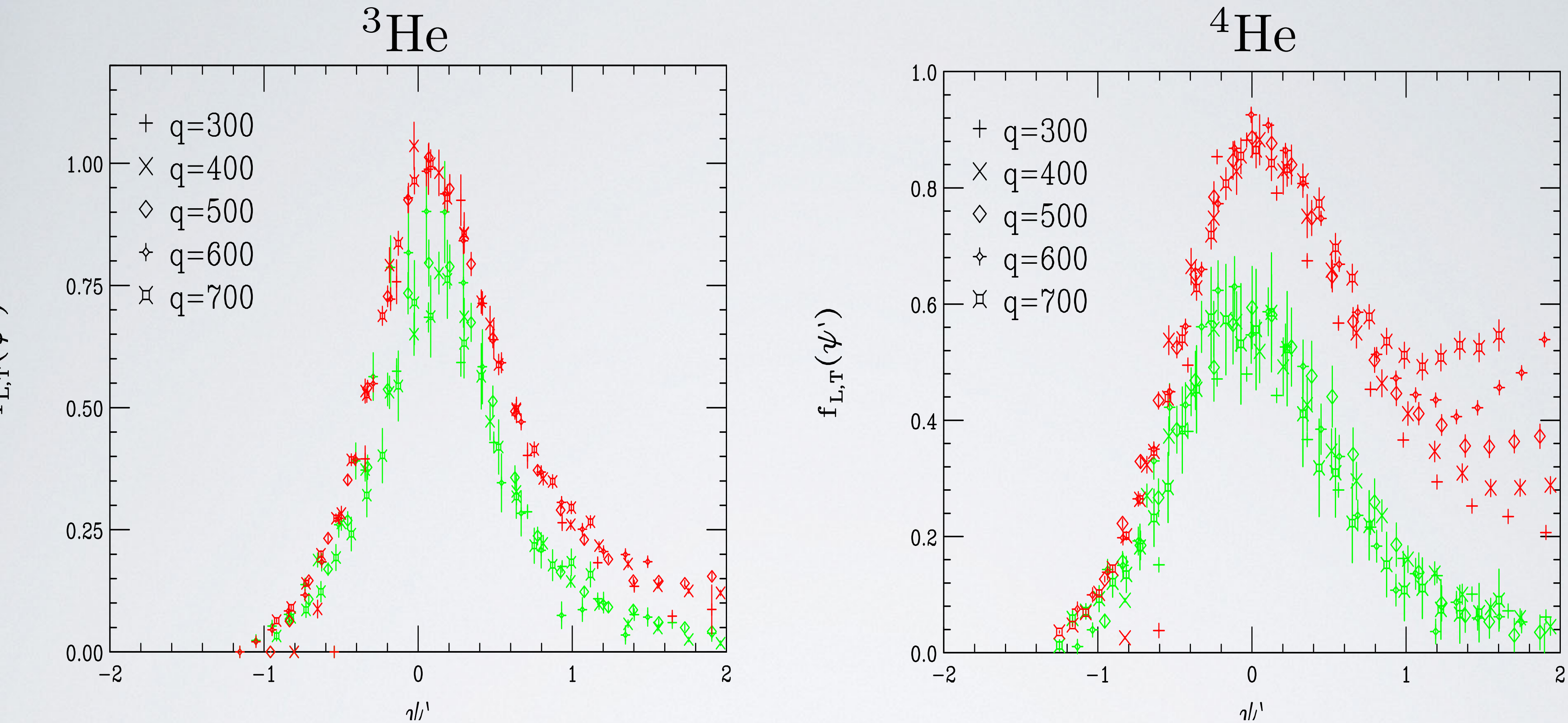


E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

P=0 pair momentum distributions

(e, e') Inclusive Response: Scaling Analysis

Donnelly and Sick (1999)



Single nucleon couplings factored out

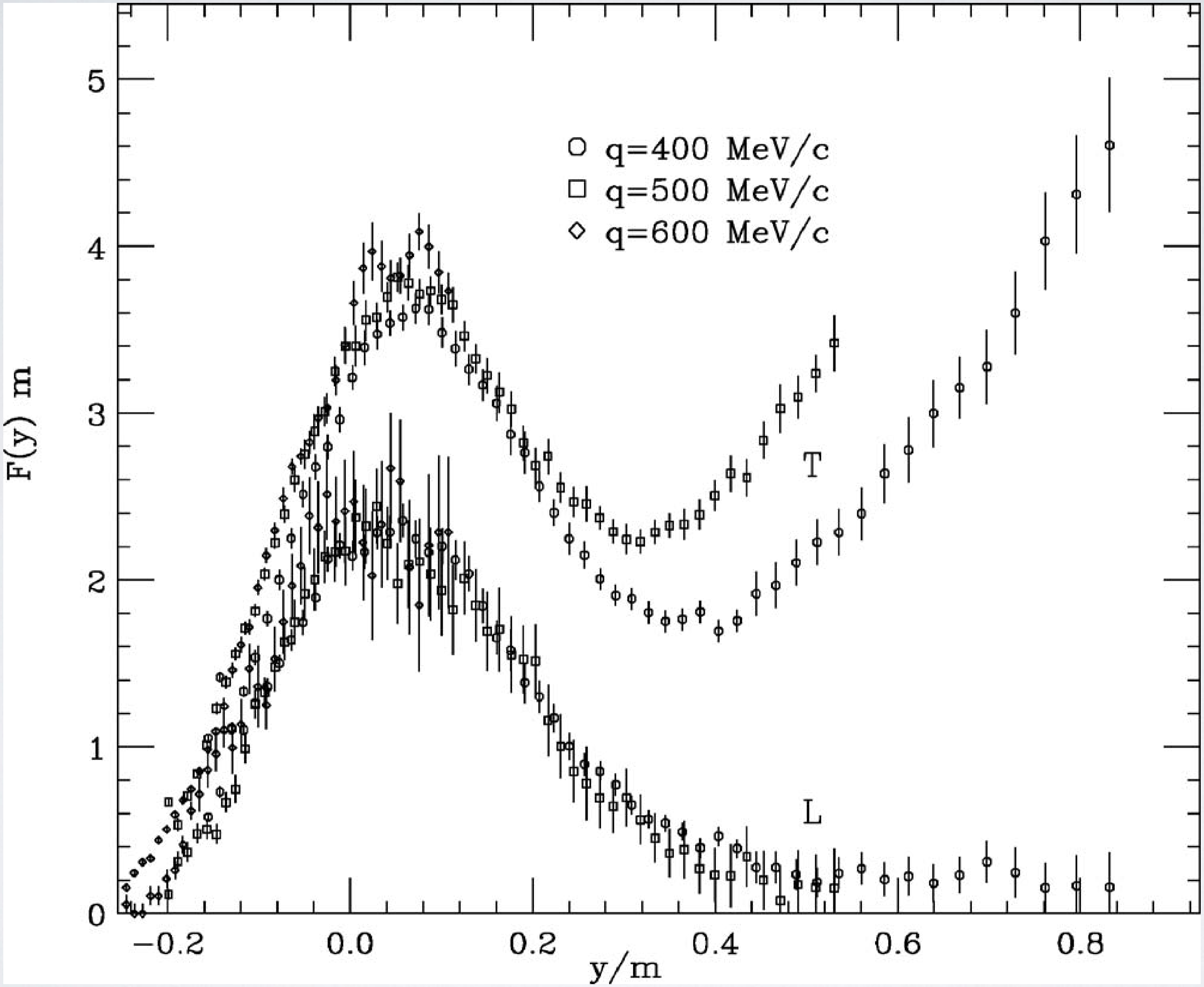
Momenta of order inverse internucleon spacing:

Large enhancement of transverse over longitudinal response

Requires beyond single nucleon physics;

spectral function alone will not work

Longitudinal/Transverse separation in ^{12}C



from Benhar, Day, Sick, RMP 2008
data Finn, et al 1984

Microscopic (non-relativistic nucleons) approach:

- Interactions fit to NN scattering data
- `Realistic' models of two-nucleon currents
- Calculate response with full inclusion of final-state interactions and two-nucleon currents

Disadvantages: (can be improved)

non-relativistic nucleons

no pion production or Δ production

Advantages:

same treatment for initial and final states

include full realistic interactions fit to NN data

with simultaneous two-nucleon currents

What we can compute reliably
(given the interaction/ current model)

$$R_{L,T}\ (q,\omega)\ = \sum_f \delta(\omega\ +\ E_0\ + E_f)\ |\ \langle\ f\ |\mathcal{O}_{\mathcal{L},\mathcal{T}}\ |\ 0\ \rangle\ |^2$$

