Spectral Densities of Three-point Correlators

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October 28, 2010

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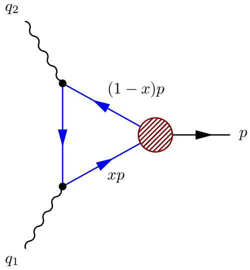
Photon-Pion Transition Form Factor



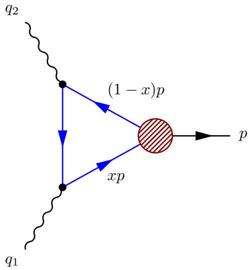
$F_{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ relates two (in general, virtual) photons with the lightest hadron, the pion.

Plays special role among exclusive processes in QCD.

For real photons $F_{\gamma^*\gamma^*\pi^0}(0, 0)$ determines rate of $\pi^0 \to \gamma\gamma$ decay, deeply related to axial anomaly.

For large photon virtualities, it has simplest structure analogous to the form factors in DIS.

In Perturbative QCD



pQCD predictions with data gives info about the shape of the pion DA $\varphi_\pi(x)$.

Since only one hadron is involved, $\gamma^*\gamma^*\pi^0$ has simplest structure for pQCD analysis.

Nonperturbative information about pion is accumulated in pion DA $\varphi_\pi(x)$.

Short-distance amplitude for $\gamma^*\gamma^* \to \pi^0$ at leading order is given by single quark propagator.

Real and a Virtual Photon $\gamma\gamma^* \to \pi^0$

For $q_1^2 = -Q^2$, $q_2^2 = 0$

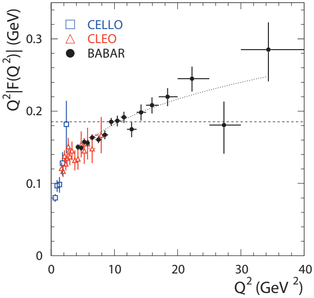
Leading-order pQCD prediction

$$F_{\gamma\gamma^*\pi}^{\mathrm{pQCD}}(Q^2) = \frac{\sqrt{2}}{3Q^2}\int_0^1 \frac{\varphi_\pi(x)}{x}\,dx \;\equiv \frac{\sqrt{2}f_\pi}{3\,Q^2}\,J$$

Information about pion DA is now accumulated in factor J:

- $J = 2$ for infinitely narrow $\sim \delta(x - 1/2)$ DA
- $J = 3$ for asymptotic $\sim 6x(1-x)$ DA
- Another measure of the width of pion DA
- $J = \infty$ for flat $\varphi_\pi(x) = f_\pi$ DA!

BaBar Data

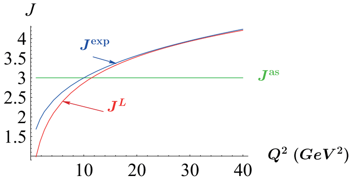


Recent BaBar data can be fitted by

$$Q^2\, F_{\gamma^*\gamma\pi^0}(Q^2) \cong \sqrt{2} f_\pi \left(\frac{Q^2}{10\,\mathrm{GeV}^2}\right)^{0.25} \equiv \frac{\sqrt{2} f_\pi}{3}\, J^{\mathrm{exp}}(Q^2)$$

$J^{\mathrm{exp}}(Q^2)$ does not flatten to a particular value!

Logarithmic Model



$J^{\exp}(Q^2)$ is very close to logarithmic function

(Radyushkin, 2009)

$$J^L(Q^2) = \ln\left(Q^2/M^2 + 1\right)$$

for $M^2 = 0.6\,\mathrm{GeV}^2$ and if $\varphi_\pi(x) = f_\pi$ and $xQ^2 \to xQ^2 + M^2$

$$J^L(Q^2) = Q^2 \int_0^1 \frac{dx}{xQ^2 + M^2}$$

M is usually treated as average intrinsic transverse momentum.

