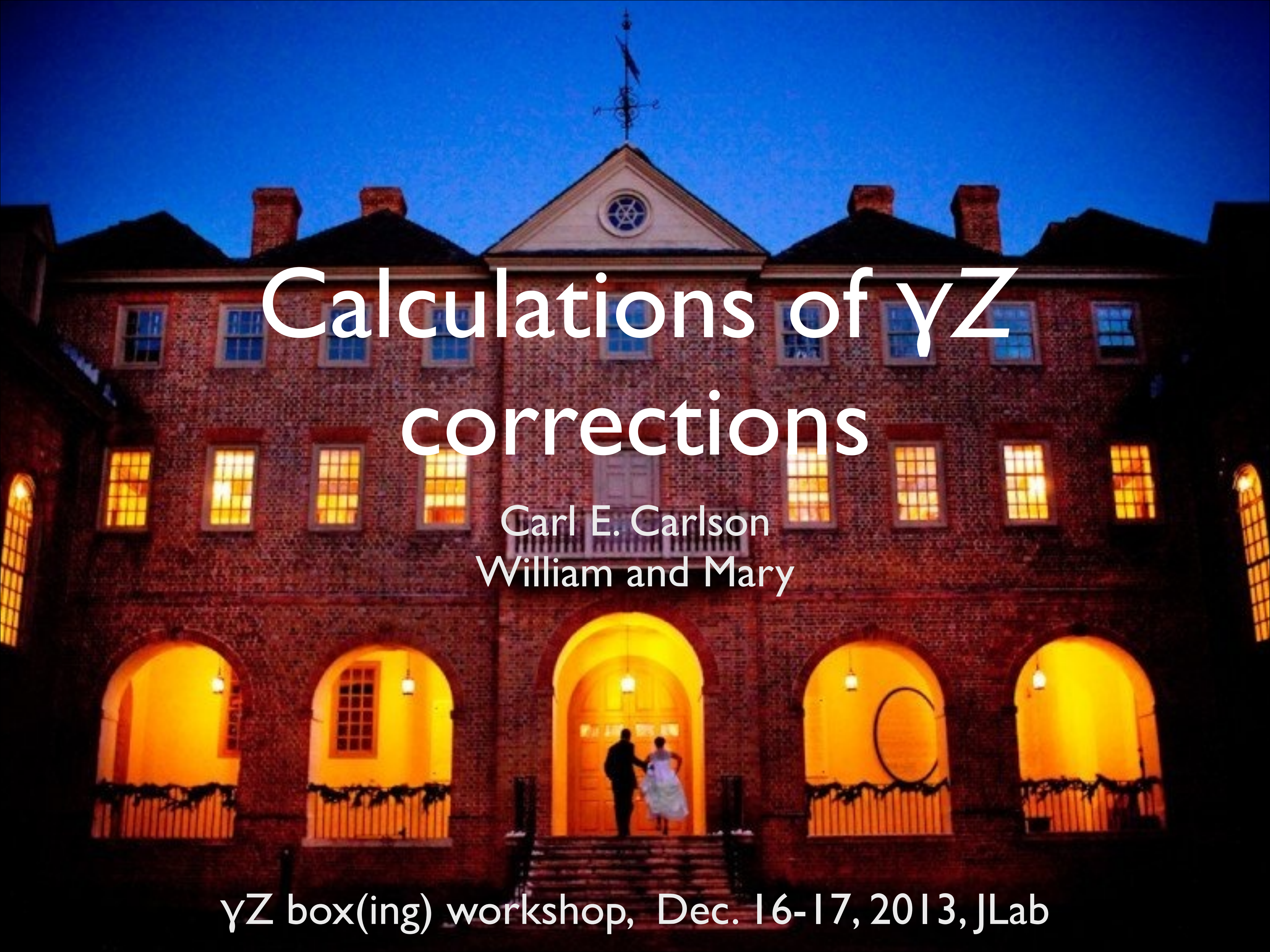


Calculations of γZ corrections

Carl E. Carlson
William and Mary

γZ box(ing) workshop, Dec. 16-17, 2013, JLab

Our relevant papers

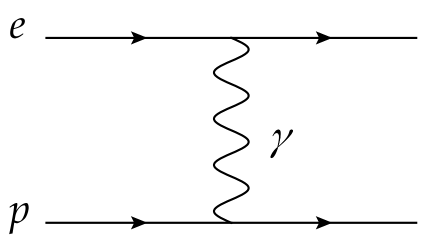
- "Contributions from γZ box diagrams to parity violating elastic ep scattering," Rislow & Carlson, Phys.Rev. D83 (2011) 113007
- "Resonance Region Structure Functions and Parity Violating Deep Inelastic Scattering," Carlson & Rislow, Phys.Rev. D85 (2012) 073002
- "Modification of electromagnetic structure functions for the γZ-box diagram," Rislow & Carlson, Phys.Rev. D88 (2013) 013018

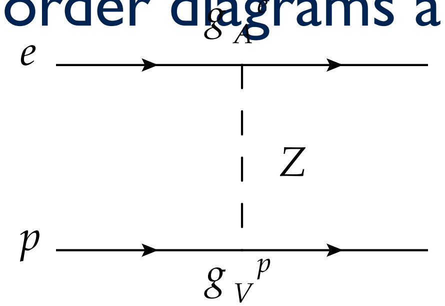
Weak Charge of the Proton

- QwP extracted from parity-violating, ep scattering:

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- Lowest order diagrams and result:



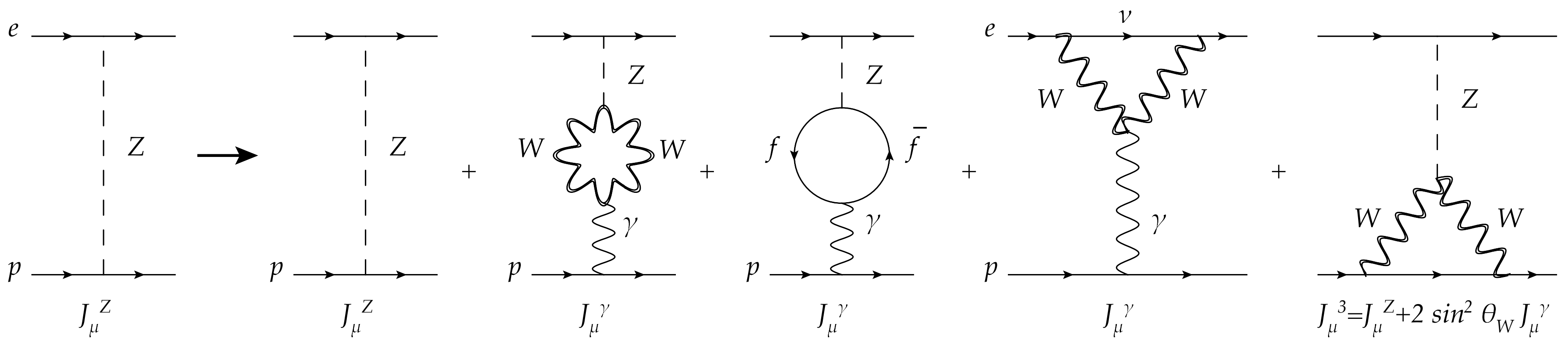


$$A_{PV} = \frac{G_F}{4\pi\alpha\sqrt{2}} Q^2 \, Q_W^p$$

- Lowest order definition of QwP :

$$Q_W^{p,LO} = -4 g_A^e g_V^p = 1 - 4\sin^2\theta_W$$

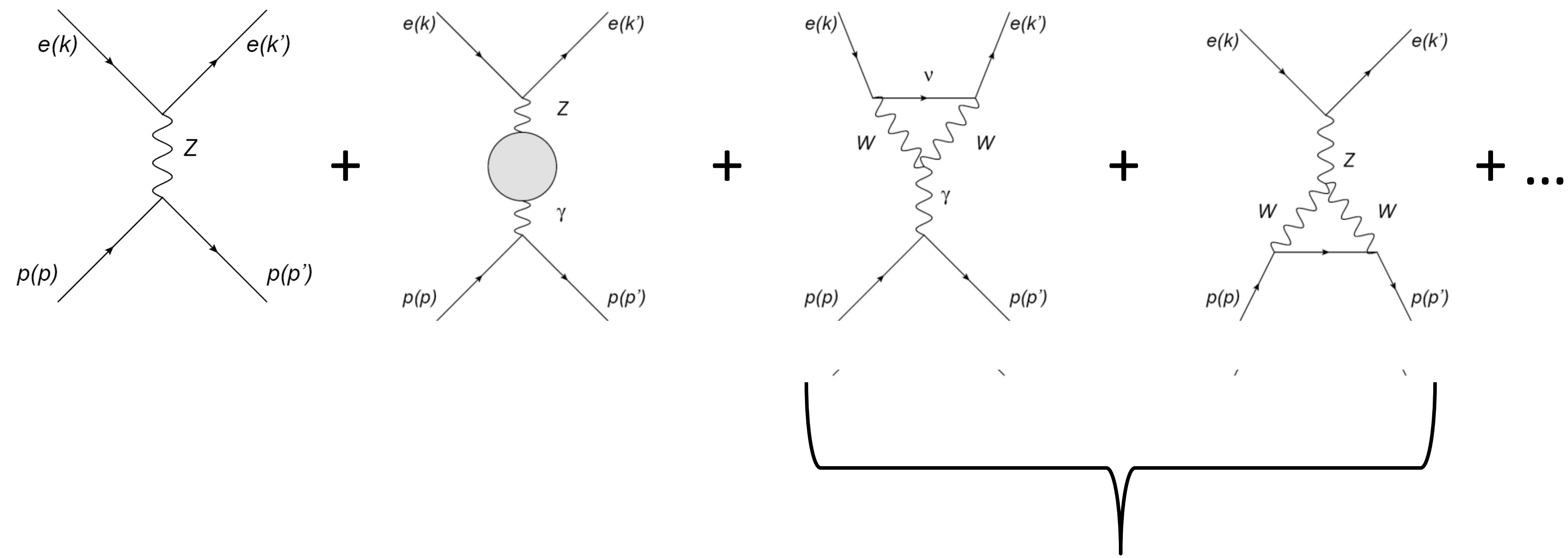
One-loop result



$$Q_W^p = (1 + \Delta\rho + \Delta_e)\left(1 - 4\sin^2\theta_W + \Delta'_e\right) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