Easy to calculate Sum Rules: ground-state observable

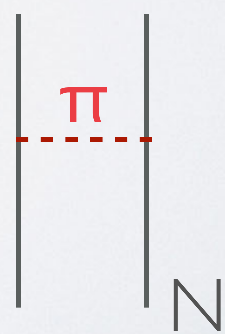
$$S(q)\ =\ \int d\omega\ R(q,\omega) = \langle 0|O^\dagger(q)\ O(q)|0\rangle$$

Imaginary Time (Euclidean Response)
statistical mechanics

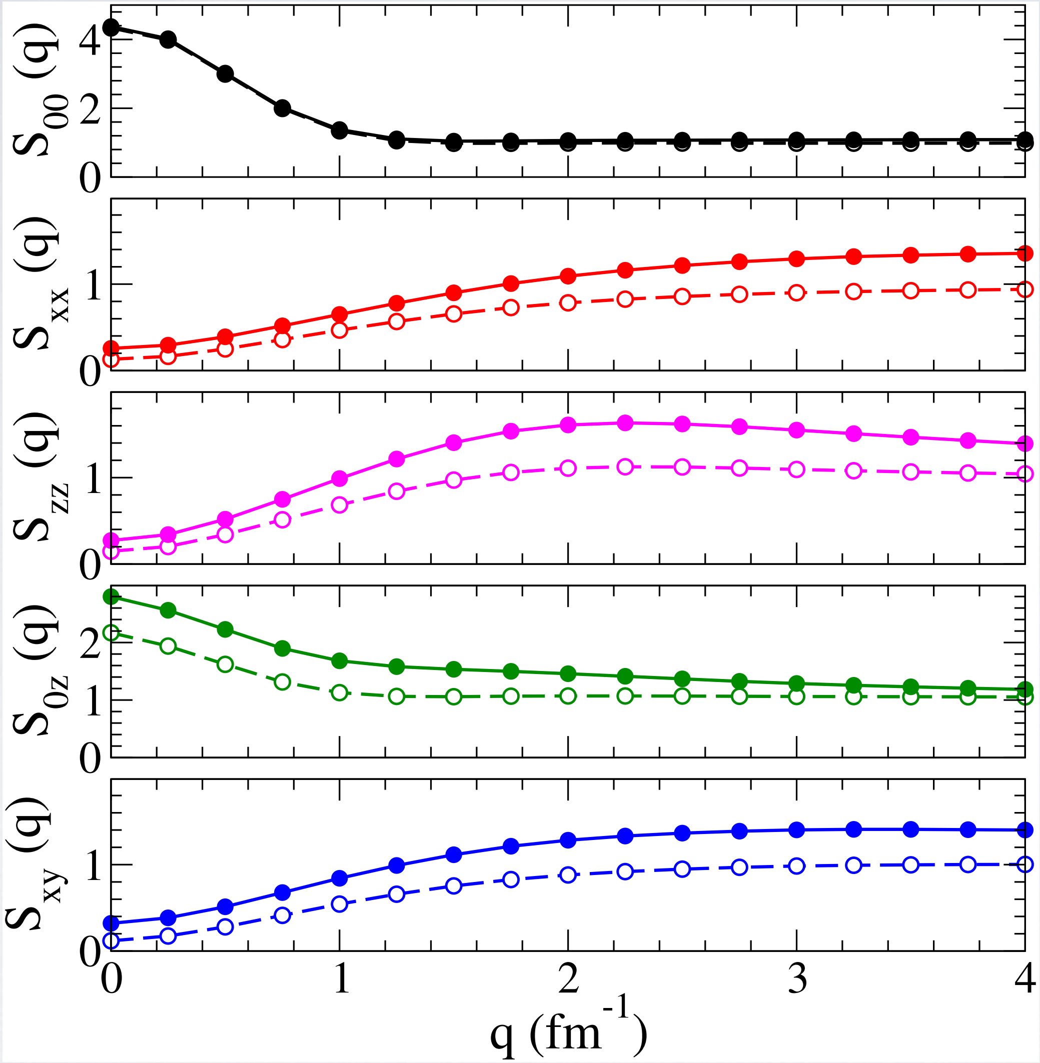
$$\tilde{R}(q,\tau)\ =\ \langle 0|\ \mathbf{j}^\dagger\ \exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\ \ \mathbf{j}\ |\mathbf{0}\rangle >$$

$$H\ = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}\ +\ \sum_{i<j<k} V_{ijk}$$

$$\mathbf{j}\ = \sum_i \mathbf{j}_i\ +\ \sum_{i<j} \mathbf{j}_{ij} + ...$$



Sum rules in 12C



EM

Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus
Full currents (filled symbols)

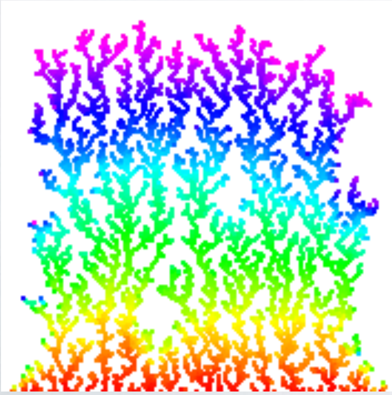
Euclidean Response

$$\tilde{R}(q,\tau) \;=\; \langle 0|\; \mathbf{j}^\dagger \;\exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\;\; \mathbf{j}\;|\mathbf{0}\rangle >$$

short `time' τ - high energy

- Exact given a model of interactions, currents
- Full final-state interactions
- `Local' Operator
- Can apply to large A; no assumptions about final states