Spectral Density: "Sudakov" Transverse Momentum

Sudakov parametrization
of momentum integration

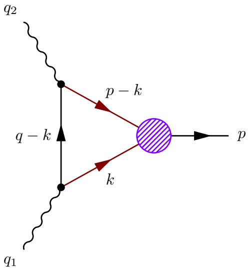
$$q_1^2 = 0; \quad q_2^2 = -Q^2$$

$$q_1 = q; \quad p = P + \frac{s}{\sigma} q;$$

$$q_2 = q_1 - p = q\left(1 - \frac{s}{\sigma}\right) - P$$

$$\sigma = 2Pq = Q^2 + s$$

$$k = xp + yq_1 + k_\perp$$



$$\begin{aligned} T &= \int \frac{dx\,dy\,dk_\perp^2}{(k^2 - i\varepsilon)\left((p-k)^2 - i\varepsilon\right)\left((q-k)^2 - i\varepsilon\right)} \\ &= \int \frac{dx\,dk_\perp^2}{x\bar{x}\sigma} \frac{1}{s - \frac{k_\perp^2}{x\bar{x}}} \frac{1}{xQ^2 + \frac{k_\perp^2}{\bar{x}}} = \int \frac{dx\,ds\,dk_\perp^2}{x\bar{x}\sigma} \delta\left(s - \frac{k_\perp^2}{x\bar{x}}\right) \frac{1}{xQ^2 + \frac{k_\perp^2}{\bar{x}}} \\ &= \int \frac{dx\,ds\,dk_\perp^2}{x\bar{x}\sigma} \delta\left(s - \frac{k_\perp^2}{x(1-x)}\right) \frac{1}{xQ^2 + \frac{k_\perp^2}{\bar{x}}} = \int dx \int \frac{dk_\perp^2}{x\bar{x}\sigma} \frac{\Psi(x, k_\perp)}{xQ^2 + \frac{k_\perp^2}{(1-x)}} \end{aligned}$$

Wave Function: "Sudakov" Transverse Momentum

$$F(Q^2) \sim \int_0^1 dx \int d^2k_\perp \frac{\Psi(x, k_\perp)}{xQ^2 + k_\perp^2/(1-x)}$$

or with,

$$\Psi(\kappa) = e^{-\frac{\kappa^2}{\lambda^2}}; \quad \kappa^2 = \frac{k_\perp^2}{x(1-x)} \Rightarrow$$

$$F\left(Q^2\right) \sim \int dx \int d\kappa^2 \frac{e^{-\frac{\kappa^2}{\lambda^2}}}{x\left(Q^2+\kappa^2\right)}$$

- Ψ-functions depending on $k_\perp$ through $k_\perp^2/x(1-x)/\sigma$ give $k_\perp^2(x) \sim x(1-x)\sigma$ and $1/x$ singularity remains.

Spectral Density: Light-Front variables

$$q_1 = P \Rightarrow q_1^2 = P^2 = 0; \quad q_2 = n(Q^2 + s) + q_\perp$$
$$k = (1-x)P + \alpha n + k_\perp; \quad q_2^2 = -q_\perp^2 = -Q^2$$

$$\int \frac{d^4 k}{(k^2 - i\varepsilon)\left((q_1 - k)^2 - i\varepsilon\right)\left((p-k)^2 - i\varepsilon\right)} \to \int \frac{ds}{s - p^2}\rho(s)$$

$$\rho q_\perp^\alpha = \int \frac{dx}{\bar{x}} \frac{k_\perp^\alpha \, d^2 k_\perp}{k_\perp^2 - i\varepsilon} \delta\left(s - \frac{(k_\perp - x q_\perp)^2}{x\bar{x}}\right)$$