Running Weinberg Angle (Czarnecki and Marciano)

$$\sin^2\theta_W \to \sin^2\theta_W(Q^2) = \kappa(Q^2)\sin^2\theta_W(M_Z^2)$$

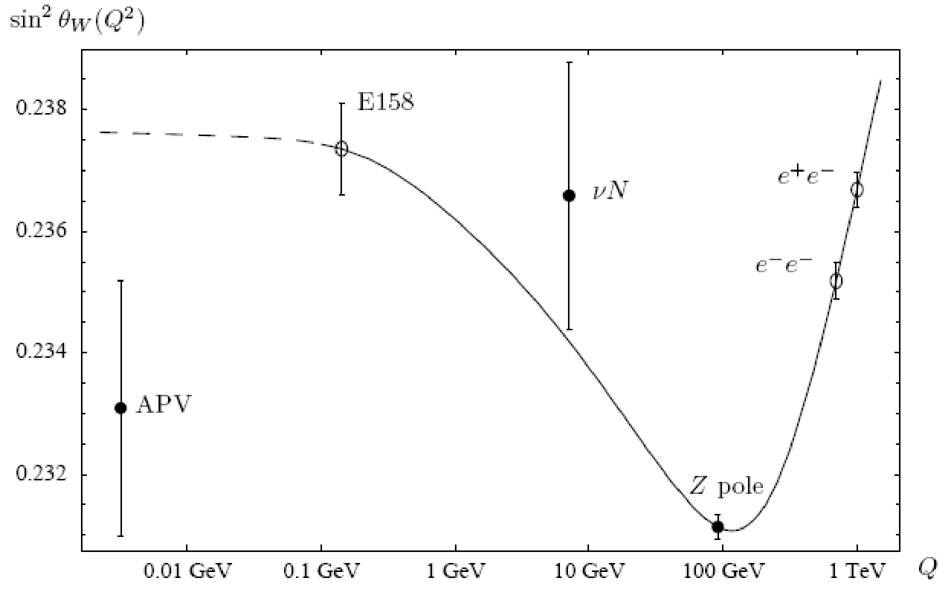


Only "Pinched" Part

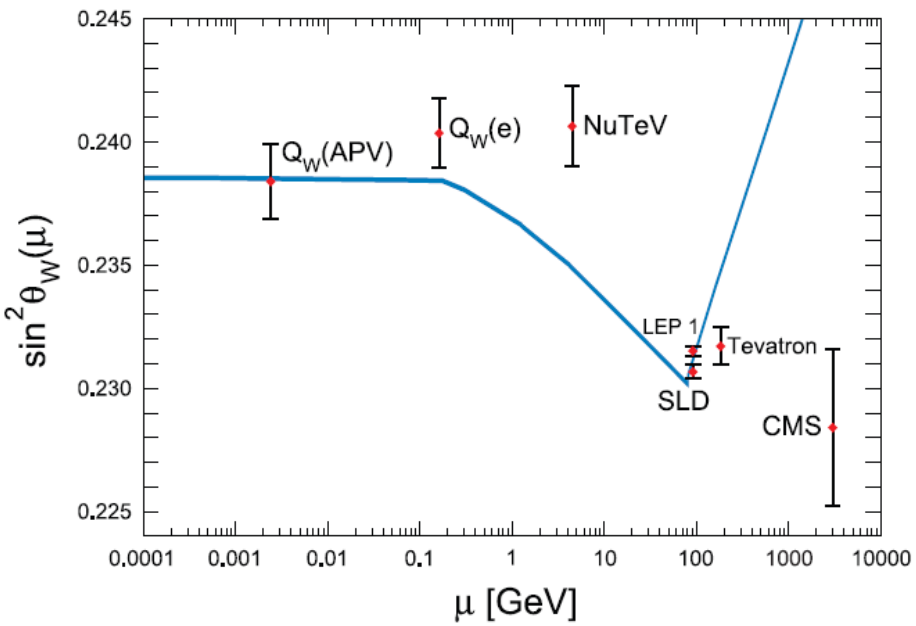
Degrassi and Sirlin, PRD 46, 3104 (1992)

Running Weinberg Angle

Czarnecki and Marciano Running
Int. J. Mod. Phys. A15, 2365 (2000)

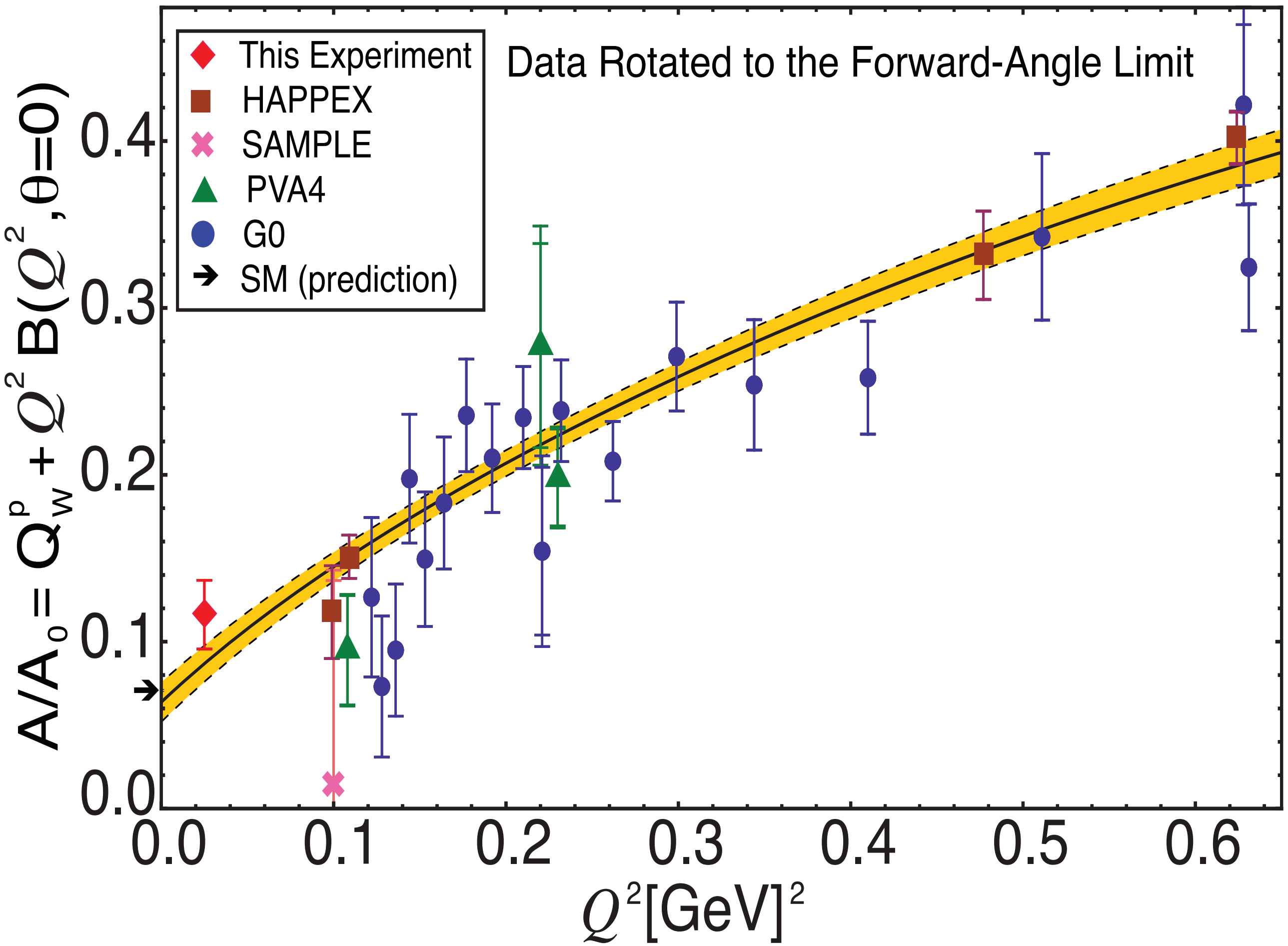


Erler and Langacker
Particle Data Group (2012)