$$\exp[-H\tau] \;\approx\; \exp[-V\tau/2]\exp[-T\tau]\exp[-V\tau/2]$$

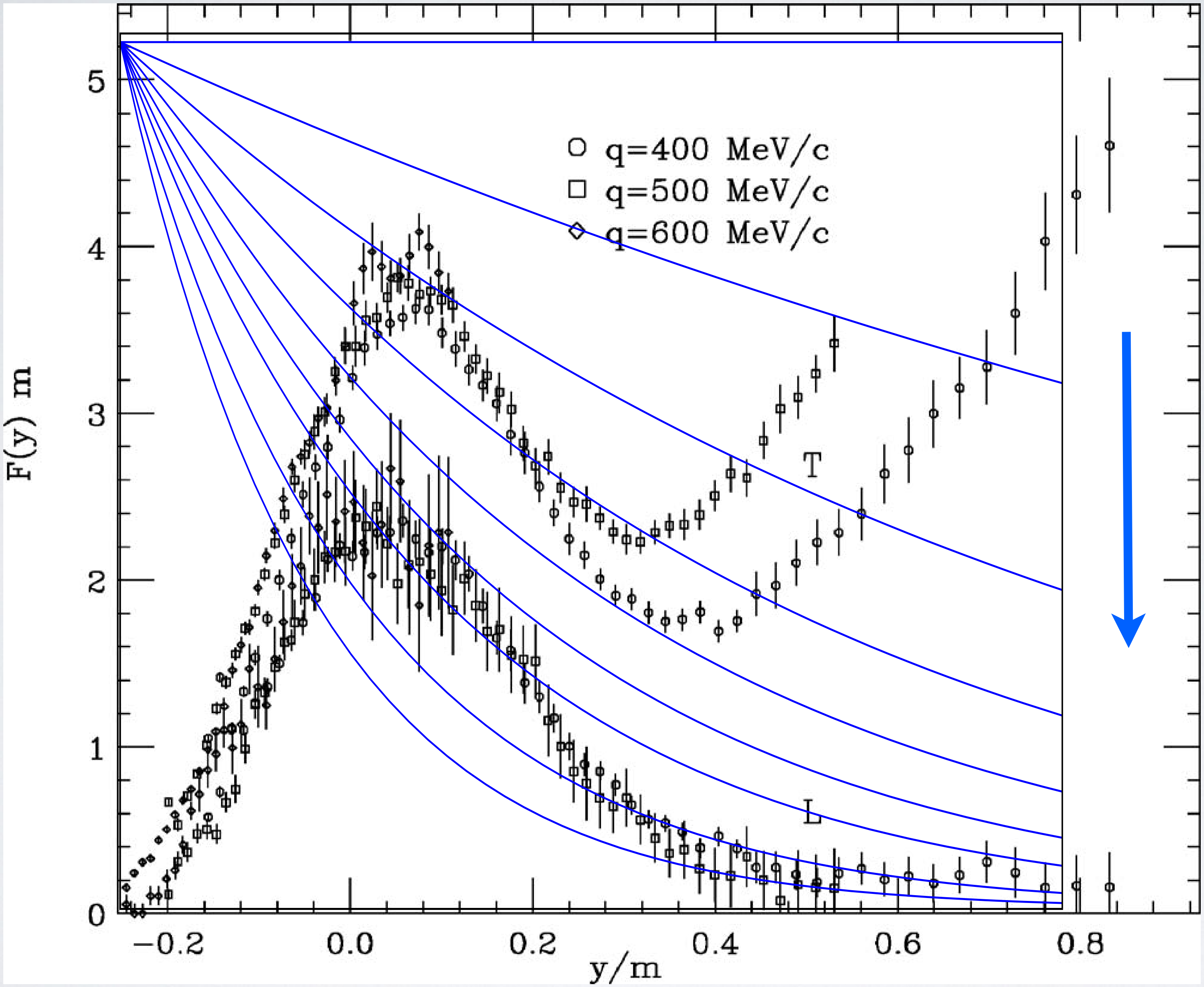


$\exp[-T\tau]$ gaussian - momentum dist

$\exp[-V\tau]$ branching, spin & isospin rotation

can introduce equivalent of spectral function

Euclidean Response



τ = inverse T

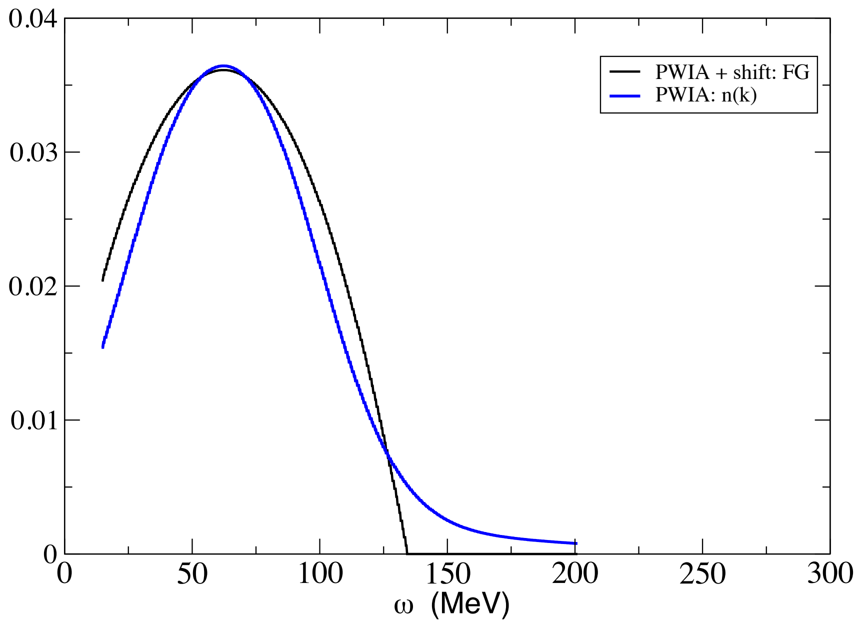