Moving it $k_\perp \to k_\perp + x q_\perp \Rightarrow$

$$= \int \frac{dx}{\bar{x}} \frac{(x q_\perp^\alpha + k_\perp^\alpha)\, d^2 k_\perp}{(x q_\perp + k_\perp)^2 - i\varepsilon} \delta\left(s - \frac{k_\perp^2}{x\bar{x}}\right)$$

$$(\epsilon_\perp \times q_\perp) F_{\gamma^* \gamma \pi^0}^{\bar{q}q}(Q^2) \sim \int_0^1 dx \int \frac{(\epsilon_\perp \times (x q_\perp + k_\perp))}{(x q_\perp + k_\perp)^2 - i\epsilon} \Psi(x, k_\perp)\, d^2 k_\perp$$

(Lepage & Brodsky, 1980)

Light-Front Formula and Gaussian Model

Simplifies for wave functions of $\Psi(x, k_\perp) = \psi(x, k_\perp^2)$ type

$$F_{\gamma^*\gamma\pi^0}^{\bar{q}q}(Q^2) = \frac{1}{2\pi^2\sqrt{3}} \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ} \psi(x, k_\perp^2)\, k_\perp dk_\perp$$

(I.M. & A.R. 1997)

Gaussian ansatz for $k_\perp$-dependence (BHL 1984, JKR 1996)

$$\Psi^G(x, k_\perp) = \frac{4\pi^2}{x\bar{x}\sigma\sqrt{6}}\, \varphi_\pi(x)\, \exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right)$$

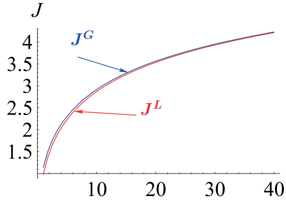
Result for form factor

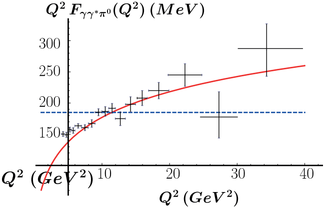
$$F_{\gamma^*\gamma\pi^0}^{G}(Q^2) = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right] dx$$

- No divergence for $x \to 0$
- Integrand is finite for $Q^2 = 0$

Properties of Gaussian Model

In fact, $J^L(Q^2, M^2 = 0.6\,\mathrm{GeV}^2)$ and $J^G(Q^2, \sigma = 0.53\,\mathrm{GeV}^2)$ practically coincide for $Q^2 > 1$ GeV^2





Average transverse momentum for Gaussian model:

$$\langle k_\perp^2 \rangle = \frac{\sigma}{3} = (0.42\,\mathrm{GeV})^2$$

$\sqrt{\langle k_\perp^2 \rangle}$ is close to folklore value of 300 MeV

Evolving Flat DA?

Light-front formula for $F_{\gamma\gamma^*\pi^0}(Q^2)$ may be written as

$$F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{2\pi^2\sqrt{3}}\int_0^1 \frac{dx}{xQ^2}\varphi(x, \mu = xQ)$$

Integral over x is dominated by $x \sim M^2/Q^2$.

- Hence $\mu \sim M^2/Q$: for large Q it corresponds to distances larger than pion radius $R \sim 1/M$.
- Evolution of pion DA should be stopped at $\mu \sim M \sim \Lambda_{\text{QCD}}$.

pQCD one-loop corrections

At one loop in perturbative QCD

$$\int_0^1 dx\,\frac{\varphi_\pi(x)}{xQ^2} \to \int_0^1 dx\,\frac{\varphi_\pi(x,\mu)}{xQ^2}\left\{1 + C_F\frac{\alpha_s}{2\pi}\left[\frac{1}{2}\ln^2 x - \frac{x\ln x}{2(1-x)} - \frac{9}{2} + \left(\frac{3}{2}+\ln x\right)\ln\left(\frac{Q^2}{\mu^2}\right)\,\right]\right\} \equiv f_\pi\,\frac{J(Q,\mu)}{Q^2}$$