Weak Charge Extrapolation

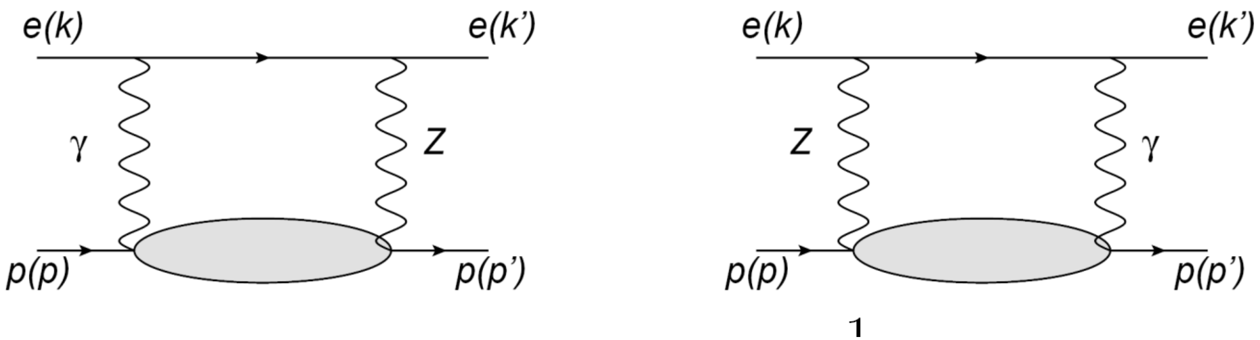
$$A_{PV}\Big|_{1\ \text{Loop}} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W^p + B_4 Q^2 + \ldots\right)$$



γZ Box

- Definition:

$$\Box_{\gamma Z} = \frac{\mathcal{M}_{\gamma Z}|_{\lambda=-1/2} - \mathcal{M}_{\gamma Z}|_{\lambda=1/2}}{\mathcal{M}_{Z}|_{\lambda=-1/2} - \mathcal{M}_{Z}|_{\lambda=1/2}} Q_W^{p,LO}$$

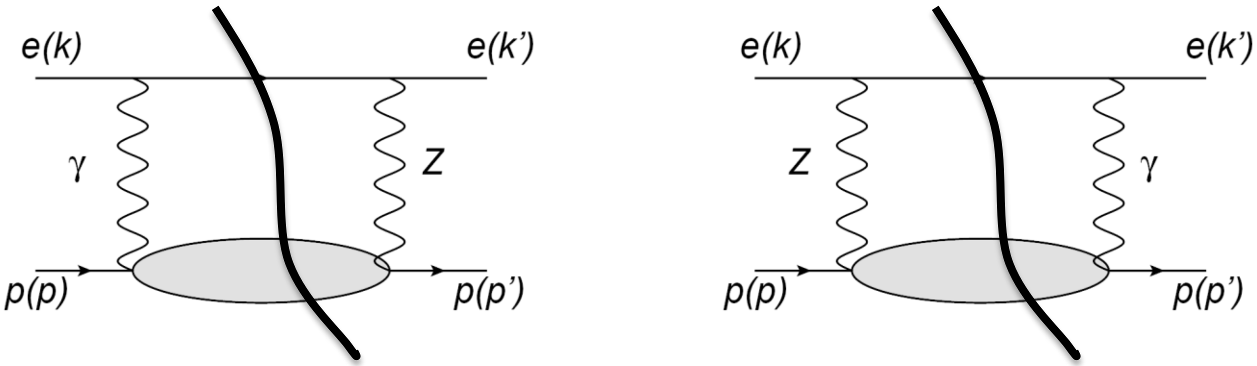


- For WW or ZZ boxes, two heavy propagators enough to ensure contributing momenta are big. Calculate w/pQCD. Here, one heavy propagator not enough. Low momenta in loop, perturbative calculation unreliable.:

$$< 0|A^\mu(x) A^\nu(y)|0 > \propto \frac{1}{q^2} \qquad < 0|Z^\mu(x) Z^\nu(y)|0 > \propto \frac{1}{q^2 - M_Z^2}$$

γZ Box

- Gorchtein and Horowitz (PRL 102, 091806 (2009)) had insight to calculate the amplitude dispersively



- Optical theorem,

$$\mathrm{Im}\,\mathcal{M}_{aa} = \frac{1}{2}\sum_b (2\pi)^4 \delta^{(4)}(p_a - p_b)\mathcal{M}_{ab}\mathcal{M}_{ba}$$

γZ Box equations

- Imaginary part

$$\mathrm{Im}\,\Box_{\gamma Z}(E_{Lab}) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \left\{ \frac{F_1^{\gamma Z}(x,Q^2) + AF_2^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} + \frac{g_V^e}{g_A^e}\frac{BF_3^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right\}$$

$$= \mathrm{Im}\,\Box^V_{\gamma Z}(E_{Lab}) + \mathrm{Im}\,\Box^A_{\gamma Z}(E_{Lab}).$$

- Dispersion relations,

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E_{Lab}) = \frac{2E_{Lab}}{\pi}\int_{\nu_\pi}^{\infty} \frac{dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^V_{\gamma Z}(E'_{Lab})$$

$$\mathrm{Re}\,\Box^A_{\gamma Z}(E_{Lab}) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} \frac{E'_{Lab}dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^A_{\gamma Z}(E'_{Lab})$$

γZ Box equations

- Imaginary part

$$\mathrm{Im}\,\Box_{\gamma Z}(E_{Lab}) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \left\{ \frac{F_1^{\gamma Z}(x,Q^2) + A F_2^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} + \frac{g_V^e}{g_A^e}\frac{B F_3^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right\}$$

$$= \mathrm{Im}\,\Box^V_{\gamma Z}(E_{Lab}) + \mathrm{Im}\,\Box^A_{\gamma Z}(E_{Lab}).$$

Structure Functions must be modeled.

- Dispersion relations,

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E_{Lab}) = \frac{2E_{Lab}}{\pi}\int_{\nu_\pi}^{\infty} \frac{dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^V_{\gamma Z}(E'_{Lab})$$

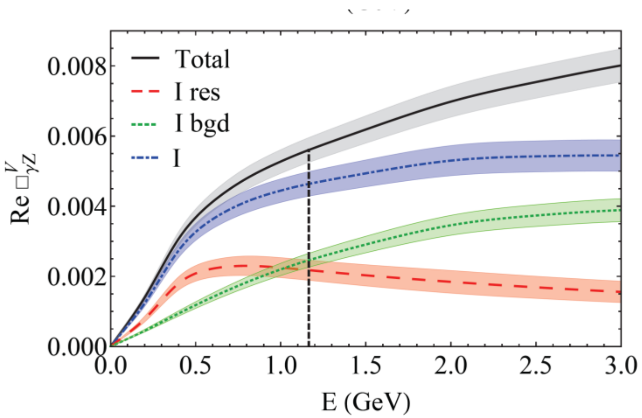
Labels proton current.

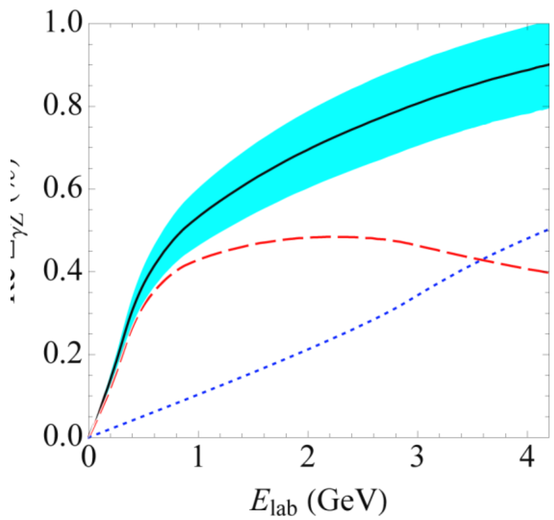
$$\mathrm{Re}\,\Box^A_{\gamma Z}(E_{Lab}) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} \frac{E'_{Lab}dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^A_{\gamma Z}(E'_{Lab})$$

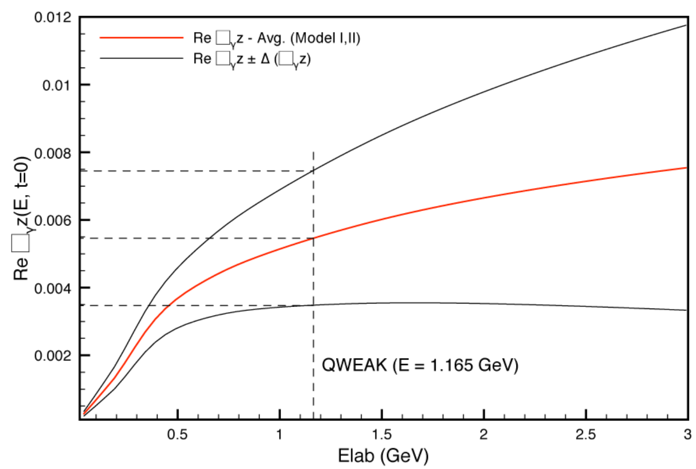
Vector Boxes

Vector box plots

Hall *et al.*	Carlson and Rislow	Gorchtein *et al.*
PRD 88, 013011 (2013)	PRD 83, 113007 (2011)	PRC 84, 015502 (2011)







