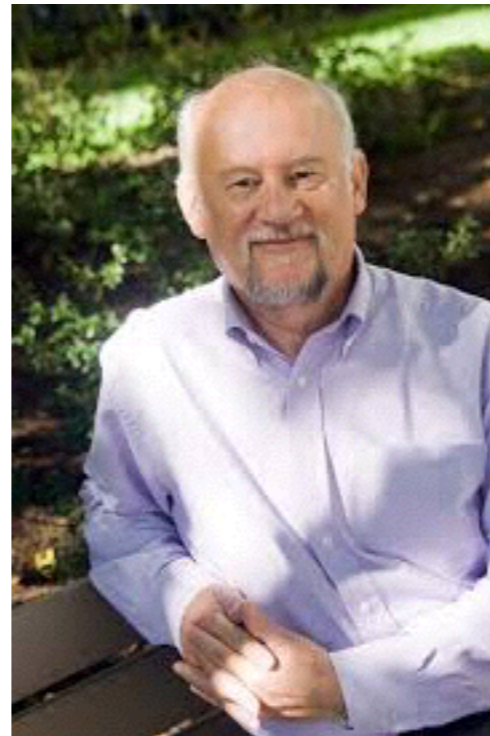


Hadron reactions and spectroscopy studies at the Joint Physics Analysis Center

In honor of Mike Pennington



To be or not to be, ... to do partial wave expansion or not ?

JPAC Mission

- JPAC was funded to support the extraction of physics results from analysis of experimental data from JLab12 and other accelerator laboratories.
- This is achieved through work on theoretical, phenomenological and data analysis tools.
- JPAC aims to facilitate close collaboration between theorists, phenomenologists, and experimentalists worldwide.
- It is engaged in education of further generation of hadron physics practitioners

JPAC People

- Postdocs:

- (past) L.Dai (Bonn), I.Danilkin (Mainz), P.Guo (Cal. State U.), C.Fernandez-Ramires (UNAM), D.Schott (Med. Coll. of Wis.)
- (current) V.Mathieu (IU), I.Lorentz (IU), A.Pilloni, (JLab) V.Pauk (JLab), D.Ronchen (Bonn U.)

- Students:

- (past) M.Shi (Pekin U.)
- (current) E.Alexeev (IU), A.Blin (Valencia), B. Hu (GWU), A.Jackura (IU), M.Mikhasenko (Bonn), J.Nis (U. Gent)

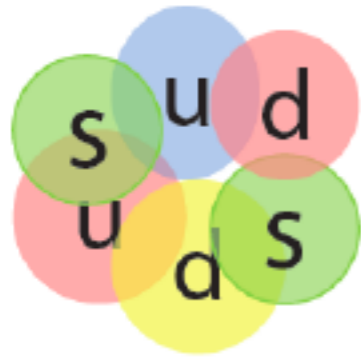
- Faculty: M.Doering (GWU), G.Fox (IU), J.T.Londergan (IU), I.Mokeev (JLab), M.Pennington (JLab), E.Passemar (IU), A.Szczepaniak (IU/JLab), R.Workman (GWU)



Products

- > 40 Research Papers (Phys. Rev., Phys. Lett, Eur. J. of Phys.)
- ~120 Invited Talks and Seminars
- Several Reaction/Reference Web Pages (include summer school + database)
- ~O(10) Ongoing Analyses
- 1 Summer School on Reaction Theory (IU,2015), 1 Workshop (Future Directions in Hadron Spectroscopy, (JLab, 2014)
- JPAC Review May-3-4, 2014

There may be hadrons that look like ...



dibaryon



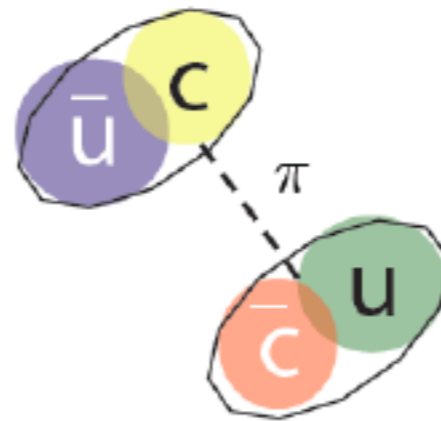
pentaquark



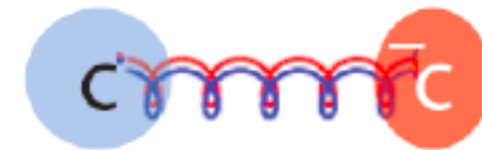
glueball



diquark + di-antiquark



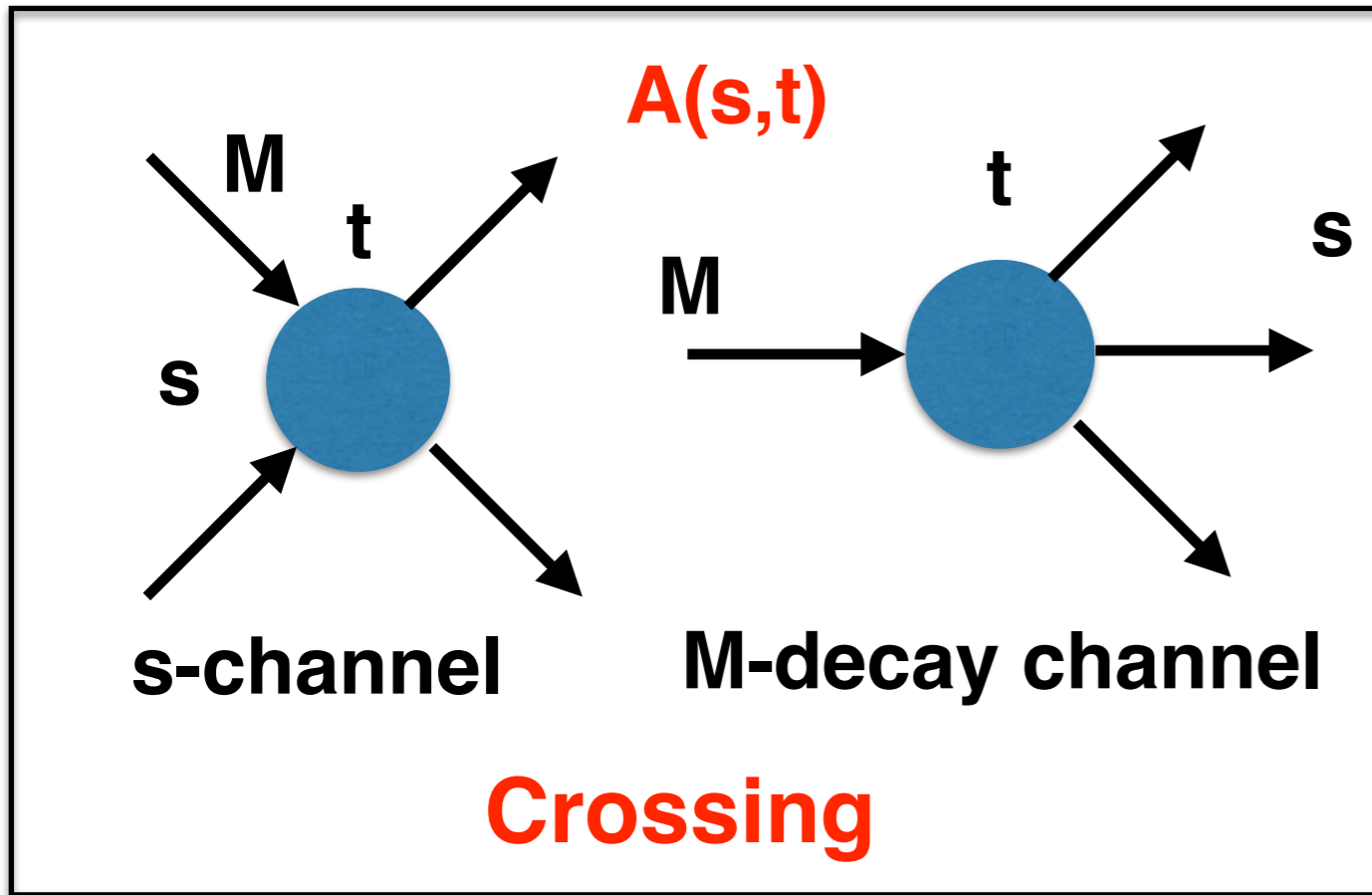
dimeson molecule



$q \bar{q} g$ hybrid

...before we know these exist it is necessary to identify resonances

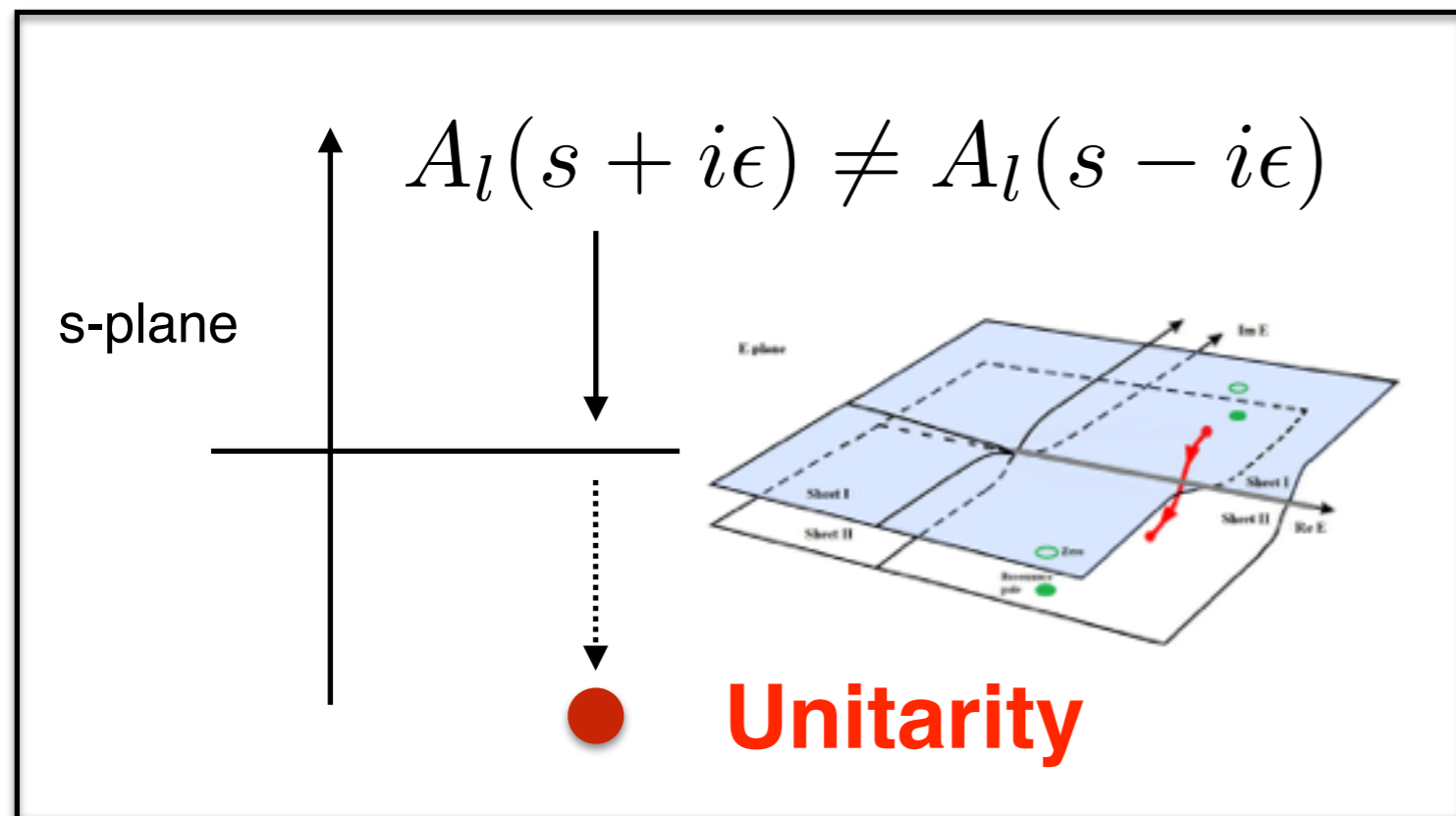
S-matrix principles: Crossing, Analyticity, Unitarity



$$A(s, t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

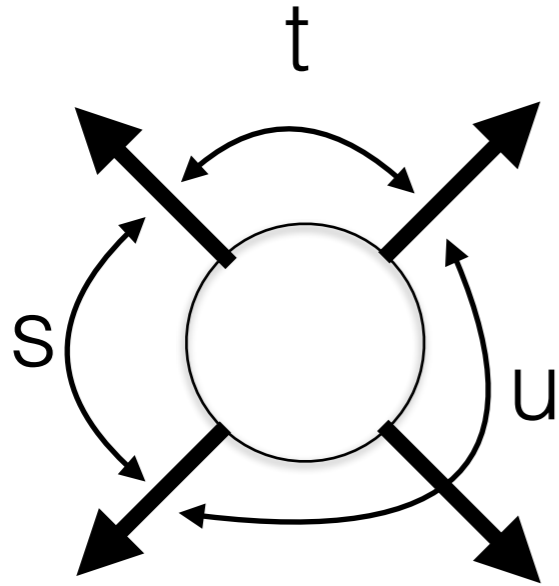
$$A_l(s) = \lim_{\epsilon \rightarrow 0} A_l(s + i\epsilon)$$



Resonances : bumps/peaks on the real axis (experiment) come from singularities in unphysical sheets

These singularities come from QCD

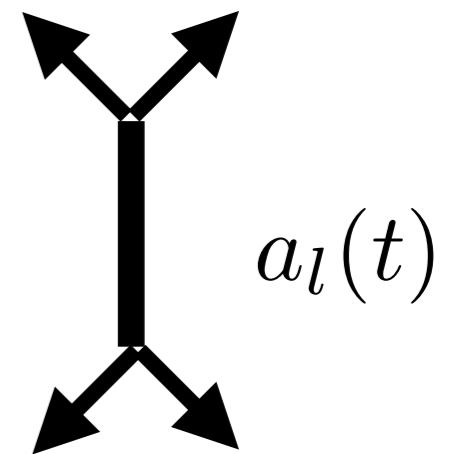
To do partial wave expansion or not ?



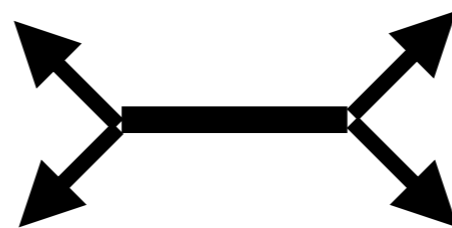
$A(s,t)$ has singularities in s , t and u

but partial wave expansions selects a specific channel, e.g. the t-channel:

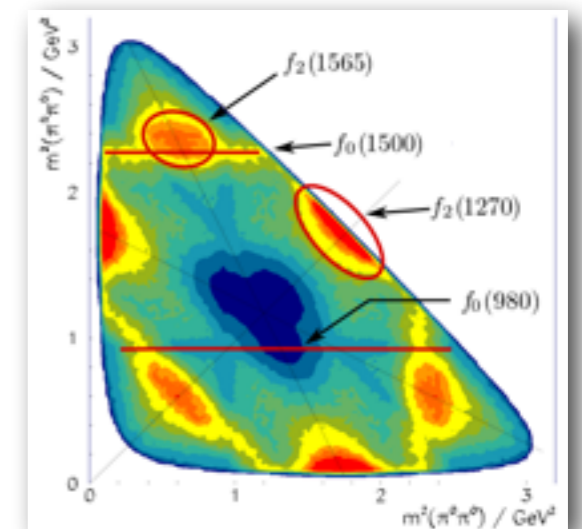
$$A(s, t) = \sum_l^{L_{max}} (2l + 1) a_l(t) P_l(z_t) \quad z_t = \frac{s - u}{t - 4\mu^2}$$



valid (convergent) in the t-channel physical region



$L_{max} = \infty$ is needed to reproduce s and u channel singularities (e.g. resonance)



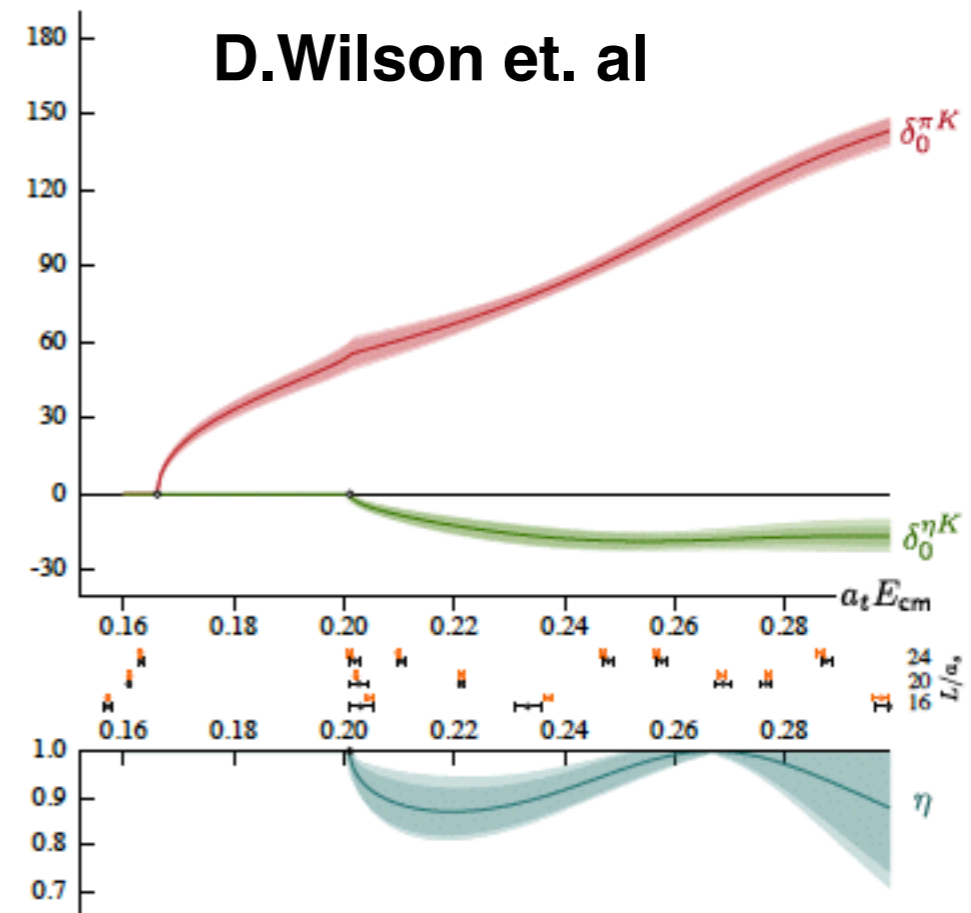
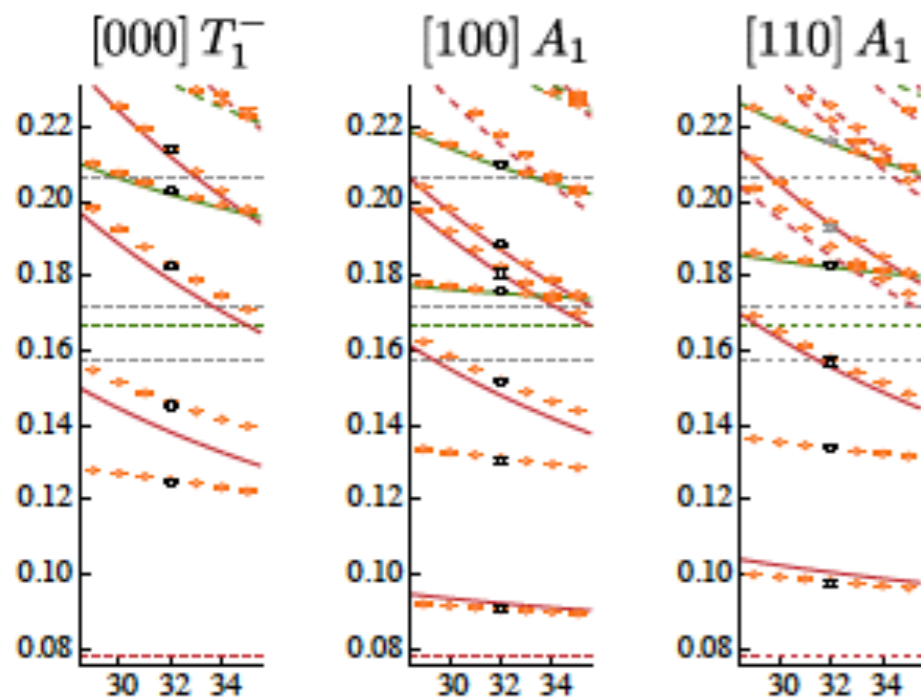
QCD on the Lattice : simulated scattering experiment

