Semi-inclusive neutrino-nucleus reactions

Oscar Moreno

Laboratory for Nuclear Science,
Massachusetts Institute of Technology
Semi-inclusive

neutrino-nucleus reactions

T. W. Donnelly
Laboratory for Nuclear Science, Massachusetts Institute of Technology

J. W. Van Orden
Department of Physics, Old Dominion University and Jefferson Lab

W. P. Ford
Department of Physics, University of Southern Mississippi
SUMMARY

• Definition of semi-inclusive
• Motivation
• Kinematics: lepton and transfer variables, the space of residual nucleus variables
• Dynamics: generalized Rosenbluth factors and hadronic response functions, tensor contraction and cross-sections
• Remarks on incident neutrino kinematics and nuclear dynamics
• Summary
In semi-inclusive charged current neutrino-nucleus reactions (anti)neutrinos interact with a nuclear target and the final charged lepton is detected in coincidence with another particle.

\[ X(\nu_\ell, \ell^- x) \quad X(\bar{\nu}_\ell, \ell^+ x) \]
In semi-inclusive charged current neutrino-nucleus reactions, (anti)neutrinos interact with a nuclear target and the final charged lepton is detected in coincidence with another particle.

**DEFINITION**

- Case of most interest: nucleon emission
MOTIVATION

- An increasing number of neutrino experiments allow for semi-inclusive measurements (ArgoNeuT, MicroBooNE: $\mu$ and $p$).
An increasing number of neutrino experiments allow for semi-inclusive measurements (ArgoNeuT, MicroBooNE: $\mu$ and p).

Semi-inclusive measurements provide information on the incident neutrino kinematics.
MOTIVATION

• An increasing number of neutrino experiments allow for semi-inclusive measurements (ArgoNeuT, MicroBooNE: $\mu$ and p).

• Semi-inclusive measurements provide information on the incident neutrino kinematics.

• Hadronic structure studies.
KINEMATICS

- Leptonic variables: $k, k', \theta$
- Exchange variables: $q, \omega$
- Detected hadron variables: $p_N, \theta_N, \phi$
• Detected hadron variables:
  \( p_N, \theta_N, \phi \)

\[
\begin{align*}
p & \equiv -p_{A-1} = p_N - q \\
p & = |p_m| \\
\mathcal{E} & \equiv E_{A-1} - E_{A-1}^0 = \omega - E_s + m_N - \sqrt{m_N^2 + p^2 + q^2 + 2pq \cos \theta_{pq}} - \sqrt{W_{A-1}^0}^2 + p^2 + W_{A-1}^0 \\
\mathcal{E} & \approx E_m - E_s
\end{align*}
\]
KINEMATICS

$(\varepsilon, p)$ plane: allowed region

For $q, \omega$ fixed, with $\omega < \omega_{QE} \quad (\Rightarrow y < 0, x > 1)$
KINEMATICS

$(\varepsilon, p)$ plane: allowed region

For $q, \omega$ fixed, with $\omega > \omega_{QE}$ ($\Rightarrow y > 0, x < 1$)
KINEMATICS

$(\varepsilon, p)$ plane: residual system distribution
DYNAMICS

Leptonic and hadronic tensors:

\[ d\sigma \sim \eta_{\mu\nu} W_{\mu\nu} = \eta_{\mu\nu}^s W_{s\mu\nu}^s + \chi \eta_{\mu\nu}^a W_{a\mu\nu}^a \]

\[ \eta_{\mu\nu} \equiv 2mm' \sum_{if} j^* \mu j^\nu \]

\[ W_{\mu\nu} \equiv \sum_{if} J^*_{fi}(q) J^\nu_{fi}(q) \]
Leptonic and hadronic tensors contraction:

$$\eta^s_{\mu\nu} W^s_{\lambda\sigma} = v_0 \left\{ \left[ \hat{V}_{CC} W^{CC} + \hat{V}_{CL} W^{CL} + \hat{V}_{LL} W^{LL} \right. \right.$$  
$$+ \hat{V}_{T} W^{T} + \hat{V}_{TT} W^{TT} + \hat{V}_{TC} W^{TC} + \hat{V}_{TL} W^{TL} \right\}$$  
$$+ \left[ \hat{V}_{TT} W^{TT} + \hat{V}_{TC} W^{TC} + \hat{V}_{TL} W^{TL} \right] \right\}$$

