Quantum Monte Carlo calculations for neutrino-nucleus scattering

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NUCLEI
Nuclear Computational Low-Energy Initiative

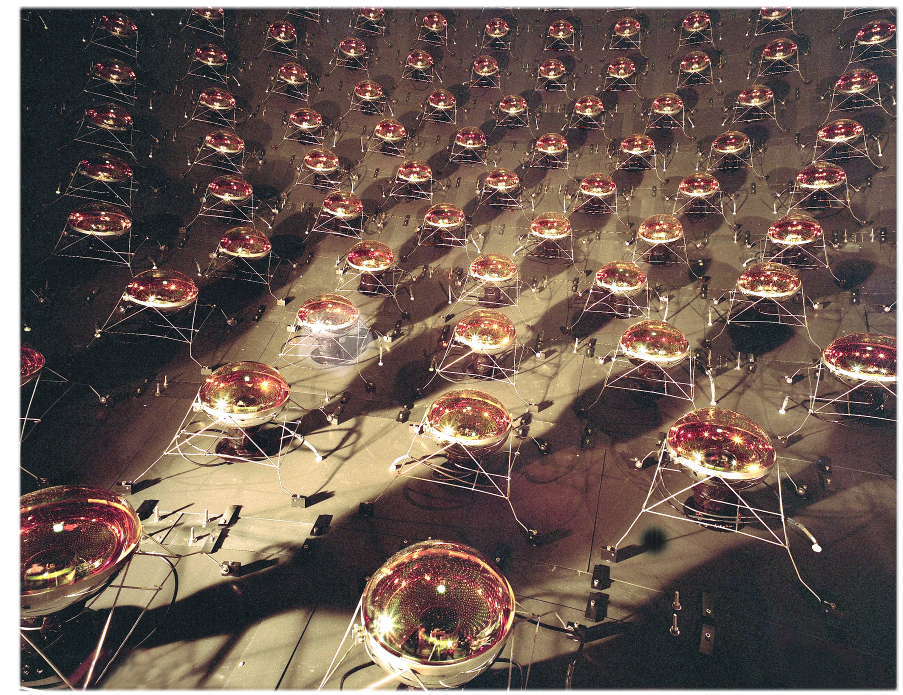
Introduction

- The electroweak response is a fundamental ingredient to describe neutrino - ^{12}C scattering.
- Excess, at relatively low energy, of measured cross section relative to oversimplified theoretical calculations.

<u>Neutrino experimental communities need accurate theoretical calculations</u>

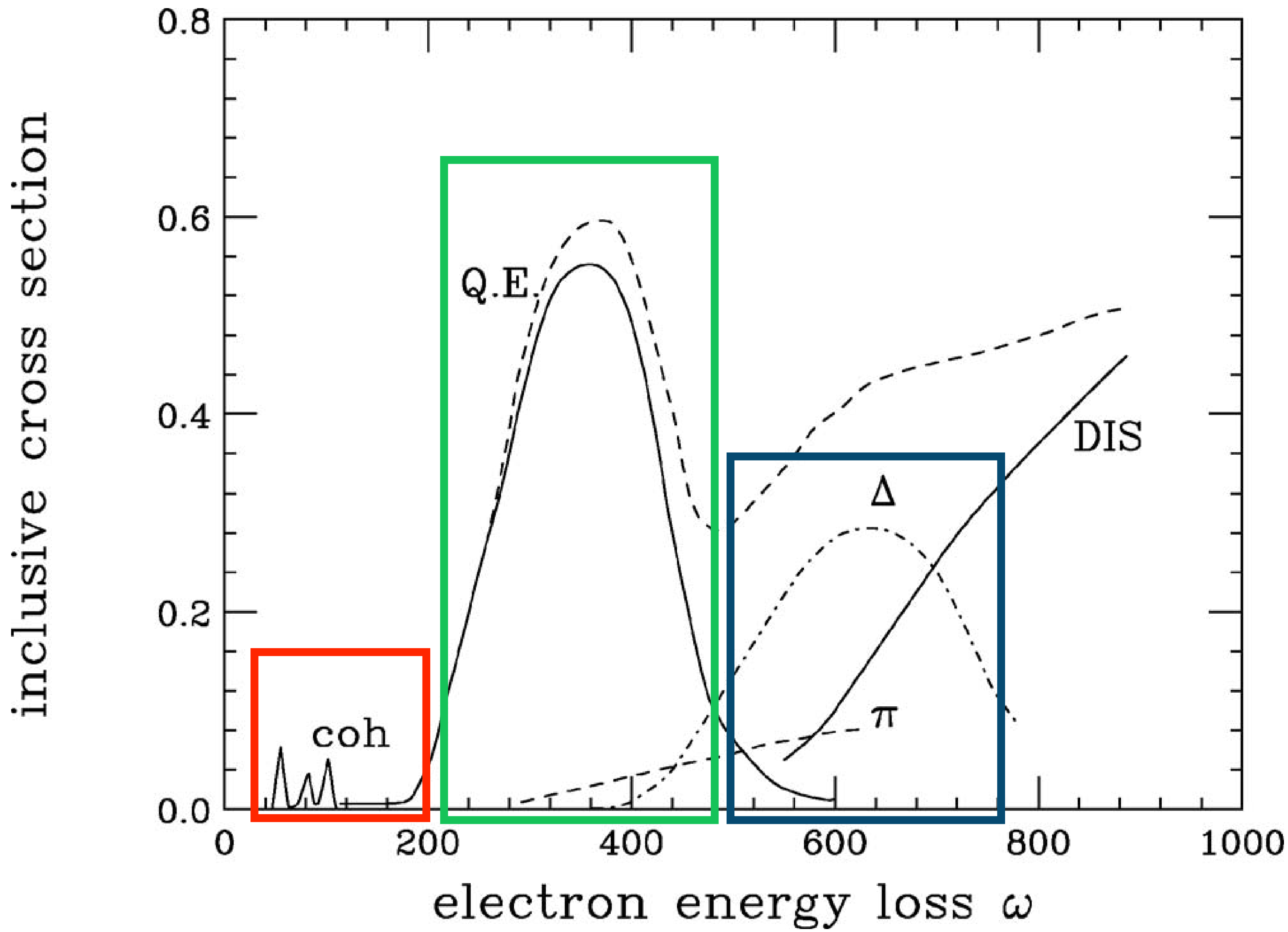
- We have first studied the electromagnetic response of ^{4}He and ^{12}C for which precise experimental data are available.

<u>A model unable to describe electron-nucleus scattering is unlikely to describe neutrino-nucleus scattering.</u>



Electron-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.

Electron-nucleus scattering

The electromagnetic inclusive cross section of the process

$$e +^{12}\mathrm{C} \rightarrow e' + X$$

where the target final state is undetected, can be written as

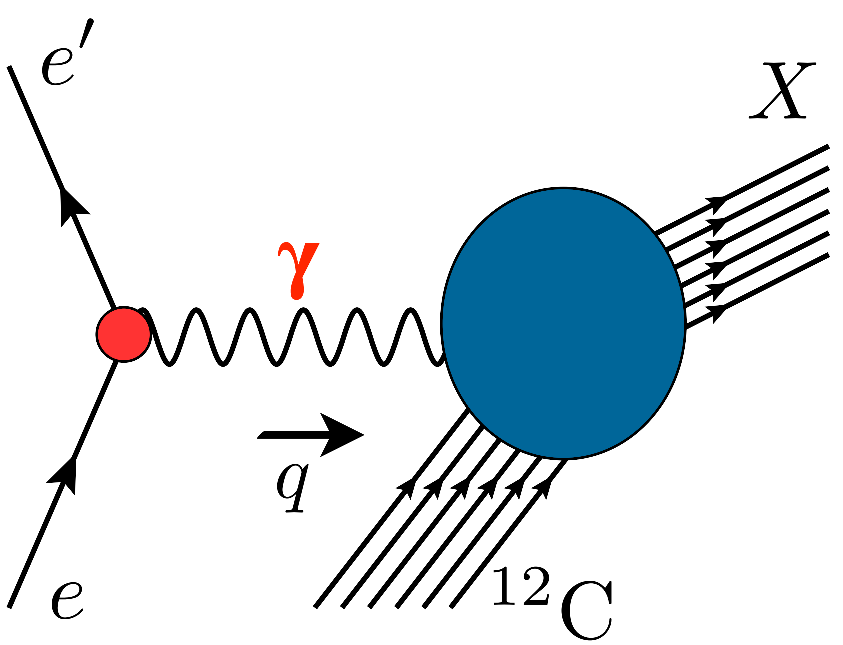
$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = -\frac{\alpha^2}{q^4}\frac{E_{e'}}{E_e} L^{EM}_{\mu\nu} W^{\mu\nu}_{EM} \ ,$$

The leptonic tensor is fully specified by the measured electron kinematic variables

$$L^{EM}_{\mu\nu} = 2[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk')]$$

The Hadronic tensor contains all the information on target structure.

$$W^{\mu\nu}_{EM} = \sum_X \langle \Psi_0 | J^{\mu\,\dagger}_{EM} | \Psi_X \rangle \langle \Psi_X | J^\nu_{EM} | \Psi_0 \rangle \delta^{(4)}(p_0 + q - p_X)$$



Neutrino-nucleus scattering

The neutral current inclusive cross section of the process

$$\nu_\ell + A \to \nu_{\ell'} + X$$

where the target final state is undetected, can be written as

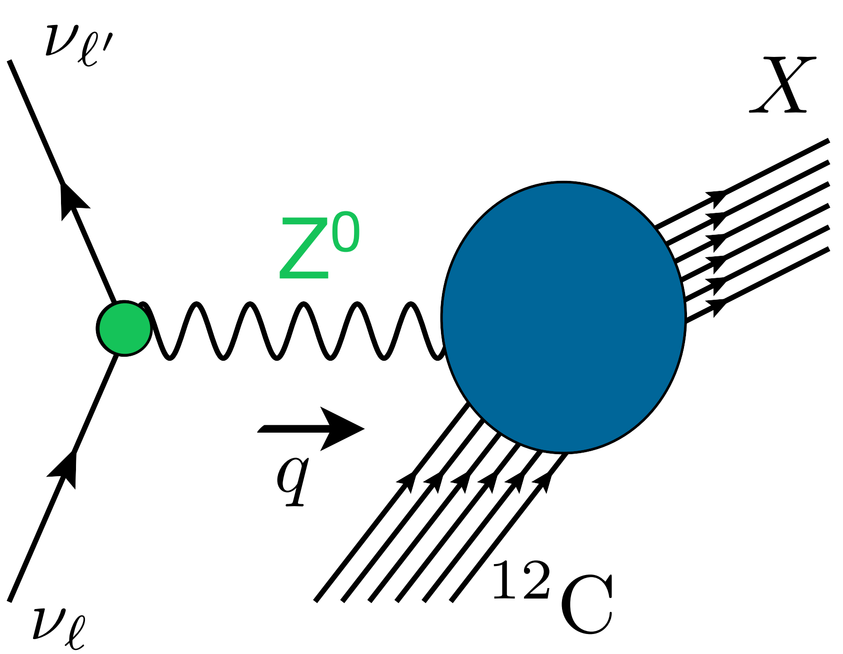
$$\frac{d^2\sigma}{d\Omega_{\nu'} dE_{\nu'}} = \frac{G_F^2}{4\pi^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{NC}_{\mu\nu} W^{\mu\nu}_{NC}$$

The leptonic tensor is fully specified by the measured neutrino kinematic variables

$$L^{NC}_{\mu\nu} = 8\left[k'_\mu\, k_\nu + k'_\nu\, k_\mu - g_{\mu\nu}(k\cdot k') + i\,\varepsilon_{\mu\nu\alpha\beta}\, k'^\beta\, k^\alpha\right]$$

The Hadronic tensor contains all the information on target structure.

$$W^{\mu\nu}_{NC} = \sum_X \langle\Psi_0|J^{\mu\,\dagger}_{NC}|\Psi_X\rangle\langle\Psi_X|J^\nu_{NC}|\Psi_0\rangle\delta^{(4)}(p_0+q-p_X)$$



Neutrino-nucleus scattering

The neutral current operator can be written as

$$J^{\mu}_{NC} = -2\sin^2\theta_W J^{\mu}_{\gamma,S} + (1 - 2\sin^2\theta_W) J^{\mu}_{\gamma,z} + J^{\mu\,5}_{z}$$

- Weinberg angle $\sin^2\theta_W = 0.2312$

- Isoscalar and isovector terms of the electromagnetic current.

$$J^{\mu}_{EM} = J^{\mu}_{\gamma,S} + J^{\mu}_{\gamma,z}$$

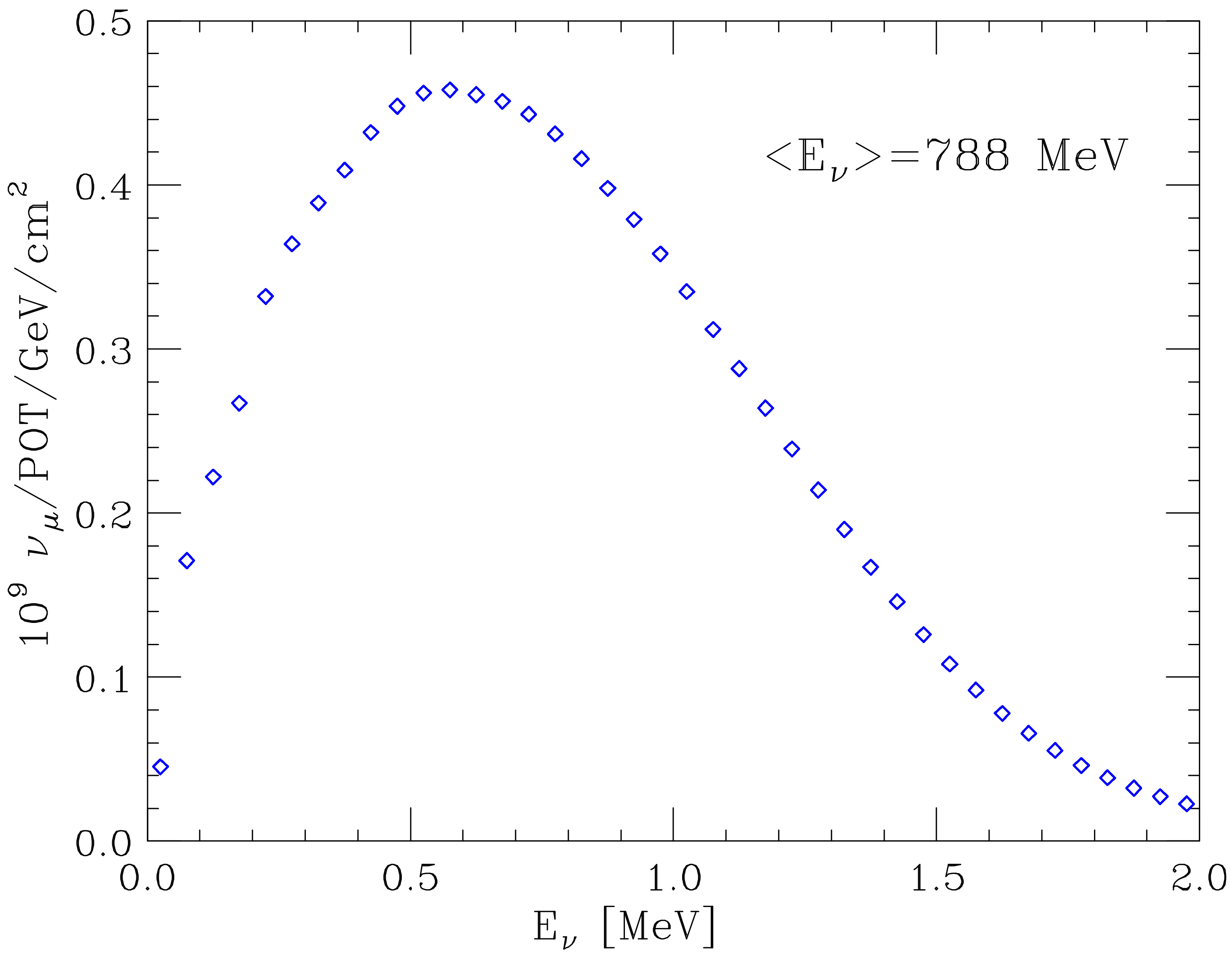
- Isovector term of the axial current, the one-body contributions of which are proportional to the axial form factor, often written in the simple dipole form

$$J^{\mu\,5}_{z} \propto G_A(Q^2) = \frac{g_A}{(1 + Q^2/\Lambda_A^2)^2}$$

The value of the axial mass obtained on neutrino-deuteron and neutrino-proton scattering data is $\Lambda_A \sim 1.03\,\mathrm{GeV}$.