Sum rule → elastic FF^2 w/ increasing τ

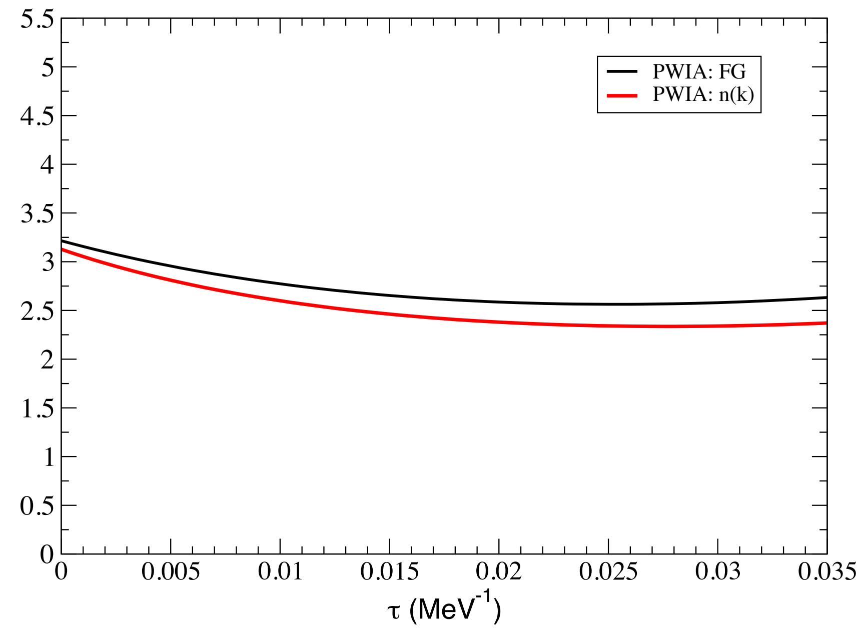
Transverse Response ^{12}C

Low Momentum Transfer

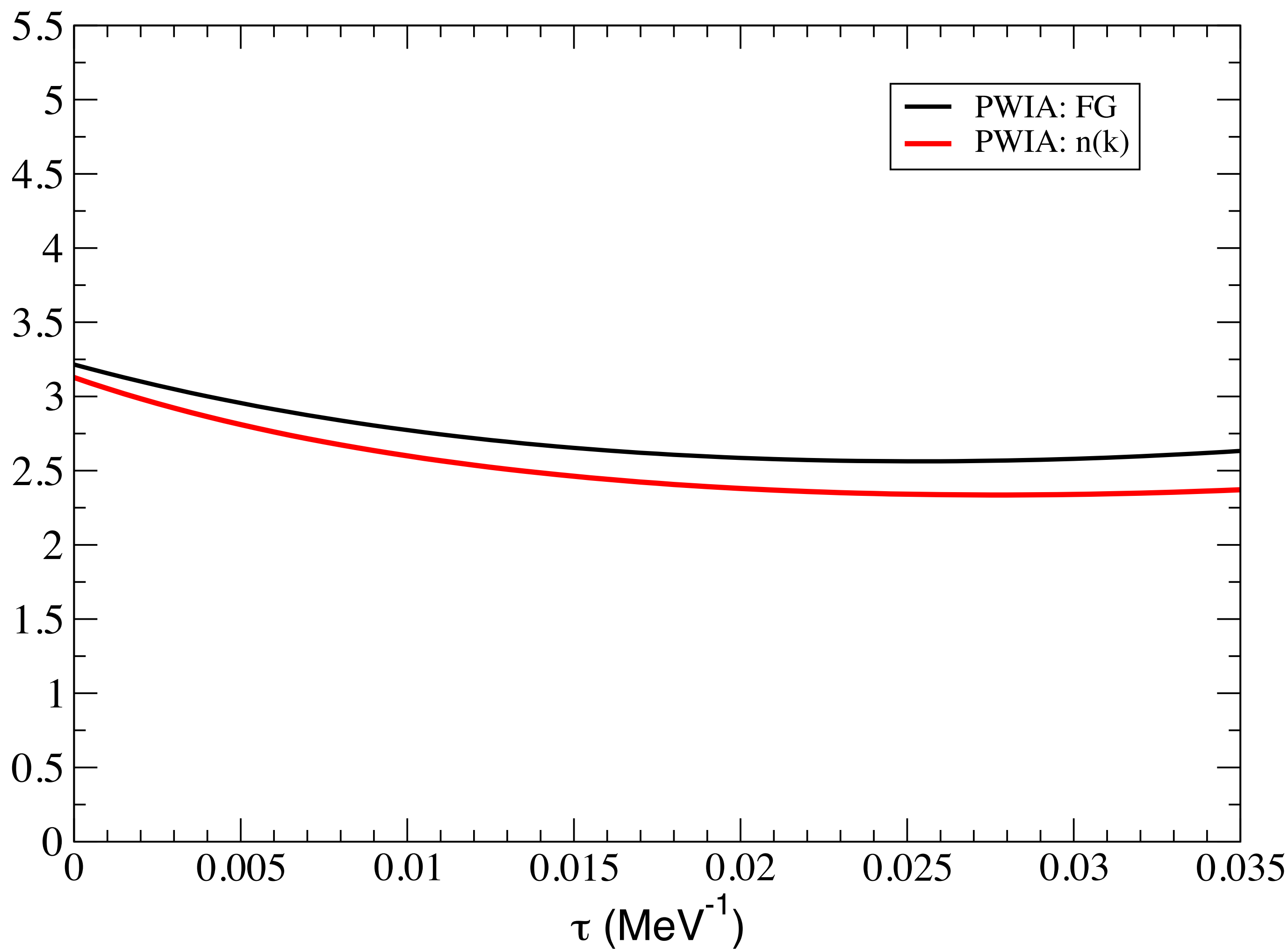
q = 300 MeV/c

PWIA models

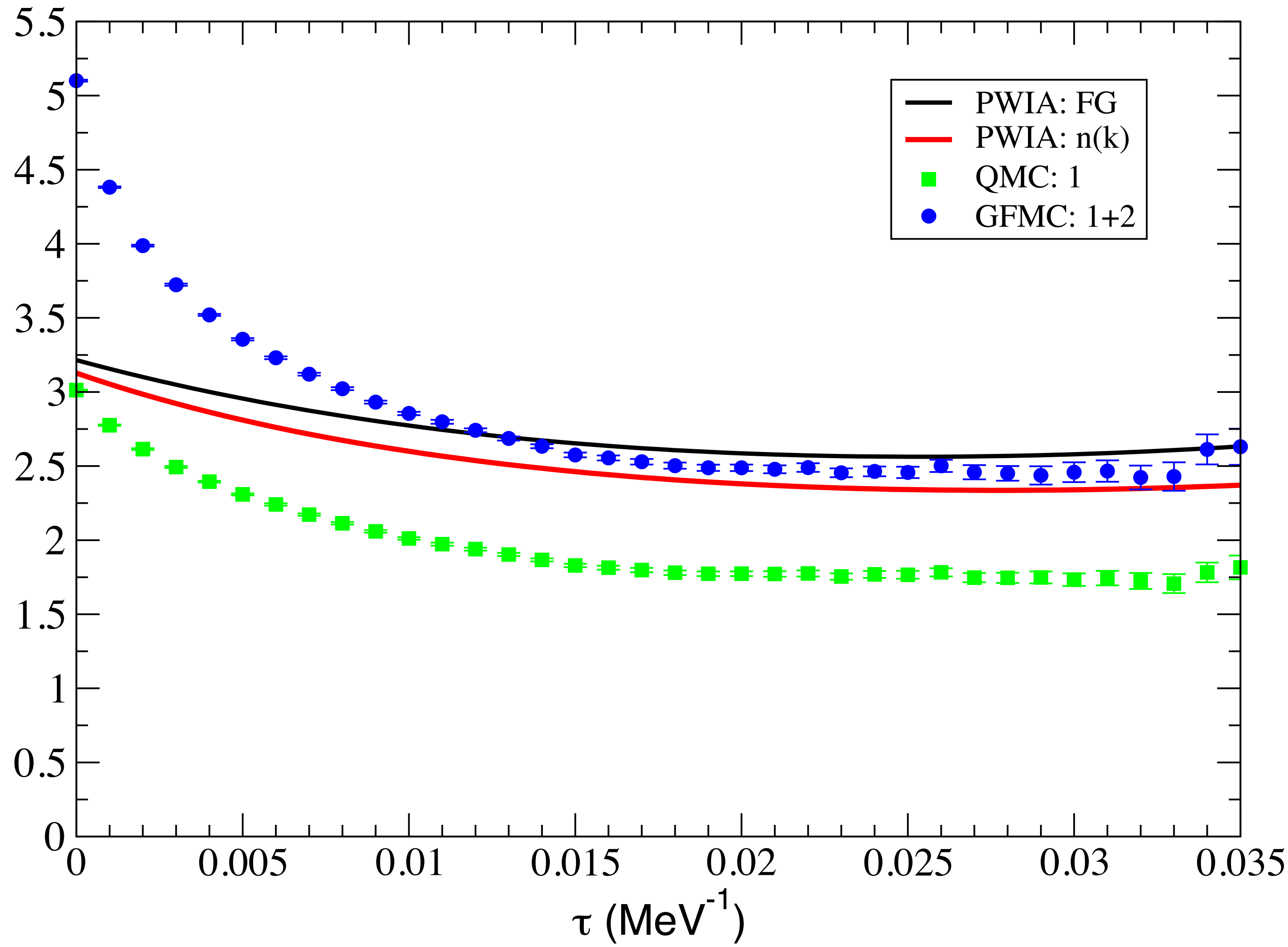




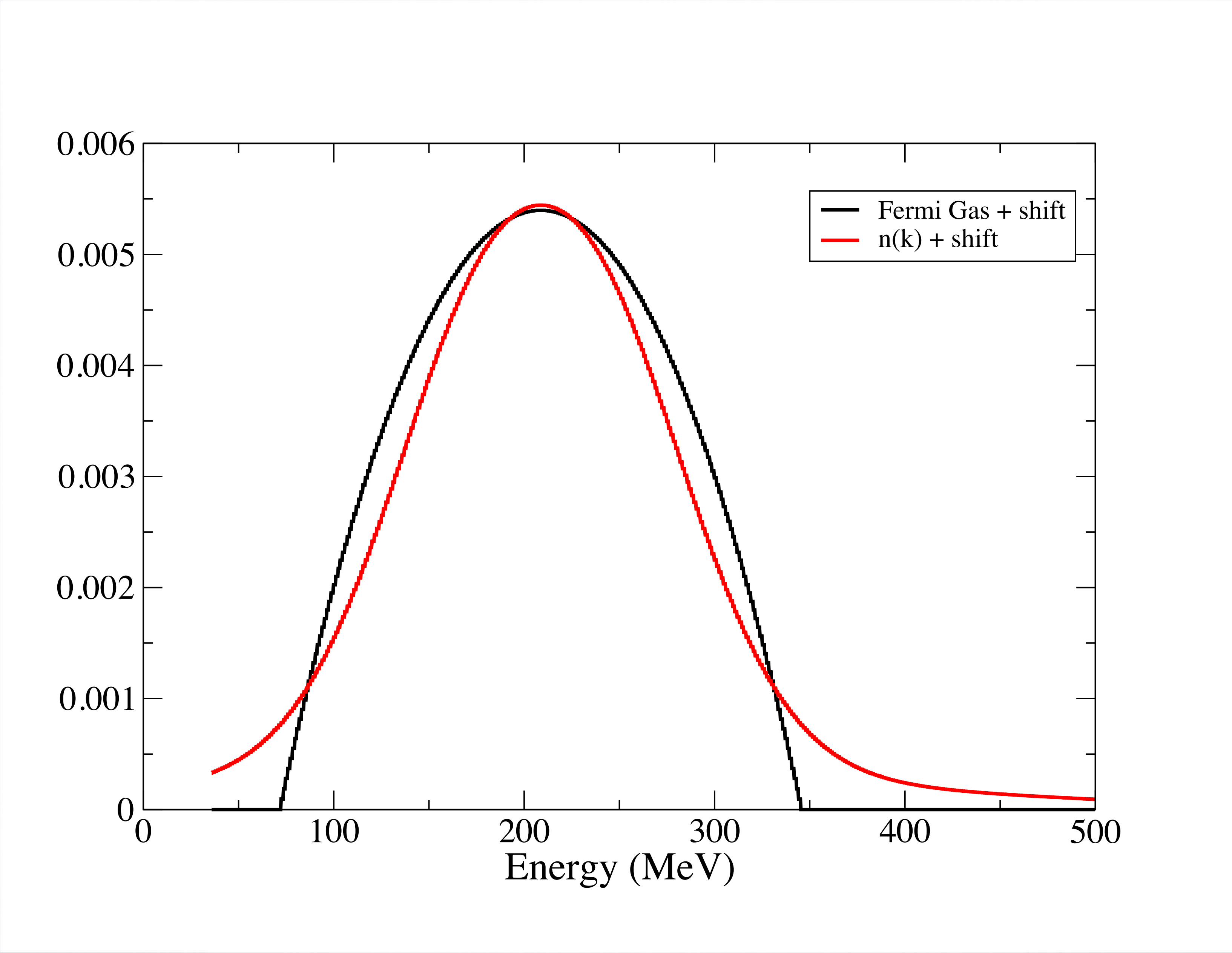
Transverse Response ^{12}C
q = 300 MeV/c