$$\eta^a_{\mu\nu} W^a_{\lambda\sigma} = v_0 \left\{ \left[ \hat{V}_{T'} W^{T'} + \hat{V}_{TC'} W^{TC'} + \hat{V}_{TL'} W^{TL'} \right. \right.$$  
$$+ \left[ \hat{V}_{CL'} W^{CL'} + \hat{V}_{TC'} W^{TC'} + \hat{V}_{TL'} W^{TL'} \right] \right\}$$
DYNAMICS

Generalized Rosenbluth factors from the leptonic tensor:

$\hat{V}_{CC}$  $\hat{V}_{CL}$  $\hat{V}_{LL}$  $\hat{V}_{T}$  $\hat{V}_{TT}$  $\hat{V}_{TC}$  $\hat{V}_{TL}$

$\hat{V}_{T'}$  $\hat{V}_{TC'}$  $\hat{V}_{TL'}$
DYNAMICS

Generalized Rosenbluth factors from the leptonic tensor:

\[ \hat{V}_{CC} = \frac{1}{2} \left\{ (a_V^2 + a_A^2) - [a_V^2(\delta - \delta')^2 + a_A^2(\delta + \delta')^2] \tan^2 \tilde{\theta}/2 \right\} \]

\[ \delta \equiv \frac{m}{\sqrt{|Q^2|}} \quad \delta' \equiv \frac{m'}{\sqrt{|Q^2|}} \]
DYNAMICS

Response functions from the hadronic tensor:

\[ \begin{align*}
W^{CC} & \quad W^{CL} & \quad W^{LL} & \quad W^{T} & \quad W^{TT} & \quad W^{TC} & \quad W^{TL} \\
W^{T'} & \quad W^{TC'} & \quad W^{TL'}
\end{align*} \]
DYNAMICS

Response functions from the hadronic tensor:

For inclusive cross-sections:
DYNAMICS

General differential cross-section:

\[
d\sigma_\chi = \frac{G^2 \cos^2 \theta_c}{2(2\pi)^5} \frac{m_N W_{A-1} v_0}{k \epsilon'} \frac{F^2_{\chi}}{E_N E_{A-1}} \mathcal{F}^3 \mathcal{p}_N \mathcal{p}_{A-1} \delta^4(K + P_A - K' - P_{A-1} - P_N).
\]

Integration over the unobserved residual system variables:

\[
d\sigma_\chi \frac{d\Omega_{k'}}{dk' d\Omega_{k'} d\Omega_{pN}} = \frac{G^2 \cos^2 \theta_c}{2(2\pi)^5} \frac{m_N W_{A-1}}{M_A^0} \frac{p_N k'^2 v_0}{\epsilon' F_{\text{rec}}} \mathcal{F}^2_\chi.
\]
In practice, one performs an integration of the cross-section over the incoming neutrino energy weighted with the neutrino flux.

For a given set of values of the semi-inclusive variables \((k', \theta, p_N, \theta_N, \phi)\), a range of incident neutrino energy corresponds to a curve in the \((\epsilon, p)\)-plane.

The nuclear target modeling is more demanding than in the inclusive case: \((\epsilon, p)\)-dependence needed. The possibility of kinematic discrimination would allow for the calibration of nuclear models.
REMARKS

Determination of the incident neutrino momentum

- From momentum conservation:

\[ k = k' \cos \theta \pm \left\{ \left[ p_N \cos \theta_N \pm (p^2 - p_N^2 \sin^2 \theta_N)^{1/2} \right]^2 - (k'^2 \sin^2 \theta) \right\}^{1/2} \]
Determination of the incident neutrino momentum

- From momentum conservation:

\[ k = k' \cos \theta \pm \left\{ p_N \cos \theta_N \pm \left( p^2 - p_N^2 \sin^2 \theta_N \right)^{1/2} \right\}^2 - k'^2 \sin^2 \theta \right\}^{1/2} \]