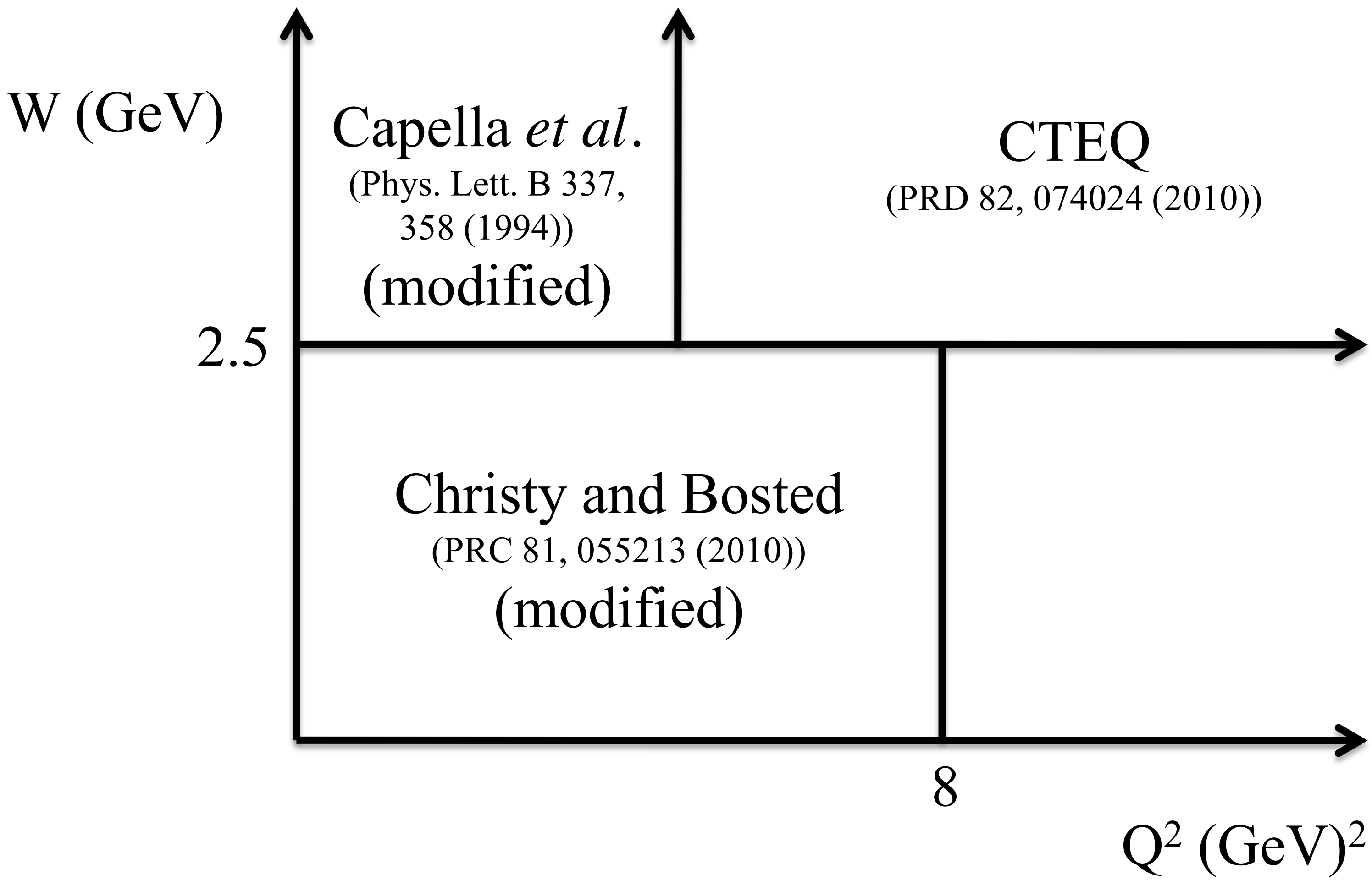
$$\mathrm{Re}\,\Box^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$$(5.6 \pm 0.36) \times 10^{-3} \quad (5.7 \pm 0.9) \times 10^{-3} \quad (5.4 \pm 2.0) \times 10^{-3}$$

- Differences come from the treatment of the structure functions

Us

- We divided up the energy regions and modified the structure functions.



Evaluation in scaling region

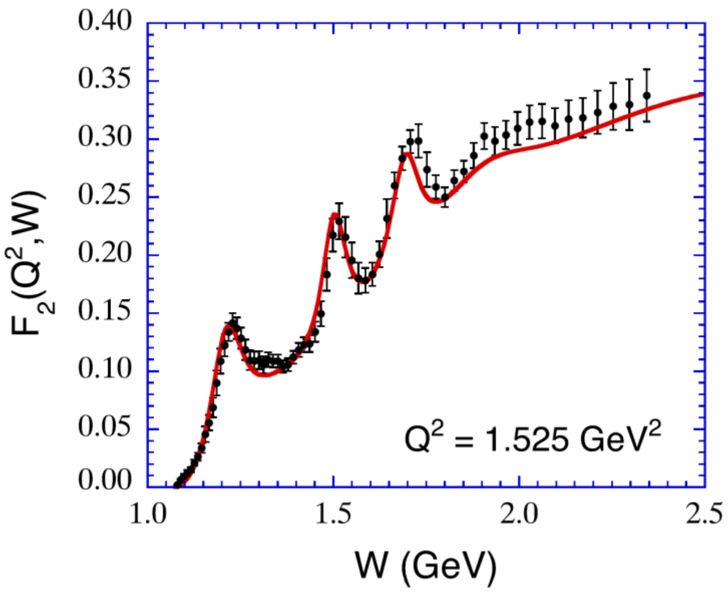
- Calculated directly using PDFs

$$F_2^{\gamma Z}(x, Q^2) = 2xF_1^{\gamma Z}(x, Q^2) = x \sum_{q,\bar{q}} 2e_q g_V^q (q(x, Q^2) + \bar{q}(x, Q^2))$$

- We use CTEQ
- Alternative
  - Hall et al. use ABM11 (PRD 86, 054009 (2012))

Evaluation in resonance region

- All later calculations modify Christy-Bosted electromagnetic fits. (May also use MAID.)
- CB fit have 7 resonances and a smooth background



- Resonances modified by corrective ratio:

$$F_1^{\gamma Z} = \sum_{res} C_{res} \times F_1^{\gamma\gamma}\big|_{res} \quad ; \qquad C_{res} = \frac{F_1^{\gamma Z}}{F_1^{\gamma\gamma}}\bigg|_{res}$$

Vector C_{res}

- Definition of structure functions:

$$F_1^{\gamma\gamma(\gamma Z)}\Big|_{N\to res} = \varepsilon_+^{\mu *}\varepsilon_+^{\nu} W_{\mu\nu}^{\gamma\gamma(\gamma Z)}$$
$$= (2)\sum_\lambda \int \mathrm{d}^4 z e^{iqz} \langle N,s \big| \varepsilon_+^* \cdot J^{\gamma(Z,V)\dagger}(z) \big| res,\lambda \rangle$$
$$\times \langle res,\lambda \big| \varepsilon_+ \cdot J^\gamma(0) \big| N,s \rangle$$

- C_{res} in terms of helicity amplitudes:

$$C_{res} = \frac{2\sum_\lambda A_\lambda^\gamma A_\lambda^Z}{\sum_\lambda (A_\lambda^\gamma)^2}$$

Vector C_{res}

- We constructed helicity amplitudes using SU(6) wave functions:

$$\langle res, \lambda | \epsilon_+ \cdot J^{\gamma(Z,V)} | N, s \rangle = 3 \times e_q^{(3)} \left(g_V^{q(3)} \right)$$
$$\times \langle \psi_{res} \phi_{res} \chi_\lambda | \underbrace{\bar{u}(k', \lambda') \epsilon_+ \cdot \gamma u(k, s')}_{= AL_+ + BS_+} | \psi_N \phi_N \chi_s \rangle$$

- Phenomenological constraints used to fit A and B:

$$\frac{|A_{1/2}^\gamma|^2 - |A_{3/2}^\gamma|^2}{|A_{1/2}^\gamma|^2 + |A_{3/2}^\gamma|^2} = \begin{cases} -1 \text{ for } Q^2 = 0 \\ +1 \text{ for high } Q^2 \end{cases}$$

C_{res} for D13(1520)

- SU(6) wave function for proton:

$$|2^8, 56\rangle = \frac{1}{\sqrt{2}} \psi^S_{L=0,L_Z=0} \left(\phi^{M,S} \chi^{M,S}_{S_Z=\pm 1/2} + \phi^{M,A} \chi^{M,A}_{S_Z=\pm 1/2} \right)$$

Spatial Flavor Spin

- Helicity amplitudes

$$A^{\gamma(Z)}_{\lambda=1/2} = 3 \times e_q^{(3)} (g_V^{q(3)}) \langle \psi_{\rm res} \phi_{\rm res} \chi_{+1/2} \times |[AL_+ + BS_+]| \psi_N \phi_N \chi_s \rangle = \frac{1}{\sqrt{6}} \Big(-A_{10} [e_u (g_V^u) - e_d (g_V^d)] - \sqrt{2} B_{10} \Big[\frac{5}{3} e_u (g_V^u) + \frac{1}{3} e_d g_V^d) \Big] \Big)$$

$$A^{\gamma(Z)}_{\lambda=3/2} = 3 \times e_q^{(3)} (g_V^{q(3)}) \langle \psi_{\rm res} \phi_{\rm res} \chi_{+3/2} \times |[AL_+ + BS_+]| \psi_N \phi_N \chi_s \rangle = -\frac{1}{\sqrt{2}} A_{10} [e_u (g_V^u) - e_d (g_V^d)].$$