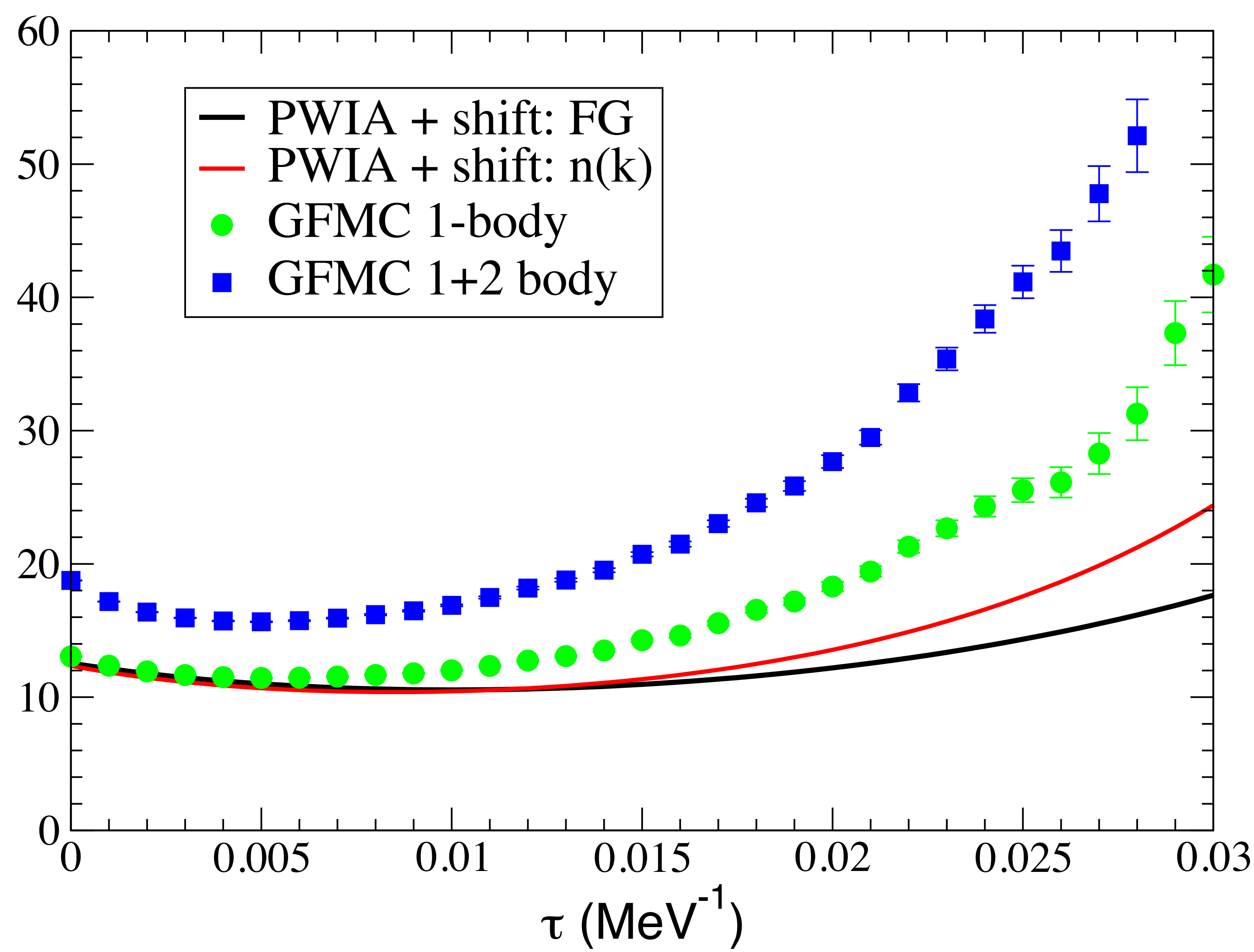
Transverse Response ^{12}C
q = 300 MeV/c



Transverse Response ^{12}C
q = 570 MeV/c



Transverse Response ^{12}C
q = 570 MeV/c



Outstanding Issues:

Larger Nuclei: (Argon)

QMC for larger nuclei
Two-nucleon factorization models

Higher (and Lower) Energy:

Include pion (and Delta production)
Relativistic Kinematics
Include Interference
More feasible for factorization models

Backup Slides

Two (complimentary) approaches:

Quantum Monte Carlo for Larger Nuclei (AFDMC, sample spins and isospins)

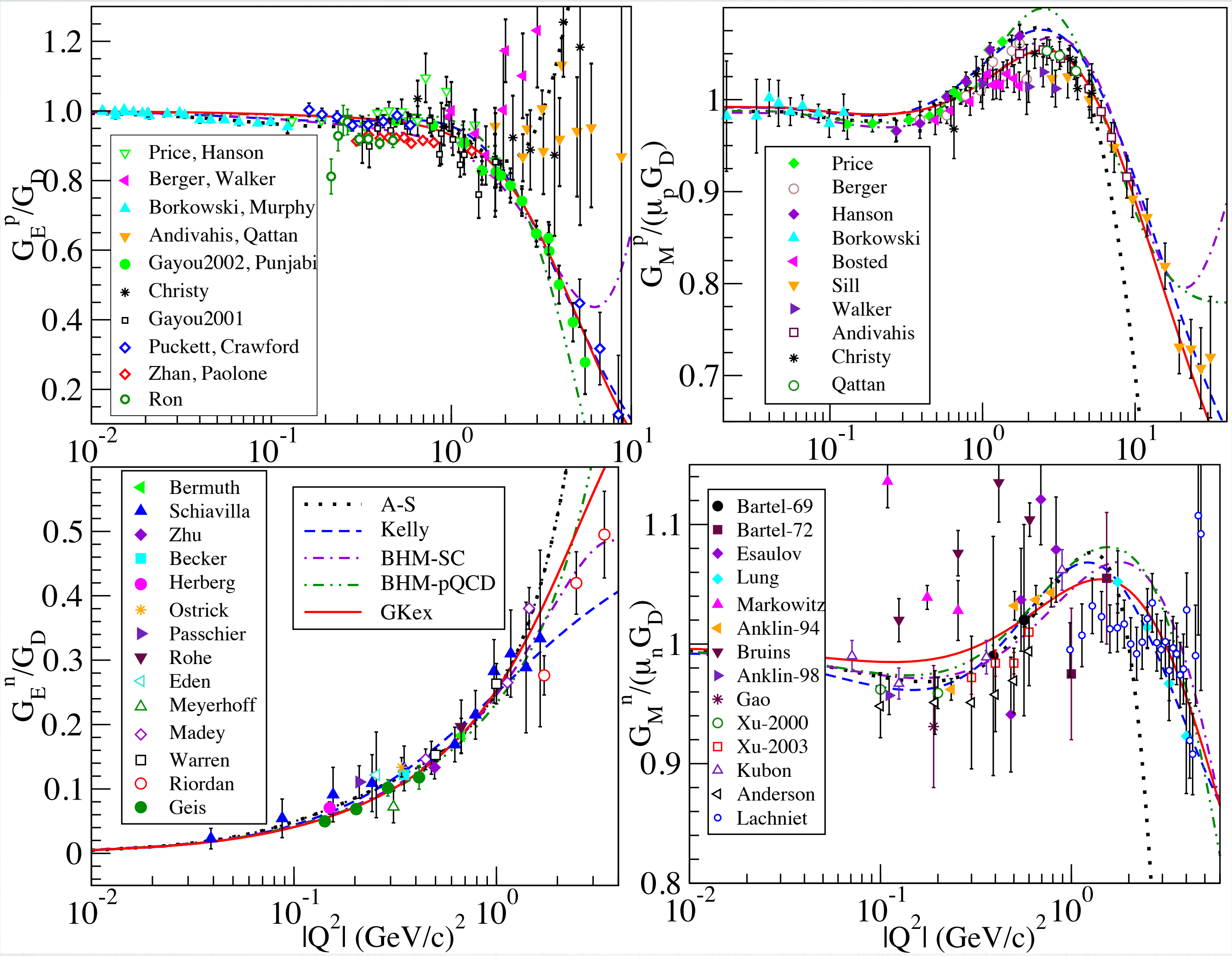
Ground states, momentum distributions,
sum rules, Euclidean Response

Factorization Approaches at two-nucleon level

keep two-nucleon dynamics exactly (interactions, current)
global constraints from QMC approaches (sum rules, Euclidean)
improvable: relativistic kinematics, Deltas, …