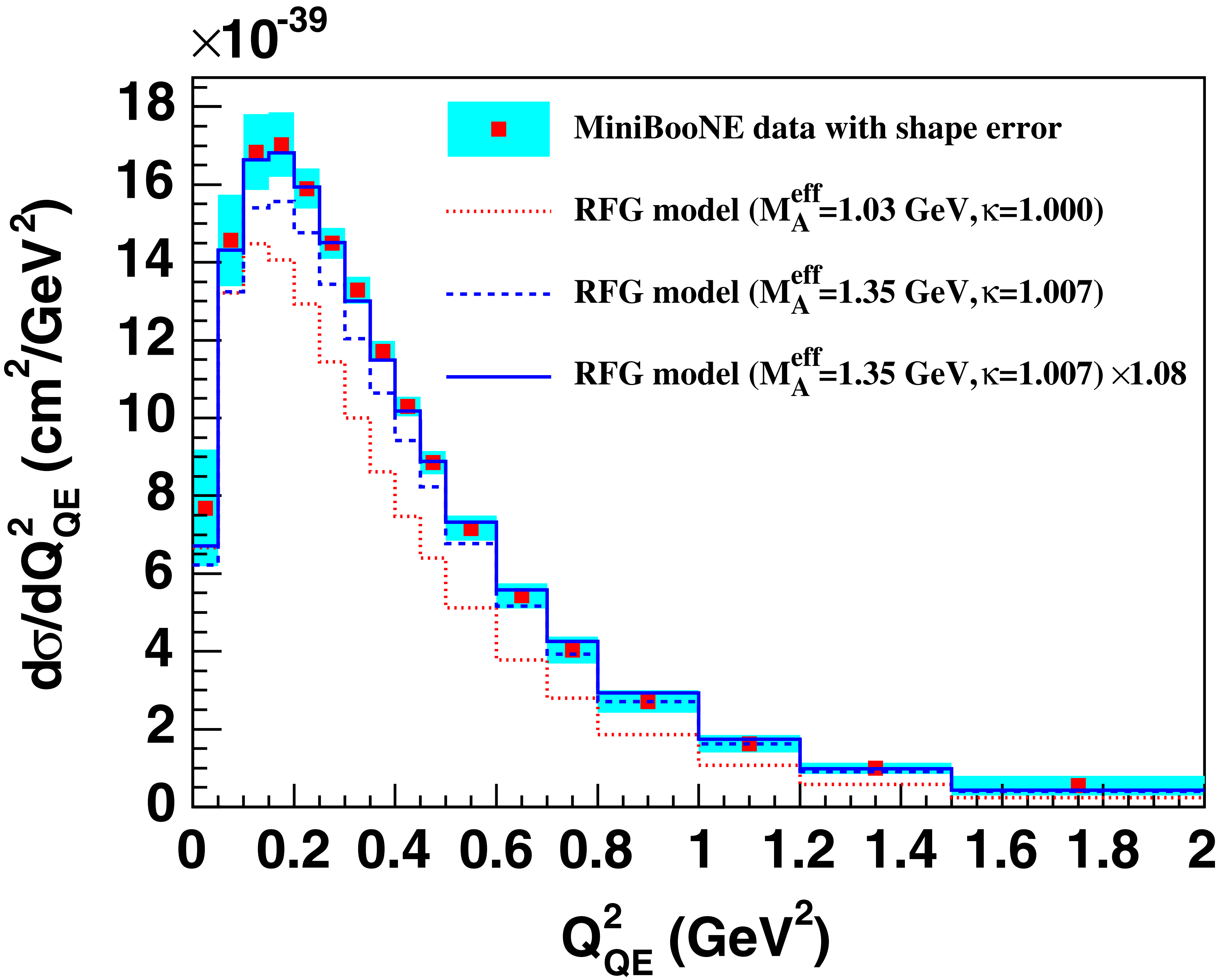
Neutrino-nucleus scattering

Because neutrino beams are always produced as secondary decay products, their energy is not sharply defined, but broadly distributed.



The problem

Relativistic Fermi gas calculations require an artificially large nucleon axial mass to reproduce the data.



- Two-body currents?
- Nuclear correlations?

Two-body MEC currents and correlations are fully accounted for in *ab initio* calculations of response functions and sum rules

Towards a unified approach

Moderate momentum transfer regime

- Ab initio Green's Function Monte Carlo calculation of the nuclear response from threshold up to the quasielastic region, initially for nuclei as large as ^{12}C (extension to larger nuclei requires further development of our AFDMC method)

Large momentum transfer regime

- Development and implementation of the factorization approximation, in which the hadronic final state is written as a product of a state representing the high-momentum particles produced in the interaction process, and a state representing the spectator nucleons, described by spectral functions.

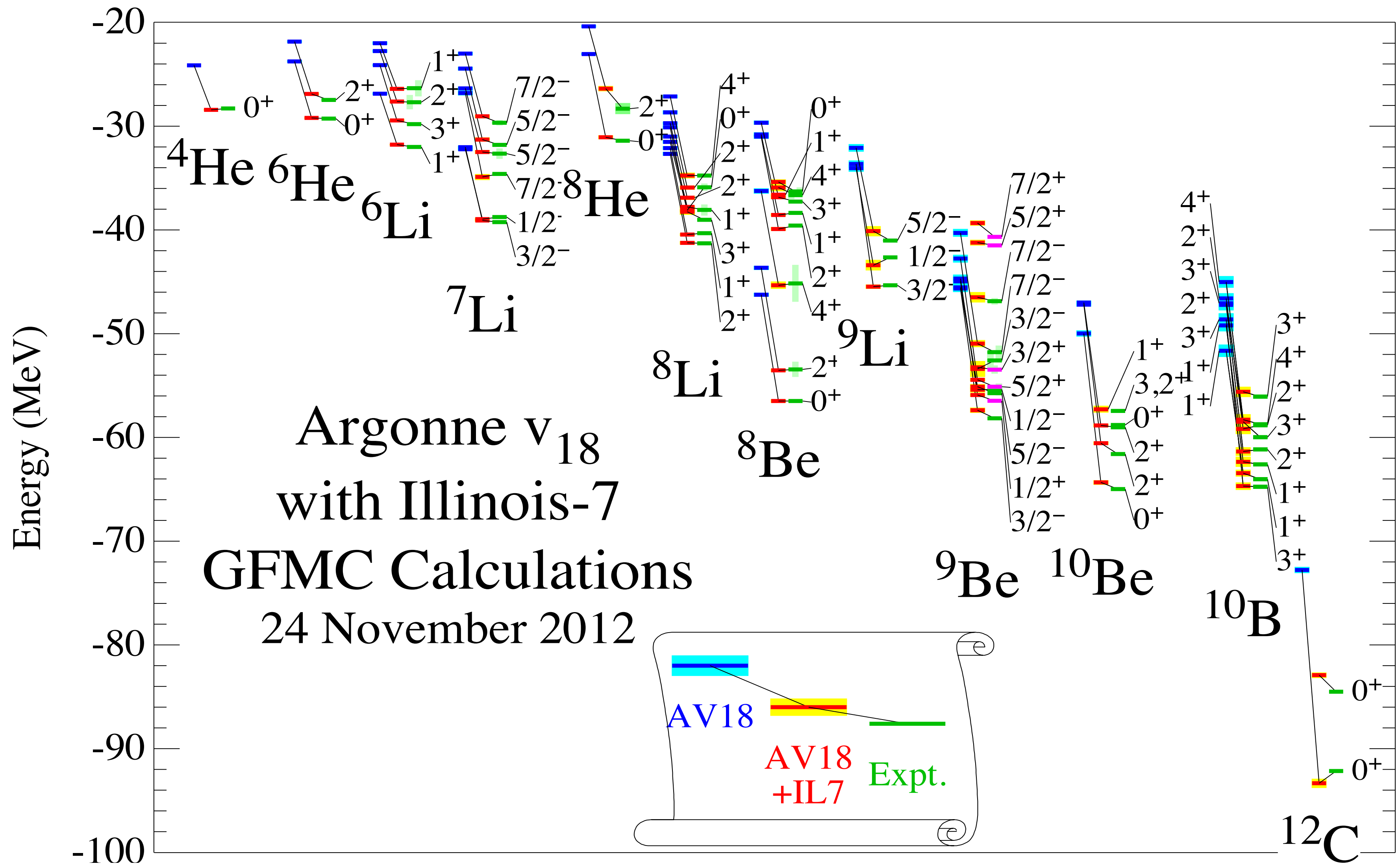
Both approaches are based on the same dynamical framework!

- The nucleus consists of a collection of A nucleons whose dynamics are described by the nonrelativistic Hamiltonian

$$H = -\frac{\hbar}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{ijk} V_{ijk}$$

Nuclear hamiltonian

- Argonne v_{18} two-body potential reproduces the ~4300 np and pp scattering data below 350 MeV of the Nijmegen database with $\chi^2 \simeq 1$.
- Illinois 7 three-body potential is needed to accurately describe the spectrum of light nuclei



Green's Function Monte Carlo

Solving the many body Schrödinger equation is made particularly difficult by the complexity of the interaction, which is spin-isospin dependent and contains strong tensor terms

$$\hat{H}\Psi_0(x_1 \ldots x_A) \;= E_0\Psi_0(x_1 \ldots x_A)$$

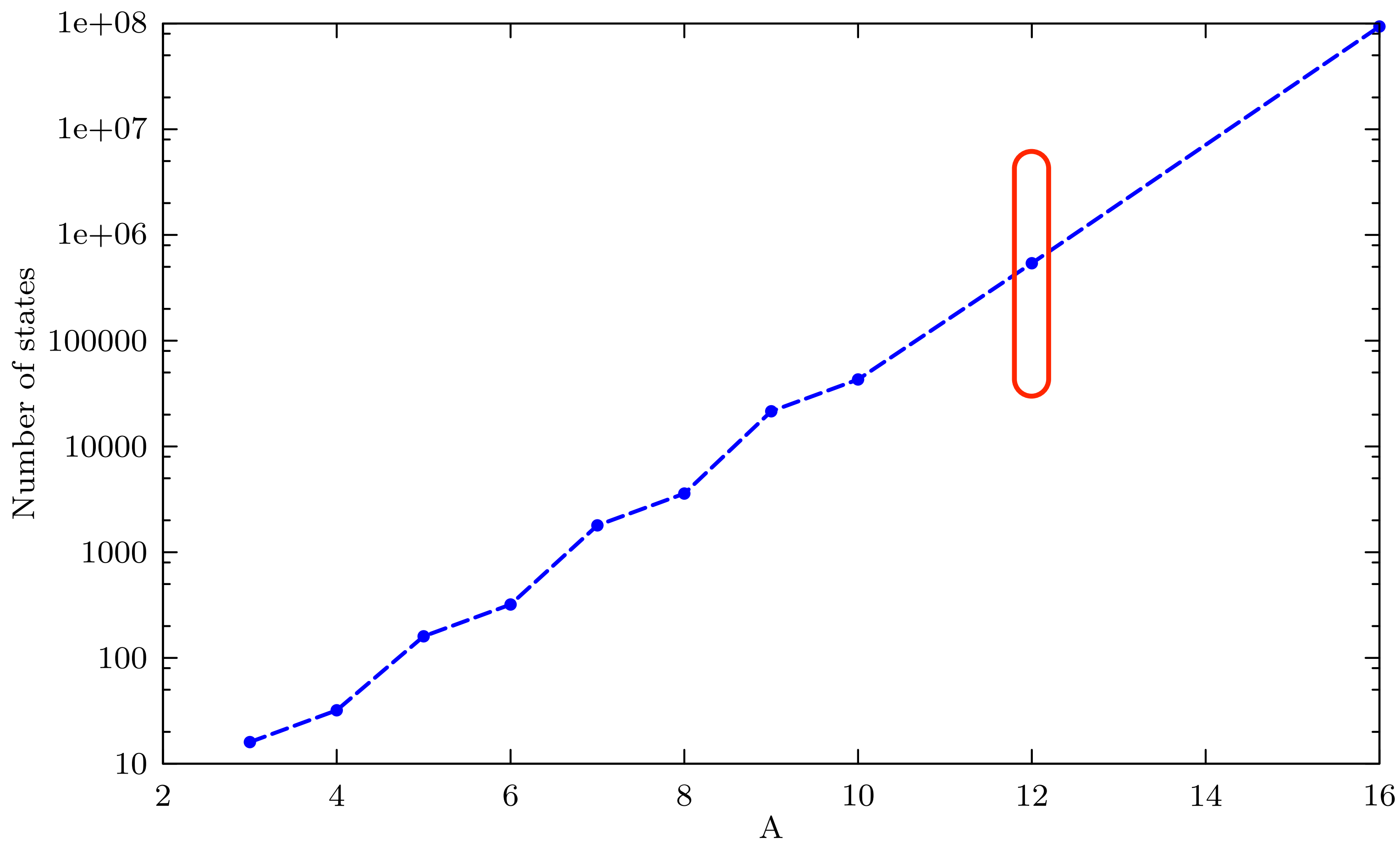
The wave function can be expressed as a sum over spin-isospin states

$$\Psi_0(x_1 \ldots x_A) = \sum_{\alpha=1}^{N} \Psi_0^\alpha(\mathbf{r}_1 \ldots \mathbf{r}_A)|\alpha\rangle$$

the number of which grows exponentially with the number of particles

$$N = 2^A \times \binom{A}{Z}$$

For ^{12}C **270,336** second order coupled differential equations in 36 variables !!!



Moderate momentum transfer regime

Moderate momentum-transfer regime

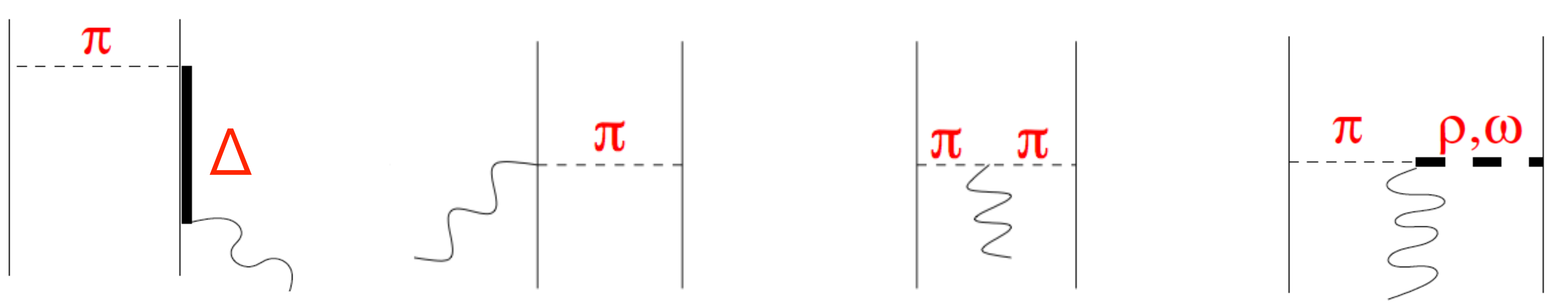
At moderate momentum transfer, the inclusive cross section of the process $\ell +^{12} C \to \ell' + X$ can be written in terms of the response functions

$$R_{\alpha\beta}(q,\omega) = \sum_f \langle \Psi_0 | J^{\dagger\,\alpha}(\mathbf{q},\omega) | \Psi_f \rangle \langle \Psi_f | J^{\beta}(\mathbf{q},\omega) | \Psi_0 \rangle \delta(\omega + E_0 - E_f),$$

Nuclear current includes one-and two-nucleon contributions

$$J^\alpha = \sum_i j_i^\alpha + \sum_{i<j} j_{ij}^\alpha$$

- j_i^α describes interactions involving a single nucleon,
- j_{ij}^α accounts for processes in which the vector boson couples to the currents arising from meson exchange between two interacting nucleons.



Moderate momentum-transfer regime

- At moderate momentum transfer, both initial and final states are eigenstates of the nonrelativistic nuclear hamiltonian

$$\hat{H}|\Psi_0\rangle = E_X|\Psi_0\rangle \qquad \hat{H}|\Psi_f\rangle = E_f|\Psi_f\rangle$$

- As for the electron scattering on ^{12}C, among the possible states there are

$$|\Psi_f\rangle = |^{11}B, p\rangle, |^{11}C, n\rangle, |^{10}B, pn\rangle, |^{10}Be, pp\rangle$$

- Relativistic corrections are included in the current operators and in the nucleon form factors.