C_{res} for $D_{13}(1520)$

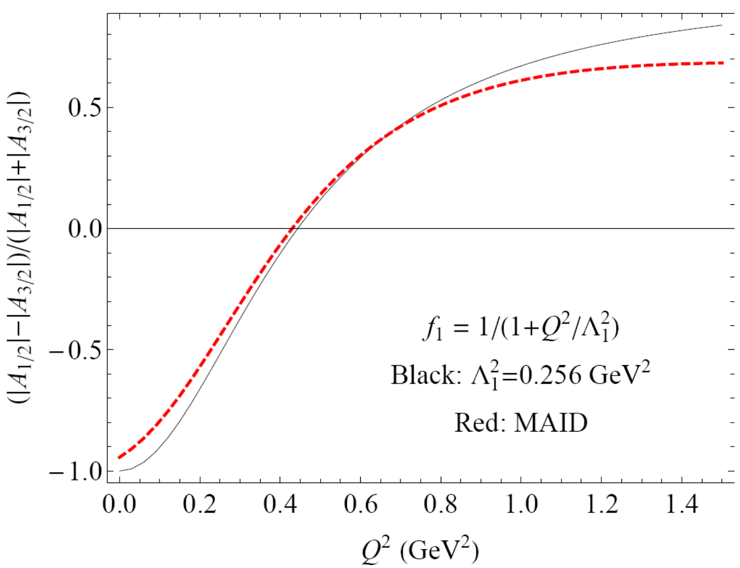
- *A* and *B* relations for $D_{13}(1520)$:

$$A_{10}(Q^2=0) = -\sqrt{2}B_{10}(Q^2=0)$$

$$\Downarrow$$

$$\frac{A_{10}(Q^2)}{B_{10}(Q^2)} = -\sqrt{2}f_1(Q^2) = -\sqrt{2}\frac{1}{1+Q^2/\Lambda_1^2}$$

- Λ found by comparing amplitudes to MAID:



Our vector C_{res}

resonance	proton electroproduction amplitudes	C^p_{res}	C^d_{res}
$P_{33}(1232)$	$A^{\gamma}_{1/2} \propto (e_u - e_d)$	$1 + Q_W^{p,LO}$	$1 + Q_W^{p,LO}$
$S_{11}(1535)$	$A^{\gamma}_{1/2} = \frac{1}{\sqrt{6}}\left(\sqrt{2}A_{10}(e_u - e_d) - B_{10}\left(\frac{5}{3}e_u + \frac{1}{3}e_d\right)\right)$	$\frac{1/3+2f_1}{1+2f_1} + Q_W^{p,LO}$	$2\frac{(1+2f_1)(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2} + Q_W^{p,LO}$
$D_{13}(1520)$	$A^{\gamma}_{1/2} = \frac{1}{\sqrt{6}}\left(A_{10}(e_u - e_d) + \sqrt{2}B_{10}\left(\frac{5}{3}e_u + \frac{1}{3}e_d\right)\right)$ $A^{\gamma}_{3/2} = \frac{1}{\sqrt{2}}A_{10}(e_u - e_d)$	$\frac{(1-f_1)(1/3-f_1)+3f_1^2}{(1-f_1)^2+3f_1^2} + Q_W^{p,LO}$	$\frac{2(1-f_1)(1/3-f_1)+6f_1^2}{(1-f_1)^2+(1/3-f_1)^2+6f_1^2} + Q_W^{p,LO}$
$F_{15}(1680)$	$A^{\gamma}_{1/2} = \sqrt{\frac{2}{5}}A_{20}(2e_u + e_d) + \sqrt{\frac{3}{5}}B_{20}\left(\frac{4}{3}e_u - \frac{1}{3}e_d\right)$ $A^{\gamma}_{3/2} = \frac{2}{\sqrt{5}}A_{20}(2e_u + e_d)$	$\frac{2/3(1-f_2)}{(1-f_2)^2+2f_2^2} + Q_W^{p,LO}$	$4\frac{1-f_2}{3(1-f_2)^2+6f_2^2+4/3} + Q_W^{p,LO}$
$S_{11}(1650)$	$A^{\gamma}_{1/2} = -\sqrt{\frac{2}{27}}B_{10}(e_u + 2e_d)$	$\frac{1/3+2f_1}{1+2f_1} + Q_W^{p,LO}$	$2\frac{(1+2f_1)(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2} + Q_W^{p,LO}$
$P_{11}(1440)$	$A^{\gamma}_{1/2} = B_{00}\left(\frac{4}{3}e_u - \frac{1}{3}e_d\right)$	$2/3 + Q_W^{p,LO}$	$12/13 + Q_W^{p,LO}$
$F_{37}(1950)$	$A^{\gamma}_{1/2} \propto (e_u - e_d)$	$1 + Q_W^{p,LO}$	$1 + Q_W^{p,LO}$
Background		$\frac{5}{6} + Q_W^{p,LO}$	$\frac{9}{10} + Q_W^{p,LO}$

R & C Background Correction

- In limit where all light quarks (u, d, s) are equally likely:

$$\frac{F_2^{\gamma Z}}{F_2^{\gamma\gamma}} = \frac{\sum_{q=u,d,s} 2e_q g_V^q x f(x)}{\sum_{q=u,d,s} (e_q)^2 x f(x)} = 1 + Q_W^{p,LO}$$

- In valence quark limit (d and 2 u's):

$$\frac{F_2^{\gamma Z}}{F_2^{\gamma\gamma}} = \frac{\sum_{q=d,u,u} 2e_q g_V^q x f(x)}{\sum_{q=d,u,u} (e_q)^2 x f(x)} = \frac{2}{3} + Q_W^{p,LO}$$

- We took their average as the correction:

$$Avg = \frac{5}{6} + Q_W^{p,LO}$$

Alternative resonance analysis

- Isospin rotate neutron amplitudes:

$$\langle N_n^*|J_\mu^\gamma|n\rangle = e_u\langle N_n^*|\bar{u}\gamma_\mu u|n\rangle + e_d\langle N_n^*|\bar{d}\gamma_\mu d|n\rangle$$

$$\langle N_n^*|J_\mu^\gamma|n\rangle = e_u\langle N_p^*|\bar{d}\gamma_\mu d|p\rangle + e_d\langle N_p^*|\bar{u}\gamma_\mu u|p\rangle$$

- Rewrite neutral current

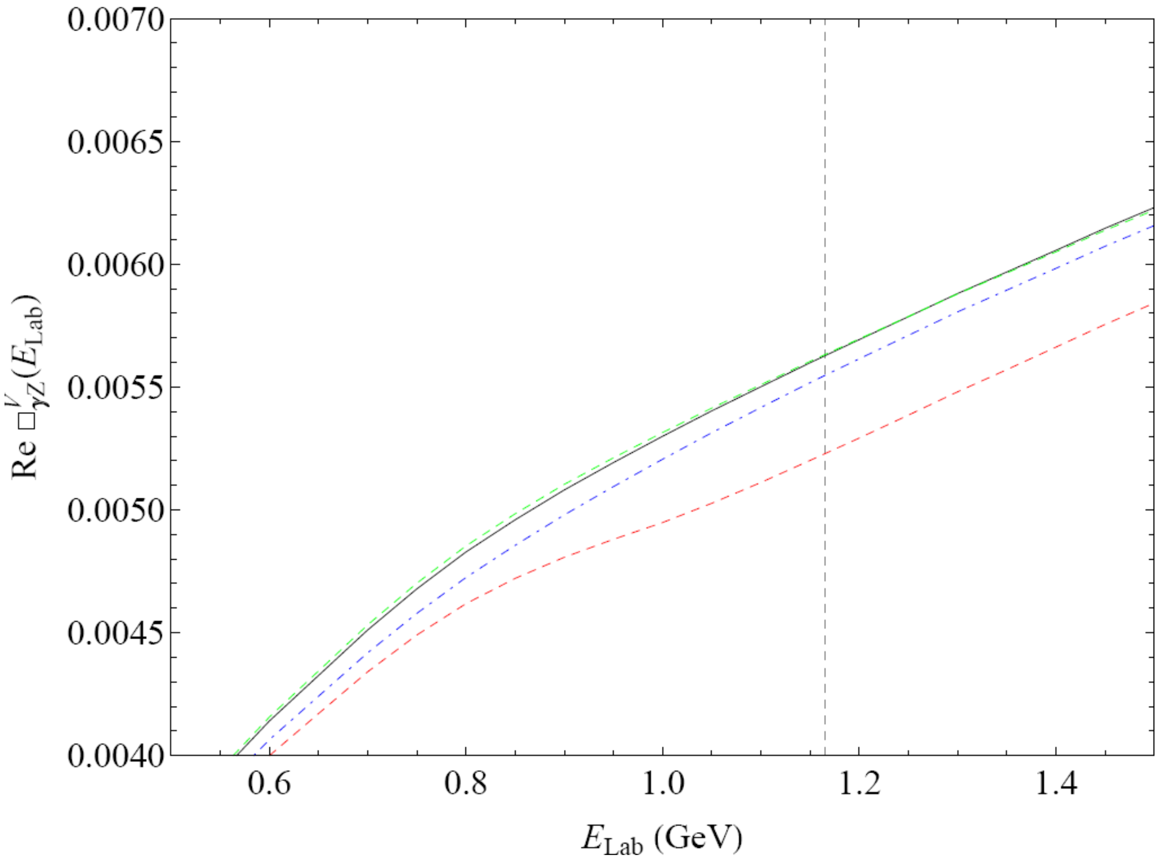
$$\langle N_p^*|J_\mu^{\gamma(Z,V)}|p\rangle = \frac{2}{3}(g_u^V)\langle N_p^*|\bar{u}\gamma_\mu u|p\rangle - \frac{1}{3}(g_d^V)\langle N_p^*|\bar{d}\gamma_\mu d|p\rangle$$

$$\langle N_p^*|J_\mu^{Z,V}|p\rangle = \frac{1}{2}(1-4\sin^2\theta_W(0))\langle N_p^*|J_\mu^\gamma|p\rangle - \frac{1}{2}\langle N_n^*|J_\mu^\gamma|n\rangle$$

$$C_{res} = 2\frac{\sum_\lambda A_\lambda^{\gamma,p} A_\lambda^{Z,p}}{\sum_\lambda (A_\lambda^{\gamma,p})^2}$$
$$= Q_W^{p,LO} - \frac{\sum_\lambda A_\lambda^{\gamma,p} A_\lambda^{\gamma,n}}{\sum_\lambda (A_\lambda^{\gamma,p})^2}$$

- C_{res} calculated using PDG photoproduction data
- GHRM used PDG data at $Q^2 = 0$, dropped relative Q^2 dependence.
- Can also use MAID to obtain neutron amplitudes, at all Q^2.