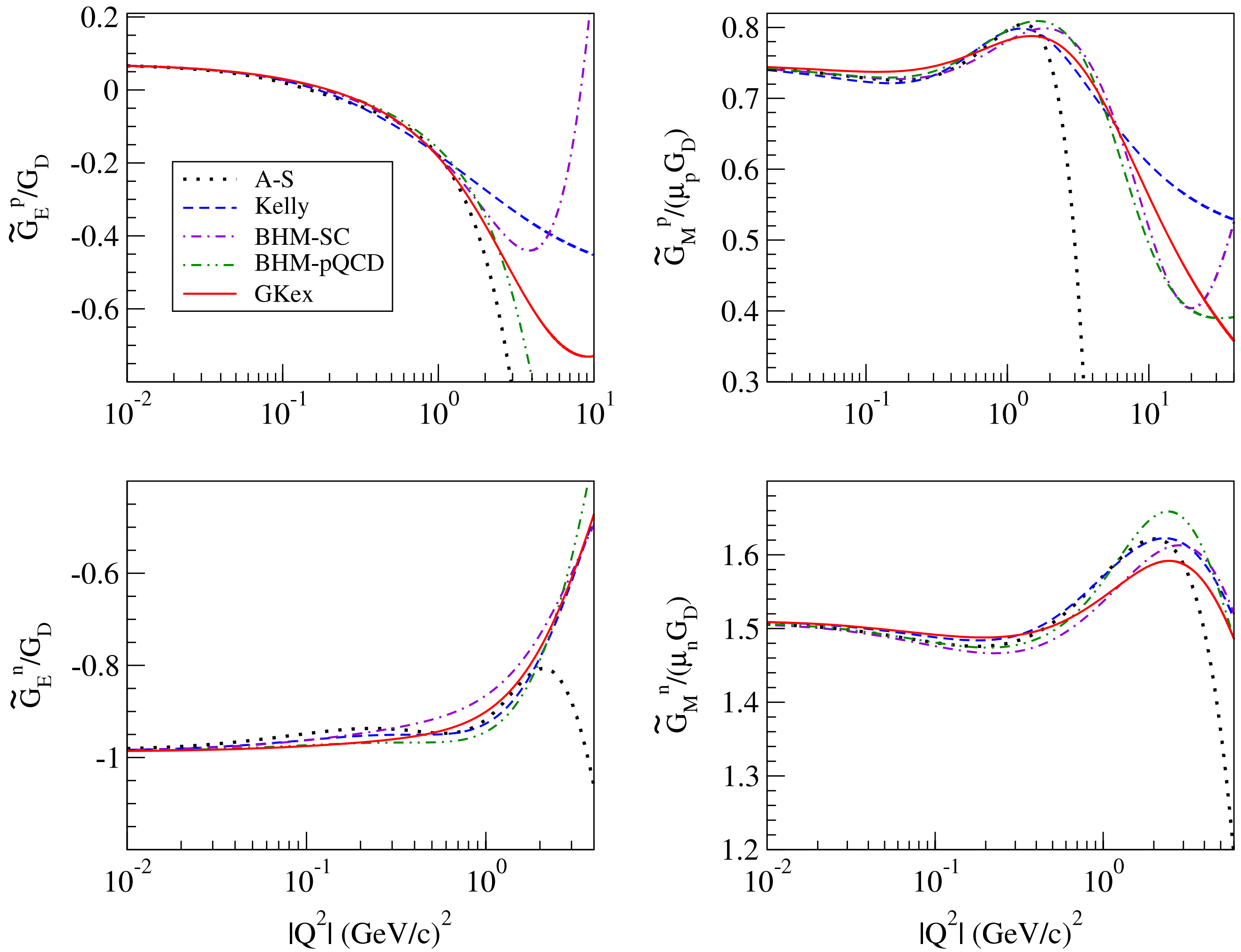
Thanks to:
ANL devoting ~100M core-hours to this project plus staff/postdoc time
NULEI SciDAC-3 project (computingnuclei.org)
INCITE award to NUCLEI project amount largest in country
 - neutrino scattering is an important goal
LANL support through LDRD-DR and LDRD-ER Projects

Nucleon Form Factors: Electromagnetic

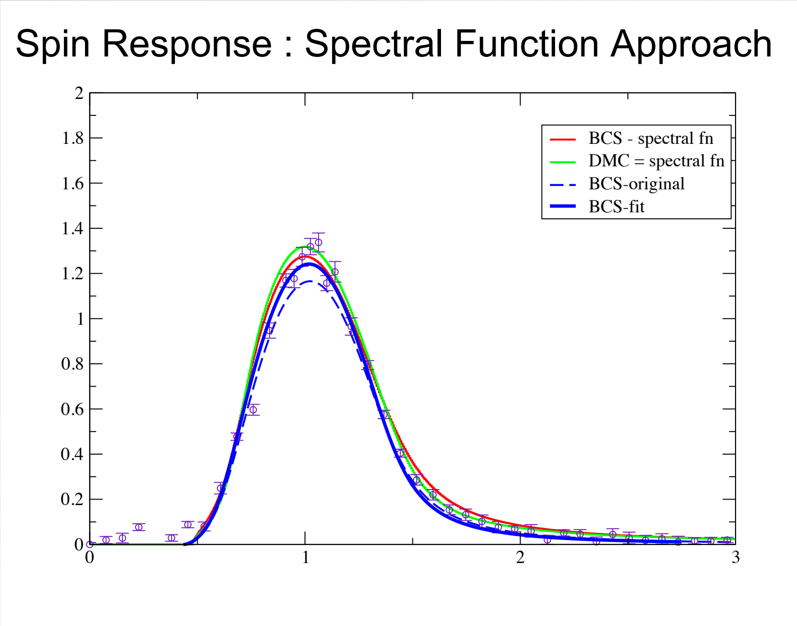


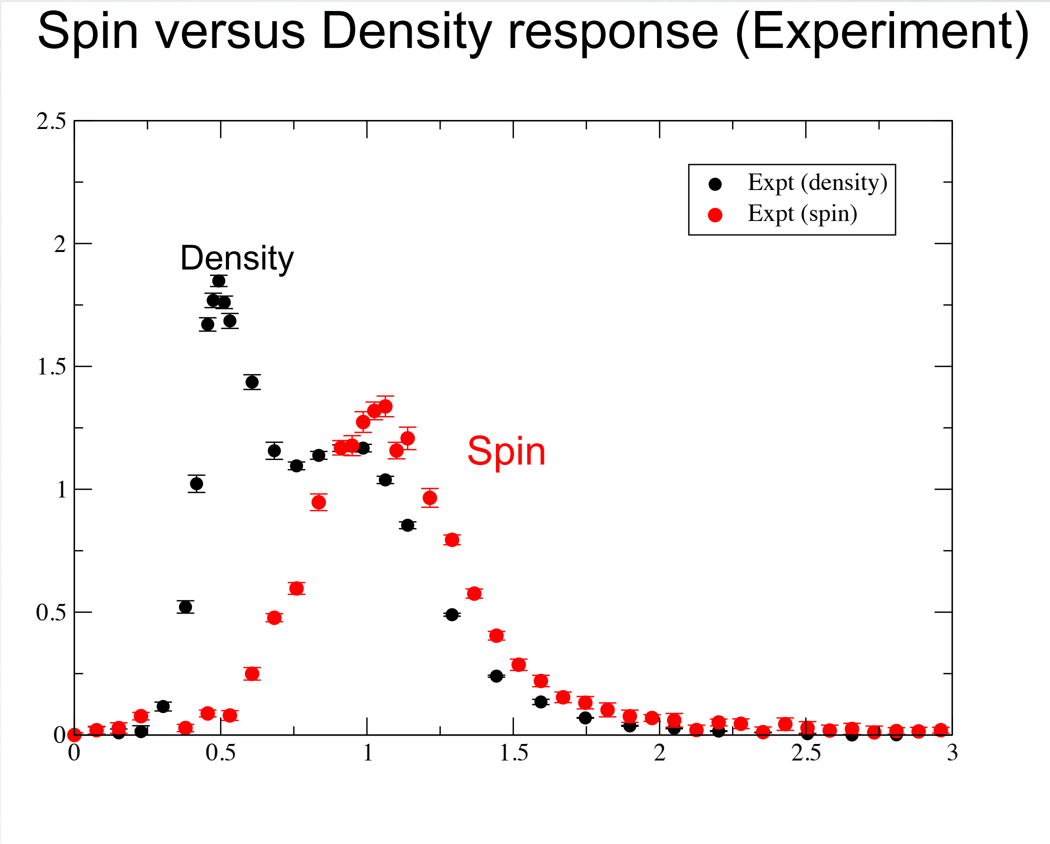
Gonzalex-Jiminez, Caballero, Donnelly, Phys. Reports 2013