(known kinematical function)

$$Z(E_i(L), L) = T(E_i)$$

(infinite volume amplitude)

E_i = discrete energy spectrum of states in the lattice



in general “solution” of the Luscher condition requires an analytical model for T

COCKTAILS

Whisky SOUR

(WHISKY, ZUCCHERO, LIMONE)

€

MANHATTAN

(WHISKY, MARTINI ROSSO, ANGOSTURA)

€

Old fashioned

(AMERICAN WHISKY, ANGOSTURA, SODA)

€

Rusty nail

(WHISKY, DRAMBUIE)

€

Stinger

(COGNAC, CREMA di MENTA bianca)

€

Sidecar

(COGNAC, COINTREAU, LIMONE)

€

Daiquiri

(RHUM, LIMONE, ZUCCHERO)

€

BANANA daiquiri

(RHUM BACARDI, BANANA frullata, LIMONE)

€

Palm beach

(RHUM, GIN, ANANAS)

€

Shanghai

(RHUM BACARDI, POMPELMO, GRANATINA)

€

Mojito

(RHUM, ZUCCHERO, LIMONE, MENTA FRESCA)

€

X.Y.Z.

(RHUM, ARANCIA, COINTREAU)

€

Margherita

(TEQUILA, LIMONE, COINTREAU)

€

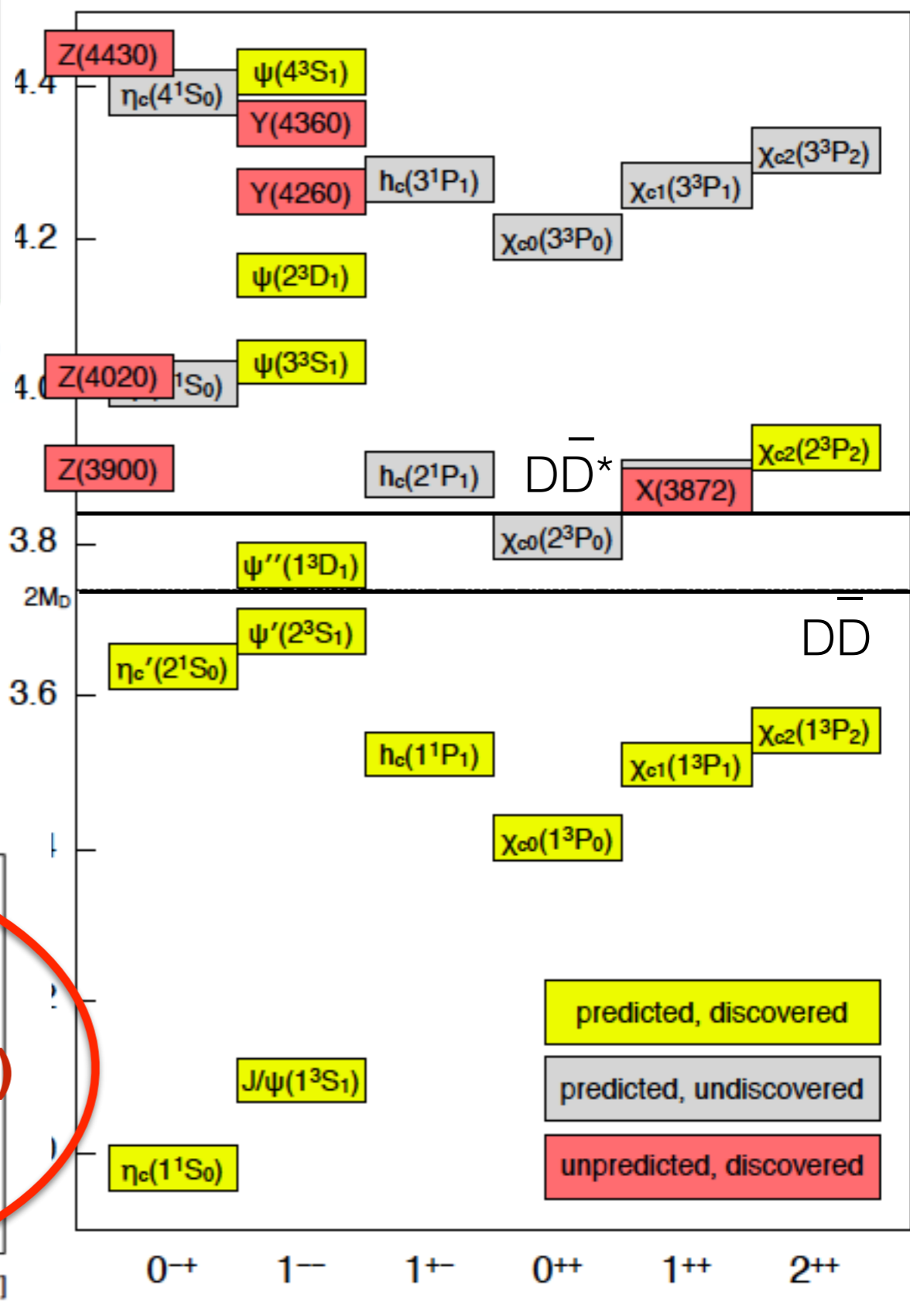
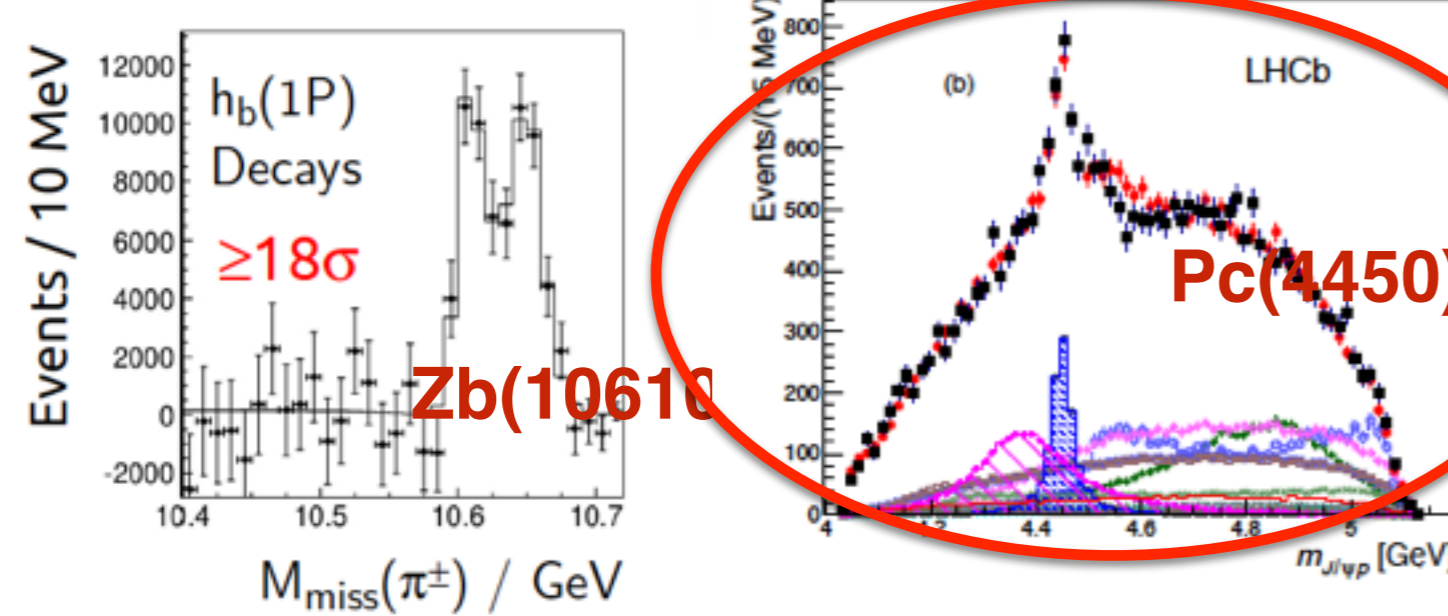
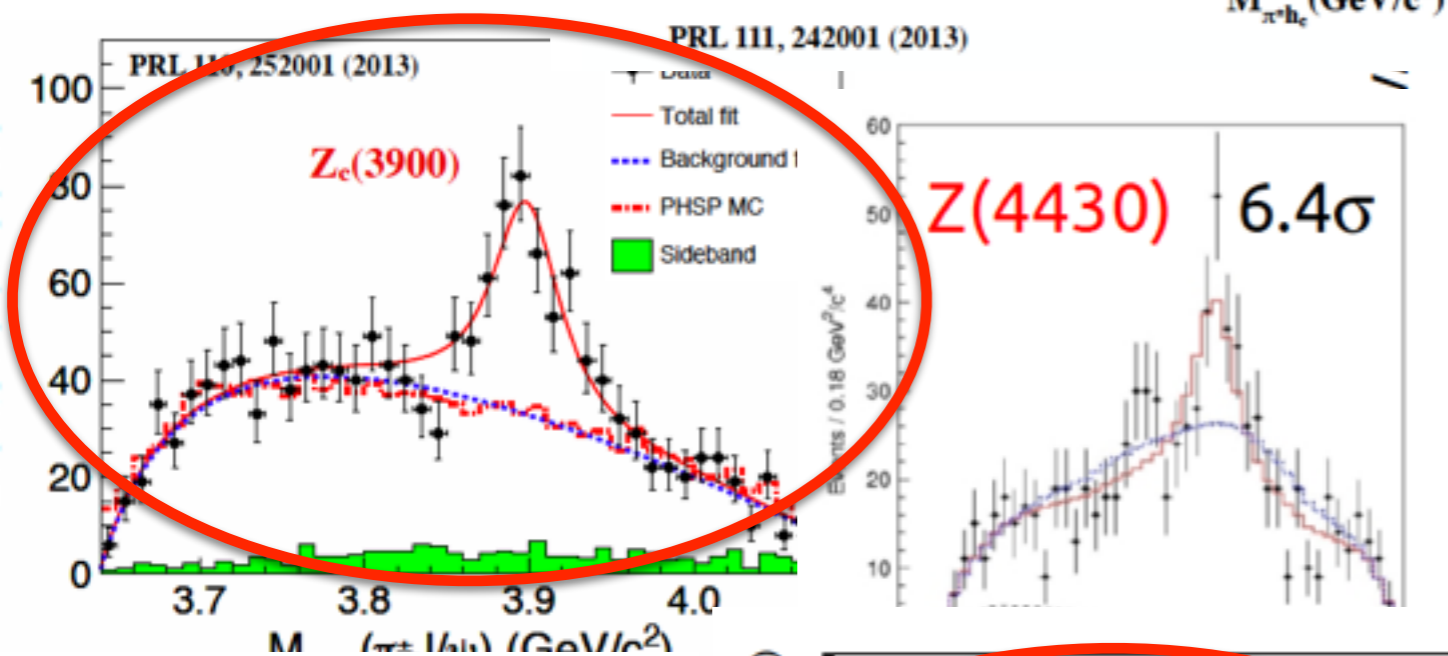
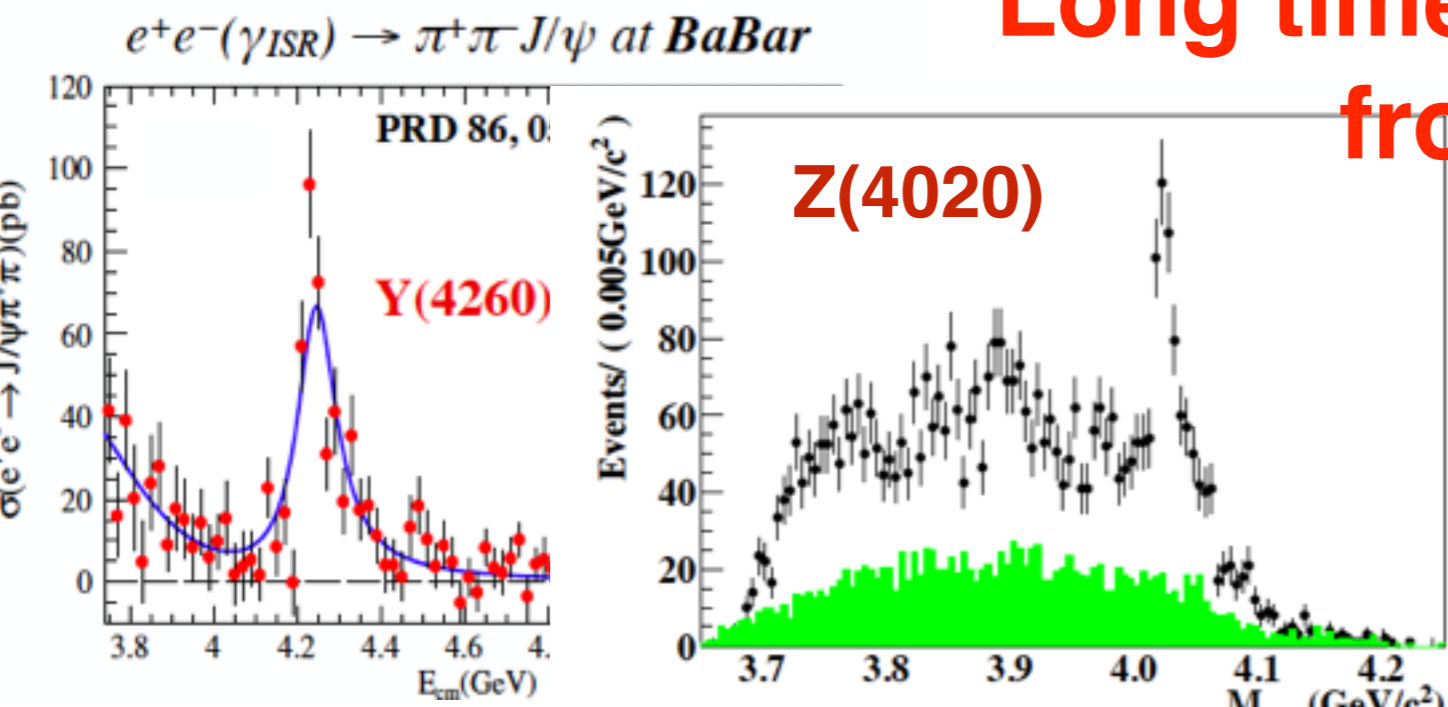
Mexico 76