- GFMC allows for "exactly" solving the nonrelativistic many-body Schrödinger equation for nuclei as large as ^{12}C.

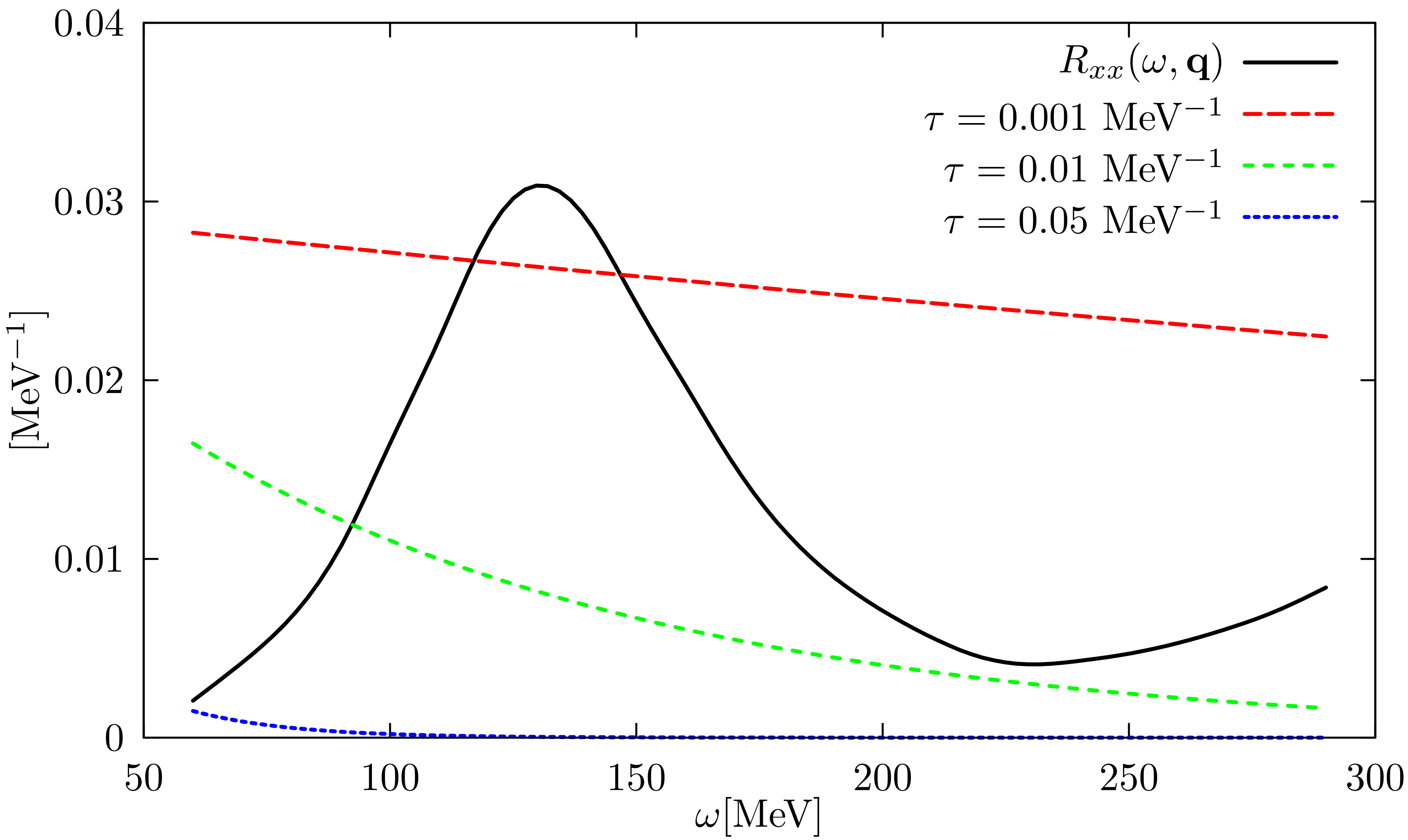
- GFMC also allows for extracting dynamical observables from ground-state properties.

Euclidean response function

The Euclidean response at finite imaginary time

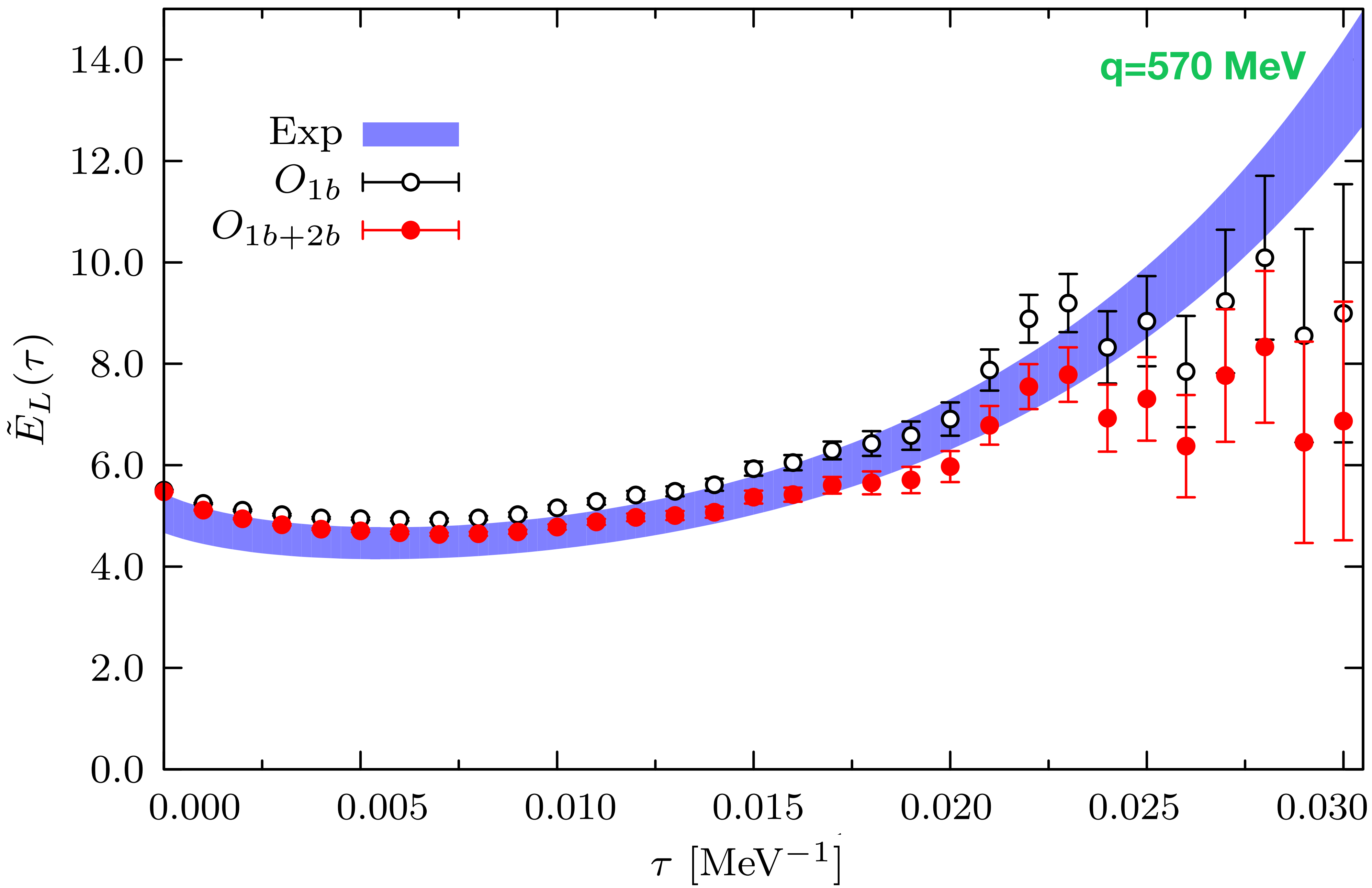
$$E_{\alpha\beta}(\tau,\mathbf{q}) = C_{\alpha\beta}(q)\int_{\omega_{el}}^{\infty} d\omega e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q},\omega) \; = \frac{\langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) e^{-(H-E_0)\tau} J_\beta(\mathbf{q}) | \Psi_0 \rangle}{\langle \Psi_0 | e^{-(H-E_0)\tau} | \Psi_0 \rangle}$$

very quickly suppresses the contribution from large energy transfer.



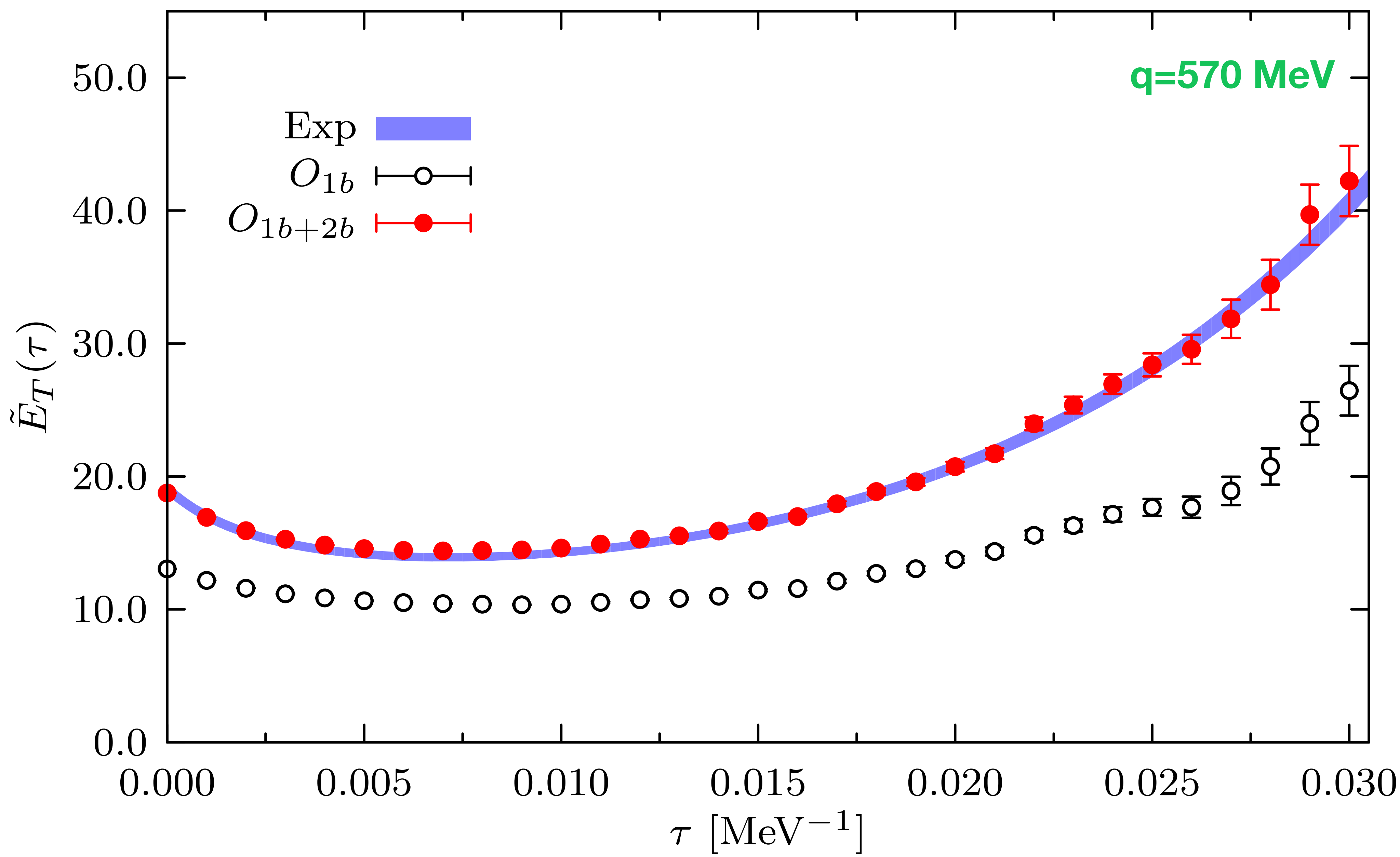
^{12}C electromagnetic Euclidean response

In the electromagnetic longitudinal case, destructive interference between the matrix elements of the one- and two-body charge operators reduces, albeit slightly, the one-body response.



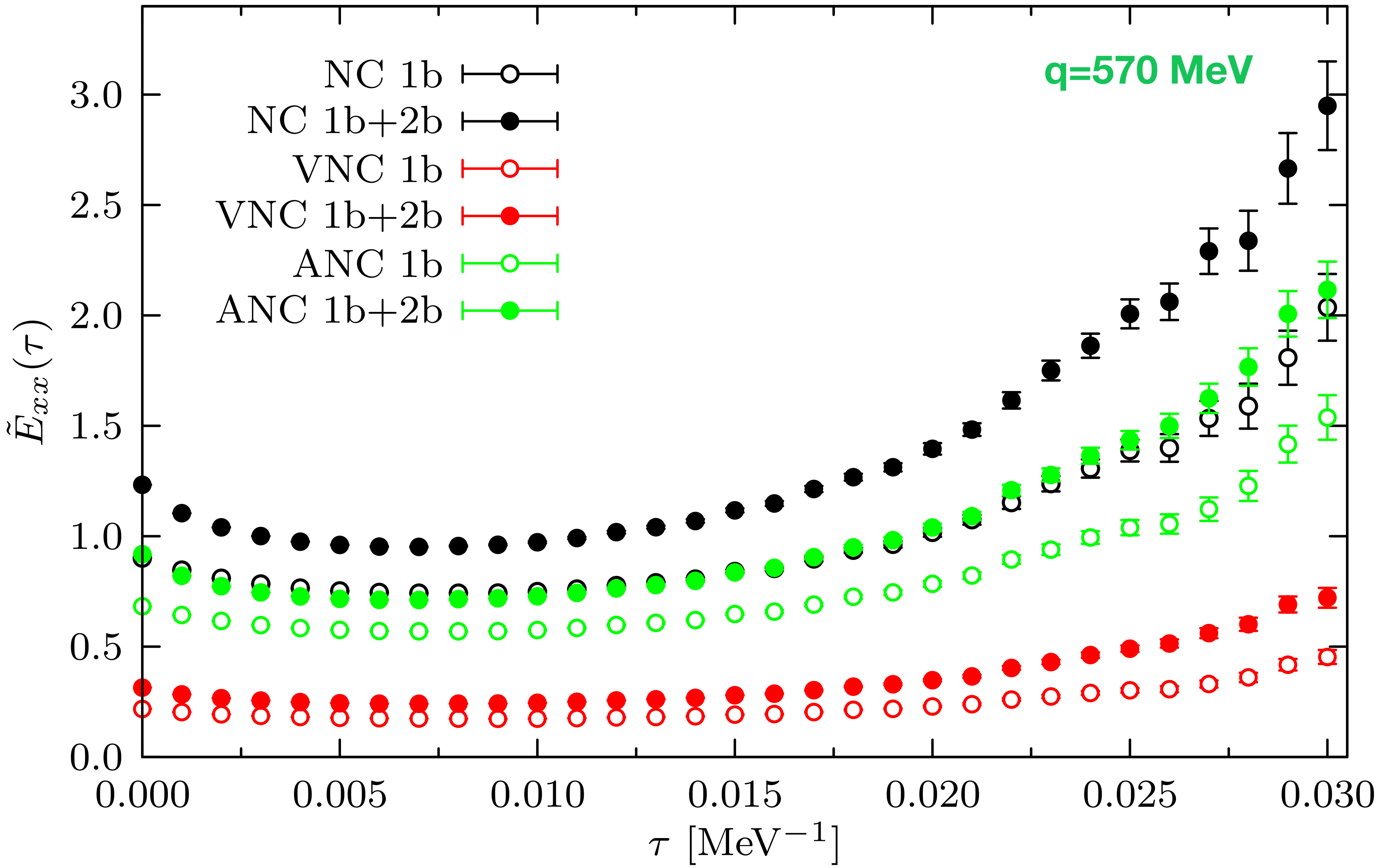
^{12}C electromagnetic Euclidean response

In the electromagnetic transverse case, two-body current contributions substantially increase the one-body response. This enhancement is effective over the whole imaginary-time region we have considered.



^{12}C neutral-current Euclidean response

Both the vector neutral current and the axial neutral current transverse responses are substantially enhanced over the entire imaginary-time region we considered.