- From energy conservation:

\[ \epsilon = \epsilon' + E_N - M_A^0 + E_{A-1} \Rightarrow \]

\[ k = \left\{ \left[ (k'^2 + m^2)\right]^{1/2} + \left( p_N^2 + m_N^2 \right)^{1/2} - M_A^0 + E + (p^2 + W_{A-1}^{(0)})^{1/2} \right\}^2 - m^2 \right\}^{1/2} \]
Determination of the incident neutrino momentum

- From momentum conservation:

\[ k = k' \cos \theta \pm \left\{ \left[ p_N \cos \theta_N \pm (p^2 - p_N^2 \sin^2 \theta_N)^{1/2} \right]^2 - k'^2 \sin^2 \theta \right\}^{1/2} \]

- From energy conservation:

\[ \epsilon = \epsilon' + E_N - M_A^0 + E_{A-1} \Rightarrow \]

\[ k = \left\{ \left[ (k'^2 + m^2)^{1/2} + (p_N^2 + m_N^2)^{1/2} - M_A^0 + \mathcal{E} + (p^2 + W_{A-1}^{(0)})^{1/2} \right]^2 - m^2 \right\}^{1/2} \]

For the particular case of deuteron target:

\[ W_{A-1}^{(0)} = m_p, \quad \mathcal{E} = 0 \]
SUMMARY

SEMI-INCLUSIVE CROSS-SECTION FORMALISM

• The masses of the incoming and outgoing leptons are kept.
SUMMARY

SEMI-INCLUSIVE CROSS-SECTION FORMALISM

• The masses of the incoming and outgoing leptons are kept.

• Scattering of both neutrinos and antineutrinos (different sign of V-A interference contributions).
SUMMARY

SEMI-INCLUSIVE CROSS-SECTION FORMALISM

• The masses of the incoming and outgoing leptons are kept.

• Scattering of both neutrinos and antineutrinos (different sign of V-A interference contributions).

• General hadronic target and hadronic products (detected particle and residual system).
SUMMARY

SEMI-INCLUSIVE CROSS-SECTION FORMALISM

• The masses of the incoming and outgoing leptons are kept.

• Scattering of both neutrinos and antineutrinos (different sign of V-A interference contributions).

• General hadronic target and hadronic products (detected particle and residual system).

• Components of the leptonic and hadronic tensors are given in terms of chargelike, longitudinal and transverse projections of the electroweak current and organized into VV, AA and VA contributions.
**SUMMARY**

**SEMI-INCLUSIVE CROSS-SECTION FORMALISM**

- The masses of the incoming and outgoing leptons are kept.

- Scattering of both neutrinos and antineutrinos (different sign of V-A interference contributions).

- General hadronic target and hadronic products (detected particle and residual system).

- Components of the leptonic and hadronic tensors are given in terms of chargelike, longitudinal and transverse projections of the electroweak current and organized into VV, AA and VA contributions.

- Formalism valid for charged-current and neutral current weak interactions; in the latter case, integrate over the unobserved outgoing neutrino variables (u-channel inclusive, different from t-channel inclusive).
SUMMARY

REMARKS

• Transformation of hadronic variables to the \((\varepsilon,p)\) variables, which are best suited to characterizing the nuclear dynamics.
• Transformation of hadronic variables to the $(\varepsilon, p)$ variables, which are best suited to characterizing the nuclear dynamics.

• Translation of a neutrino energy range onto the $(\varepsilon, p)$-plane: kinematical discrimination of nuclear dynamics.
SUMMARY

REMARKS

• Transformation of hadronic variables to the \((\varepsilon,p)\) variables, which are best suited to characterizing the nuclear dynamics.

• Translation of a neutrino energy range onto the \((\varepsilon,p)\)-plane: kinematical discrimination of nuclear dynamics.