€

XYZ

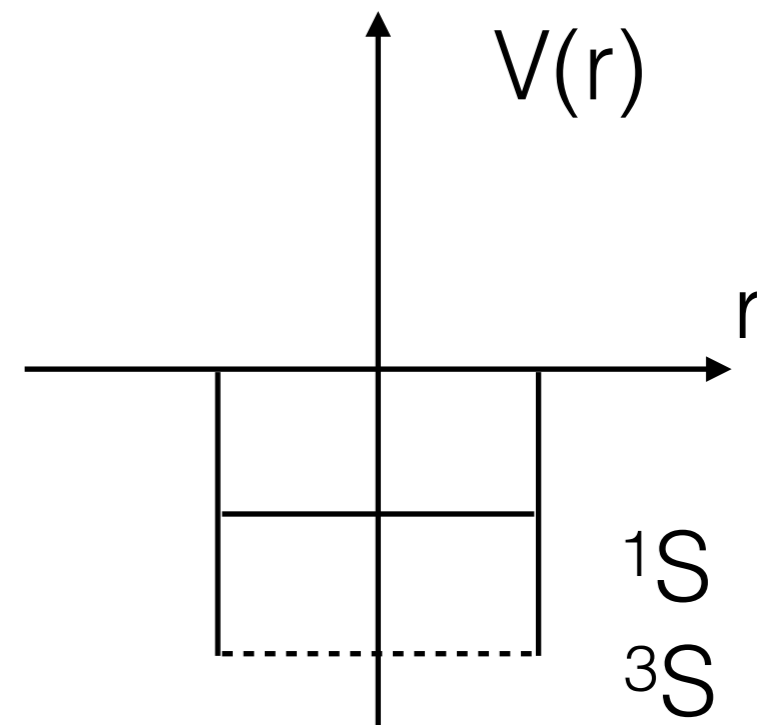
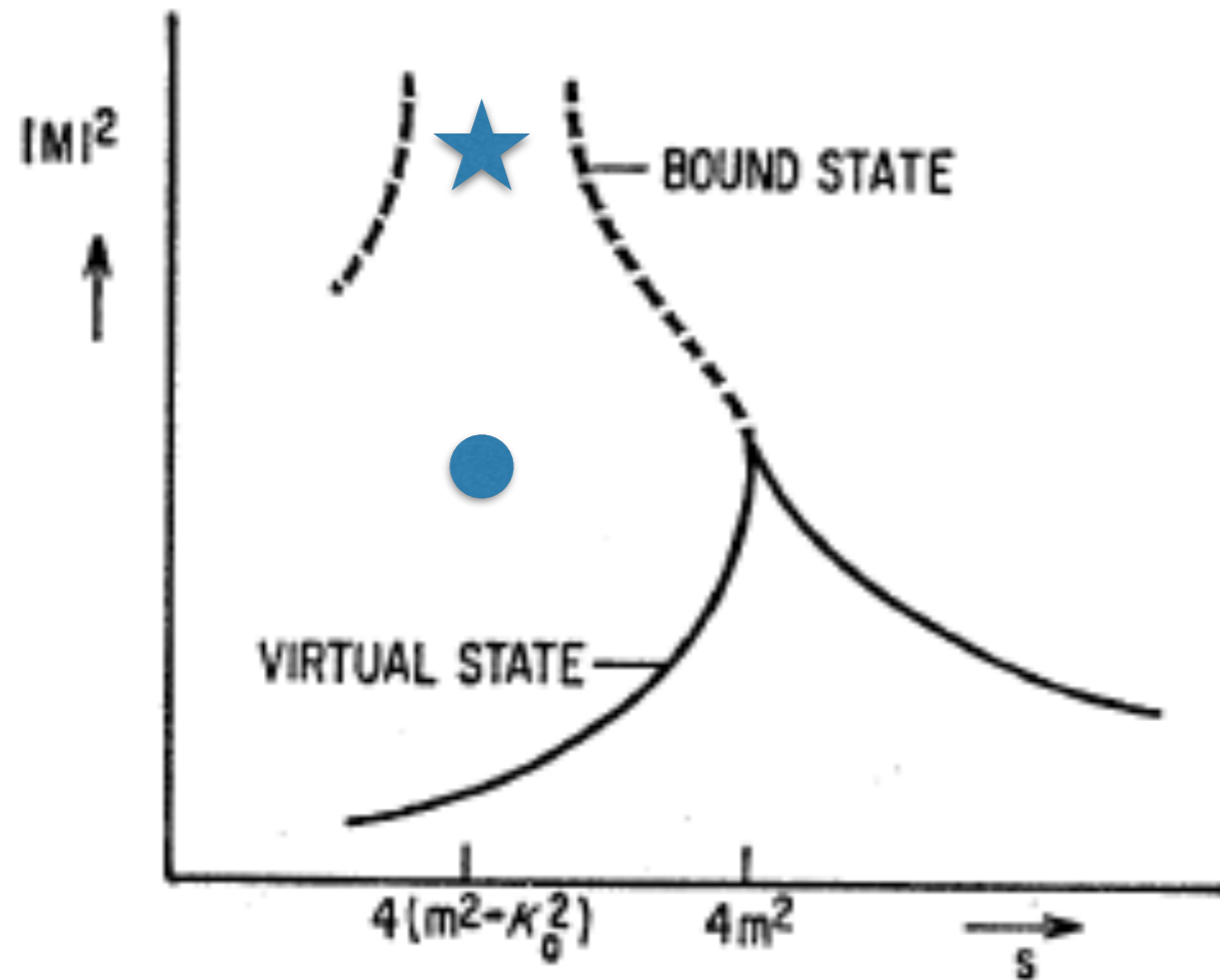
“dynamics”

Long time ago hadrons were made from valence quarks

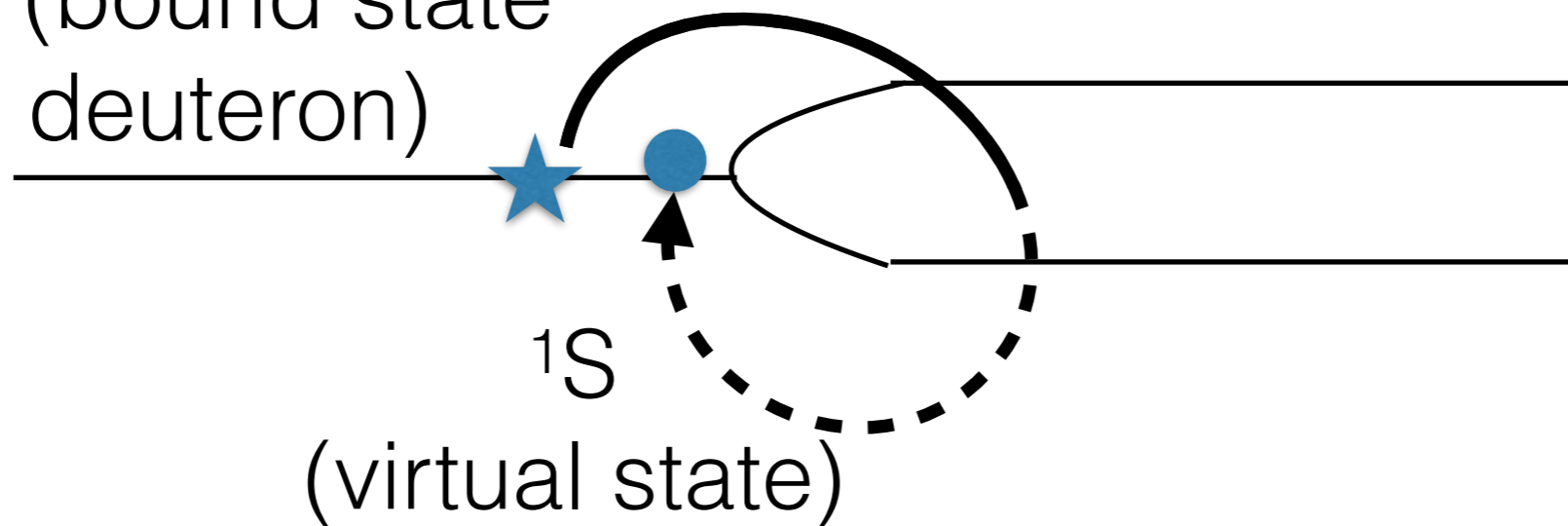


O(10) open flavor decay thresholds

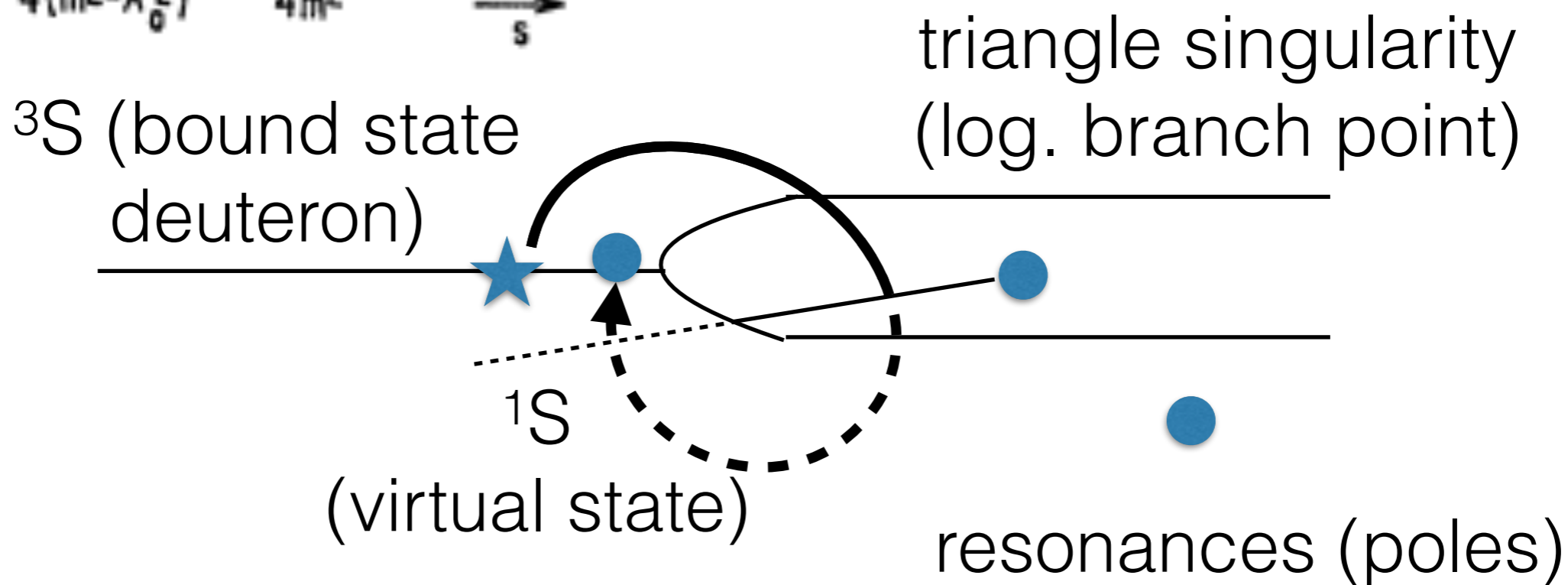
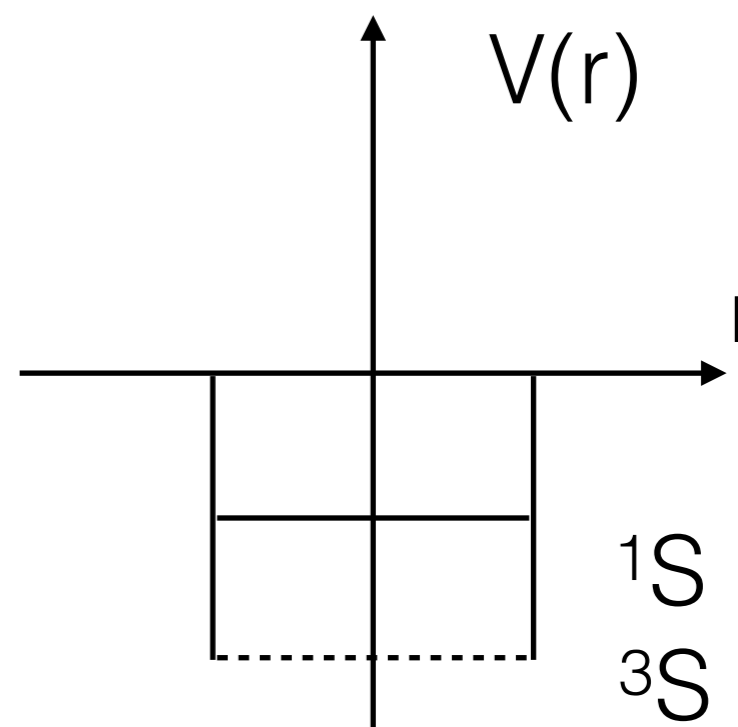
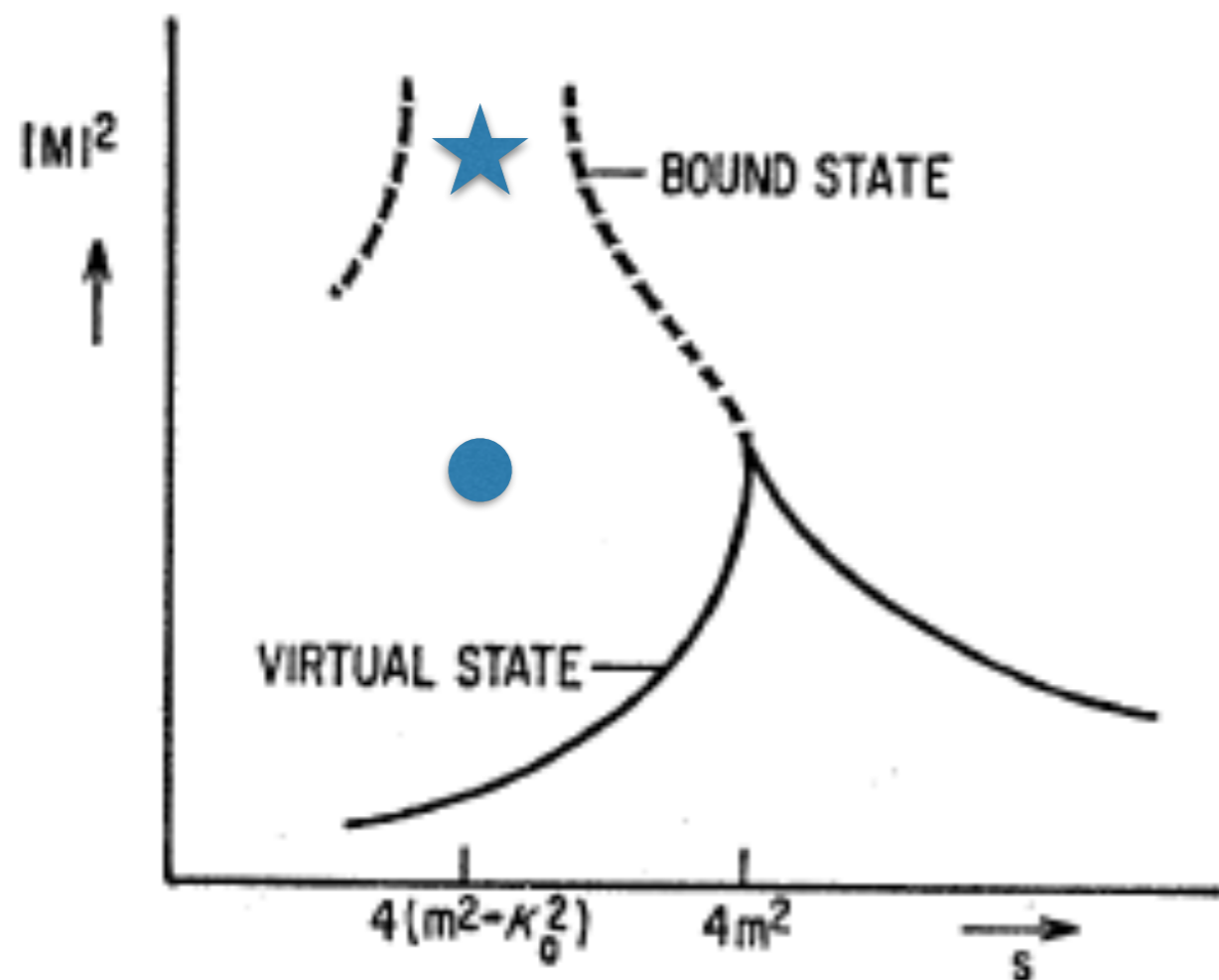
Singularities, is all that matters: cusps, poles and more cusps



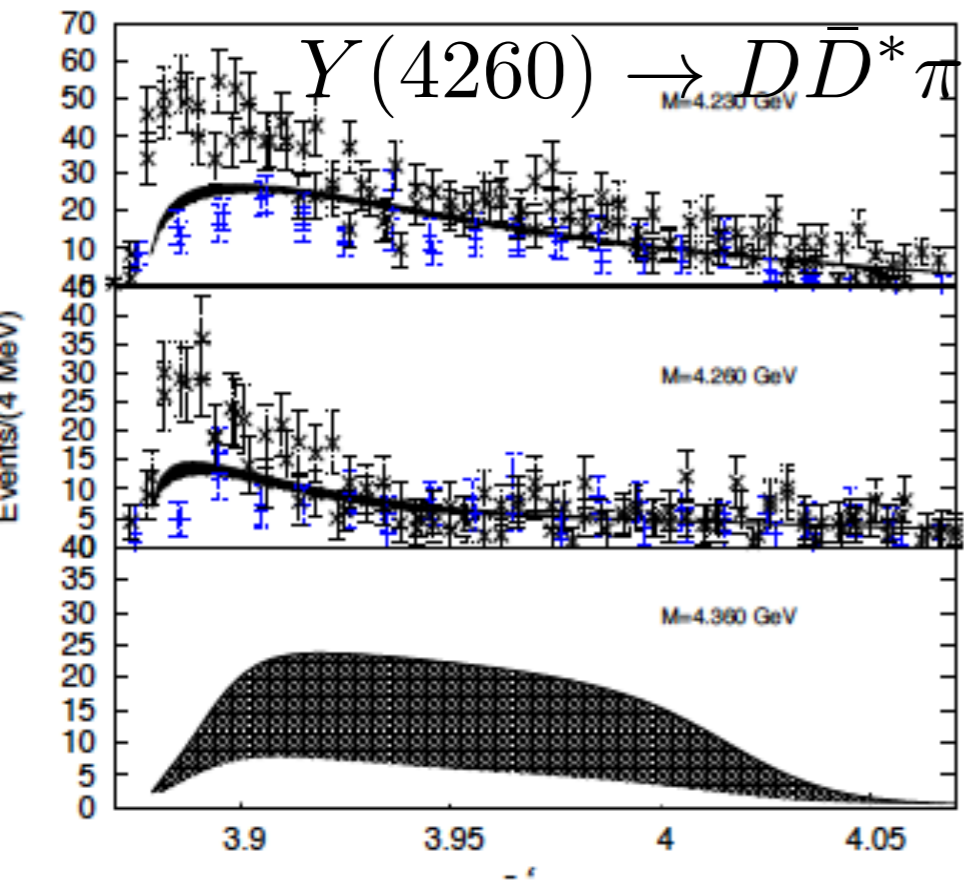
$3S$ (bound state
deuteron)