Inversion of the Euclidean response

The Euclidean response formalism allows one to extract dynamical properties of the system from its ground-state.

- Best suited for Quantum Monte Carlo approaches
- Wide range of applicability: atomic physics, cold atoms, neutrino scattering, neutron star cooling…

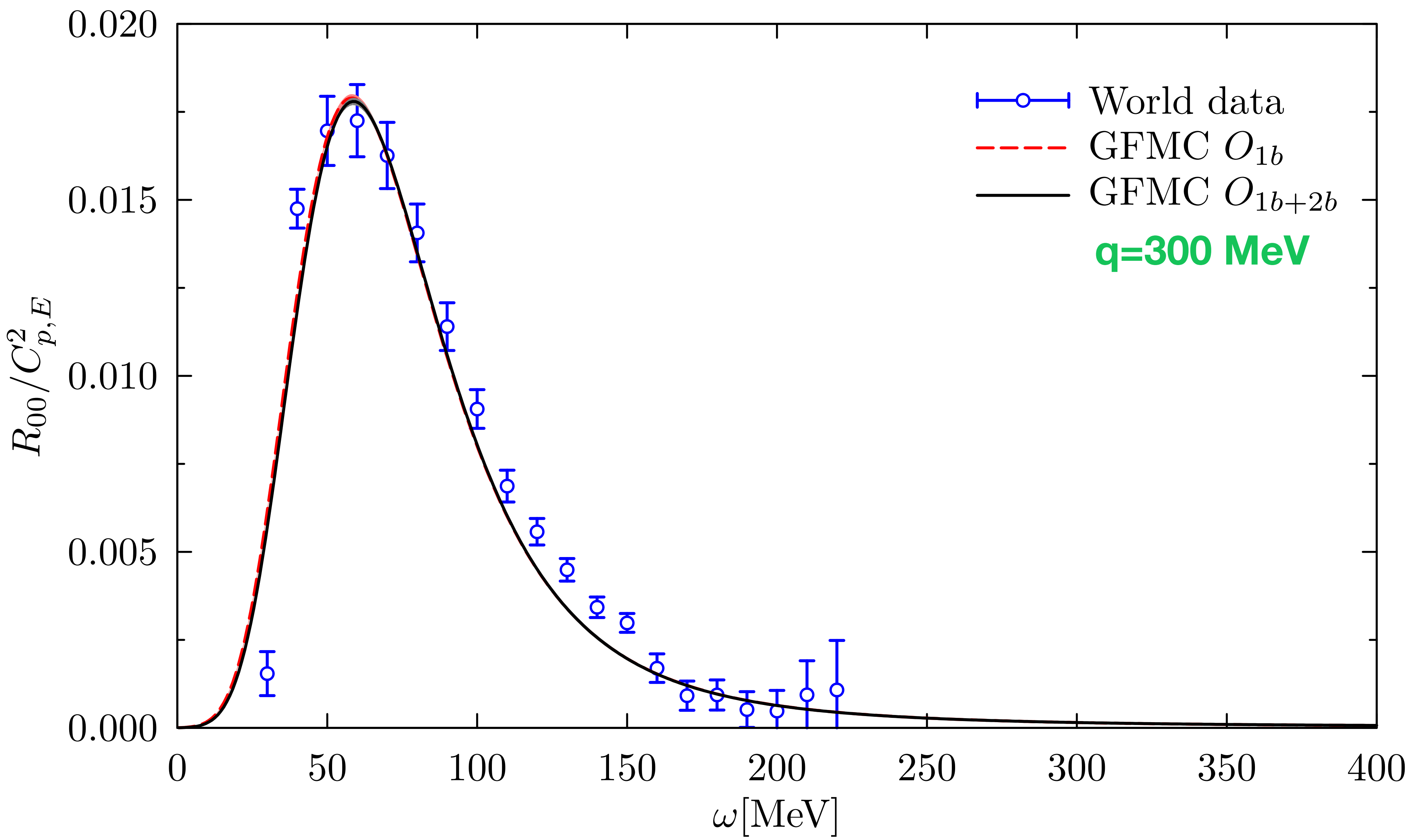
Inverting the Euclidean response is an ill posed problem: any set of observations is limited and noisy and the situation is even worse since the kernel is a smoothing operator.

$$E_{\alpha\beta}(\tau, \mathbf{q}) \longrightarrow R_{\alpha\beta}(\omega, \mathbf{q})$$

We found **historic maximum entropy** to be simple to implement and adequate for our purposes.

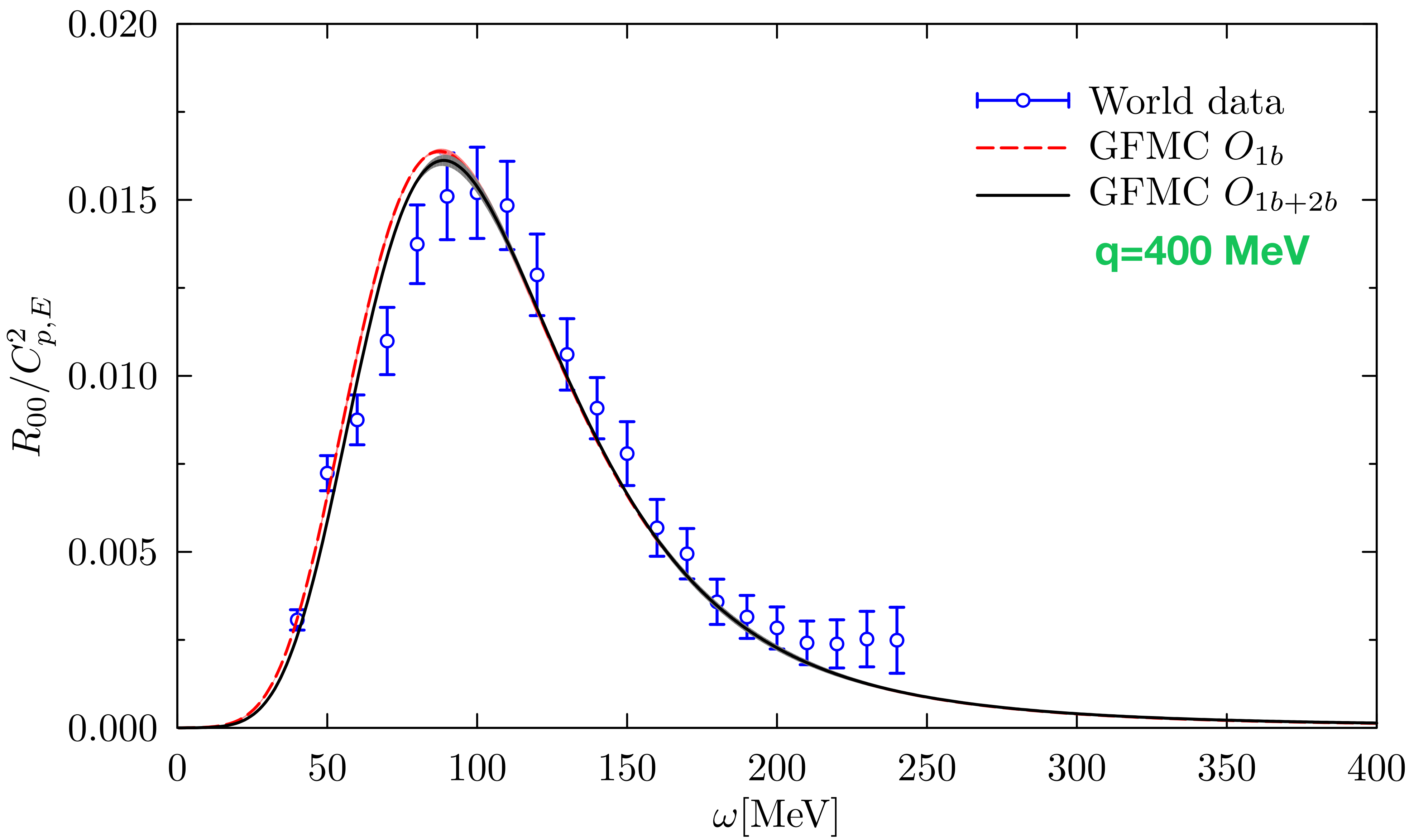
^{4}He electromagnetic response

Preliminary results indicate that the two-body currents do not provide significant changes in the longitudinal response.



^{4}He electromagnetic response

Two-body currents do not provide significant changes in the longitudinal response. The agreement with experimental data appears to be remarkably good.



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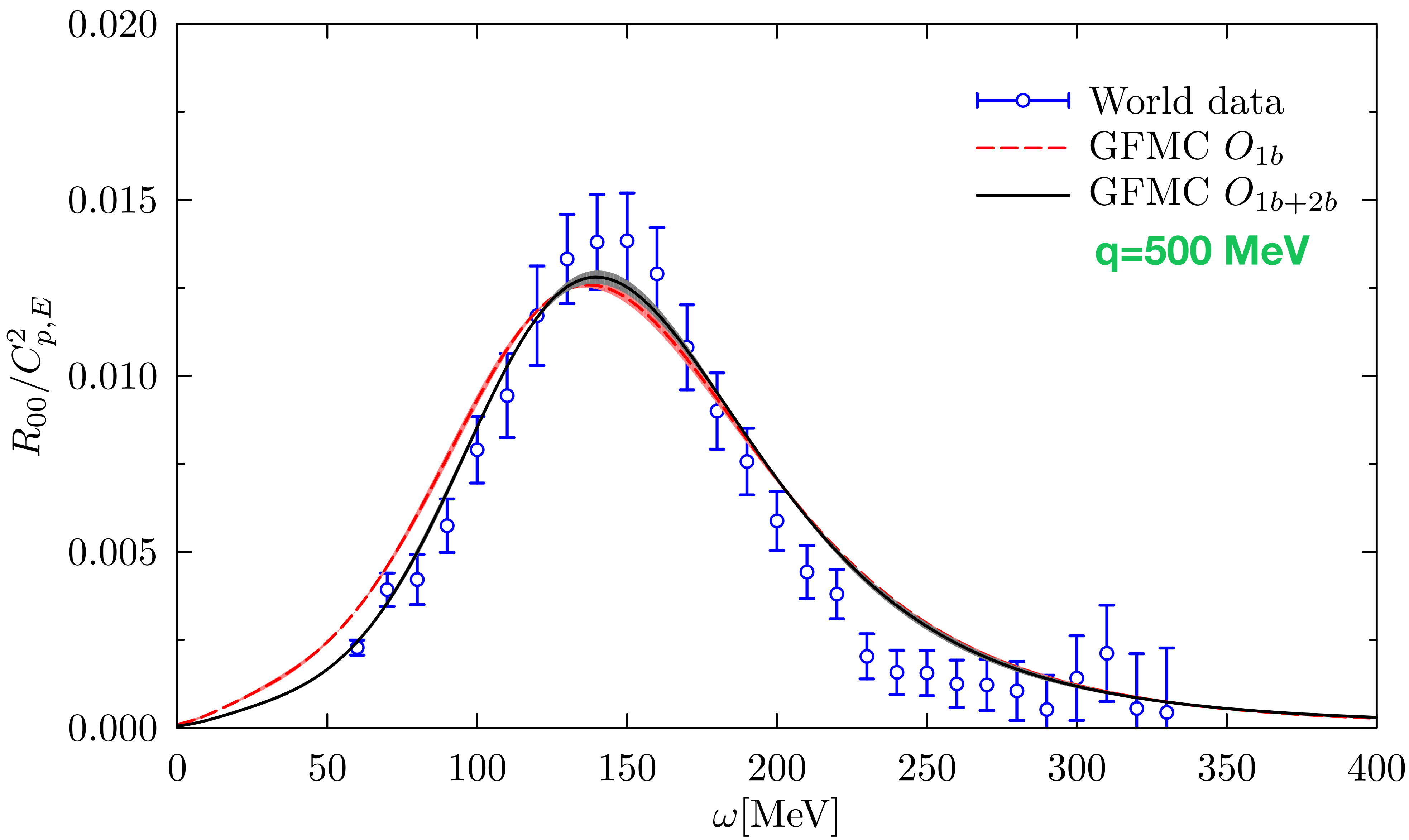

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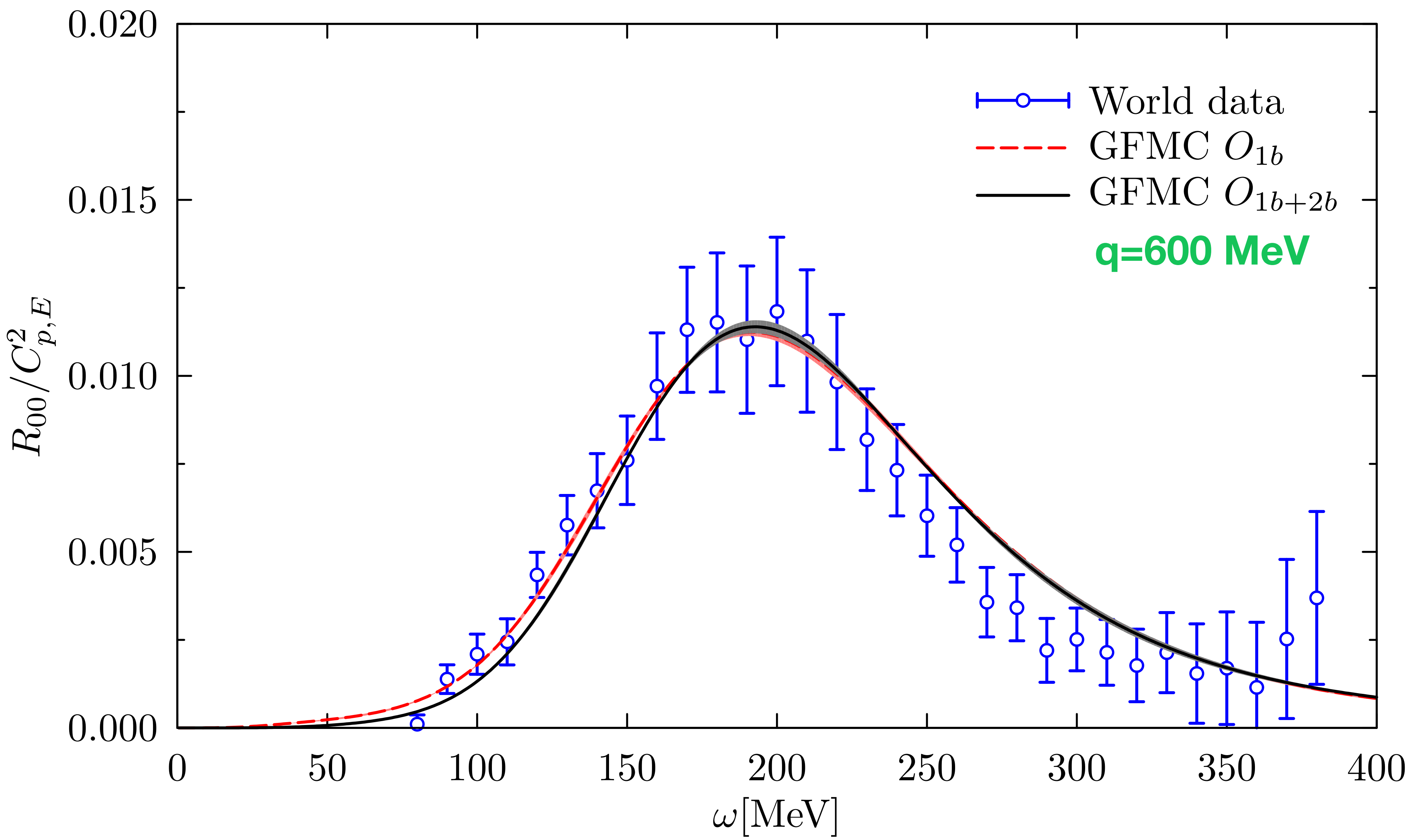