• Determination of the incident neutrino kinematics in semi-inclusive measurements (special case of deuteron).
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

- From semi-inclusive to inclusive
- From (mainly) charged-current to neutral-current
- From quasi-elastic to elastic
- From detection of ejected particles to detection of target recoil
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

- From semi-inclusive to inclusive
- From (mainly) charged-current to neutral-current
- From quasi-elastic to elastic
- From detection of ejected particles to detection of target recoil

In elastic neutrino scattering the target nucleus remains in its ground state. In general, the main contribution is the **coherent scattering**, proportional to $N^2$ and valid when $q \ll l/R \approx 160 \text{ A}^{-1/3} \text{ MeV}$; it is the only contribution for even-even targets.
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

• Relationship between coherent electron-nucleus and coherent neutrino-nucleus cross-sections in PWBA:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{(\nu,\nu)} = \left[ \frac{(a_{W}^{\nu})^{2} + (a_{A}^{\nu})^{2}}{2(a_{A}^{e})^{2}} \right] A_{(e,e)}^{2} \left( \frac{d\sigma}{d\Omega} \right)_{(e,e)}
\]

where the parity-violating elastic electron scattering asymmetry is defined as

\[
A_{(e,e)} = \frac{(\frac{d\sigma}{d\Omega})_{h=+1}^{h=+1} - (\frac{d\sigma}{d\Omega})_{h=-1}^{h=-1}}{(\frac{d\sigma}{d\Omega})_{h=+1}^{h=+1} + (\frac{d\sigma}{d\Omega})_{h=-1}^{h=-1}}
\]
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

\[
\left( \frac{d\sigma}{d\Omega} \right)_{(\nu,\nu)} = \left[ \frac{(a_V^\nu)^2 + (a_A^\nu)^2}{2 (a_A^e)^2} \right] A_{(e,e)}^2 \left( \frac{d\sigma}{d\Omega} \right)_{(e,e)}
\]

- Deviations from this prediction:
  - Coulomb distortion.
  - Effect of higher order corrections.
  - Different coupling of $Z^0$ to neutrinos and charged leptons.
  - Other effects affecting differently neutrinos and charged leptons.
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

• Relationship between relative uncertainties:

\[ \mathcal{E} \left( \frac{d\sigma}{d\Omega} \right)_{(\nu,\nu)} \approx 2 \mathcal{E} A_{(e,e)} \]
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

- Relationship between relative uncertainties:

\[ \mathcal{E} \left( \frac{d\sigma}{d\Omega} \right)_{(\nu,\nu)} \approx 2 \mathcal{E} \mathcal{A}_{(e,e)} \]

to which the PV experiment statistical contribution is:

\[ \mathcal{E}^{\text{stat.}} \left( \frac{d\sigma}{d\Omega} \right)_{(\nu,\nu)} \approx 2 \chi_{PV}^{-\frac{1}{2}} \mathcal{F}_{PV}^{-\frac{1}{2}} \]
A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

- For measurements at different kinematic conditions:

\[
\left( \frac{d\sigma(k_\nu, \theta_\nu)}{d\Omega} \right)_{(\nu, \nu)} = \mathcal{K}(k_\nu, k_e, \theta_e) \mathcal{A}_{(e,e)}^2(\hat{k}_e, \hat{\theta}_e) \left( \frac{d\sigma(k_e, \theta_e)}{d\Omega} \right)_{(e,e)}
\]

\[
\mathcal{K} = \frac{k_e^2 (k_\nu - \omega_e)^2 [2 k_\nu^2 - \omega_e (2 k_\nu + M_A)]}{k_\nu^2 (k_e - \omega_e)^2 [2 k_e^2 - \omega_e (2 k_e + M_A)]}
\]
ELASTIC NEUTRINO SCATTERING

Preliminary results, starting point
Semi-inclusive neutrino-nucleus reactions

Oscar Moreno

Laboratory for Nuclear Science,
Massachusetts Institute of Technology
EXTRA MATERIAL
Generalized Rosenbluth factors

\[ \hat{V}_{CC} = \frac{1}{2} \left\{ (a_v^2 + a_A^2) - [a_v^2(\delta - \delta')^2 + a_A^2(\delta + \delta')^2] \tan^2 \tilde{\theta}/2 \right\} \]

\[ \hat{V}_{CL} = -\frac{1}{2} (a_v^2 + a_A^2) \left[ \nu - \frac{1}{\rho'} (\delta^2 - \delta'^2) \tan^2 \tilde{\theta}/2 \right] \]