Models of Electroweak Form Factors



Cold Atoms (Fermions at Unitarity)





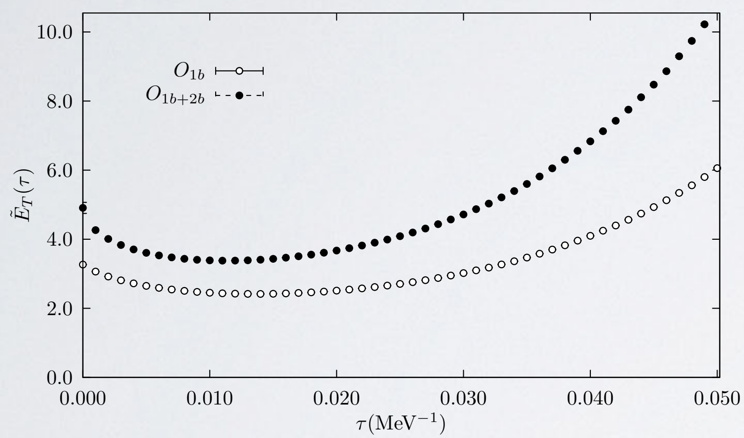
Both at q = 4.5 kF

Density and Spin Response Identical for PWIA or Spectral Function

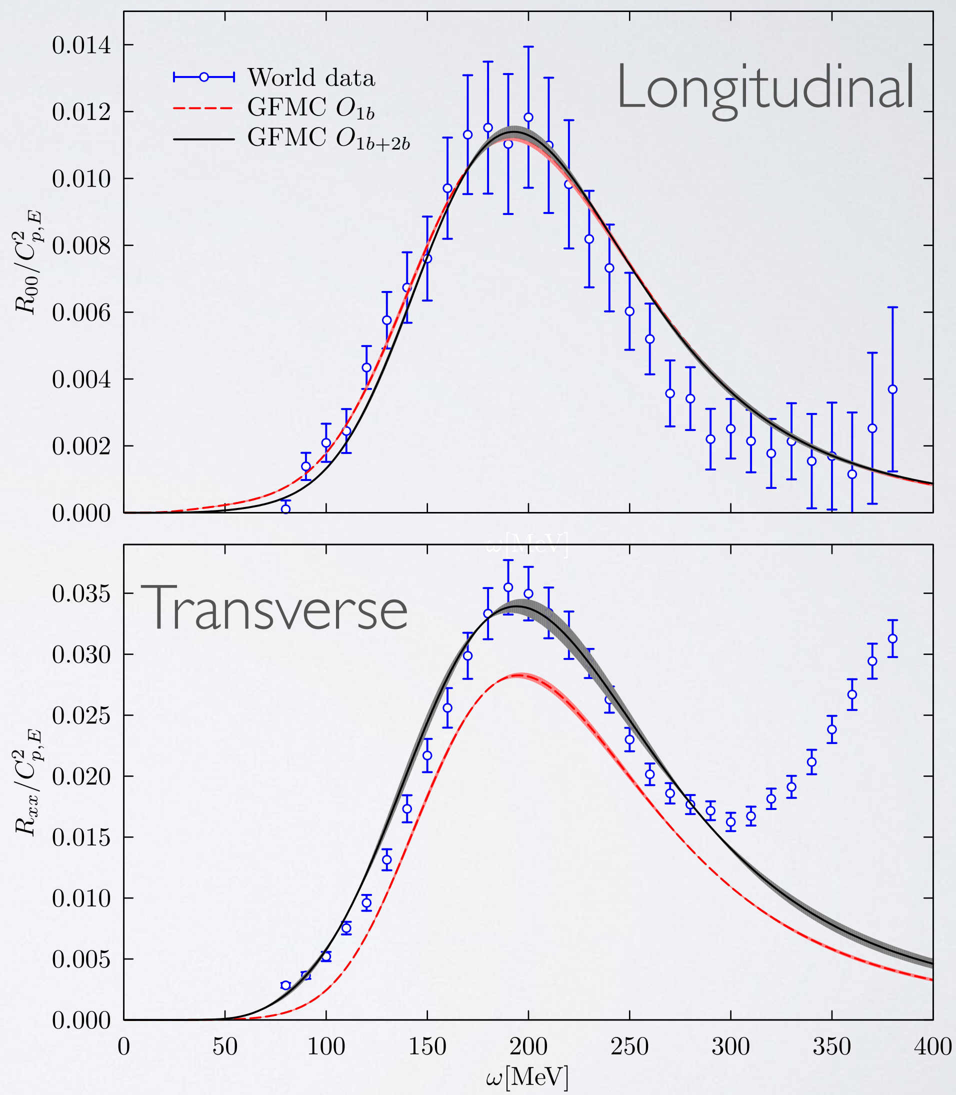
A=4 EM response

Euclidean

q = 500 MeV



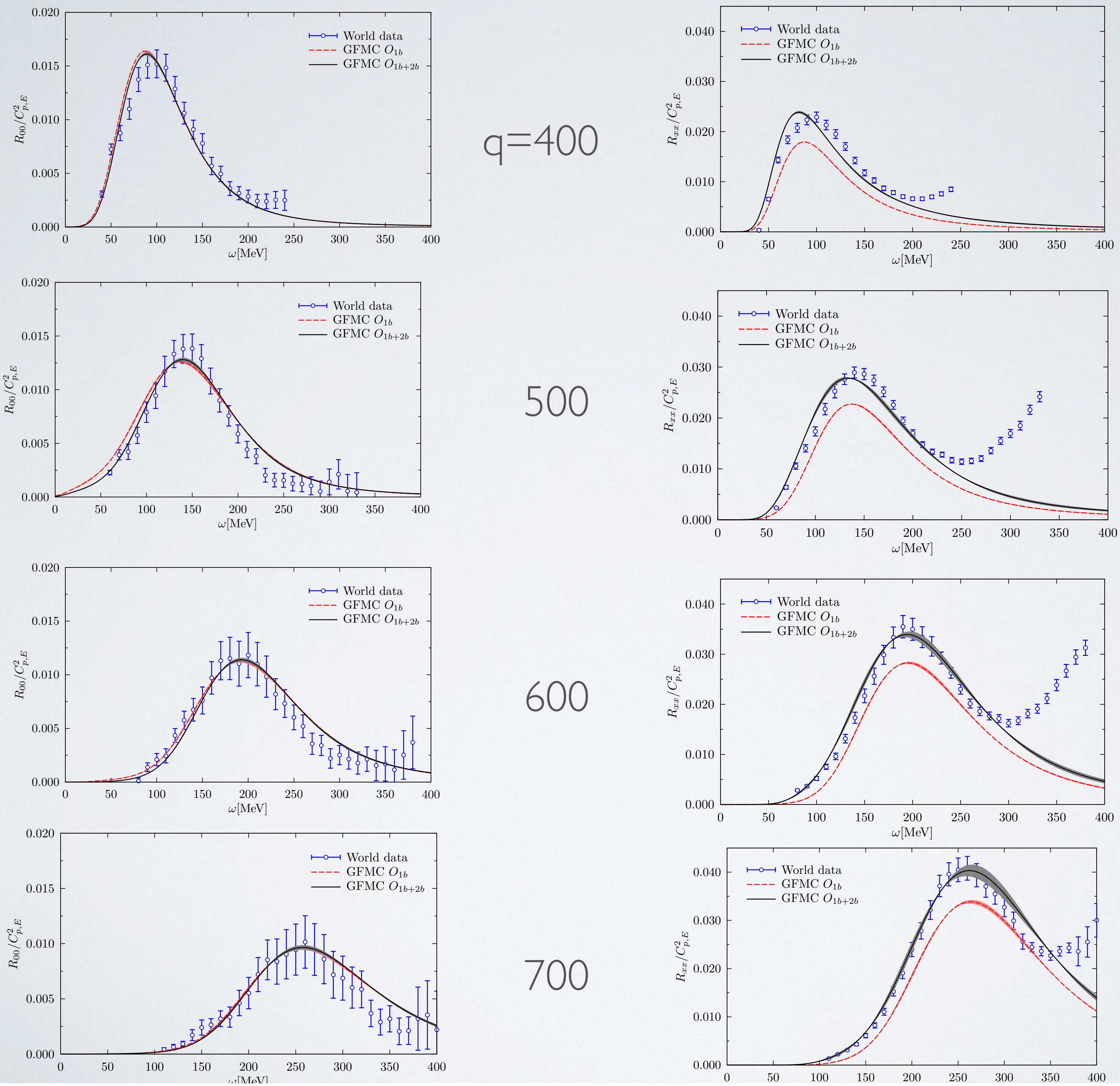