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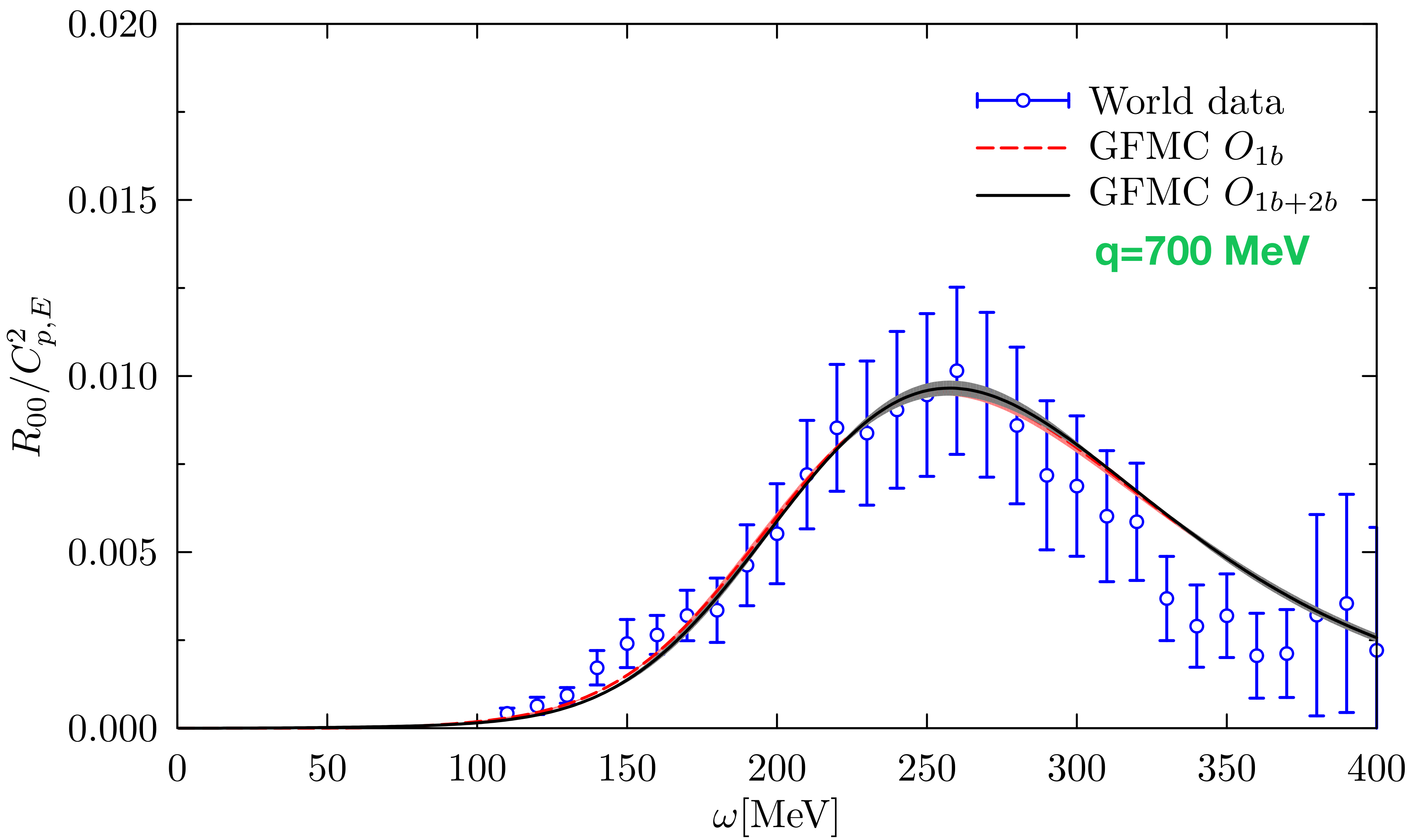
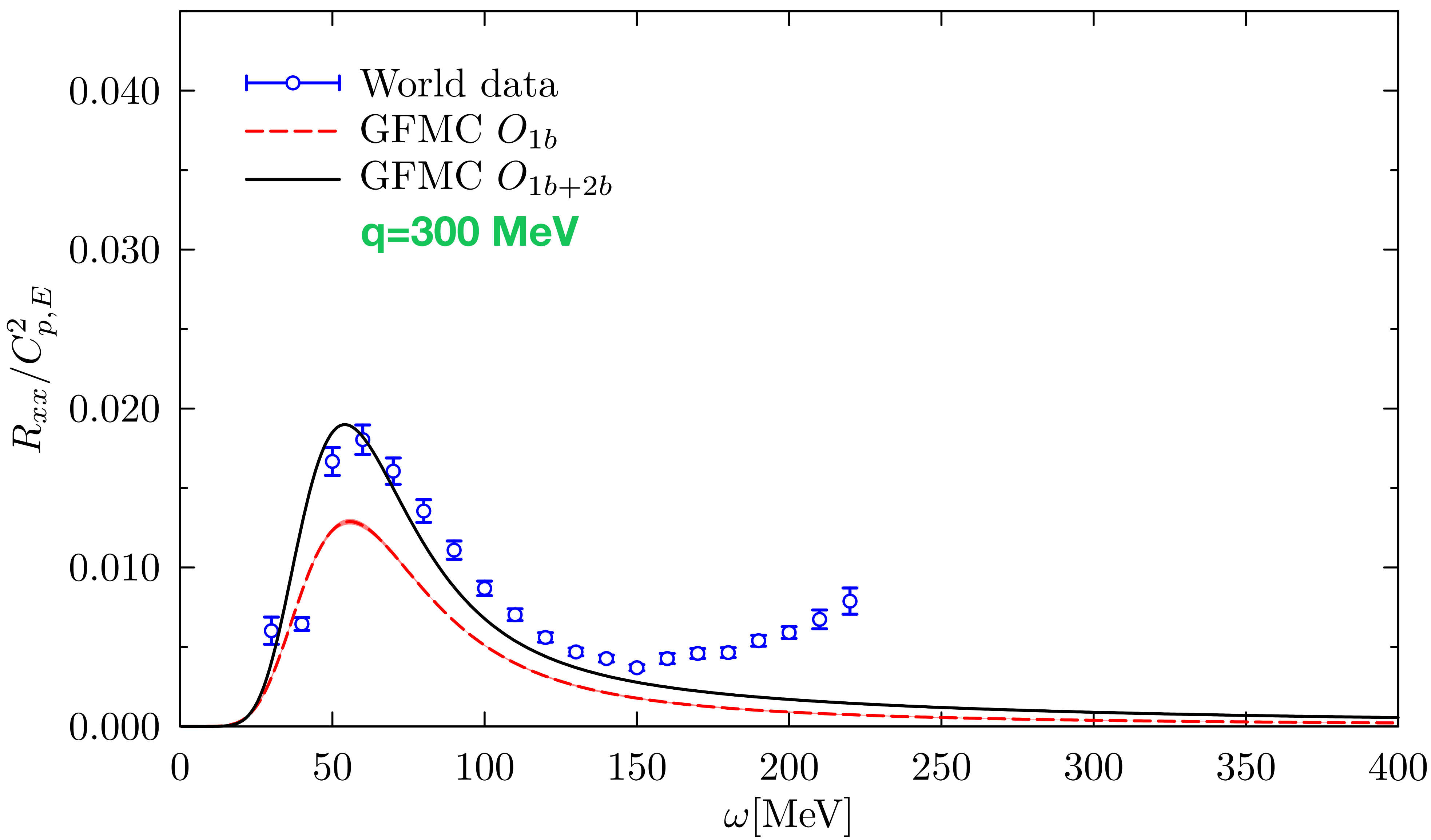


^{4}He electromagnetic response

Two-body currents significantly enhance the transverse response function, not only in the dip region, but also in the quasielastic peak and threshold regions.



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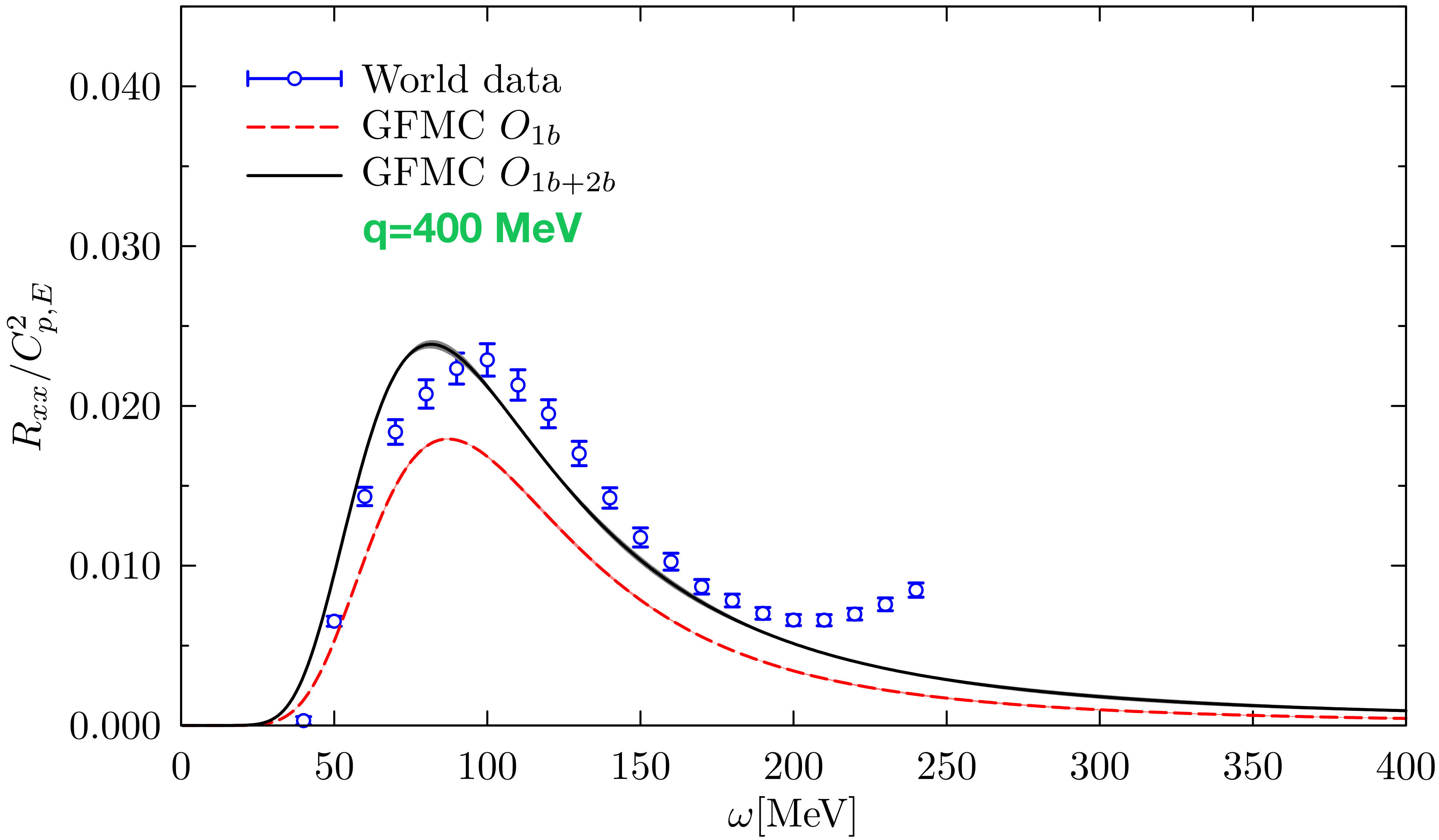

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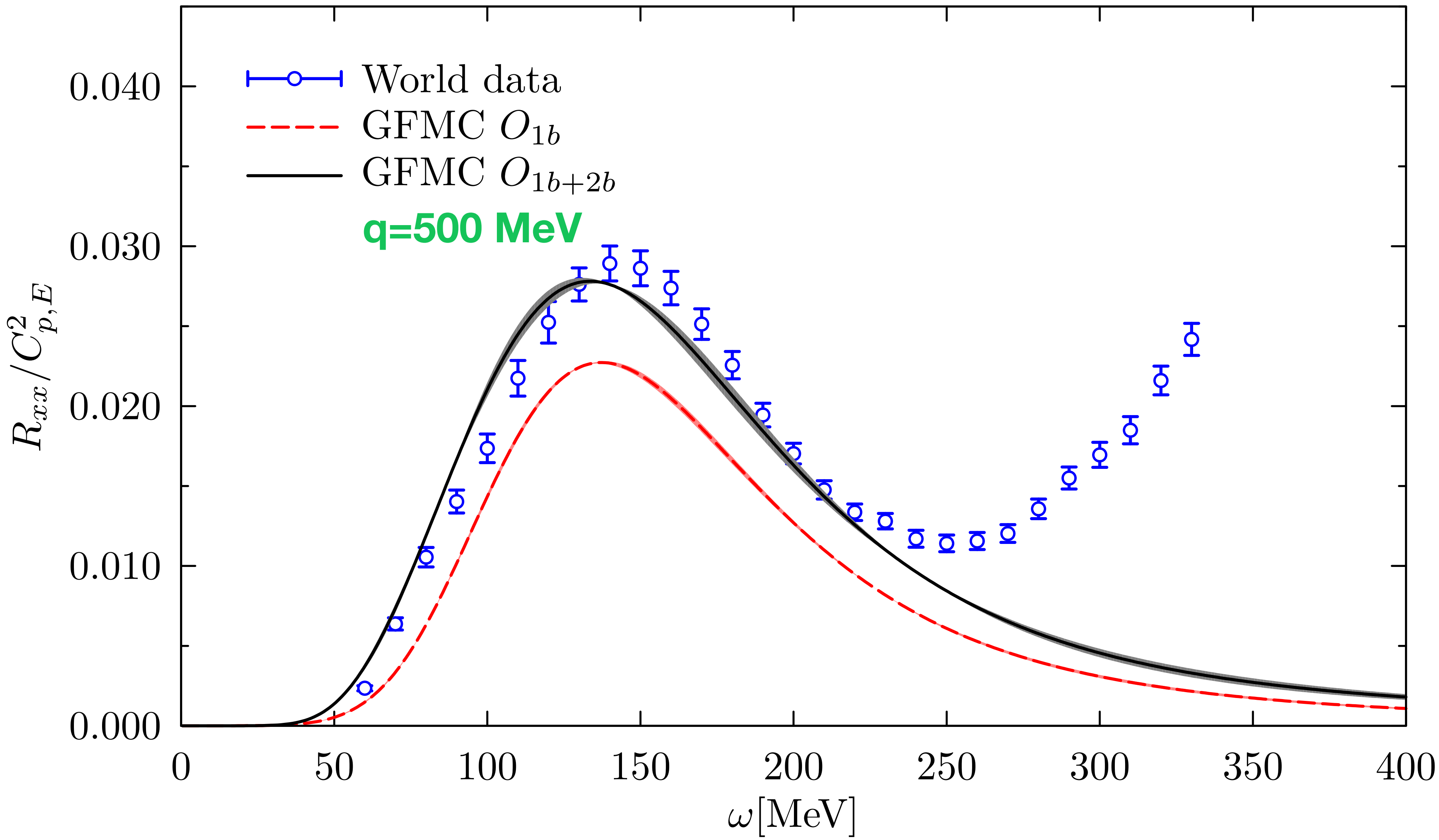