C_{res} Effect on γZ Box

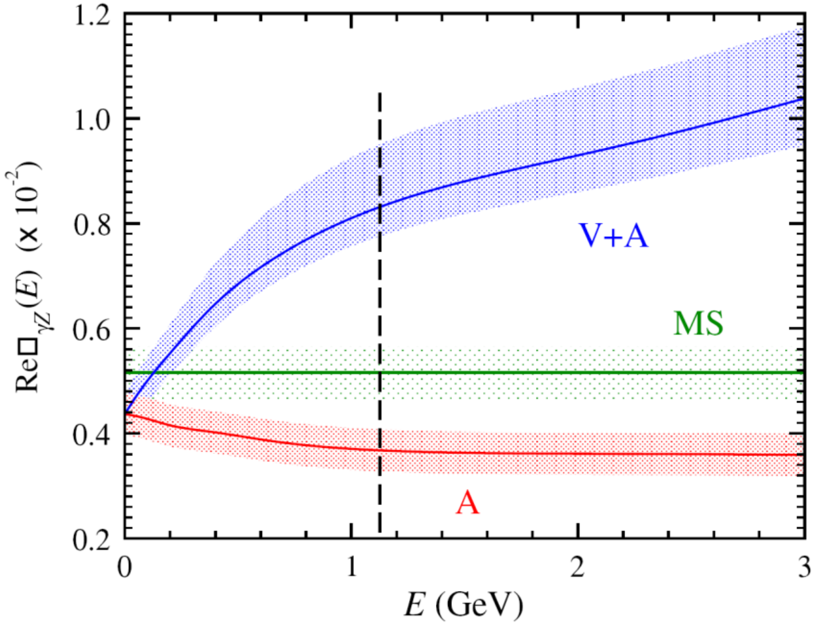


- Solid: Constituent Quark Model of Carlson and Rislow
- Blue: PDG (used by Gorchtein et al. and Hall et al.)
- Red: MAID (Eur.Phys.J.ST 198, 141 (2011))
  – Green: MAID without Roper Resonance

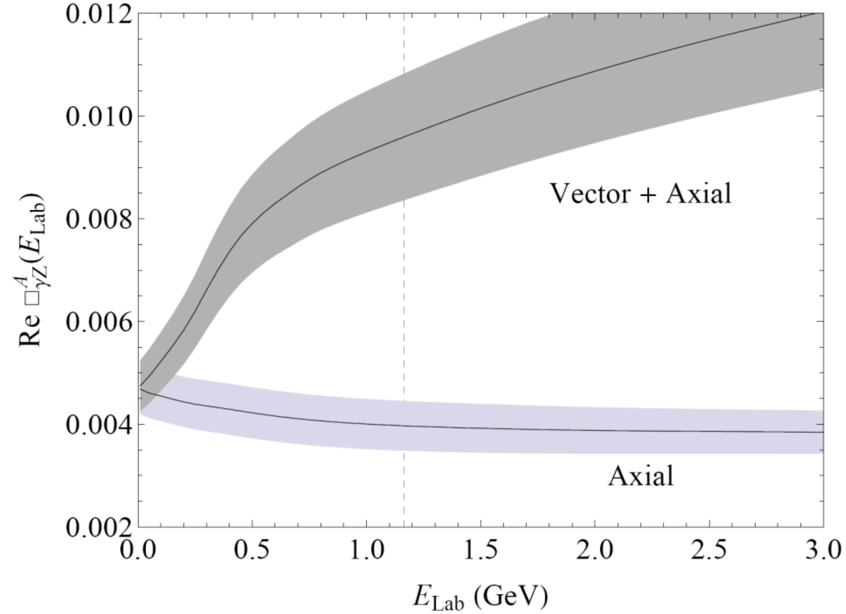
Axial Box Analyses

Axial Box Calculations

Blunden *et al.*
PRL 107, 081801 (2011)



Carlson and Rislow
PRD 88, 013018 (2013)



$$\mathrm{Re}\Box^{A}_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$(3.7 \pm 0.4) \times 10^{-3}$ $\qquad$ $(4.0 \pm 0.5) \times 10^{-3}$

Comments on $\Box_{\gamma Z}^{A}$

- The $\Box_{\gamma Z}^{V} \approx 0.005$ just discussed compares to $\Box_{\gamma Z} \approx 0.0051$ quoted on "old days". Pure coincidence. This was just for $\Box_{\gamma Z}^{A}$.

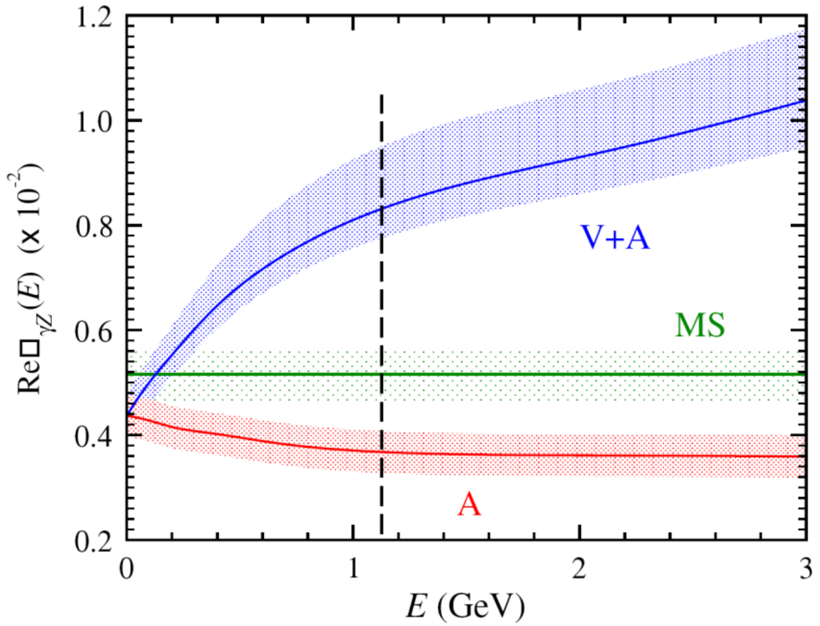
- $\Box_{\gamma Z}^{A}$ can be calculated anew. With the DR treatment there are no logs to guess arguments of.

$$\Box^{A}_{\gamma Z}(E) \quad = \quad \frac{2}{\pi} \int_{E'>0} \frac{E'\, dE'}{E'^2 - E^2} \, \mathrm{Im}\, \Box^{A}_{\gamma Z}(E')$$

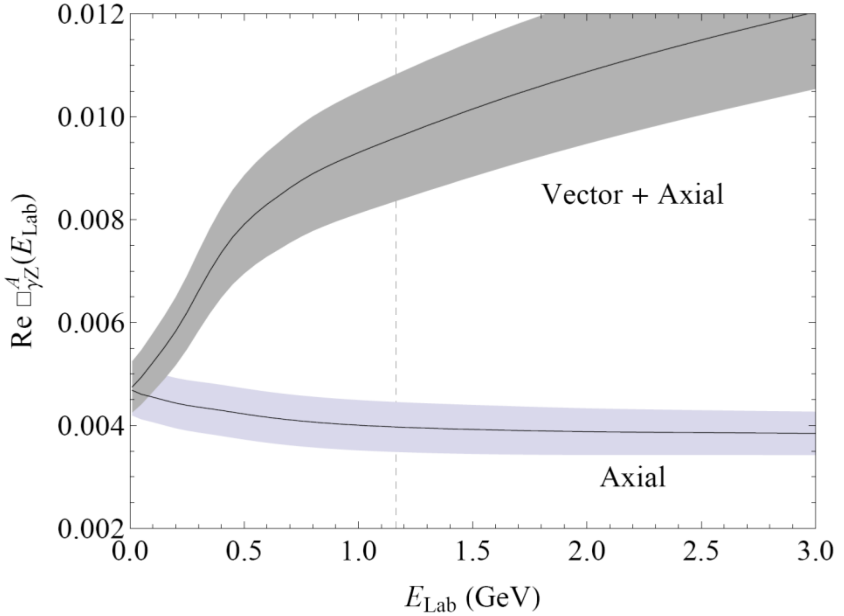
- Notes
  - Not zero at threshold
  - The E' makes high energies important
  - Most of result comes from scaling region for $F_3^{\gamma Z}$, where it can be obtained reliably

Current Axial Box Results

Blunden *et al.*
PRL 107, 081801 (2011)