Singularities, is all that matters: cusps, poles and more cusps



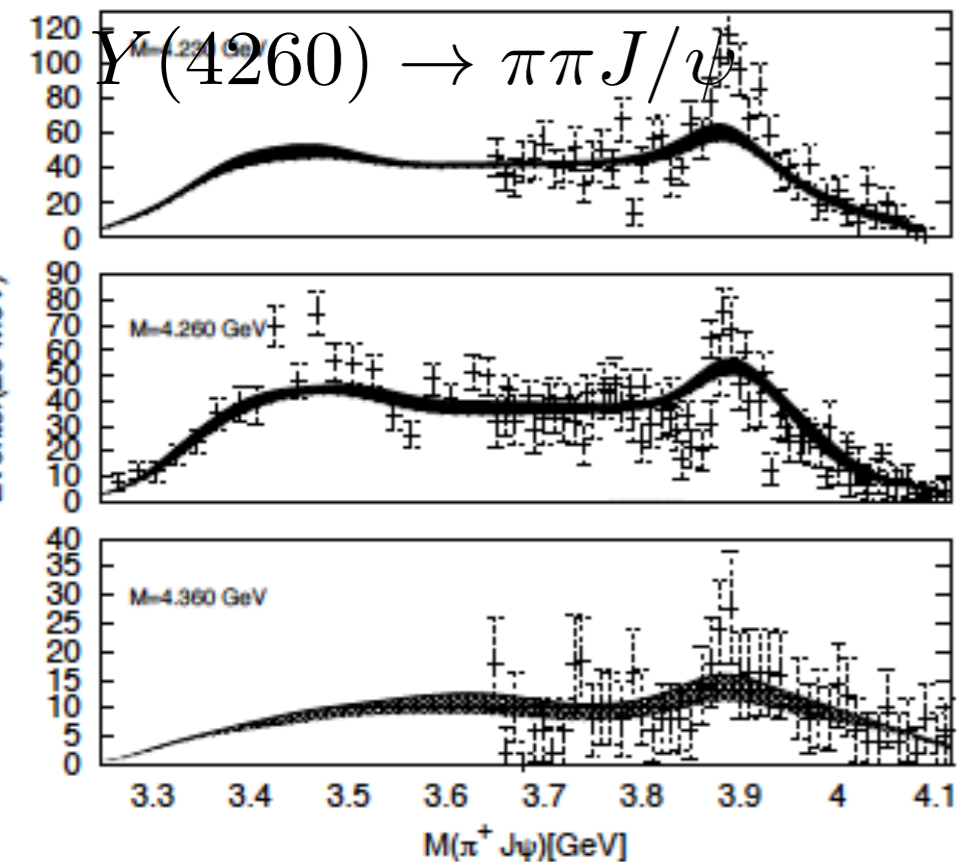
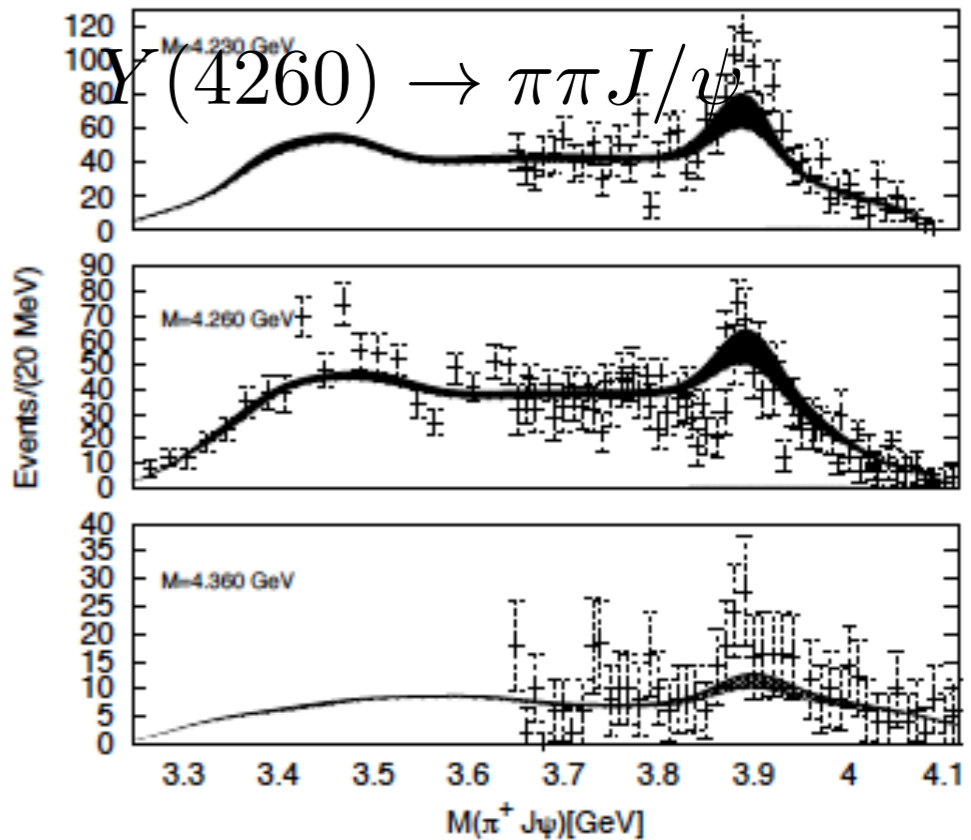
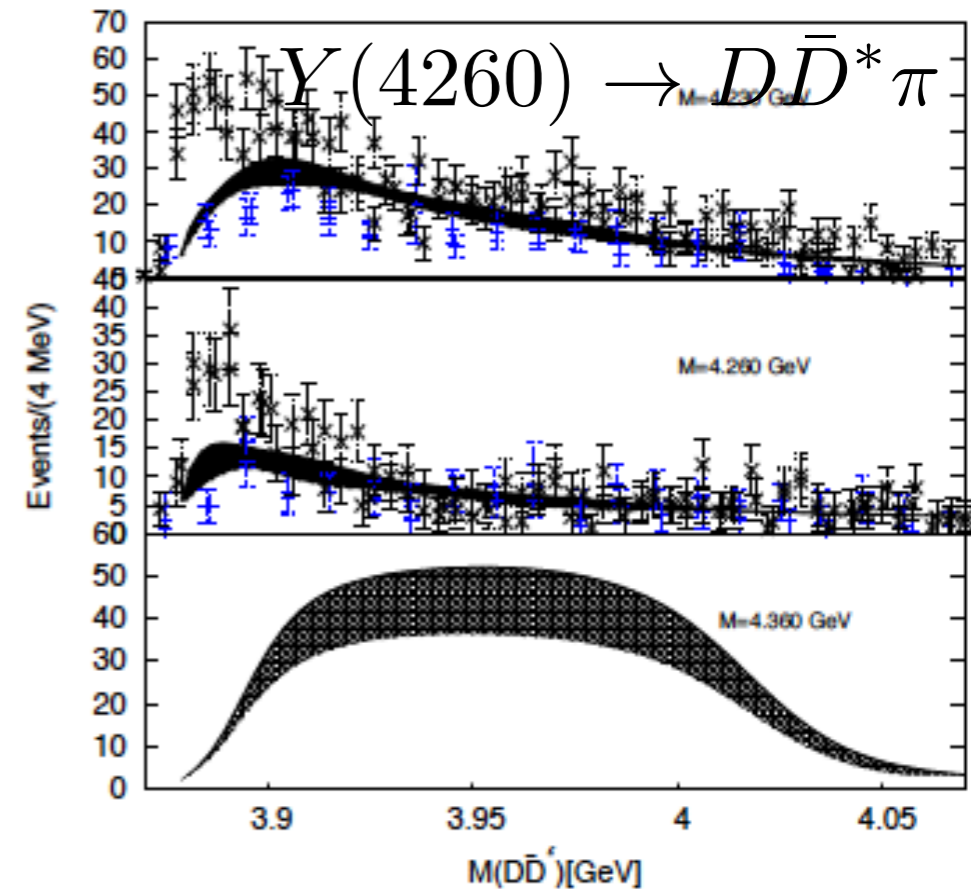
Model I



Amplitude analysis has to be applied at the event-by-event basis

Fitting projection may not be enough to discriminate between amplitude models/dynamics

Model II



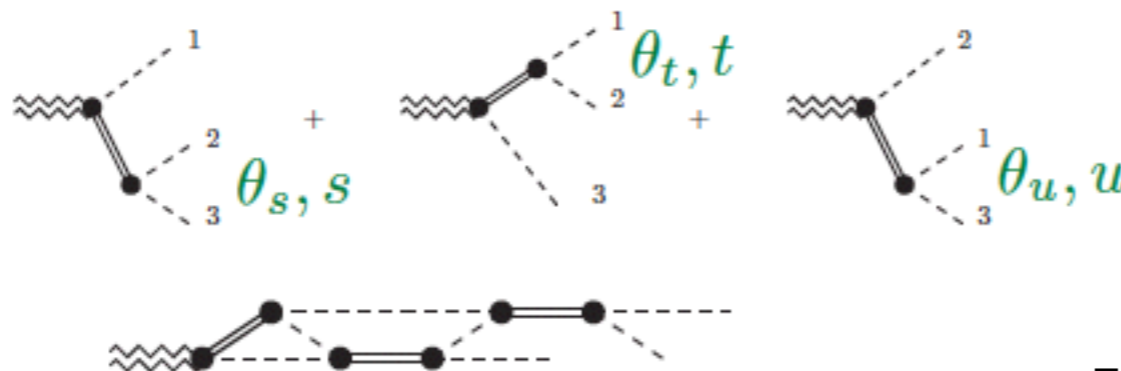
2.4 $\omega/\phi \rightarrow 3\pi$

Danilkin et al., JPAC'15

- Simple system: restricted to odd partial waves
 → P wave interactions only (neglecting F- and higher)
- Amplitude: $\mathcal{A}_\lambda(s, t, u) = \varepsilon_{\mu\nu\alpha\beta} \epsilon_\lambda^\mu p_{\pi^+}^\nu p_{\pi^-}^\alpha p_{\pi^0}^\beta \mathcal{F}(s, t, u)$
 $\mathcal{F}(s, t, u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$
- F(s) function of one variable with only a right-hand cut
- Unitarity relation: $\text{Disc } F(s) = \rho(s) \overbrace{t^*(s)}^{\pi\pi \rightarrow \pi\pi} (F(s) + \hat{F}(s))$
- Relation of dispersion to reconstruct the amplitude everywhere:

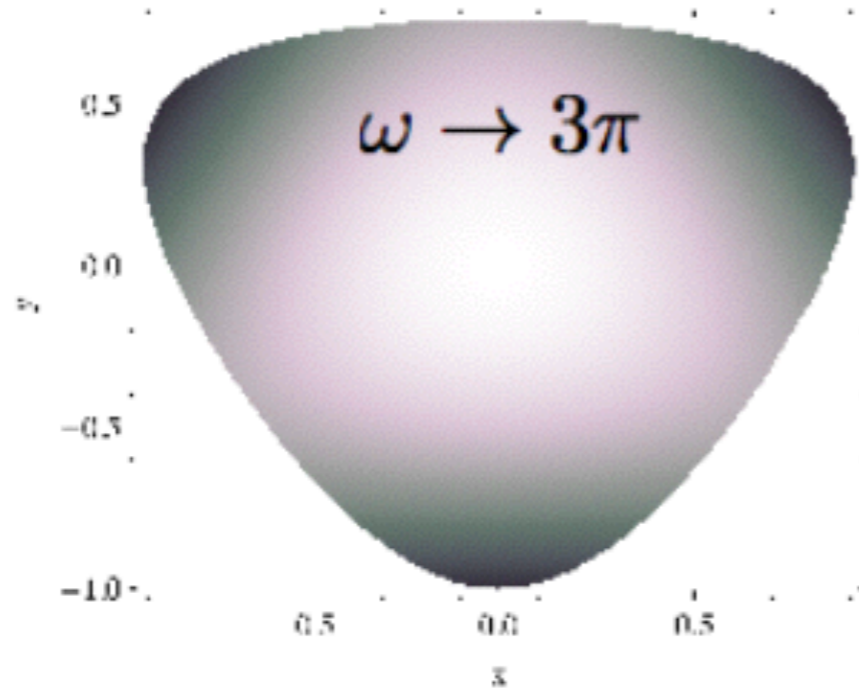
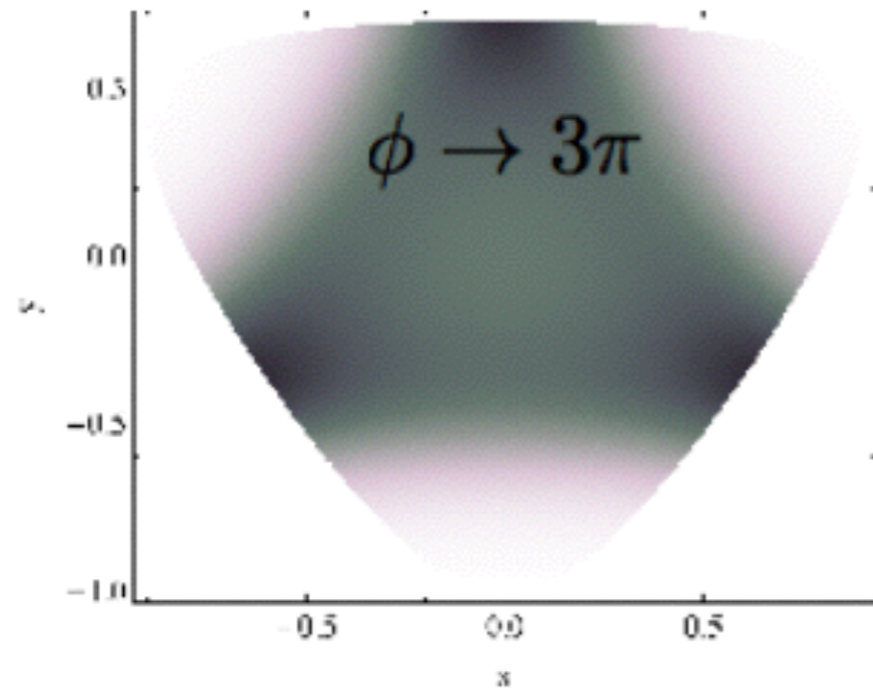
$$F(s) = \Omega(s) \left(\int_{4M_\pi^2}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc } F(s)}{\Omega^*(s')(s'-s-i\varepsilon)} + \sum_{i=0}^N a_i \omega^i(s) \right)$$

$$\omega(s) = \frac{\sqrt{s_i} - \sqrt{s_i - s}}{\sqrt{s_i} + \sqrt{s_i - s}}$$

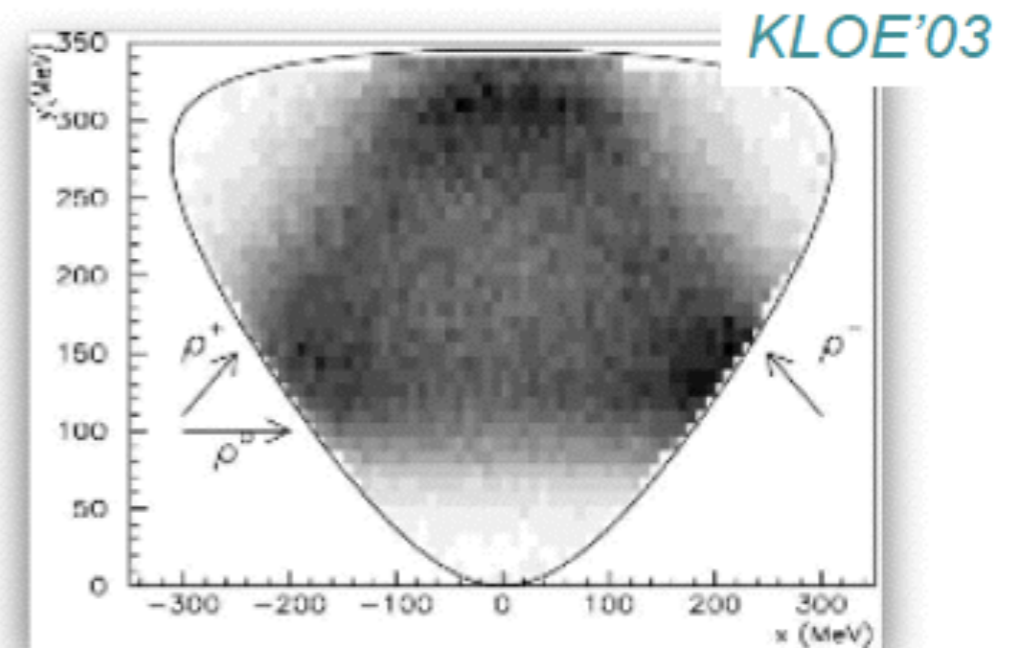


2.4 $\omega/\phi \rightarrow 3\pi$

- Dalitz plots:

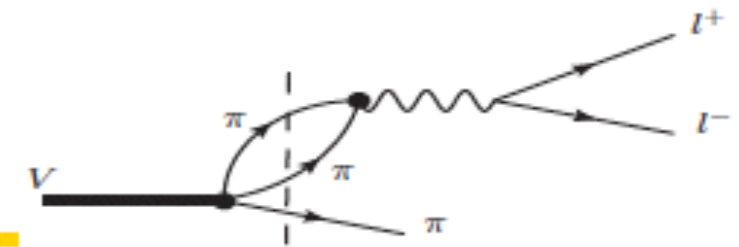


Danilkin et al., JPAC'15



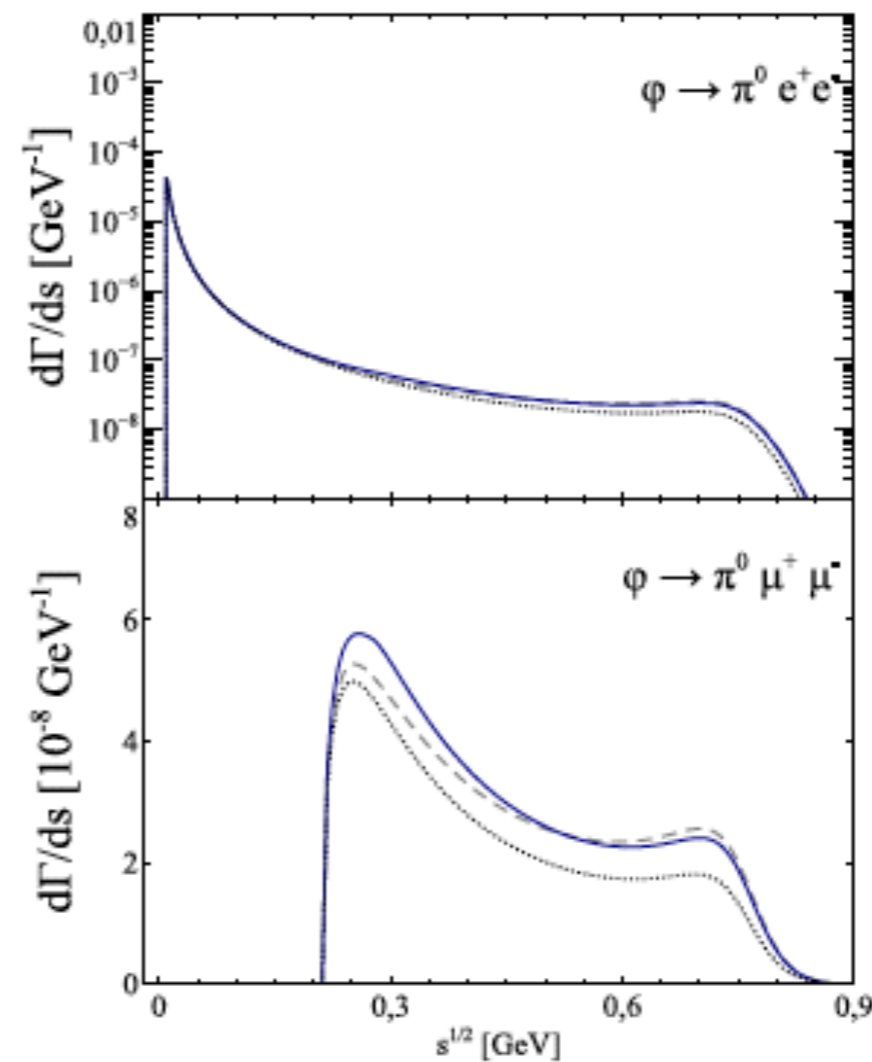
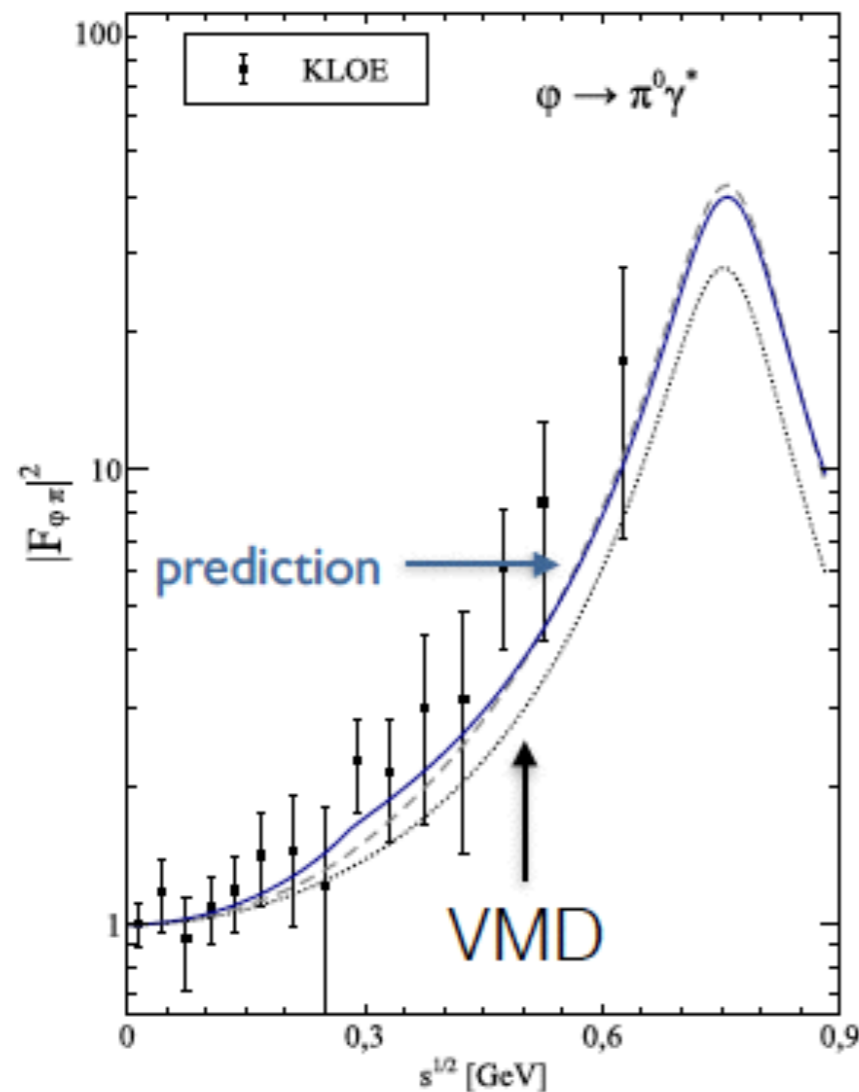
- Only one parameter (overall normalization)
→ fixed from $\Gamma_{\text{exp}}(\omega/\phi \rightarrow 3\pi)$
- $\omega \rightarrow 3\pi$: distribution is relatively flat.
Upcoming high-statistic data from *CLAS*, *WASA*, *KLOE2*
- Fit event by event by event *g12*
CLAS data in progress:
C. Salgado, V. Crede, C Zeoli

2.5 Application: $\phi \rightarrow \pi^0 \gamma^*$



Danilkin et al., JPAC

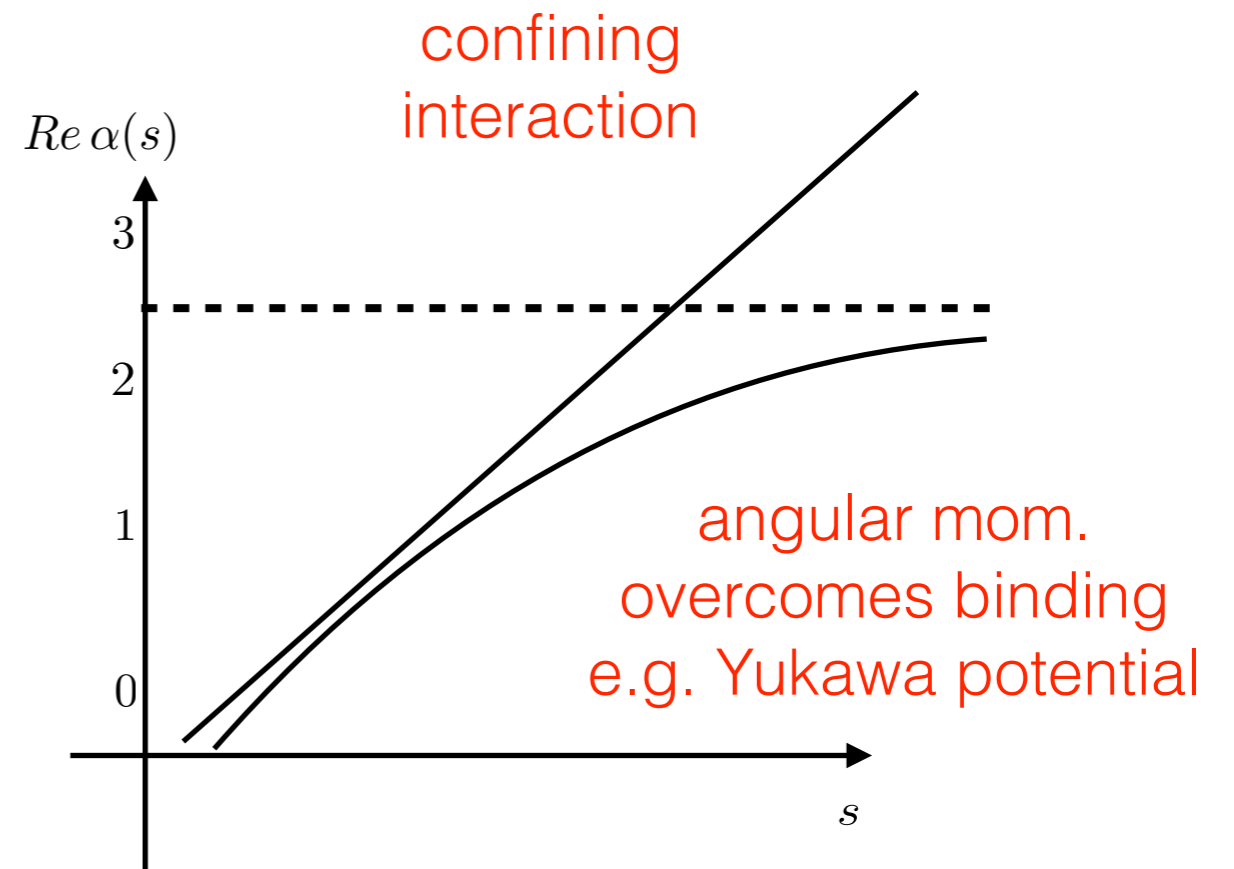
- Form factor: $f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc } f_{V\pi}(s')}{s' - s} + \sum_{i=0}^N C_i \omega(s)^i$



- C_0 fixed from $\Gamma_{\text{exp}}(\phi \rightarrow \pi\gamma)$. Grey: no 3 body effects
- Prediction consistent with new *KLOE* data'16
- Possible future data from *VEPP-2000*

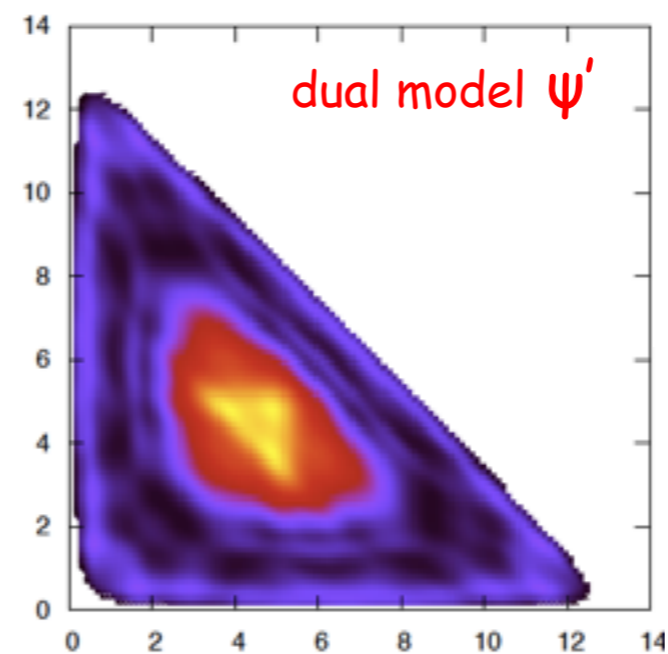
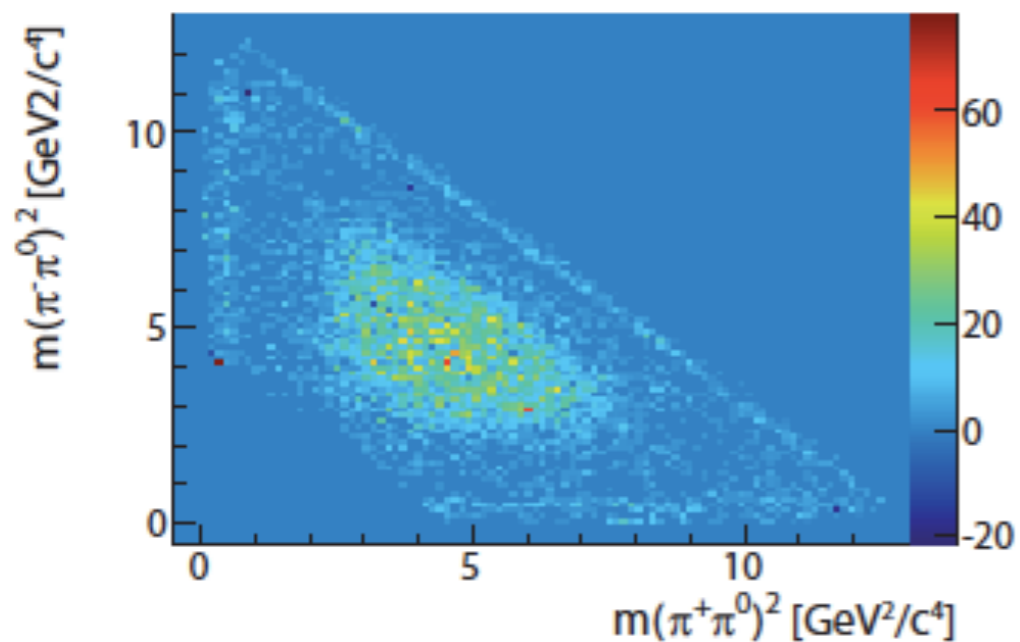
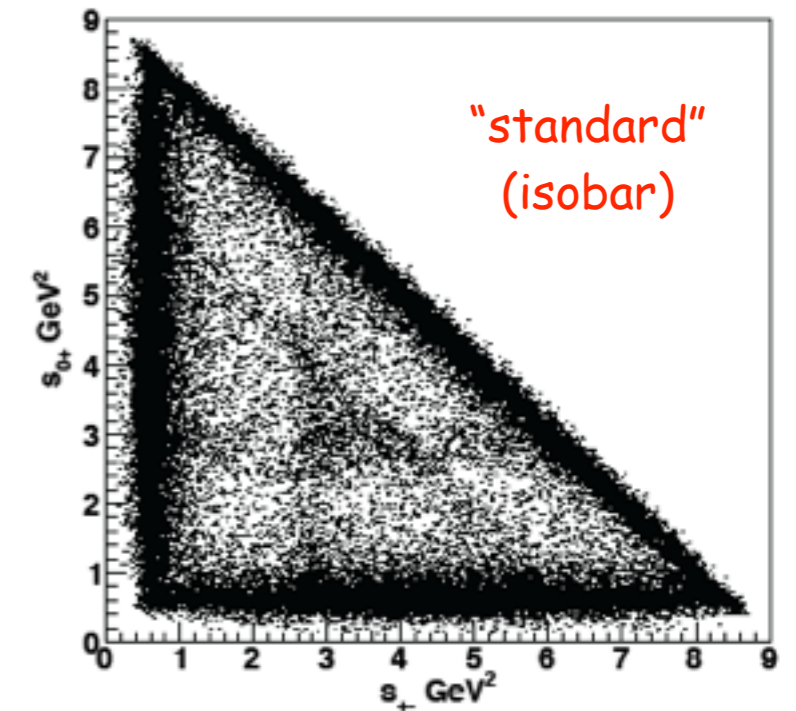
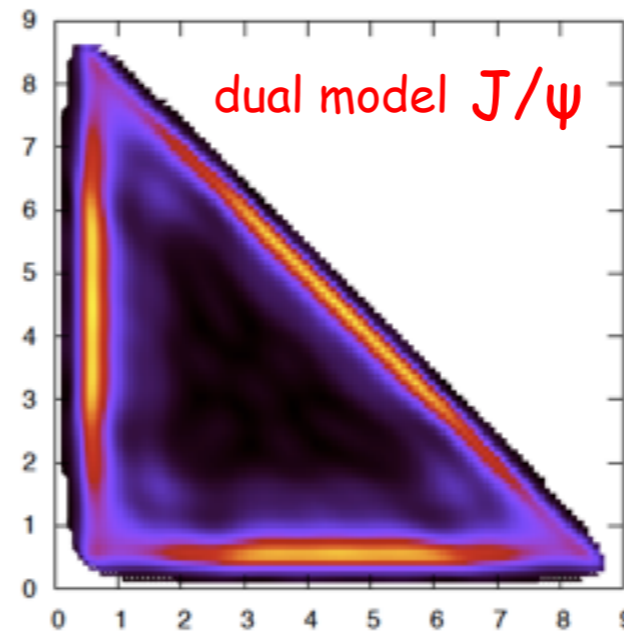
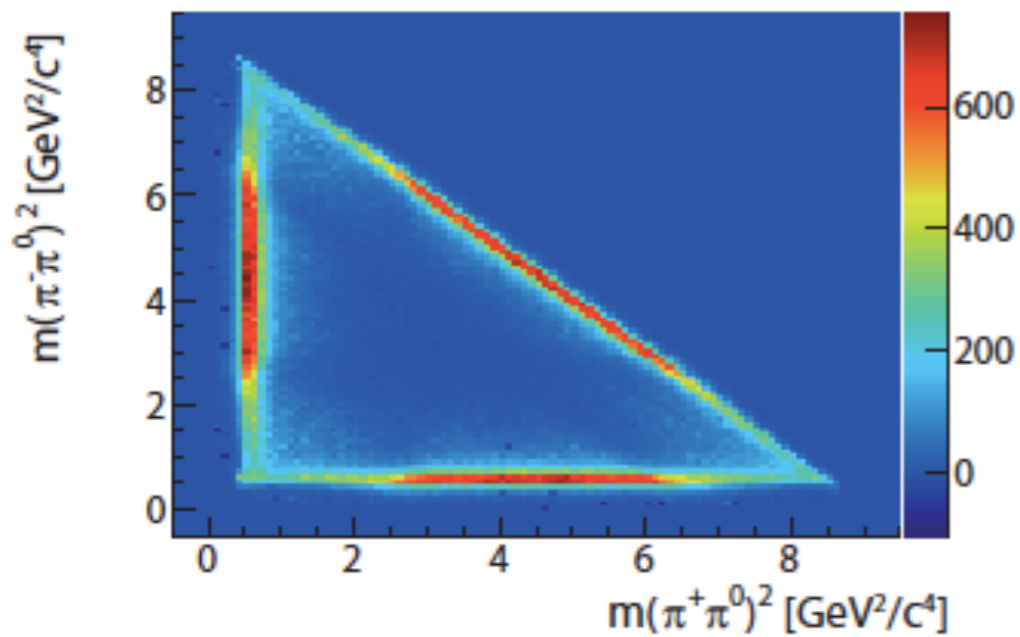
Reggeons vs Resonances

- Resonances are not constrained by S-matrix principles -> need priori knowledge
- Resonances are poles in s at fixed L ; Reggeons are poles in L at fixed s



Revival of “old” ideas e.g. dual models

$$A(s, t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s) - J(t))}$$



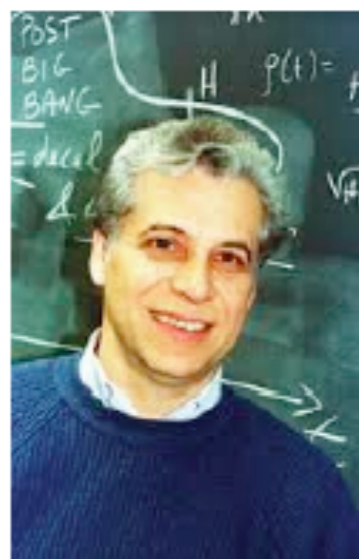
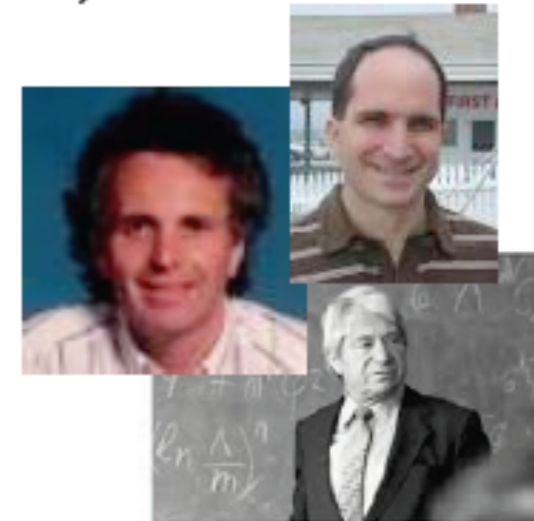


relativistic h.o.



string of relativistic oscillators

QCD, loop representation, large- N_c , AdS/CFT, ...