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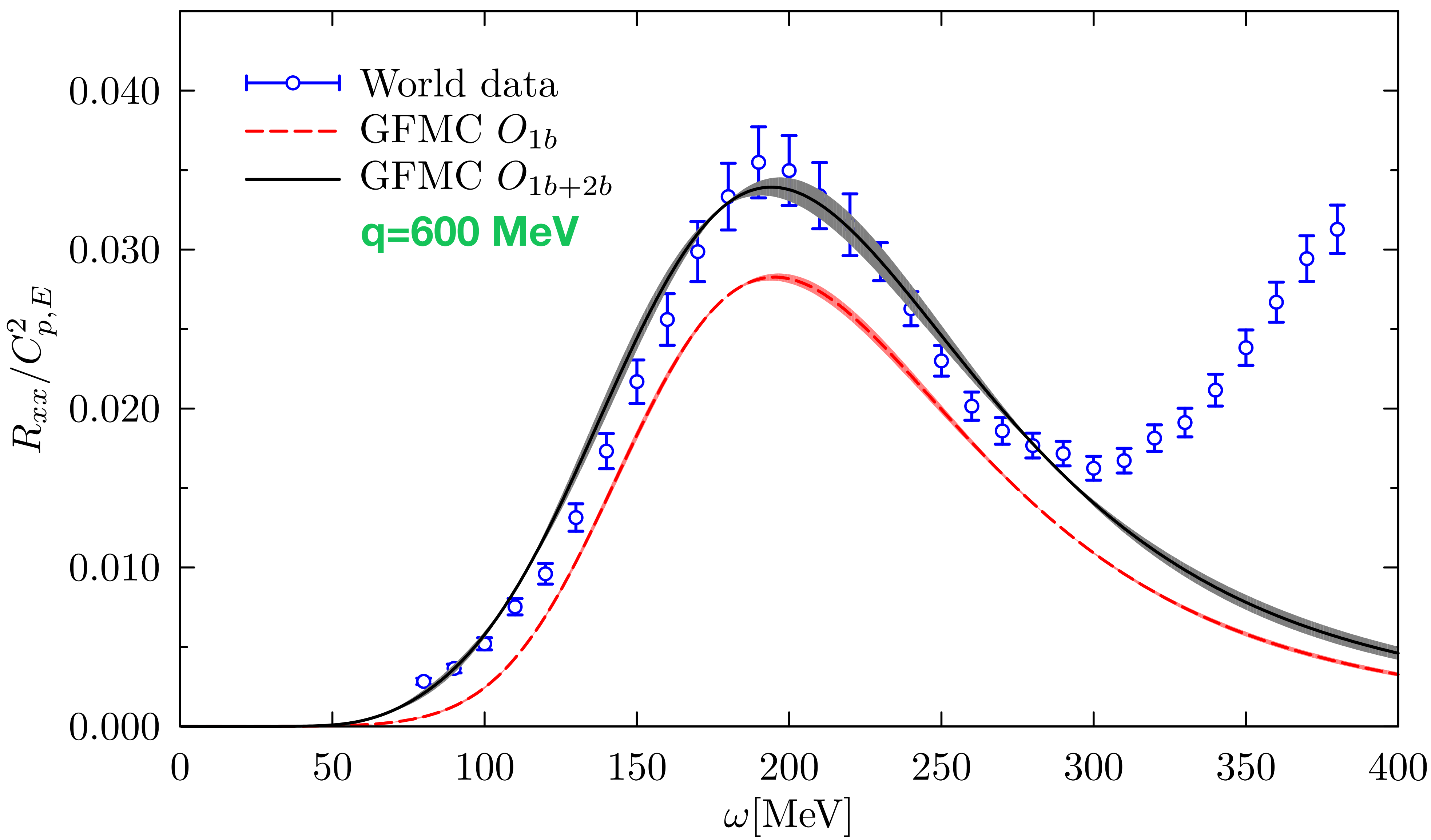

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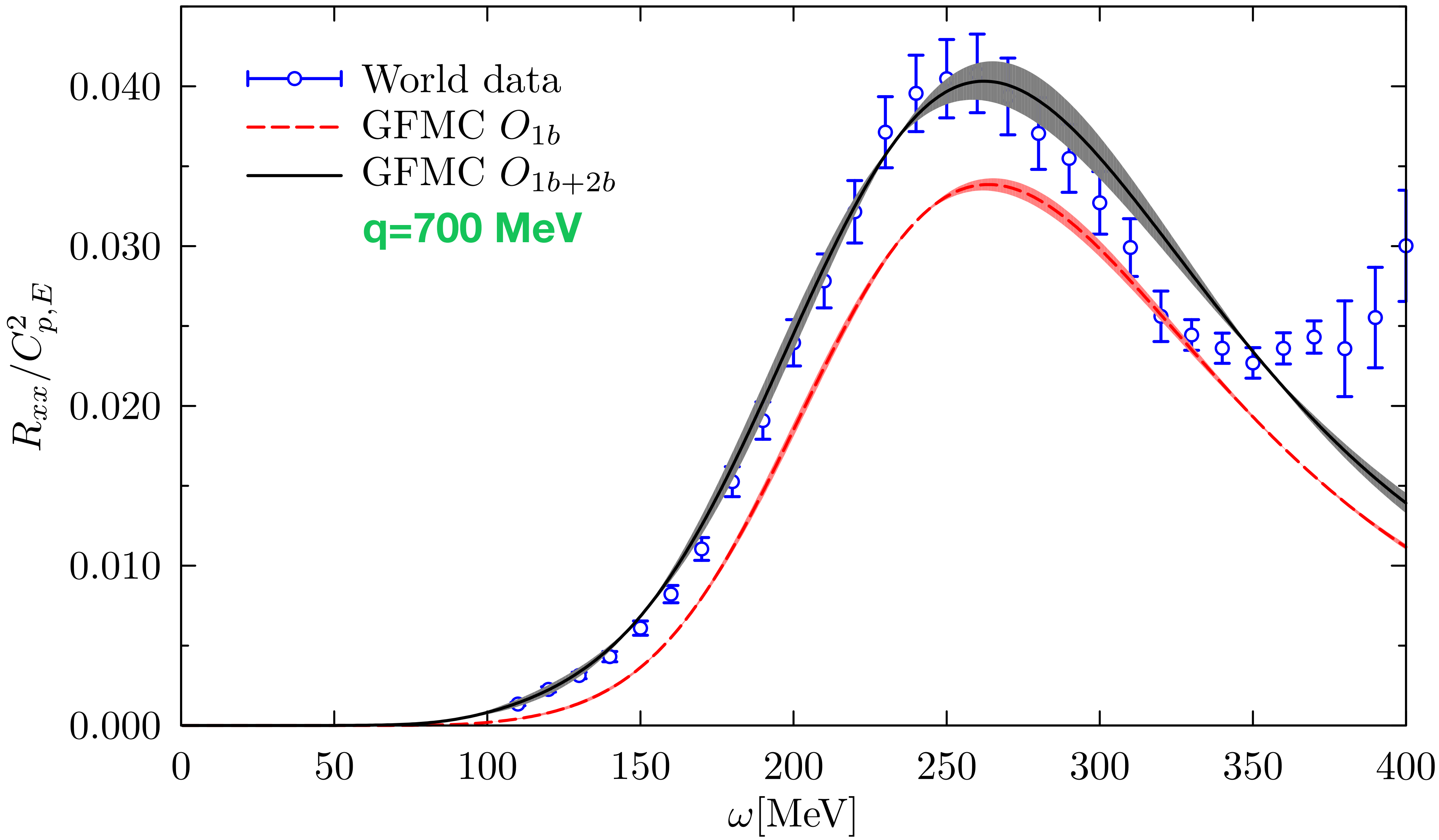
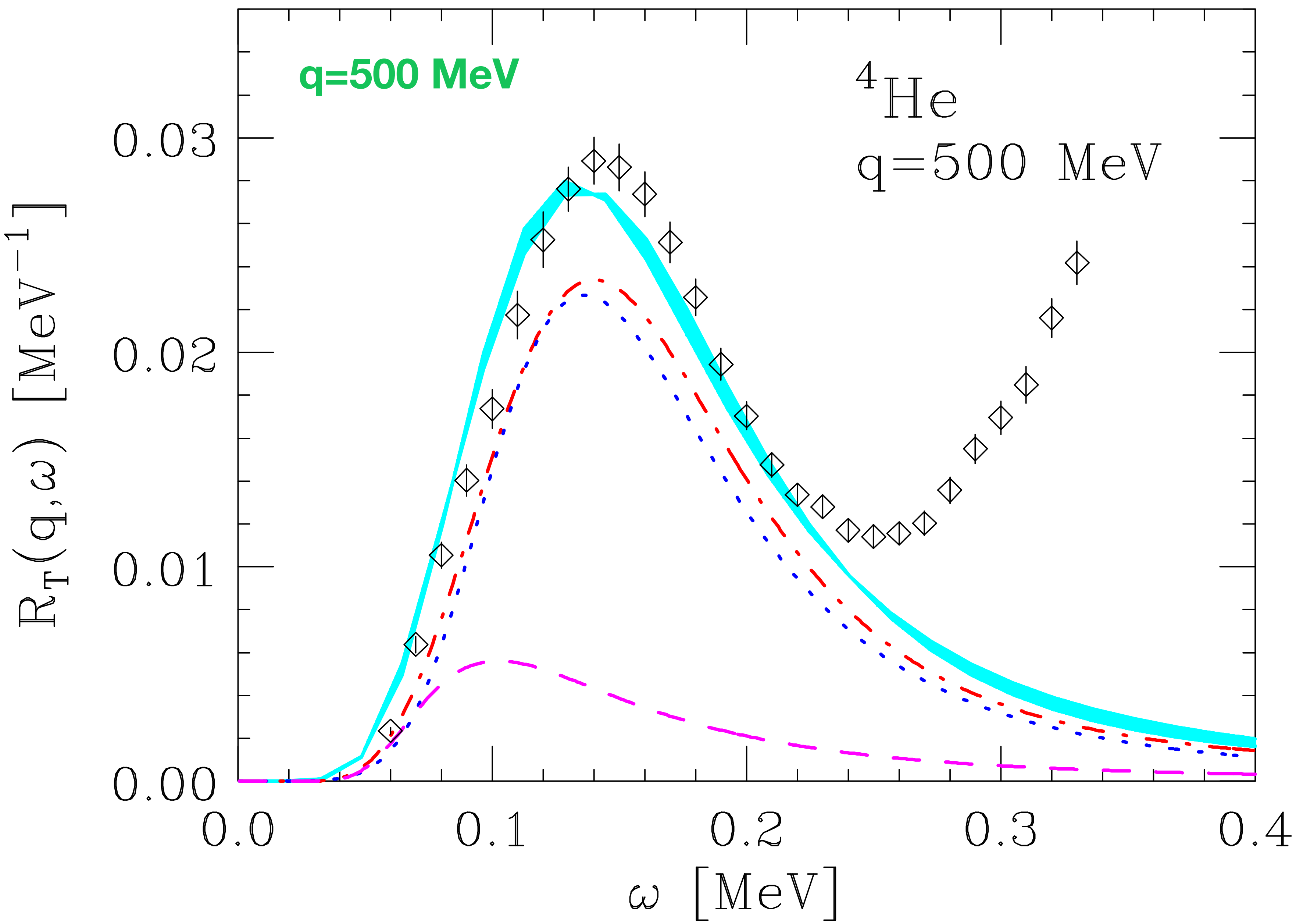


^{4}He electromagnetic response

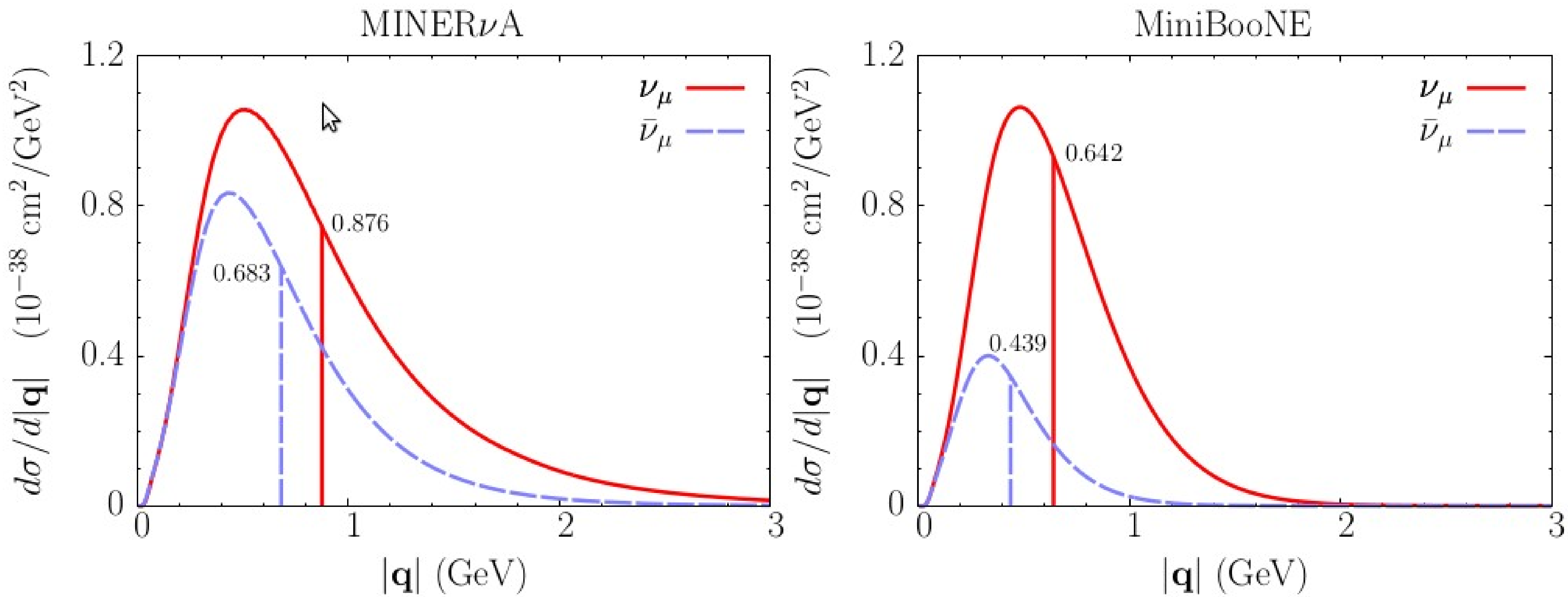
A direct consequence of nucleon-nucleon correlations is the large positive contribution of the interference term which peaks at energy loss $\omega < \omega_{QE}$.



Large momentum-transfer regime

Large momentum-transfer regime

The nuclear current operator and the nuclear final state depend on momentum transfer. At large momentum transfer non relativistic approximations become inadequate.



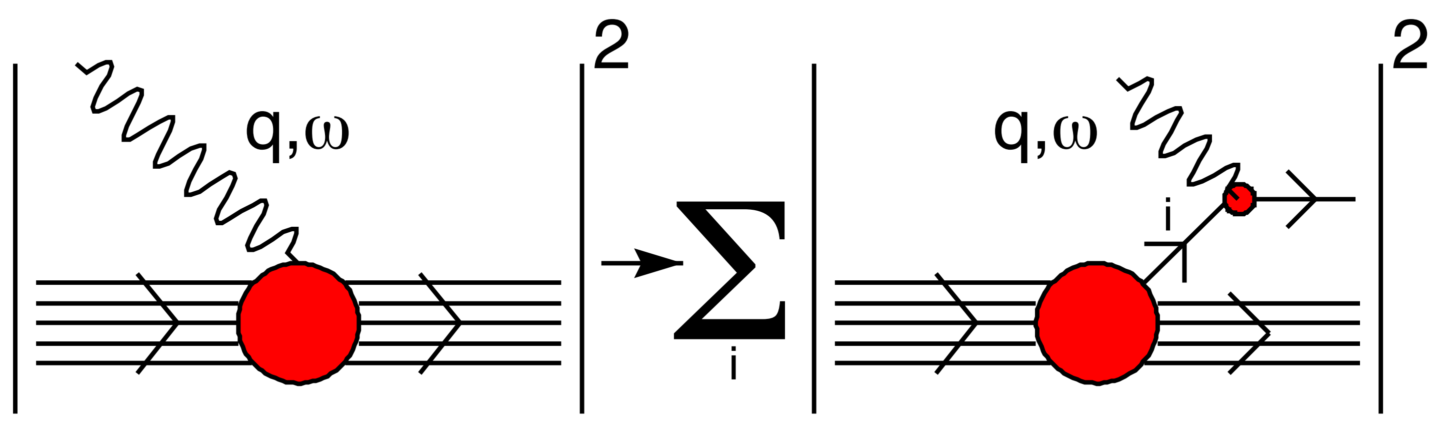
|q|-dependence of CCQE cross section averaged with the Minerva and MiniBooNE fluxes