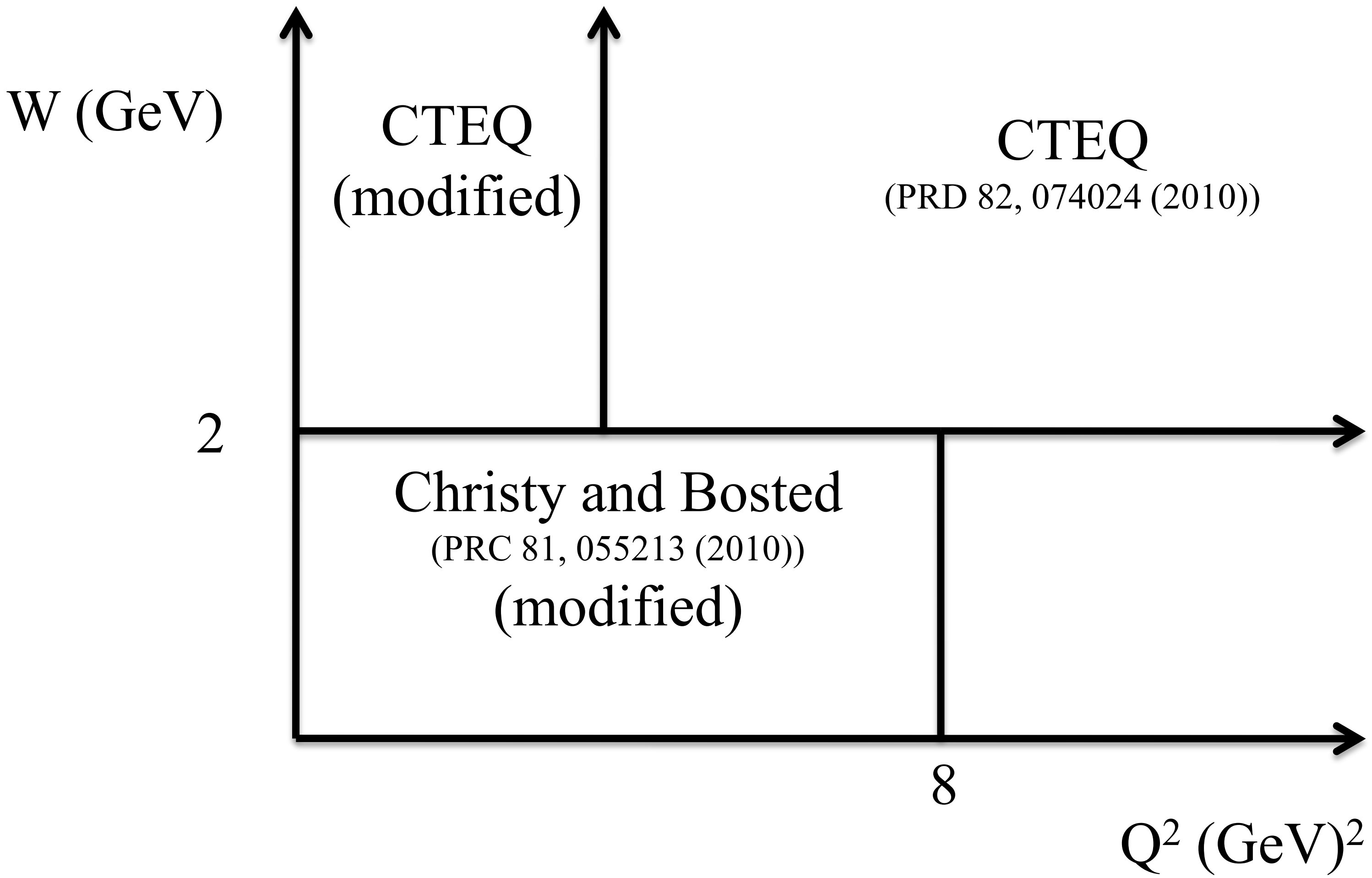
Carlson and Rislow
PRD 88, 013018 (2013)



$$\mathrm{Re}\Box^{A}_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$(3.7 \pm 0.4) \times 10^{-3}$ $\qquad$ $(4.0 \pm 0.5) \times 10^{-3}$

Same split of regions



Evaluation of Axial Structure Function

- Scaling region

$$F_3^{\gamma Z}(x,Q^2) = \sum_q 2e_q g_q^A \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

- High W, low Q^2,

$$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)\Big|_{\mathrm{CTEQ}}$$

$$Q_0^2 = 1\ \mathrm{GeV}^2$$

$$\Lambda^2 = 0.7\ \mathrm{GeV}^2$$

- Resonance Region:
- Blunden et al. use Lalakulich et al. (PRD74,014009) for F_3
  - Fewer resonances than the Christy Bosted fit: fits for $D_{13}(1520)$, $P_{11}(1440)$, $P_{33}(1232)$, and $S_{11}(1535)$
- We continue modifying the Christy Bosted fit.

$$C_{res} = \frac{F_3^{\gamma Z}}{F_1^{\gamma\gamma}}$$

$$F_3^{\gamma Z}\Big|_{N\to res} =$$

$$3\left(2g_A^{q(3)}\right)\frac{2\nu}{q_z}\langle\psi_N\phi_N\chi_s\Big|\left[\frac{2m_q}{q_z}BS_+\right]^\dagger\Big|\psi_{res}\phi_{res}\chi_\lambda\rangle$$

$$\times\, 3e_q^{(3)}\langle\psi_{res}\phi_{res}\chi_\lambda\Big|[AL_+ + BS_+]\Big|\psi_N\phi_N\chi_s\rangle,$$

Rislow & Carlson Axial C_{res}

resonance	proton axial current amplitudes	C^p_{res}	C^d_{res}
$P_{33}(1232)$	$A^{Z,A}_{1/2} \propto (g^u_A - g^d_A)\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$
$S_{11}(1535)$	$A^{Z,A}_{1/2} = -\frac{1}{\sqrt{6}}B_{10}\left(\frac{5}{3}g^u_A + \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{1}{3(2f_1+1)}\frac{16m_q\nu}{3q_z^2}$	$\frac{(1+2f_1)+(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2}\frac{16m_q\nu}{3q_z^2}$
$D_{13}(1520)$	$A^{Z,A}_{1/2} = \sqrt{\frac{2}{6}}B_{10}\left(\frac{5}{3}g^u_A + \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$ $A^{Z,A}_{3/2} = 0$	$\frac{1-f_1}{(f_1-1)^2+3f_1^2}\frac{16m_q\nu}{3q_z^2}$	$\frac{(1-f_1)-(f_1-1/3)}{(1-f_1)^2+(f_1-1/3)^2+6f_1^1}\frac{16m_q\nu}{3q_z^2}$
$F_{15}(1680)$	$A^{Z,A}_{1/2} = \sqrt{\frac{3}{5}}B_{20}\left(\frac{4}{3}g^u_A - \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$ $A^{Z,A}_{3/2} = 0$	$\frac{(1-f_2)}{(1-f_2)^2+2f_2^2}\frac{20m_q\nu}{3q_z^2}$	$\frac{(1-f_2)+2/3}{(1-f_2)^2+2f_2^2+4/9}\frac{20m_q\nu}{3q_z^2}$
$S_{11}(1650)$	$A^{\gamma}_{1/2} = -\sqrt{\frac{2}{27}}B_{10}\left(g^u_A + 2g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{1}{3(2f_1+1)}\frac{16m_q\nu}{3q_z^2}$	$\frac{(1+2f_1)+(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2}\frac{16m_q\nu}{3q_z^2}$
$P_{11}(1440)$	$A^{Z,A}_{1/2} = B_{00}\left(\frac{4}{3}g^u_A - \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{20m_q\nu}{3q_z^2}$	$\frac{100m_q\nu}{13q_z^2}$
$F_{37}(1950)$	$A^{Z,A}_{1/2} \propto (g^u_A - g^d_A \frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$
Background		$\frac{5}{3}$	$\frac{9}{5}$

R & C Background Correction

- Smooth background

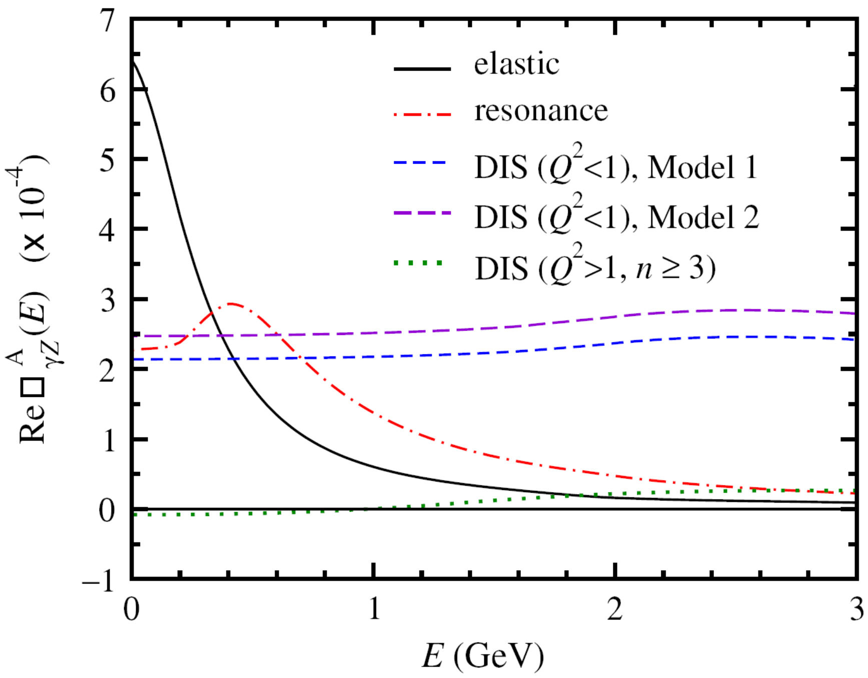
$$C_{bkgd}|_{x\to 0} = \frac{\sum_{q=u,d,s} 2e_q g_q^A f_q(x)}{\frac{1}{2}\sum_{q=u,d,s}(e_q)^2 f_q(x)} = 0$$

$$C_{bkgd}|_{\text{valence quarks}} = \frac{\sum_{q=u,u,d} 2e_q g_q^A f_q(x)}{\frac{1}{2}\sum_{q=u,u,d}(e_q)^2 f_q(x)} = \frac{10}{3}$$