$\omega \rightarrow 3\pi$



$$A(s, t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s) - J(t))}$$

string revolution



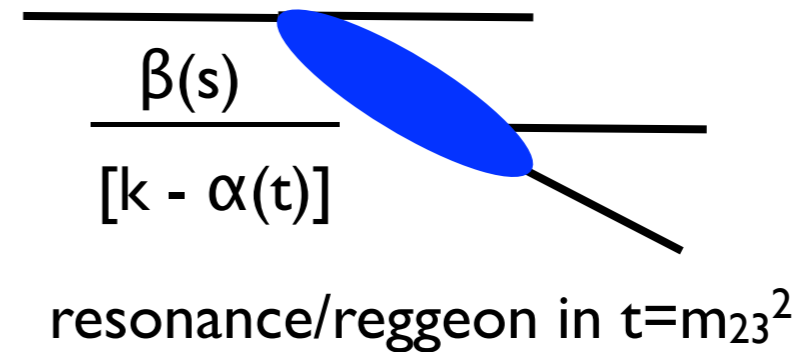
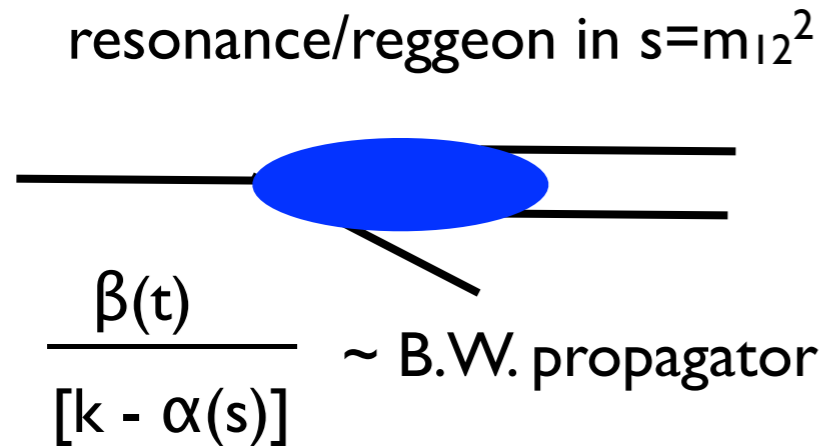
Dynamical assumptions

- **Duality:** resonances in direct channel dual to reggeons in cross channels and backgrounds are dual to the pomeron
- **All resonances are “connected”:** resonances belong to Regge trajectories (reggeons)
- **Asymptotics:** determined by Regge poles
- **Unitarity:** imaginary parts determined by decay thresholds

Veneziano amplitude satisfies all of the above except
unitarity.

Veneziano amplitude: “compact” expression for the full amplitude

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} \quad \alpha(s) = a + bs$$



$A(s,t)$ can be written as sum over resonances in either channel.

$$A(s, t) = \sum_k \frac{\beta_k(t)}{k - \alpha(s)} = \sum_k \frac{\beta_k(s)}{k - \alpha(t)}$$

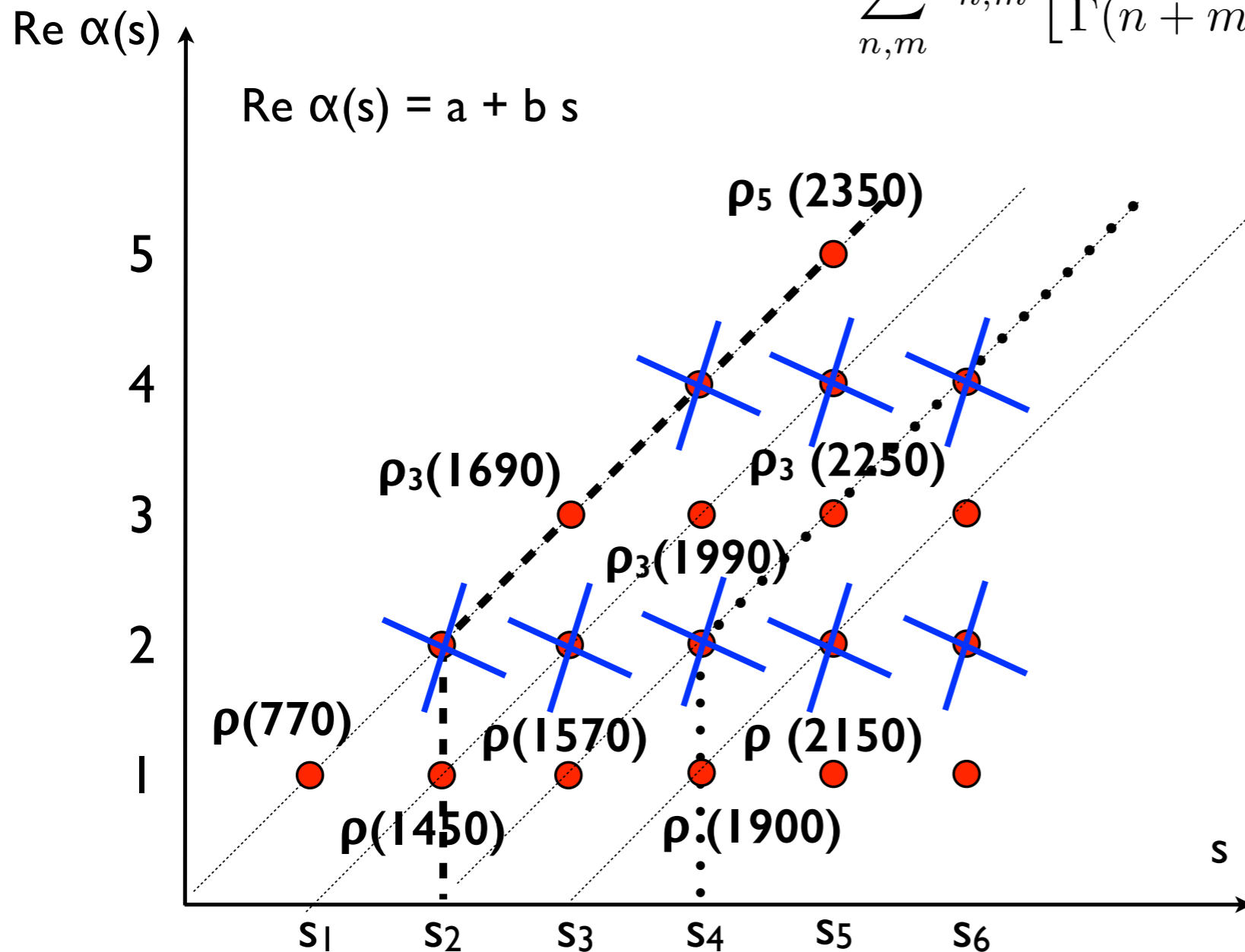
Note: in V-model resonance couplings, β , are fixed!

$$\beta_k(t) \propto (1 + \alpha(t))(2 + \alpha(t)) \cdots (k + \alpha(t))$$

Resonances couplings, β , should depend on final state particles: a linear superposition of Veneziano amplitudes can be used to suppress or enhance individual resonances or trajectories

$$M = \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha \epsilon^\beta A(s, t, u)$$

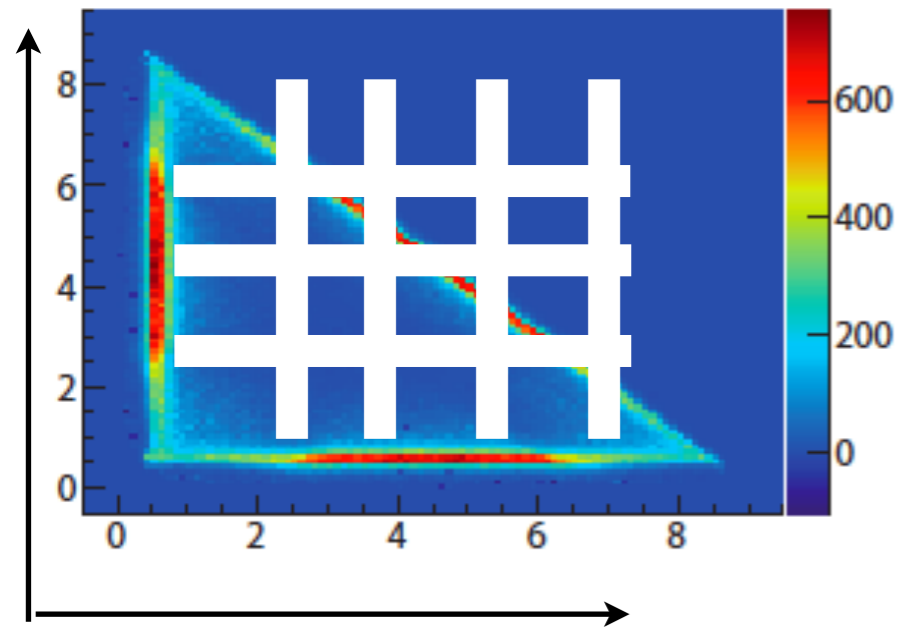
$$A = \sum_{n,m} c_{n,m} \left[\frac{\Gamma(n - \alpha(s))\Gamma(n - \alpha(t))}{\Gamma(n + m - \alpha(s) - \alpha(t))} + (s, u) + (t, u) \right]$$



- even-spin ρ 's do not couple to $\pi\pi$ and should decouple in $J/\psi \rightarrow 3\pi$
- coupling of odd-spin ρ 's depend on can depend vary depending on trajectory

$$A_{n,m}(s, t) \equiv \frac{\Gamma(n - \alpha_s)\Gamma(n - \alpha_t)}{\Gamma(n + m - \alpha_s - \alpha_t)}.$$

t how to remove (infinite) number of poles?



Use a linear combination of $A_{2,1}$ and $A_{2,2}$ to remove pole at $\alpha_s = 2$

Use a linear combination of $A_{3,1}$, $A_{3,2}$, $A_{3,3}$ to remove pole at $\alpha_s = 3$,

etc.

$$n \geq m \geq 1$$

$$A_{1,1} = \frac{\Gamma(1 - \alpha_s)\Gamma(1 - \alpha_t)}{\Gamma(2 - \alpha_s - \alpha_t)}$$

has poles at $\alpha_s = 1, 2, 3, \dots$

$$A_{2,1} = \frac{\Gamma(2 - \alpha_s)\Gamma(2 - \alpha_t)}{\Gamma(3 - \alpha_s - \alpha_t)}$$

have poles at $\alpha_s = 2, 3, 4, \dots$

$$A_{2,2} = \frac{\Gamma(2 - \alpha_s)\Gamma(2 - \alpha_t)}{\Gamma(4 - \alpha_s - \alpha_t)}$$

$$A_{3,1}, A_{3,2}, A_{3,3}$$

have poles at $\alpha_s = 3, 4, 5, \dots$

$$A_{4,1}, A_{4,2}, A_{4,3}, A_{4,4}$$

have poles at $\alpha_s = 4, 5, 6, \dots$

$$A_{n,m}(s, t) \rightarrow \mathcal{A}(s, t) = \sum_{n \geq 1, n \leq m \leq 1} c_{n,m} A_{n,m}(s, t)$$

remove all poles but the one at $\alpha=1$

$$c_{n,1} = \frac{c_{1,1}}{\Gamma(n)}, \quad c_{n,2} = -\frac{c_{1,1}}{\Gamma(n-1)}, \quad c_{n,m} = 0 \text{ for } m > 2,$$

$$\mathcal{A}_1(s, t) = c_{1,1} \frac{2 - \alpha_s - \alpha_t}{(1 - \alpha_s)(1 - \alpha_t)}.$$

... but the Regge limit is now lost !

remove all poles between $N \geq \alpha \geq 2$

$$\begin{aligned} \mathcal{A}_1(s, t; N) &= c_{1,1} \frac{2 - \alpha_s - \alpha_t}{(1 - \alpha_s)(1 - \alpha_t)} \quad \text{has Regge limit is for } s > N \\ &\times \frac{\Gamma(N + 1 - \alpha_s)\Gamma(N + 1 - \alpha_t)}{\Gamma(N)\Gamma(N + 2 - \alpha_s - \alpha_t)} \end{aligned}$$

In the past this was done by choosing an arbitrary set of n, m and fitting $c(n, m)$ to the data (e.g. Lovelace, Phys. Lett. B28, 265 (1968), Altarelli, Rubinstein, Phys. Rev. 183, 1469 (1969))

We do in a systematic way. In addition it allows for imaginary non-linear (and complex) trajectories without introducing “ancestors”

$$\mathcal{A}_n(s, t; N) = \frac{2n - \alpha_s - \alpha_t}{(n - \alpha_s)(n - \alpha_t)} \sum_{i=1}^n a_{n,i} (-\alpha_s - \alpha_t)^{i-1} \\ \times \frac{\Gamma(N + 1 - \alpha_s) \Gamma(N + 1 - \alpha_t)}{\Gamma(N + 1 - n) \Gamma(N + n + 1 - \alpha_s - \alpha_t)}.$$

n : number of Regge trajectories

$a_{n,i}$: determine resonance couplings

N : determines the onset of Regge behavior

$\alpha(s), \alpha(t) = \text{Re } \alpha + i \text{Im } \alpha$: with $\text{Im } \alpha$ related to resonance widths

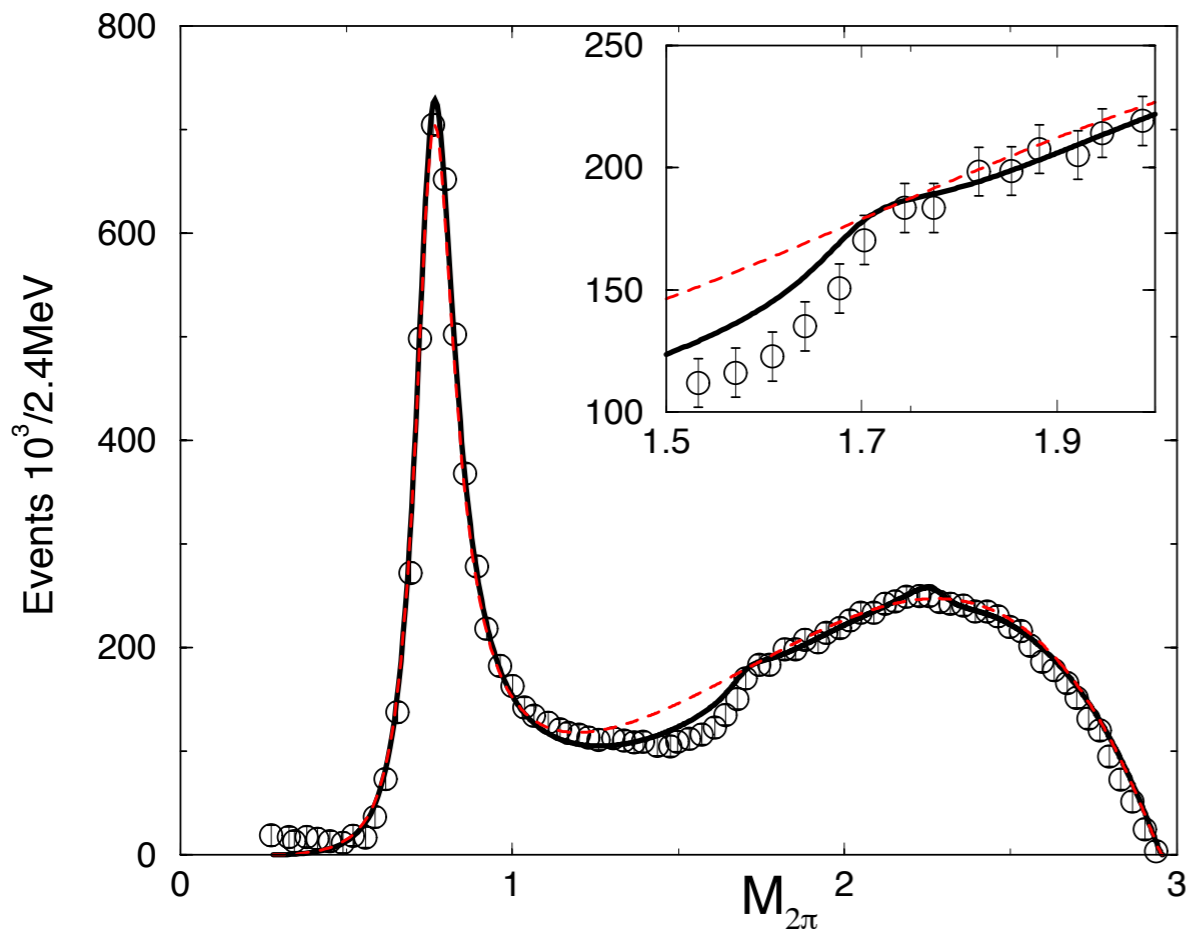


FIG. 2: Dalitz plot projection of the di-pion mass distribution from J/ψ decay. The solid is the result of the fit with three amplitudes and the dashed line with the amplitude \mathcal{A}_1 alone. The insert shows the mass region of the ρ_3 and its contribution from the fit with the full set of amplitudes (solid line) as compared. Absence of the structure at 1.7GeV from the fit with the \mathcal{A}_1 amplitude is indicated by the dashed line.