IA: Spectral function approach

At large momentum transfer, scattering off a nuclear target reduces to the incoherent sum of scattering processes involving individual bound nucleons

$$J^\mu \to \sum_i j_i^\mu$$

$$|\Psi_X\rangle \to |\mathbf{p}\rangle \otimes |n_{A-1}\rangle$$

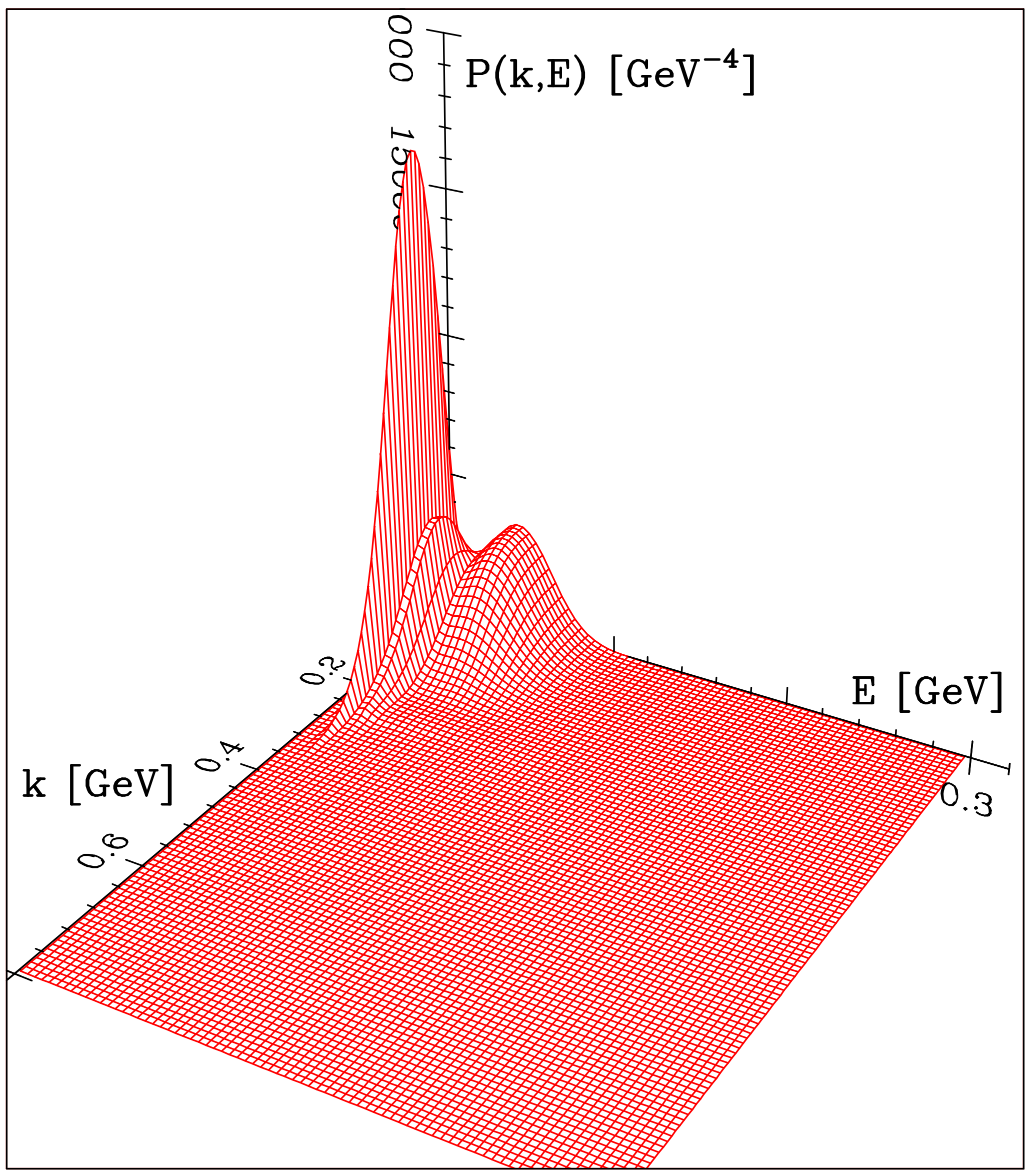


$$\frac{d\sigma_{IA}}{d\Omega_{e'}dE_{e'}} = \int d^3p\ dE\ P(\mathbf{p},E)\ \left[Z\frac{d\sigma_{ep}}{d\Omega_{e'}dE_{e'}} + (A-Z)\frac{d\sigma_{en}}{d\Omega_{e'}dE_{e'}}\right]\delta(\omega - E + m - E_x)$$

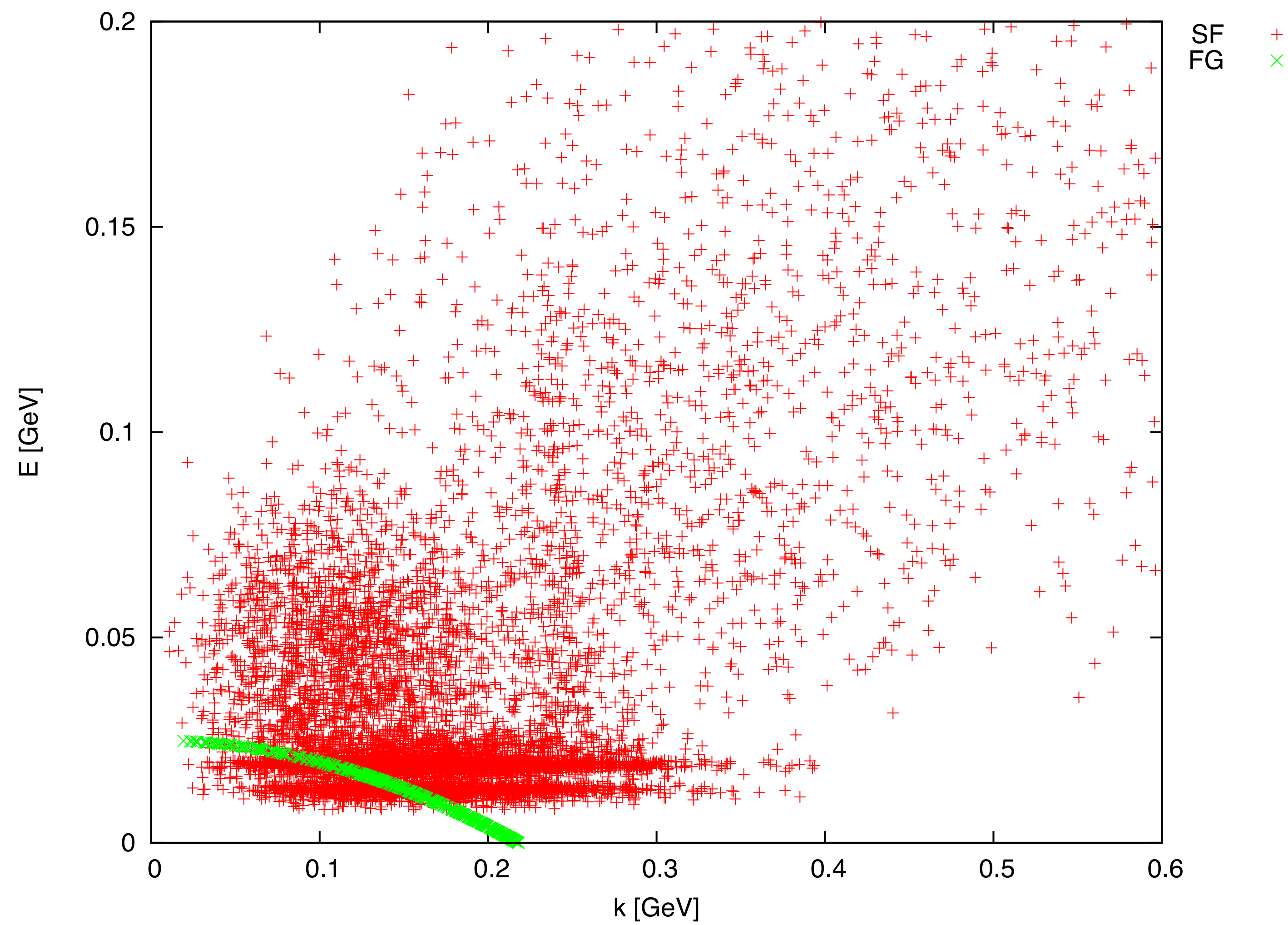
The spectral function yields the probability of removing a nucleon with momentum $\mathbf{p}$ from the target ground state leaving the residual system with excitation energy E .

IA: Spectral function approach

Oxygen spectral function



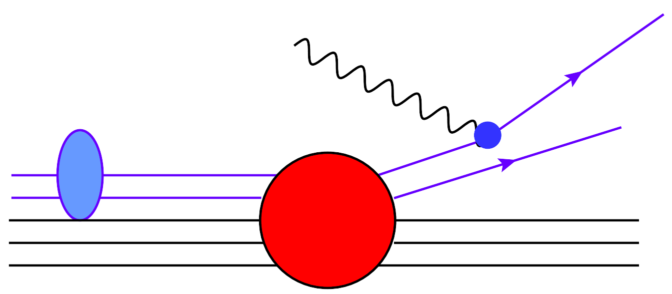
Momentum and removal energy sampled from LDA (red) and RFGM (green) oxygen spectral functions



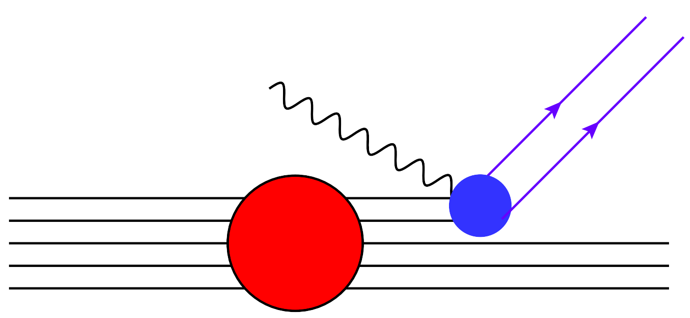
Neutrino-nucleus scattering

The observed excess of CCQE cross section may be traced back to the occurrence of events with 2p2h final states.

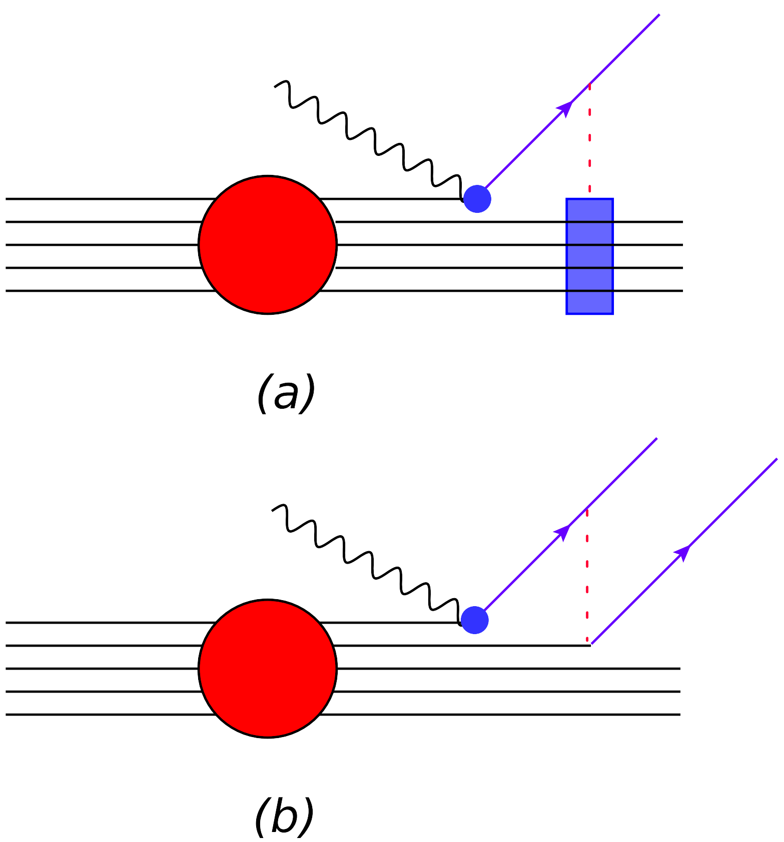
- Initial State Correlations (ISC)



- Meson Exchange Currents (MEC)



- Final State Interactions (FSI):



Neutrino-nucleus scattering

Using relativistic MEC and realistic description of the nuclear ground state requires the extension of the factorization scheme to two-nucleon emission amplitude

- Rewrite the hadronic final state in the factorized form

$$|\Psi_X\rangle \to |\mathbf{p}\mathbf{p}'\rangle \otimes |n_{A-2}\rangle$$

where $|n_{A-2}\rangle$ is the state of the spectator (A – 2)-nucleon system carrying momentum $\mathbf{p}_n$.

- The two-nucleon current simplifies to

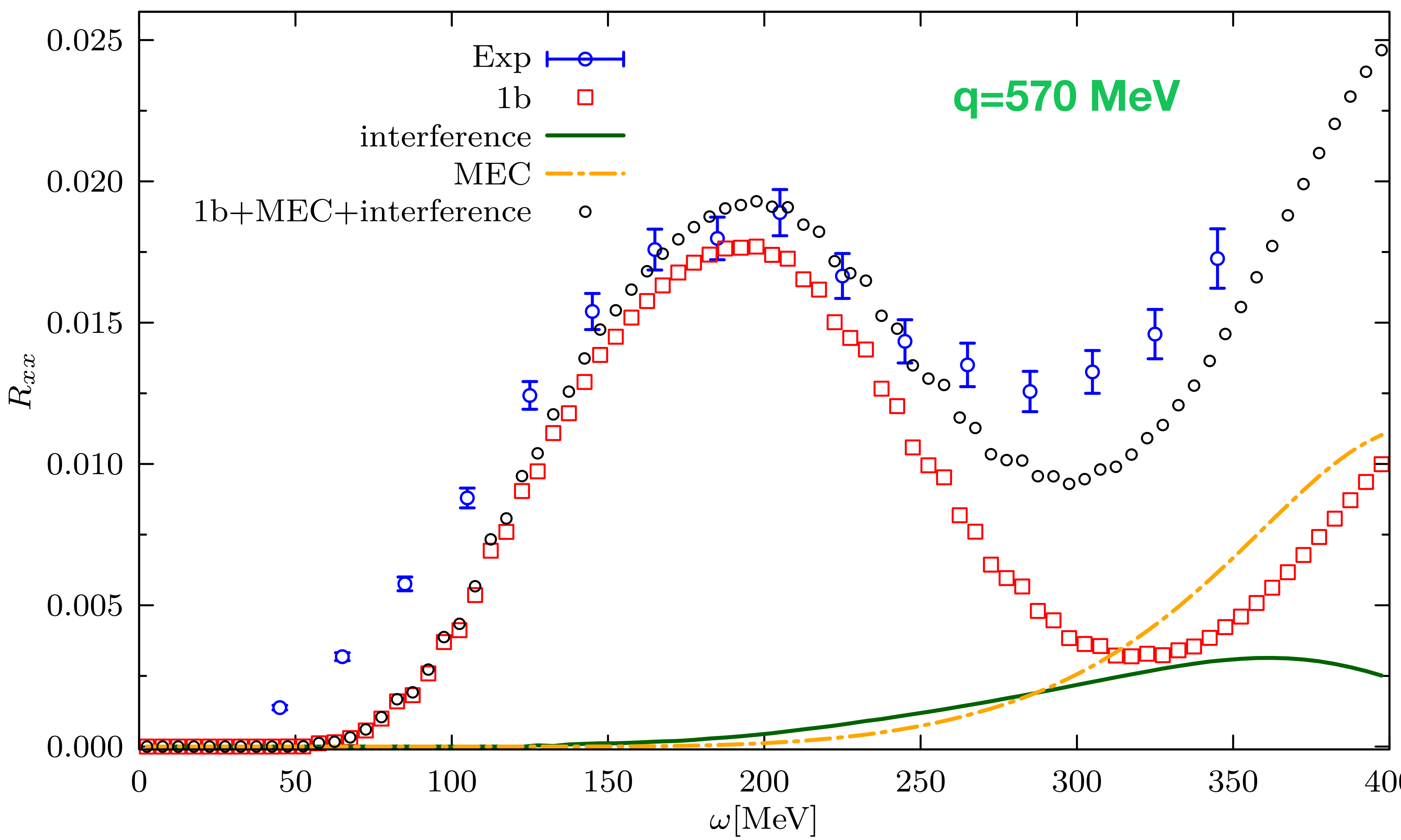
$$\langle\Psi_X|J^\mu_{ij}|\Psi_0\rangle \to \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}')\langle\mathbf{p}\mathbf{p}'|J^\mu_{ij}|\mathbf{k}\mathbf{k}'\rangle\delta(\mathbf{k}+\mathbf{k}'-\mathbf{p}_n)$$

- The nuclear amplitude $M_n(\mathbf{k},\mathbf{k}')$ is independent on $\mathbf{q}$ and can be obtained within nonrelativistic many-body theory.

Two-body currents within SF approach

Using relativistic MEC and realistic description of the nuclear ground state requires the extension of the factorization scheme to two-nucleon emission amplitude

Preliminary ^{12}C calculations show a significant enhancement of the total cross section.