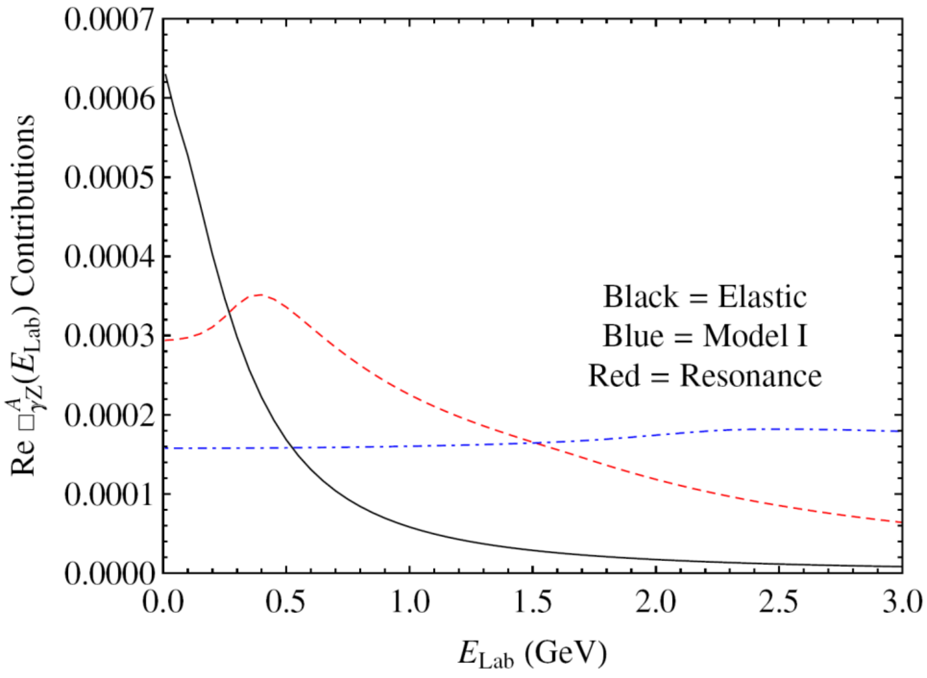
- Average 5/3

Axial Box Contributions

Blunden *et al.*
PRL 107, 081801 (2011)



Carlson and Rislow
PRD 88, 013018 (2013)



(Overall results already given)

Summary

- The world is saved (almost), regarding the γZ corr. to Q_{Weak}.
- I.e., $\Box_{\gamma Z}^{V}$ now calculated.
- About (8.1±1.4)% of Q_W^p at E_{elec}=1.165 GeV. Proportional to E_{elec}.
- $\Box_{\gamma Z}^{A}$ also now calculated w/o guesswork on logs
- About (6.3±0.6%) of Q_W^p at E_{elec} threshold. Small dependence on E_{elec}.
- For goal of 1% or better measurement of Q_{Weak} (P2 at Mesa), energy is about 1/6 of JLab experiment, and corrections and error in $\Box_{\gamma Z}^{V}$ scale with energy.
- Would like to improve $\Box_{\gamma Z}^{A}$ but believe this is very manageable

Post summary: A_{PV}

- With one-photon exchange,

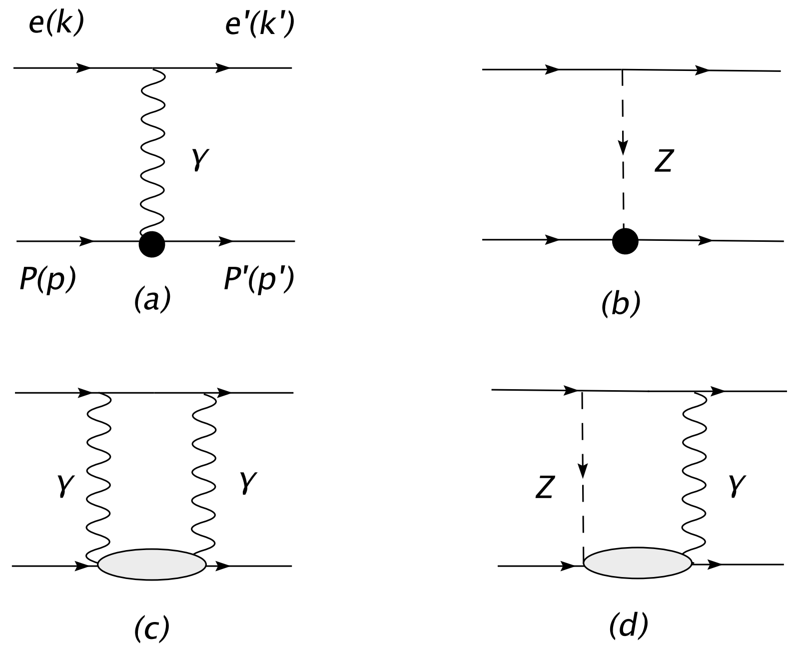
$$A_{\rm PV}^{\rm Born} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E^{\rm Born} + A_M^{\rm Born} + A_A^{\rm Born}}{[\epsilon(G_{Ep}^{\gamma})^2 + \tau(G_{Mp}^{\gamma})^2]}$$

where

$$A_E^{\rm Born} = -2g_A^e \epsilon G_{Ep}^Z G_{Ep}^{\gamma}, \qquad A_M^{\rm Born} = -2g_A^e \tau G_{Mp}^Z G_{Mp}^{\gamma}$$

$$A_A^{\rm Born} = 2g_V^e \sqrt{\tau(1+\tau)(1-\epsilon^2)} G_A^Z G_{Mp}^{\gamma}$$

May also have 2-photon exchange



- whence,

$$A_{\mathrm{PV}} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E + A_M + A_A + A'_M + A'_A}{\epsilon|G'_{Ep}|^2 + \tau|G'_{Mp}|^2 + 2\sqrt{\tau(1+\tau)(1-\epsilon^2)}G^{\gamma}_{Mp}\,\mathrm{Re}(G'_{Ap})}$$

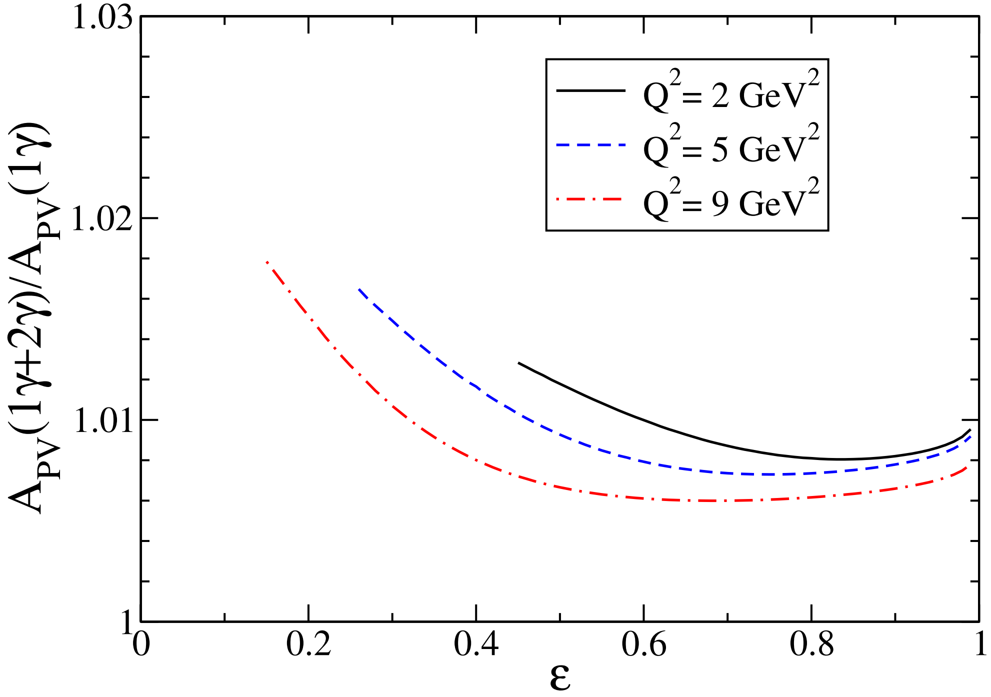
where

$$A'_A = 2g^e_V(1+\tau)G^Z_A\,\mathrm{Re}(G'_{Ap})$$

$$A'_M = -2g^e_A\sqrt{\tau(1+\tau)(1-\epsilon^2)}G^Z_M\,\mathrm{Re}(G'_{Ap}).$$

plot

- *I*.e., there are effective modifications to electromagnetic G_E and G_M, plus a new e.m. form factor, here called G_A'
- Calculated (estimated) mid-last-decade,



- Reference: Afanasev and me, PRL **94**, 212301 (2005).