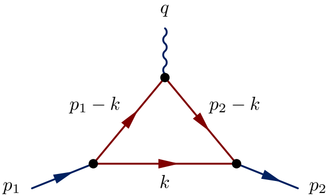
Taking regularized flat DA $\varphi_r(x) \sim f_\pi\, x^r(1-x)^r$ with small r

$$J_r(Q,\mu) = \left(\frac{1}{r}+2\right)\left\{1+\frac{\alpha_s}{3\pi}\left[\frac{2}{r^2}+\frac{\pi^2}{3}-9+\mathcal{O}(r) - \left(\frac{2}{r}-3+\frac{\pi^2}{3}\,r+\mathcal{O}(r^2)\right)\ln\left(\frac{Q^2}{\mu^2}\right)\right]\right\}$$

- To get rid of $\mathcal{O}(1/r^2)$ term one should take $\mu^2 = Q^2\,e^{-1/r}$, i.e. $\mu^2 = 10^{-4}Q^2$ for $r = 0.1$
- Very small value, even for highest BaBar point $Q^2 = 40\,\mathrm{GeV}^2$
- Need to check it in SD approach.

Drell-Yan Formula for Pion FF

Consider $T(p_1, p_2, q)$
in scalar one-loop diagram



- Infinite Momentum Frame.

Light-like vectors P and n satisfying $2(P \cdot n) = 1$. Then, p_1 and p_2 can be written as

$$\begin{aligned} p_1 &= P + p_1^2 n \\ p_2 &= P + (p_2^2 + Q^2)\, n + q_\perp \\ k &= xP + \alpha n + k_\perp \ . \end{aligned}$$

scalar part of the integral

$$\mathcal{T}_1 = \frac{1}{(2\pi)^4 i} \int \frac{d^4k}{(k^2 - i\varepsilon)((p_1 - k)^2 - i\varepsilon)((p_2 - k)^2 - i\varepsilon)}$$

Derivation of DY Formula

$$T_1 = \frac{1}{2(2\pi)^3} \int d^2k_\perp \int_0^1 \frac{dx}{x(1-x)^2} \frac{1}{p_1^2 - \frac{k_\perp^2}{x(1-x)}} \frac{1}{p_2^2 - \frac{(k_\perp - xq_\perp)^2}{x(1-x)}}$$

double dispersion relation:

$$\mathcal{T}_1 = \frac{1}{\pi^2} \int_0^\infty \int_0^\infty \frac{ds_1}{p_1^2 - s_1} \frac{ds_2}{p_2^2 - s_2} \rho_1(s_1, s_2, Q^2)$$

$$\begin{aligned} \rho_1(s_1, s_2, Q^2 = q_\perp^2) &= \frac{1}{16\pi} \int_0^1 \frac{dx}{x(1-x)^2} \\ &\times \int d^2k_\perp \delta\left(s_1 - \frac{k_\perp^2}{x(1-x)}\right) \delta\left(s_2 - \frac{(k_\perp - xq_\perp)^2}{x(1-x)}\right) . \end{aligned}$$

$$F(Q^2) \sim \int_0^1 dx \int d^2k_\perp \Psi^*(x, k_\perp - xq_\perp)\Psi(x, k_\perp)$$

Conclusions

- Discussed BaBar data and its explanation within flat DA scenario.
- Derived "WF Formula " in Sudakov Parametrization and was shown that $1/x$ divergence is not eliminated.
- Derived "WF Formula " in Light-Front formalism.
- In Light-Front formalism $1/x$ divergence is removed.
- Established link between covariant $4D$ formalism and two versions of $(x, k_\perp)$ description: Sudakov & Light-Front.
- Work in progress: α_s corrections in Spectral Density formalism.
- Flat WF is ideologically close to point-like pion vertex proposed in pion production model by Gary Goldstein and Simonetta Liuti.

Thank You!

and

Congratulations on your birthday, dear Gary!