Constraining the spectral function with QMC

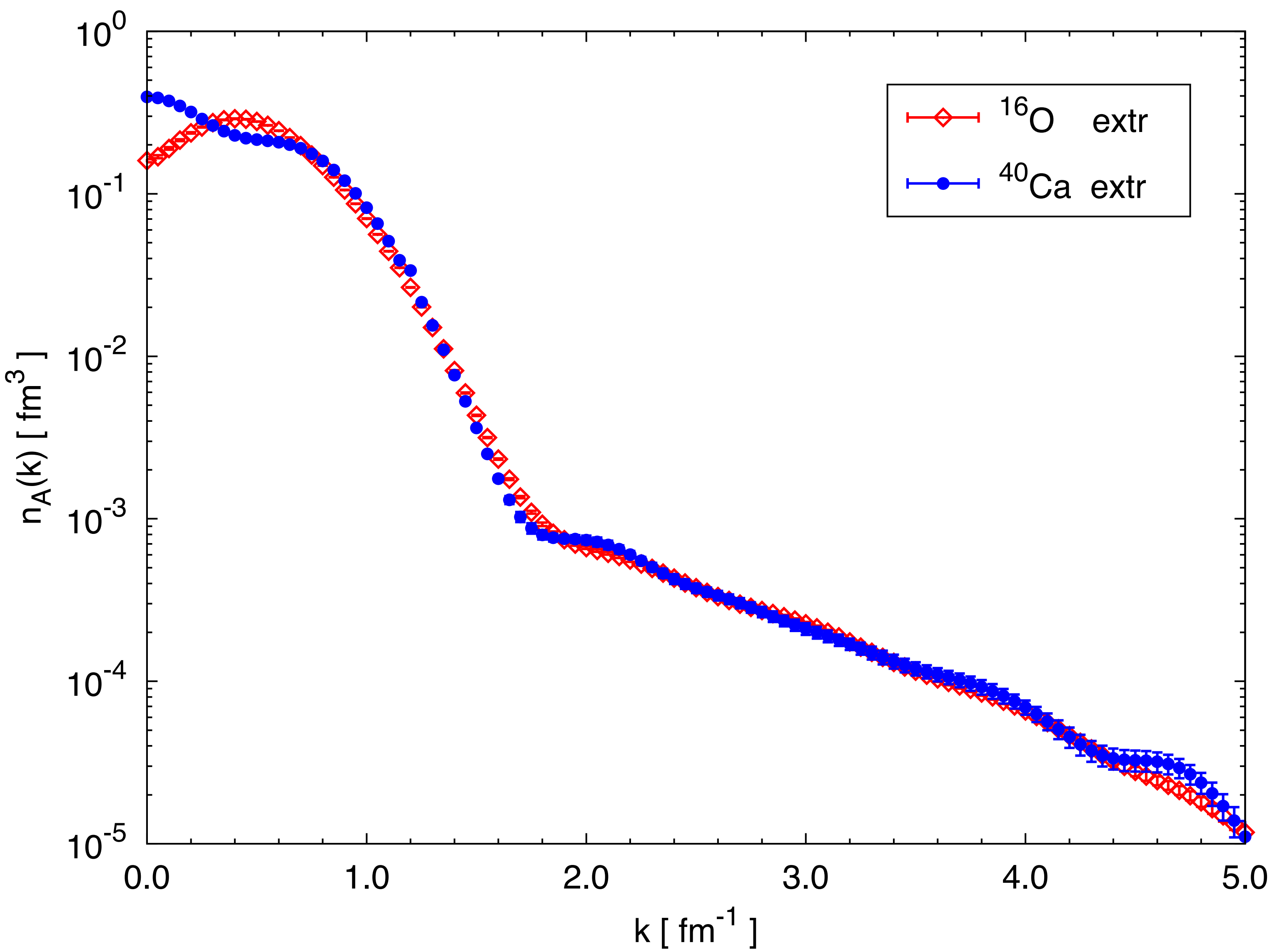
The sum rule of the spectral function corresponds to the momentum distribution

$$\int dEP(\mathbf{k},E) = \langle \Psi_0 | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | \Psi_0 \rangle$$

- Within cluster variational Monte Carlo, we have already computed the momentum distribution of nuclei as large as ^{16}O and ^{40}Ca.

- The energy weighted sum rules of the spectral function can also be computed within CVMC

$$\int dEEP(\mathbf{k},E) = \langle \Psi_0 | a^\dagger_{\mathbf{k}} [H, a_{\mathbf{k}}] | \Psi_0 \rangle$$



Conclusions

- For relatively large momentum transfer, the two-body currents enhancement is effective in the entire energy transfer domain.

- We have computed the electromagnetic and neutral-current Euclidean response of ^{12}C. The agreement of the former with experimental data is remarkably good.

- ^{4}He results for the electromagnetic response obtained using Maximum Entropy technique are in very good agreement with experimental data.

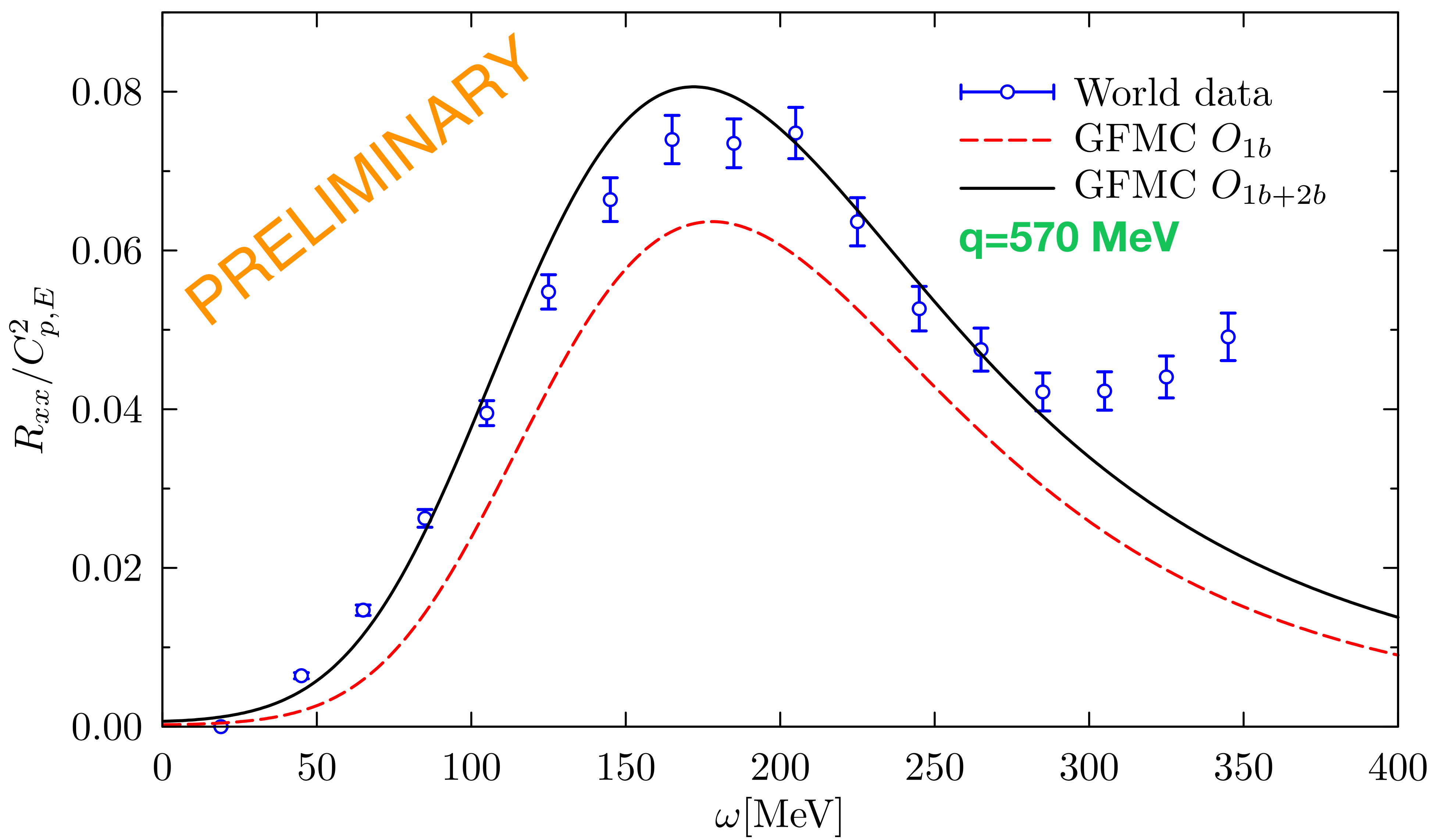
- The extension of the factorization scheme underlying the IA is a viable option for the development of a unified treatment of processes involving one- and two-nucleon currents in the region of large momentum transfer.

- We have computed the momentum distribution of ^{16}O and ^{40}Ca: these results will be used to constrain the spectral function of these nuclei

Future plans - moderate momentum transfer

- We are implementing charged-current transition transition operator in GFMC; the corresponding Euclidean response will be computed before the end of 2015.

- Preliminary results on the inversion of the ^{12}C Euclidean response are promising. Need for more statistic (and computing time) and improved inversion techniques.

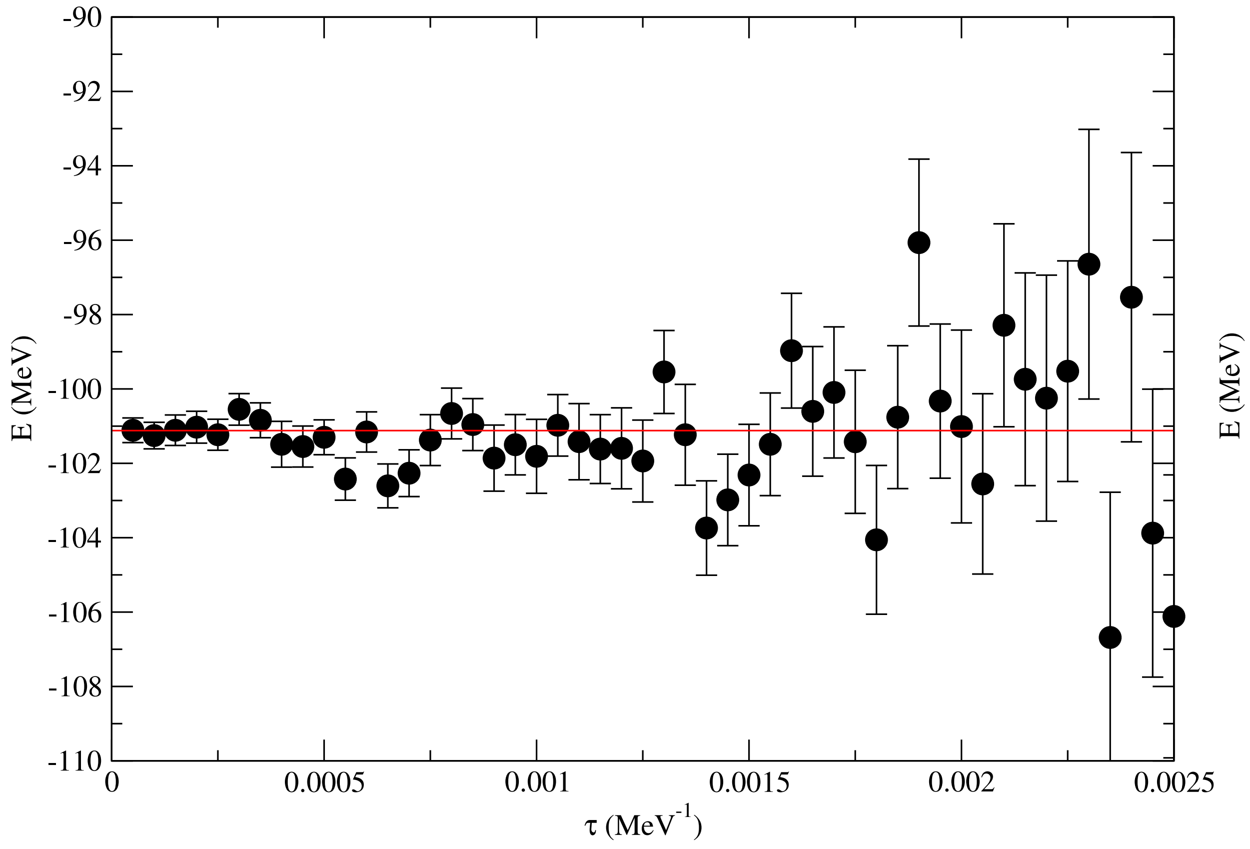


Future plans - moderate momentum transfer

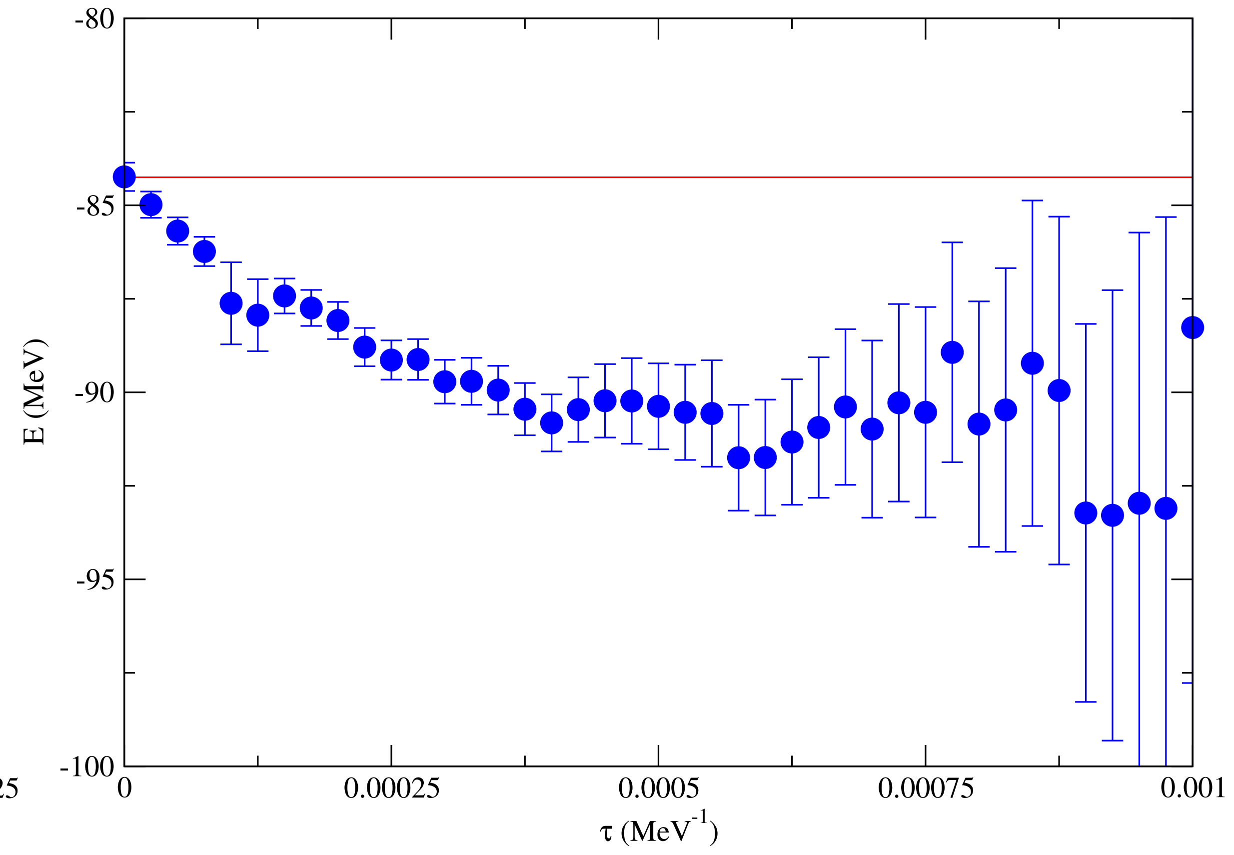
- The recently improved version of the auxiliary-field diffusion Monte Carlo method (AFDMC) has allowed us to compute the ground-state energies of nuclei as large as ^{16}O and ^{40}Ca.

- Unconstrained evolution allows for the calculation of Euclidean response functions for larger nuclei and stellar matter. Possible impact on neutron star cooling and supernovae explosion!

^{16}O Argonne v_6' + Coulomb



^{16}O Argonne v_7' + Coulomb



Future plans - large momentum transfer

- We are implementing fully-relativistic MEC currents in the spectral function approach. The interference between one- and two- body current will be fully accounted for.

- Cluster variational Monte Carlo calculations of the energy weighted sum rules of the spectral function for nuclei as large as ^{40}Ca will be carried out. Crucial interplay with (e,e') experiment on Argon at JLab.

- We plan to compute the Laplace transform of the spectral function using both GFMC and AFDMC. Maximum-entropy technique may well be used to obtain the real spectral function.

$$P^{(E)}(\mathbf{k},\tau) = \langle 0|a^\dagger(\mathbf{k})e^{-(H-E_0)\tau}a(\mathbf{k})|0\rangle$$

Thank you