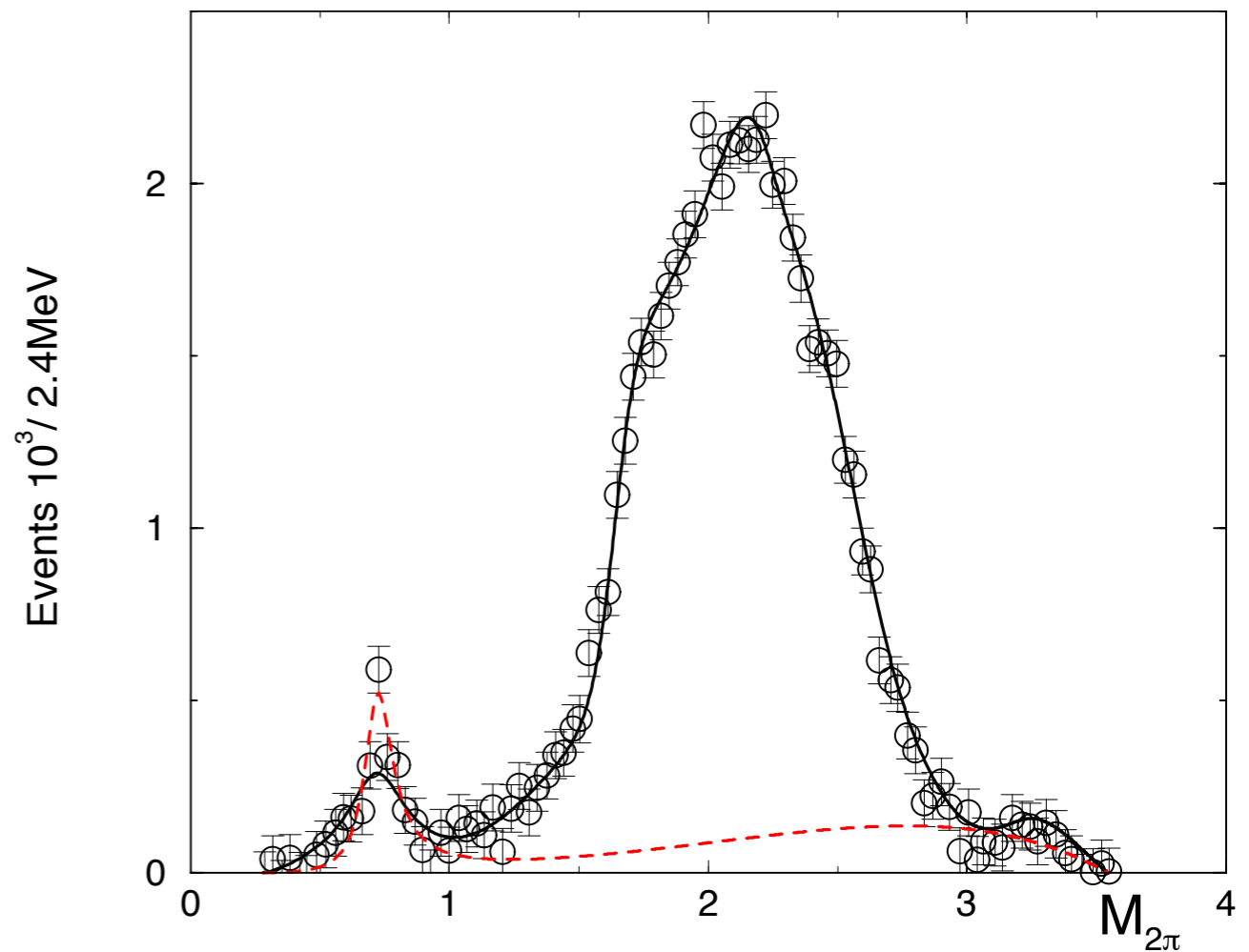


FIG. 3: Dalitz plot projection of the di-pion mass distribution from ψ' decay. The solid is the result of the fit with three amplitudes and the dashed line is the fit with \mathcal{A}_1 alone.

Ongoing analyses

$J/\psi \rightarrow 3\pi$ from BaBar (A.Palano) and BESIII (S.Fergan)

$\omega \rightarrow 3\pi$ from CLAS (g11, A.Celentano, g8, C.Zeoli)

$\phi \rightarrow 3\pi$ from CLAS (GlueX, C.Salgado)

$f_1 \rightarrow 3\pi$ from CLAS (g11, A.Rizo)

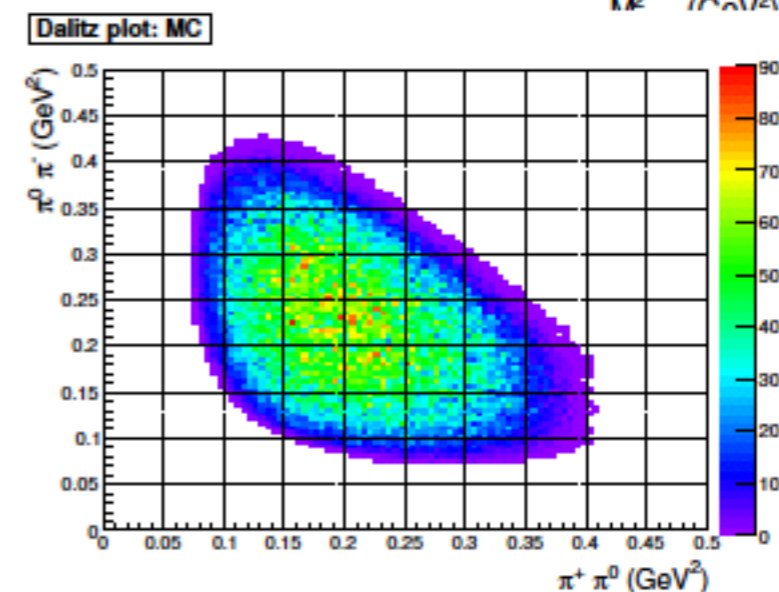
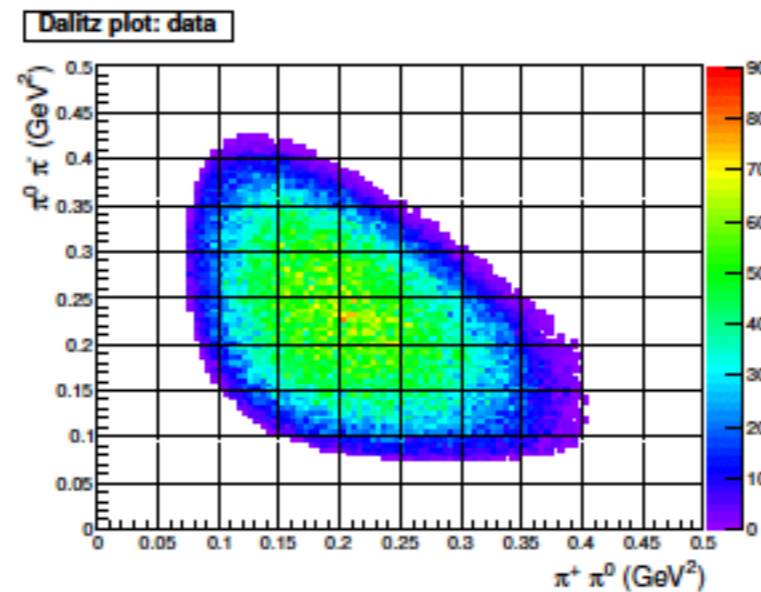
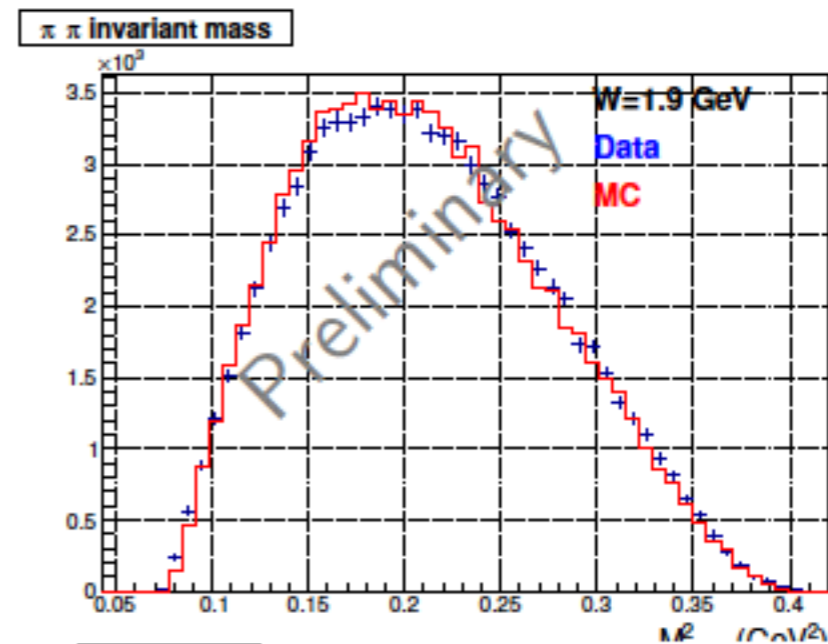
First fit: $A = A_1$ only (ρ pole), using the “nominal” $\pi\pi$ Regge trajectory parametrization:

$$\alpha_s = 0.47 + 0.9s + 0.12i\sqrt{s - 4m_\pi^2} \rightarrow s_\rho = 0.58 - 0.095i \quad (M = 0.76 \text{ GeV}, \Gamma = 0.124 \text{ GeV})$$

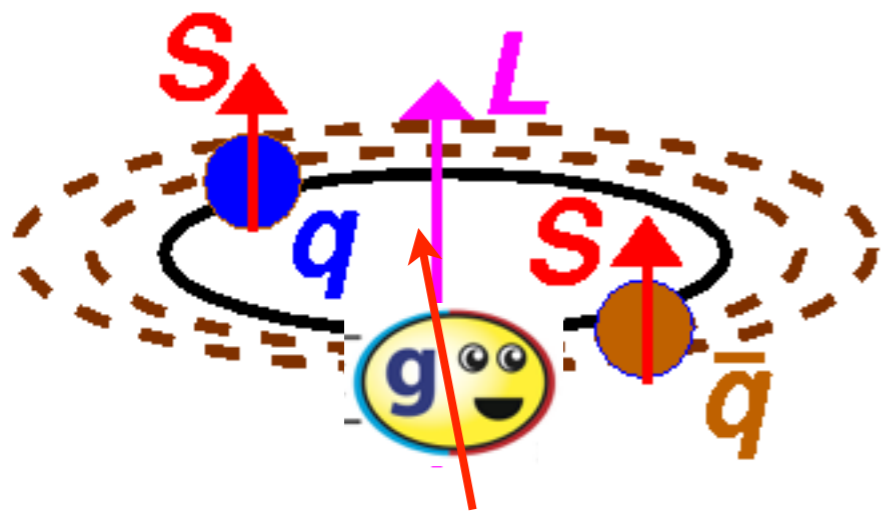
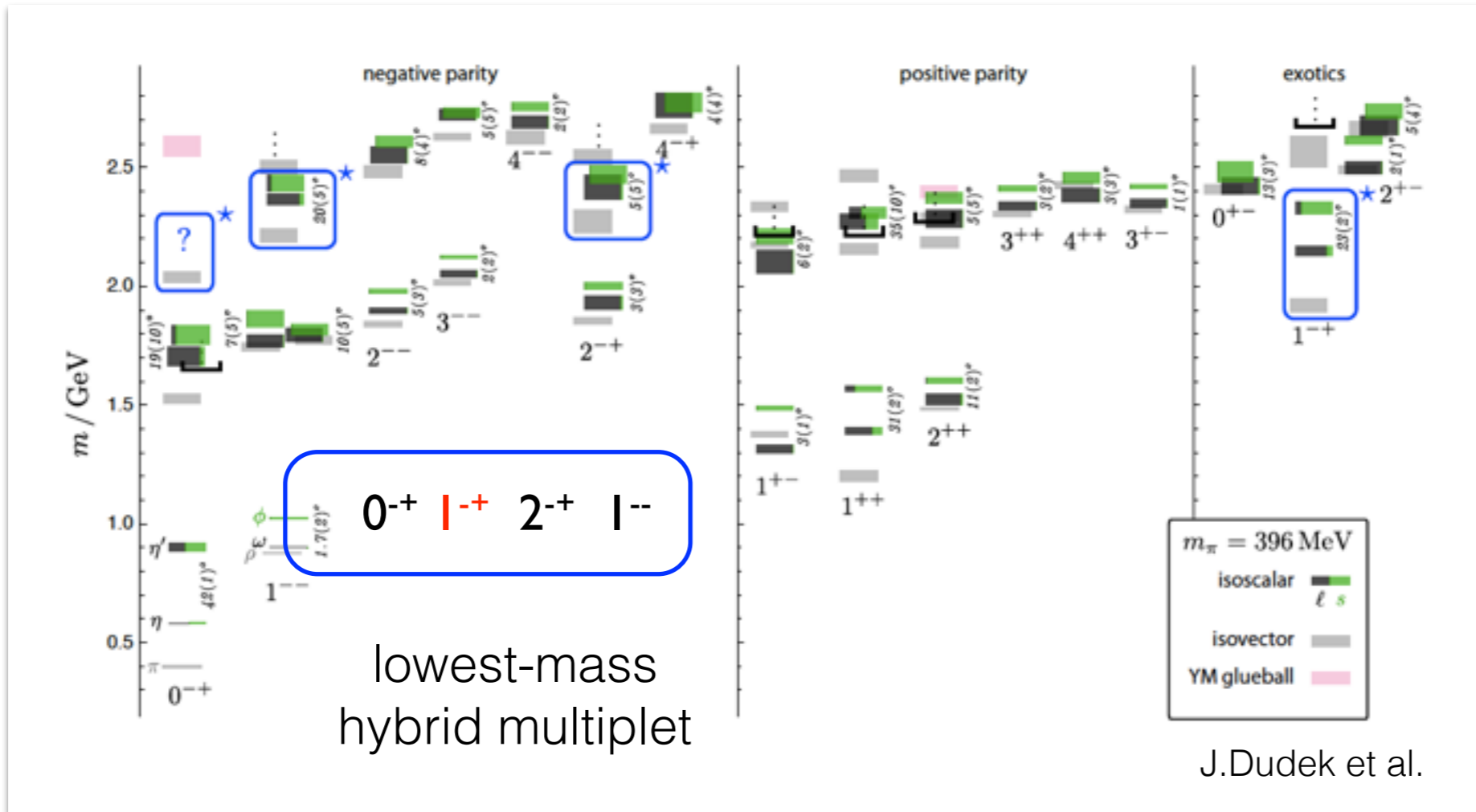
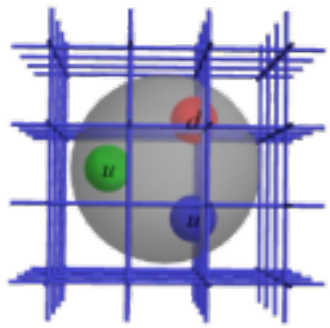
Fit parameters: a_1 (overall normalization)

Results

- Very good agreement data-weighted MC for production variables
- Veneziano model describes well the $\omega \rightarrow 3\pi$ decay
- Fit results are stable as a function of \sqrt{s}



Hybrids

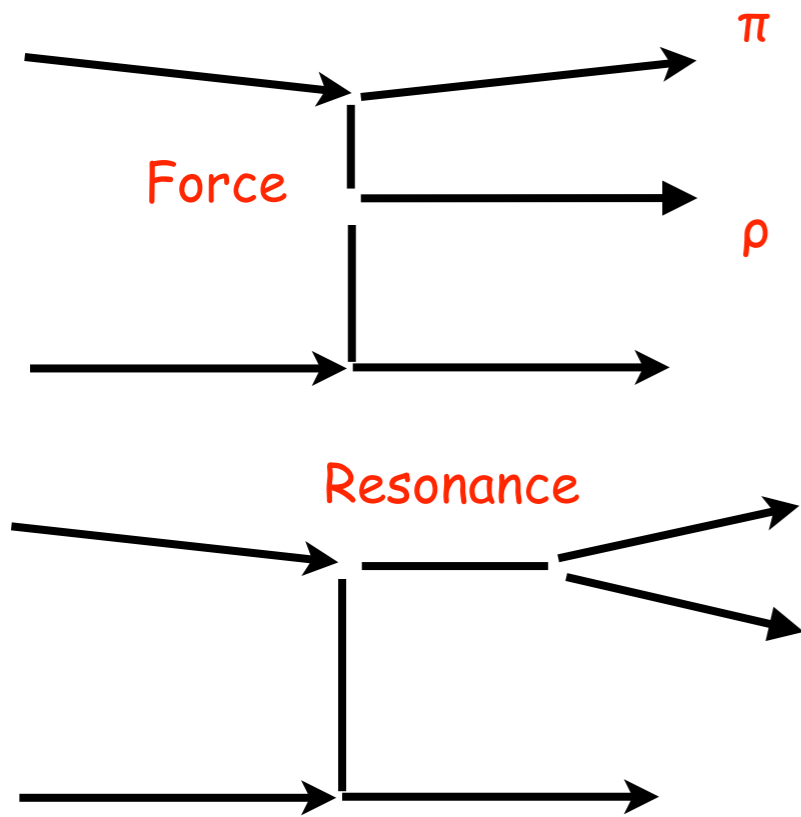


behave

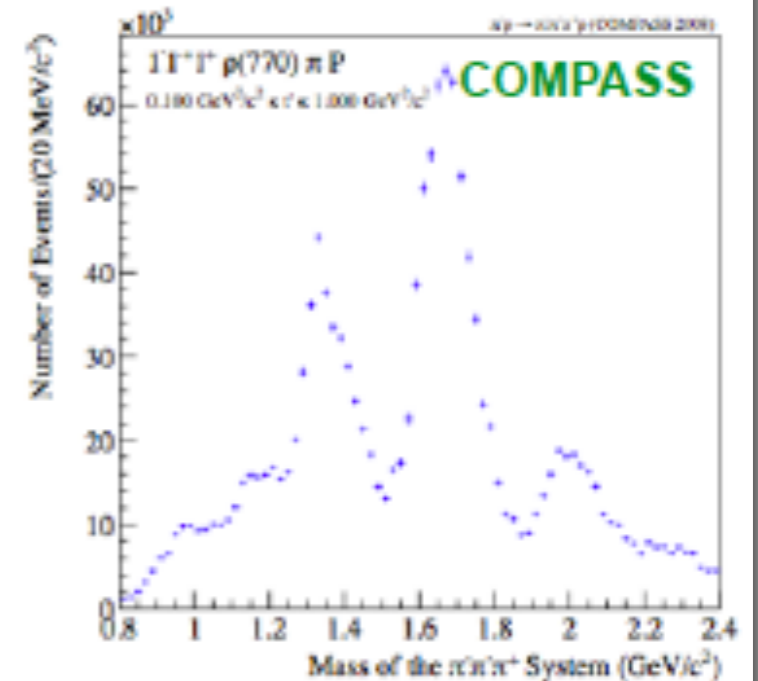
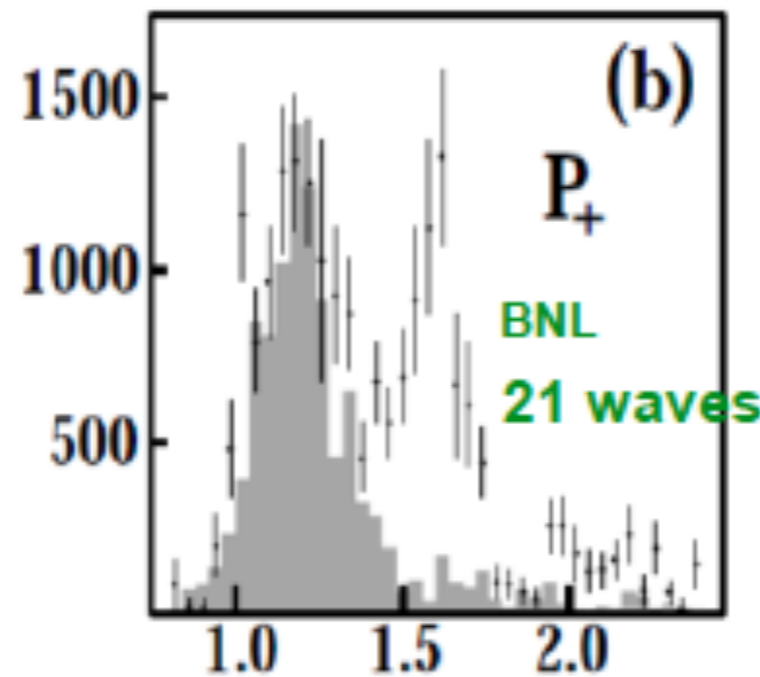
Gluons behave as 1^{+-} quasi-particle
 Prediction of (Coulomb gauge) QCD

$$\begin{array}{c}
 J^{PC} \text{ glue} \\
 \downarrow \\
 1^{+-} \times 0_{S_{Q\bar{Q}}}^{-+} = \boxed{1^{--}} \\
 \downarrow \\
 1^{+-} \times 1_{S_{Q\bar{Q}}=1}^{-+} = \boxed{0^{-+}, 1^{-+}, 2^{-+}}
 \end{array}$$