\[ \hat{V}_{LL} = \frac{1}{2} \left\{ (a_v^2 + a_A^2) \left[ \nu^2 - \frac{1}{\rho'} (2\nu - \rho \rho' (\delta^2 - \delta'^2))(\delta^2 - \delta'^2) \tan^2 \tilde{\theta}/2 \right] + [a_v^2(\delta - \delta')^2 + a_A^2(\delta + \delta')^2] \tan^2 \tilde{\theta}/2 \right\} \]

\[ \hat{V}_{T} = \frac{1}{2} (a_v^2 + a_A^2) \left\{ \left[ \frac{1}{2} \rho + \tan^2 \tilde{\theta}/2 \right] + \left( \frac{\nu}{\rho'} (\delta^2 - \delta'^2) - \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right) \tan^2 \tilde{\theta}/2 \right\} - (a_v^2 - a_A^2) \delta \delta' \tan^2 \tilde{\theta}/2, \]

\[ \hat{V}_{TT} = \frac{1}{2} (a_v^2 + a_A^2) \left\{ -\frac{1}{2} \rho + \left[ (\delta^2 + \delta'^2) - \frac{\nu}{\rho'} (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right] \tan^2 \tilde{\theta}/2 \right\}, \]

\[ \hat{V}_{TC} = -\frac{1}{2} (a_v^2 + a_A^2) \frac{1}{\rho'} \tan \tilde{\theta}/2 \times \left( \frac{1}{2} - \frac{1}{\rho} \left[ (\delta^2 + \delta'^2) - \frac{\nu}{\rho'} (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right] \tan^2 \tilde{\theta}/2 \right)^{1/2}, \]
Generalized Rosenbluth factors

\[ v_0 \equiv (\epsilon + \epsilon')^2 - q^2. \]
\[ \nu \equiv \frac{\omega}{q}, \]
\[ \rho \equiv \frac{|Q^2|}{q^2} = 1 - \nu^2; \quad \rho' \equiv \frac{q}{\epsilon + \epsilon'}, \]
\[ \delta \equiv \frac{m}{\sqrt{|Q^2|}}; \quad \delta' \equiv \frac{m'}{\sqrt{|Q^2|}}, \]
\[ \tan^2 \tilde{\theta}/2 = \frac{|Q^2|}{v_0} = \frac{\rho \rho'^2}{1 - \rho^2}. \]

\[ \hat{V}_{TL} = -(\nu - \rho \rho' (\delta^2 - \delta'^2)) \hat{V}_{TC}, \]
\[ \hat{V}_{TT} = 0, \]
\[ \hat{V}_{TC} = 0, \]
\[ \hat{V}_{TL} = 0, \]
\[ \hat{V}_{T'} = a_V a_A \frac{1}{\rho'} (1 + \nu \rho' (\delta^2 - \delta'^2)) \tan^2 \tilde{\theta}/2, \]
\[ \hat{V}_{TC'} = -a_V a_A \tan \tilde{\theta}/2 \times \left[ \frac{1}{2} - \frac{1}{\rho'} \left( (\delta^2 + \delta'^2) - \nu \rho' (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right) \tan^2 \tilde{\theta}/2 \right]^{1/2}, \]
\[ \hat{V}_{TL'} = -\nu \hat{V}_{TC'}, \]
\[ \hat{V}_{CL'} = 0, \]
\[ \hat{V}_{TC'} = 0, \]
\[ \hat{V}_{TL'} = 0. \]
Generalized Rosenbluth factors,

ERL

\begin{align*}
v_{CC} &= 1, \\
v_{CL} &= -\nu, \\
v_{LL} &= \nu^2, \\
v_T &= \frac{1}{2} \rho + \tan^2 \theta / 2, \\
v_{TT} &= -\frac{1}{2} \rho, \\
v_{TC} &= -\frac{1}{\sqrt{2\rho'}} \tan \theta / 2, \\
v_{TL} &= -\nu v_{TC}, \\
v_{T'} &= \tan \theta / 2 \sqrt{\rho + \tan^2 \theta / 2}, \\
v_{TC'} &= -\frac{1}{\sqrt{2}} \tan \theta / 2, \\
v_{TL'} &= -\nu v_{TC'}. \end{align*}