Spin and the Proton Transverse Shape

- Proton form factor, model calculation - proton not round via spin dependent density
- Model independent neutron charge density
- Measure shape of proton on lattice (impact parameter dependent GPD) coordinate-space probability, and in experiment (TMD): TMD is momentum-space probability
- GAM “Transverse Charge Densities”
Ratio of Pauli to Dirac Form Factors 1995
Frank, Jennings, Miller  
theory, data 2000

Impulse approximation

Model proton wave function $\Psi(k_\perp, K_\perp, \xi, \eta)$

Poincare invariant

Light front variables for boost: $K \rightarrow K + \eta q_\perp$

Dirac spinors carry orbital angular momentum

Flat due to orbital angular momentum
Model exists

- lower components of Dirac spinor
- orbital angular momentum
- shape of proton? Wigner Eckart
  no quadrupole moment
- spin dependent densities SDD
  non-relativistic example
Non-Rel. $p_{1/2}$ proton outside $0^+$ core

\[ \langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = \mathcal{R}(r_p) \sigma \cdot \hat{\mathbf{r}}_p | s \rangle \]

Binding potential rotationally invariant

\[ \rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r) \]

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

\[ \rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p)^{(1+\sigma \cdot \mathbf{n})/2} | \psi_{1,1/2s} \rangle \]

\[ = \frac{R^2(r)}{2} \langle s | \sigma \cdot \hat{\mathbf{r}}(1 + \sigma \cdot \mathbf{n})\sigma \cdot \hat{\mathbf{r}} | s \rangle \]

$n \parallel \hat{s}$: \[ \rho(\mathbf{r}, \mathbf{n} = \hat{s}) = R^2(r) \cos^2 \theta \]

$n \parallel -\hat{s}$: \[ \rho(\mathbf{r}, \mathbf{n} = -\hat{s}) = R^2(r) \sin^2 \theta \]

non-spherical shape depends on spin direction
How to measure?-Lattice and/or experiment

Relation between coordinate and momentum space densities? Model independent technique needed.
Model Independent Technique

• Light front coordinates, $\infty$ momentum frame, IMF

“Time” $x^+ = (ct + z)/\sqrt{2} = (x^0 + x^3)/\sqrt{2}$, “evolution” $p^- = (p^0 - p^3)/\sqrt{2}$

“Space” $x^- = (x^0 - x^3)/\sqrt{2}$, “Momentum” $p^+ = (p^0 + p^3)/\sqrt{2}$

“Transverse position, momentum, $b, p$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs, TMDS
Model independent transverse charge density

\[ J^+ (x^-, b) = \sum_{q} e_q q_+^\dagger (x^-, b) q_+ (x^-, b) \]

Charge Density operator IMF

\[ \rho_\infty (x^-, b) = \langle p^+, R = 0, \lambda | \sum_{q} e_q q_+^\dagger (x^-, b) q_+ (x^-, b) | p^+, R = 0, \lambda \rangle \]

\[ F_1 = \langle p^+, p', \lambda | J^+ (0) | p^+, p, \lambda \rangle \]

\[ \rho (b) \equiv \int dx^- \rho_\infty (x^-, b) = \int \frac{Q dQ}{2\pi} F_1 (Q^2) J_0 (Q b) \]
Transverse charge densities from parameterizations (Alberico)

Figure 4

Nucleon $\rho(b)$. (a) Proton transverse charge density. (b) Neutron transverse charge density. These densities are obtained by using the parameterization of Reference 91.

We exploit Equation 31 by using measured form factors to determine $\rho(b)$. Recent parameterizations (87–91) of $G_E$ and $G_M$ are very useful, so we use Equation 43 to obtain $F_1$ in terms of $G_E$, $G_M$. Then $\rho(b)$ can be expressed as a simple integral of known functions,

$$\rho(b) = \int_0^\infty \frac{d Q}{\pi} Q^2 \frac{1}{4} + \tau \frac{G_M(Q^2)}{\frac{Q^2}{4 M^2}} \left[ J_0(Q b) \frac{G_E(Q^2)}{Q^2} + \tau \right],$$

where $\tau = \frac{Q^2}{4 M^2}$ and $J_0$ is a cylindrical Bessel function.