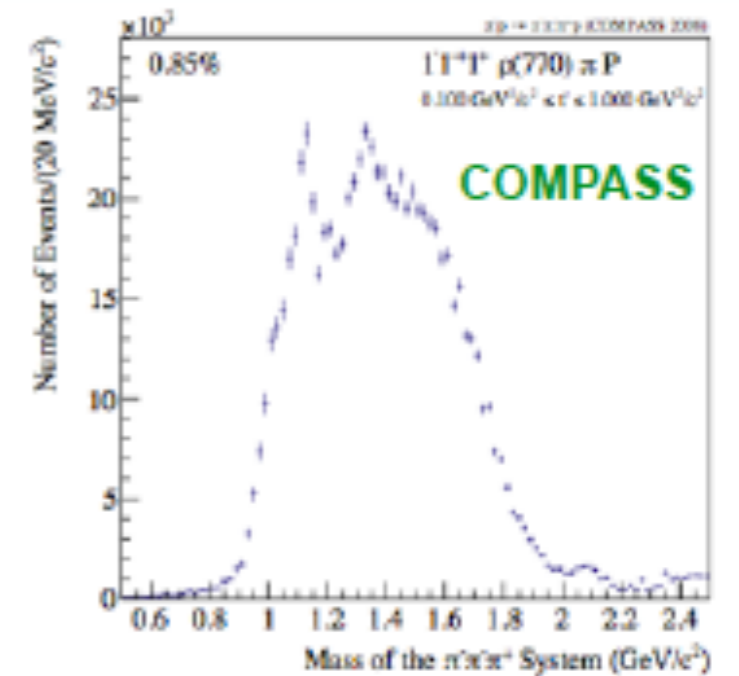
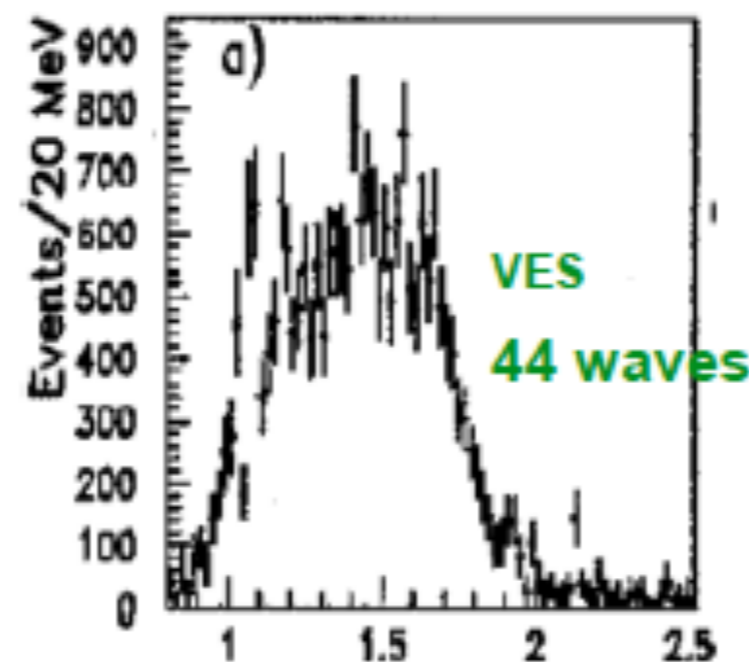
$$\pi^- p \rightarrow \pi^- \pi^+ \pi^- p : 1^{-+} \rho \pi P$$

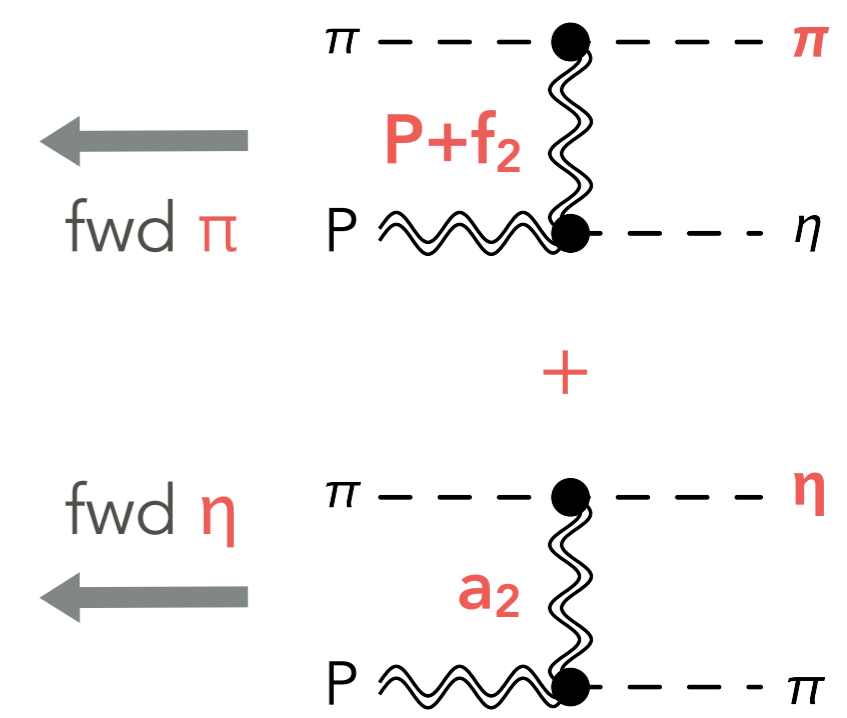
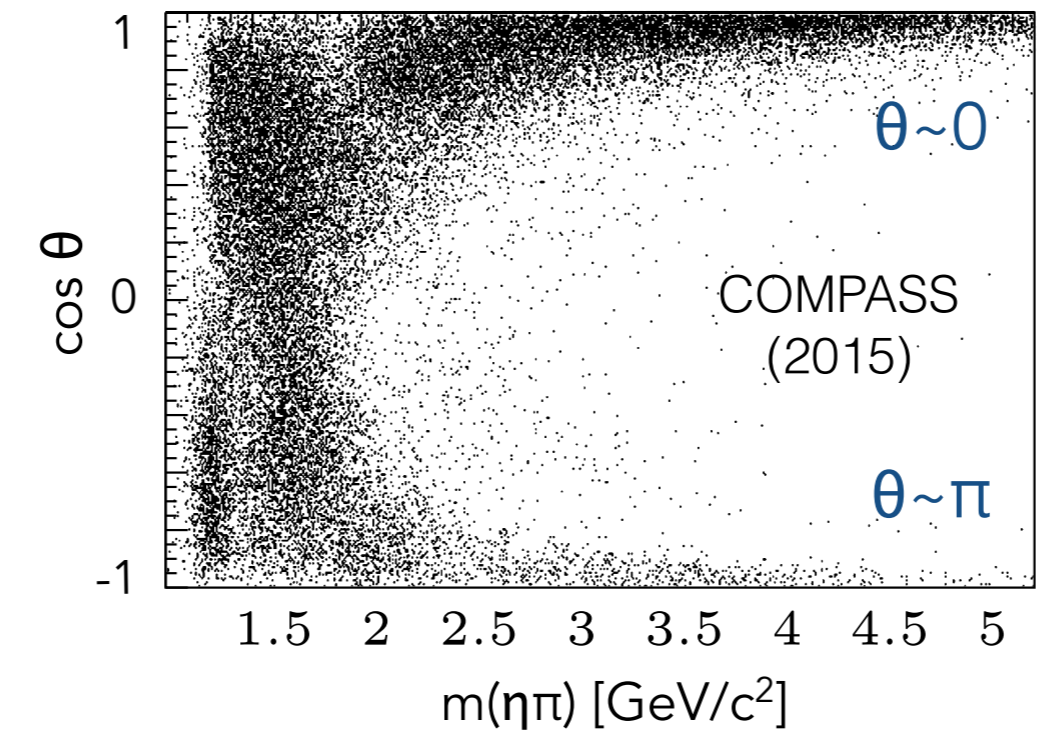
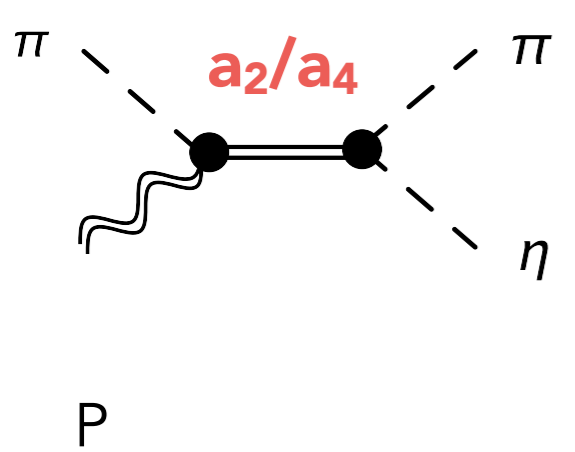
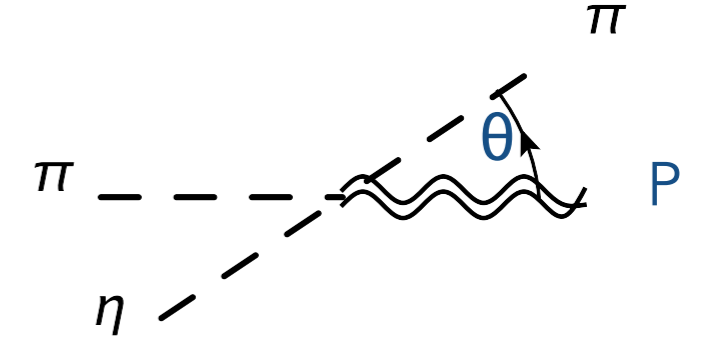
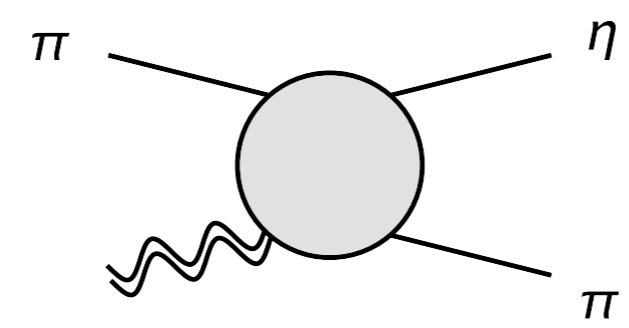
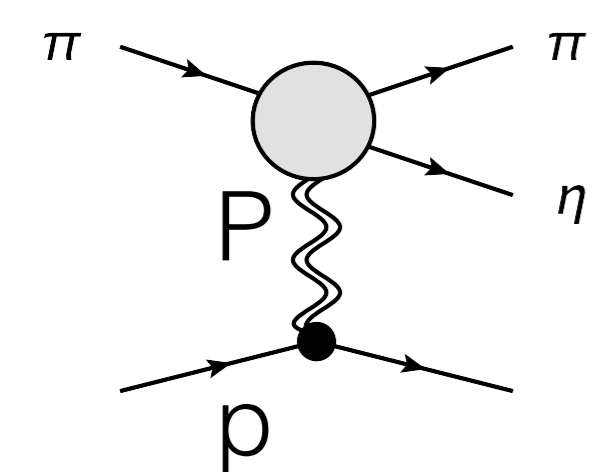


interpretation
ambiguous without
implementing force-
resonance duality
>> Finite Energy
Sum Rules



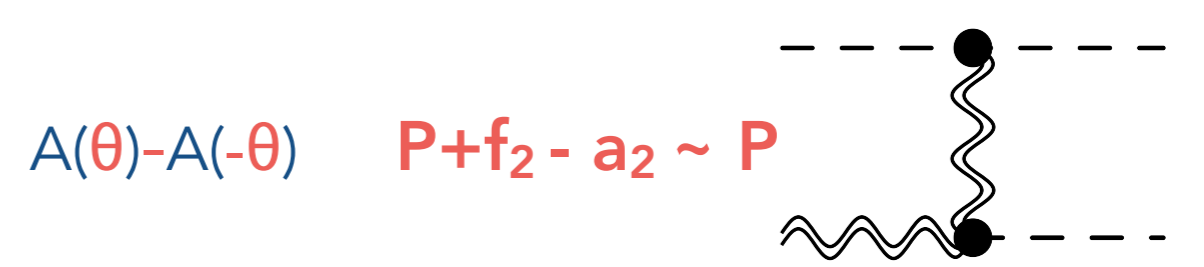
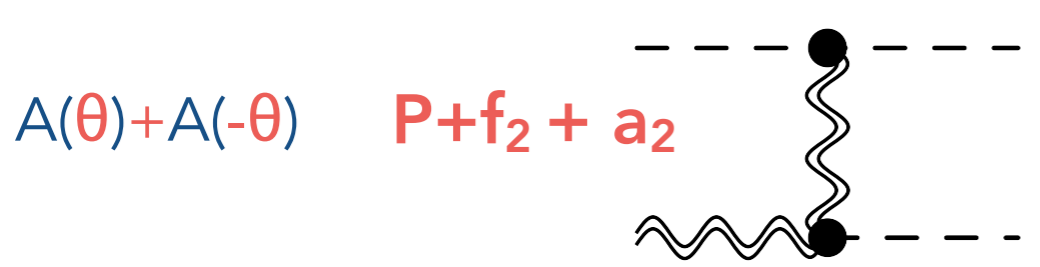
F.Haas, PhD, COMPASS





Even Waves

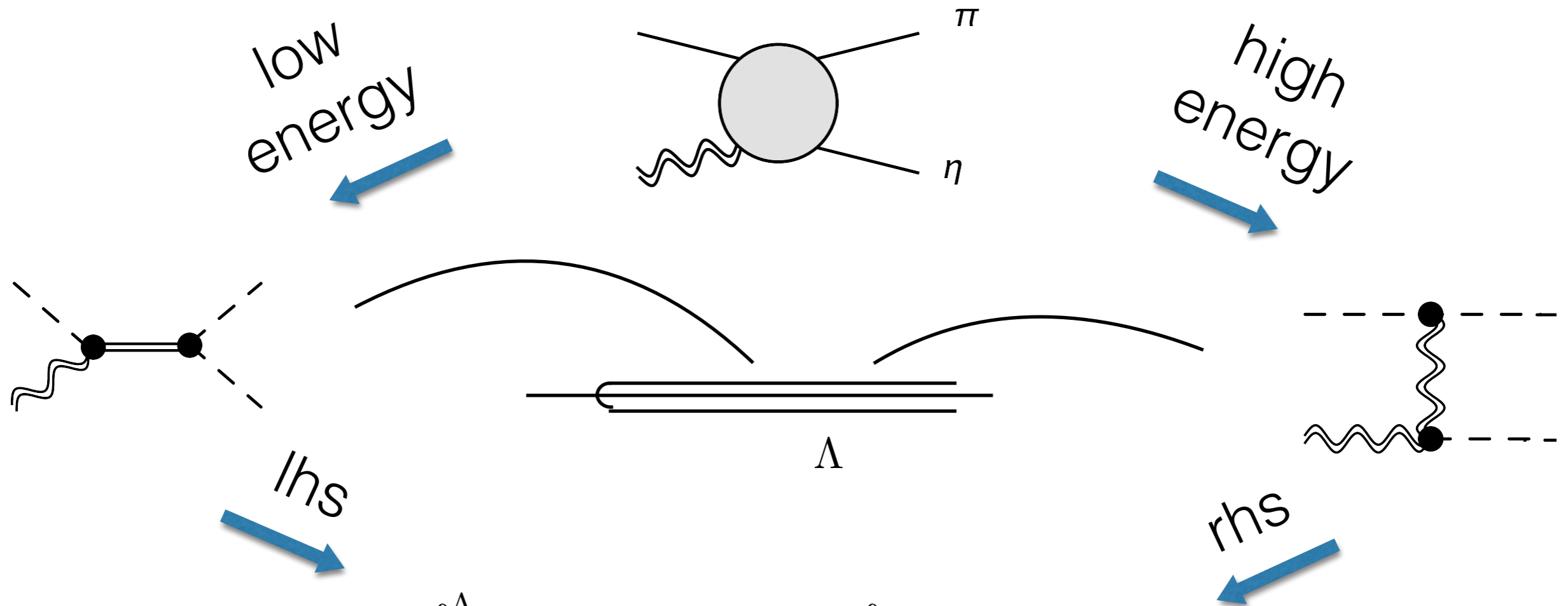
Odd (Exotic) Waves



Asymmetry due P-exchange

Finite energy sum rules

$$A(s_1, t_1, t_2, s_2/s) = \sum_i A_i(s_1, t_1, t_2) \left(\frac{s_2}{s}\right)^2$$



at fixed t_1, t_2

$$\int^{\Lambda} \text{Im} A_i(s_1, t_1, t_2) = \int_{\Lambda} \text{Im} A_i(s_1, t_1, t_2)$$

first time (ever) to be applied in analysis of 2-to-3 reactions!

Summary

- Precision in amplitude analysis is a key to a successful hadron spectroscopy program.
- This requires close collaboration between theory, phenomenology and experiment practitioners.
- Thanks to Mike's vision the foundations for such an effort are now in place through JPAC.