q = 600 MeV



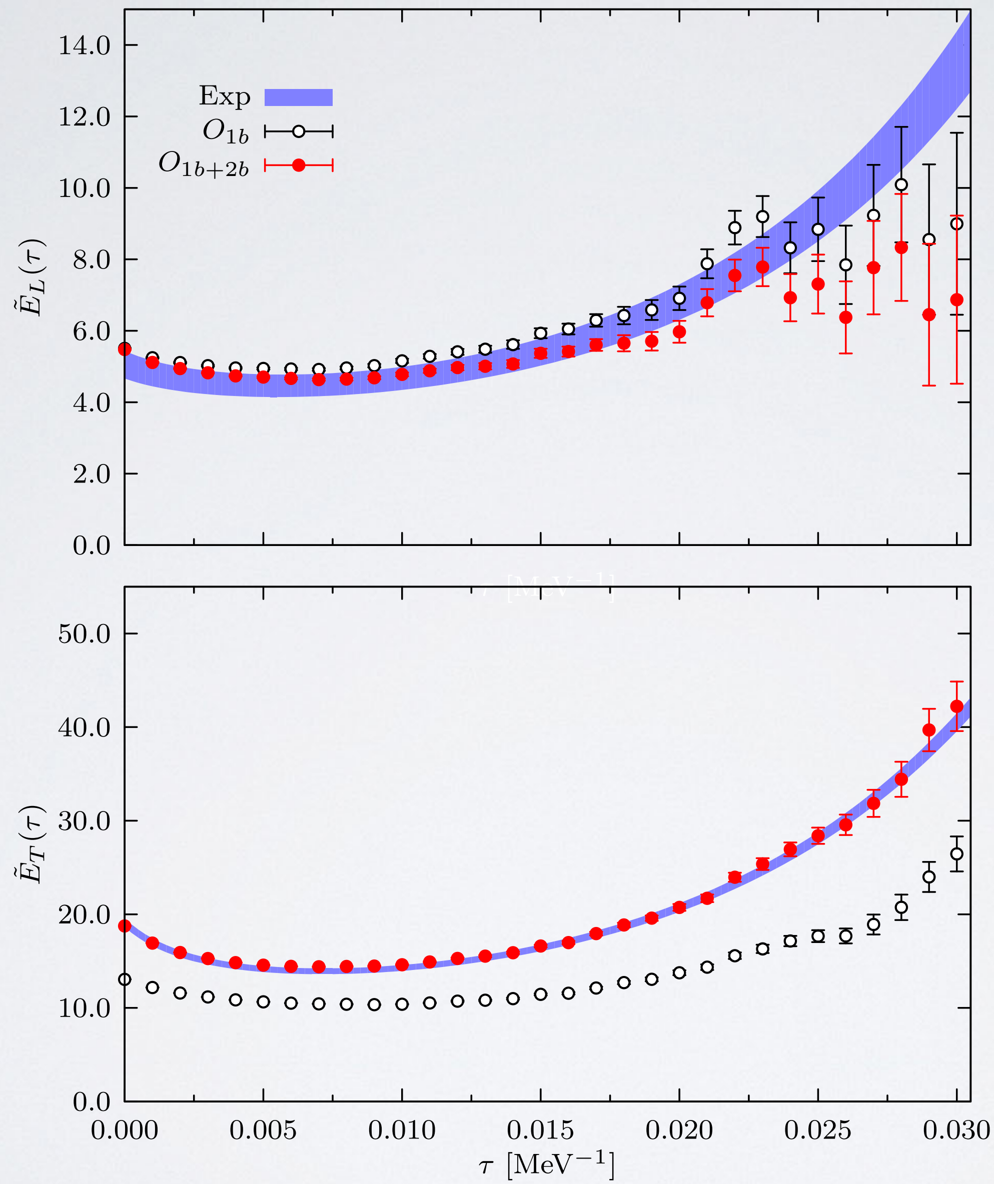
Lovato, et al, arXiv:1501.01981

^{4}He EM

Longitudinal | Transverse

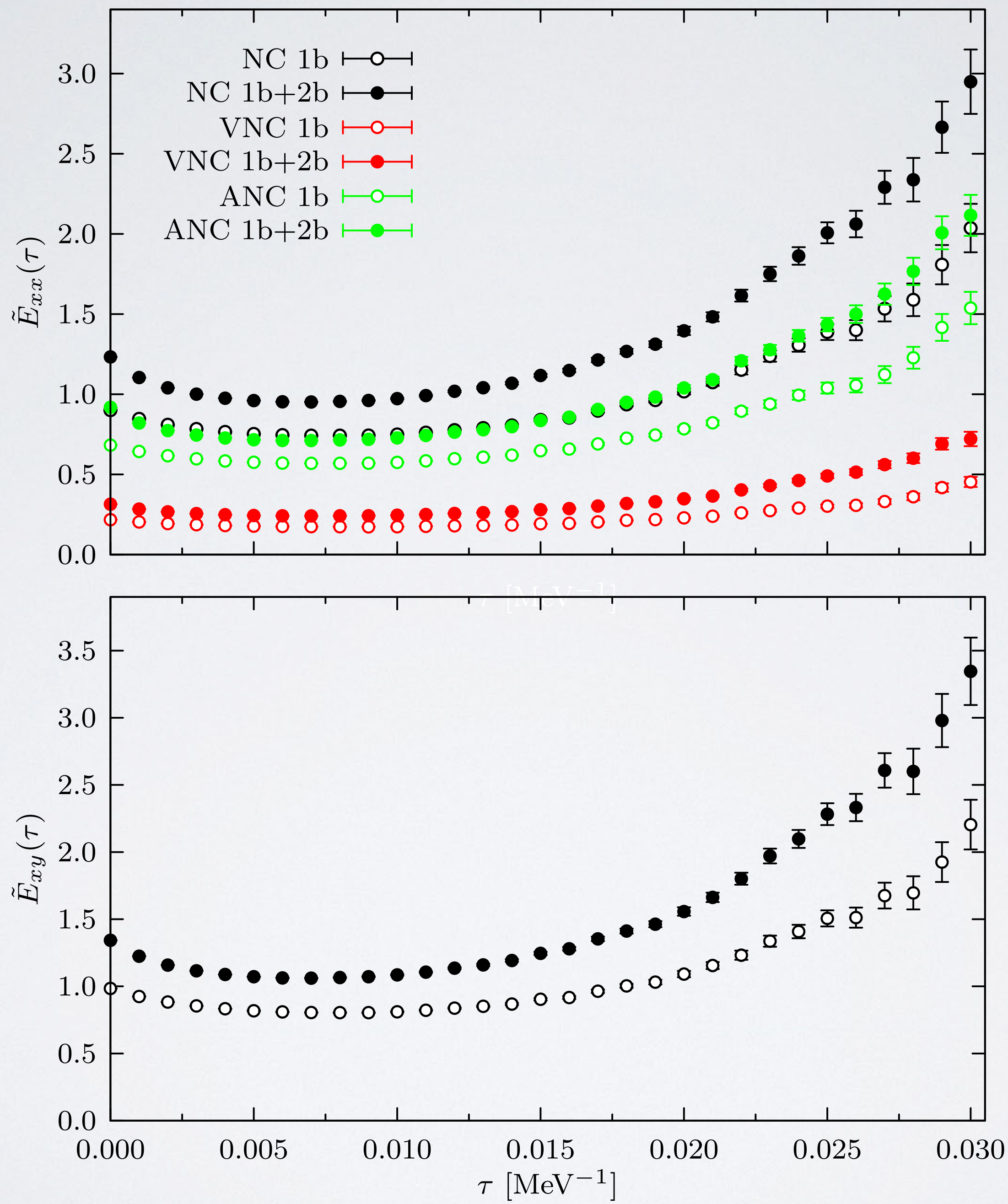


12C Euclidean Response: EM



Lovato, et al, arXiv:1501.01981

^{12}C Euclidean Response: Neutral Current



Lovato, et al, arXiv:1501.01981