A straightforward application of Equation 44 to the proton using the parameterizations of Reference 91 yields the results shown in Figure 4a. The curves obtained by using the two different parameterizations overlap. Furthermore, there seems to be negligible sensitivity to form factors at very high values of $Q^2$ that are currently unmeasured. The density is peaked at low values of $b$ but contains has a long positive tail, suggesting a long-ranged, positively charged pion cloud.

The neutron results are shown in Figure 4b. The curves obtained by using the two different parameterizations seem to overlap. Surprisingly, the central neutron charge density is negative.

Changes may still occur before final publication online and in print.

Thursday, October 28, 2010
Generalized Coordinate Space Densities

\[ \rho^{\Gamma}(b) = \sum_q e_q \int dx^- q_+(x^-, b) \gamma^+ \Gamma q_+(x^-, b) \]

\[ \Gamma = \frac{1}{2}(1 + n \cdot \gamma) \] gives spin-dep density

Local operators calculable as x moments on lattice. Göckeler et al. PRL98,222001

\[ \tilde{A}''_{T10} \sim \text{sdd} \] spin-dependent density

Schierholtz, Zanotti 2009 - this quantity is not zero, proton is not round
Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...

This is not zero! proton is not round
Shapes of the proton

Relate spin dependent density to experiment


Field-theoretic spin dependent momentum density is related to the transverse momentum distribution $h_{1T}^{-1}$

$$\Phi^{[\Gamma]}(x, K_T) = \int \frac{d\xi - d^2\xi_T}{2(2\pi)^3} e^{iK\cdot \xi} \langle P, S | \overline{\psi}(0) \Gamma \mathcal{L}(0, \xi; n) \psi(\xi) | P, S \rangle \bigg|_{\xi^+ = 0}$$

Mulders Tangerman’96

$$\Phi^{[i\sigma^i + \gamma_5]}(x, K_T) = S_T^i h_1(x, K_T^2) + \frac{(K_i^T K_j^T - \frac{1}{2} K_T^2 \delta_{ij})}{M^2} S_T^j h_{1T}^{-1}(x, K_T^2)$$

$$\sigma^i + \gamma^5 \sim \gamma^0 \gamma + \sigma^i$$,

then relate equal time to $\xi^+ = 0$ by integration over $x$
Shapes of the proton

Relate spin dependent density to experiment


Field-theoretic spin dependent \textbf{momentum} density is related to the transverse momentum distribution $h^{\perp}_{1T}$

\[
\Phi^{[\Gamma]}(x, K_T) = \left. \int \frac{d\xi - d^2\xi_T}{2(2\pi)^3} e^{iK\cdot\xi} \langle P, S|\bar{\psi}(0) \Gamma \mathcal{L}(0, \xi; n_-) \psi(\xi)|P, S \rangle \right|_{\xi^+ = 0}
\]

Mulders Tangerman’96

\[
\Phi^{[i\sigma^i + \gamma_5]}(x, K_T) = S^i_T h_{1}(x, K_T^2) + \frac{(K^i_T K^j_T - \frac{1}{2} K^2_T \delta_{ij}) S^j_T}{M^2} h^{\perp}_{1T}(x, K_T^2)
\]

\[
\sigma^i + \gamma^5 \sim \gamma^0 \gamma^+ \sigma^i,
\]

then relate equal time to $\xi^+ = 0$ by integration over $x$
Measure $h_{1T} \perp p \to e', \pi X$

H. Avakian LOI at Jlab

Cross section has term proportional to $\cos 3\phi$

Boer Mulders '98 there are other ways to see $h_{1T} \perp$
Summary

- Form factors, GPDs, TMDs, understood from unified light-front formulation, GPD-coordinate space density, TMD momentum space density
- Neutron central transverse density is negative-consistent with Cloudy Bag Model
- Proton is not round- lattice QCD spin-dependent-density is not zero
- Experiment can whether or not proton is round by measuring $h_{1T}$
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The Proton