thinking inside the box
- hadron resonances from lattice QCD

Jozef Dudek

Old Dominion University

Jefferson Lab

calculational results from the
hadron spectrum collaboration

lattice QCD

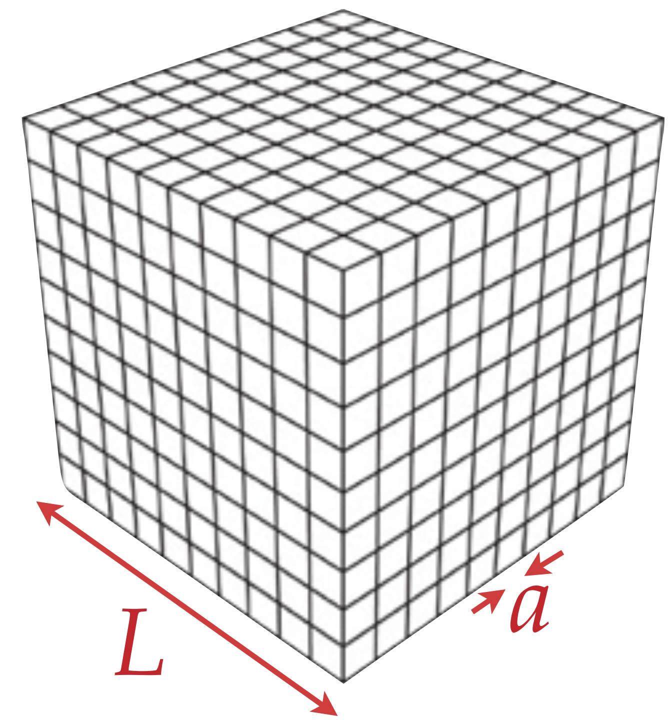
- first-principles numerical approach to the field-theory
  - evaluate **correlation functions**

$$\int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}A_\mu \;\; f(\psi, \bar{\psi}, A_\mu)\; e^{i\int d^4x\, \mathcal{L}(\psi,\bar{\psi},A_\mu)}$$

via **Monte-Carlo** sampling of path-integral

on a **finite cubic grid**

CUBIC LATTICE



lattice QCD

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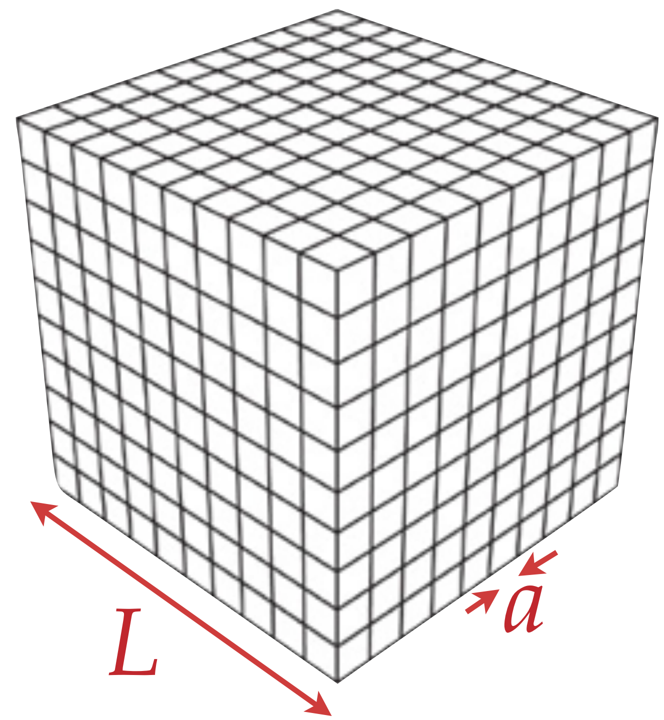

» in principle recover physical QCD as

$$a \to 0 \qquad L \to \infty$$

» practical calculations often use

$$m_q^{\text{calc.}} > m_q^{\text{phys.}}$$

lattice QCD

- first-principles numerical approach to the field-theory
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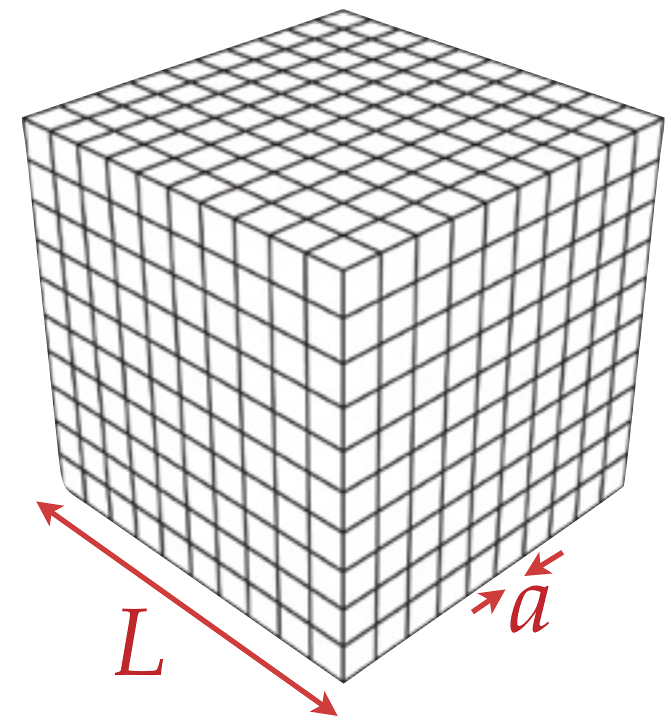
via **Monte-Carlo** sampling of path-integral

on a **finite cubic grid**

  - e.g. discrete spectrum from (euclidean) **two-point correlation functions**

$$\langle 0 | \mathcal{O}(t)\, \mathcal{O}(0) | 0 \rangle = \sum_n e^{-E_n t} \Big| \langle 0 | \mathcal{O} | n \rangle \Big|^2$$

CUBIC LATTICE



» in principle recover physical QCD as

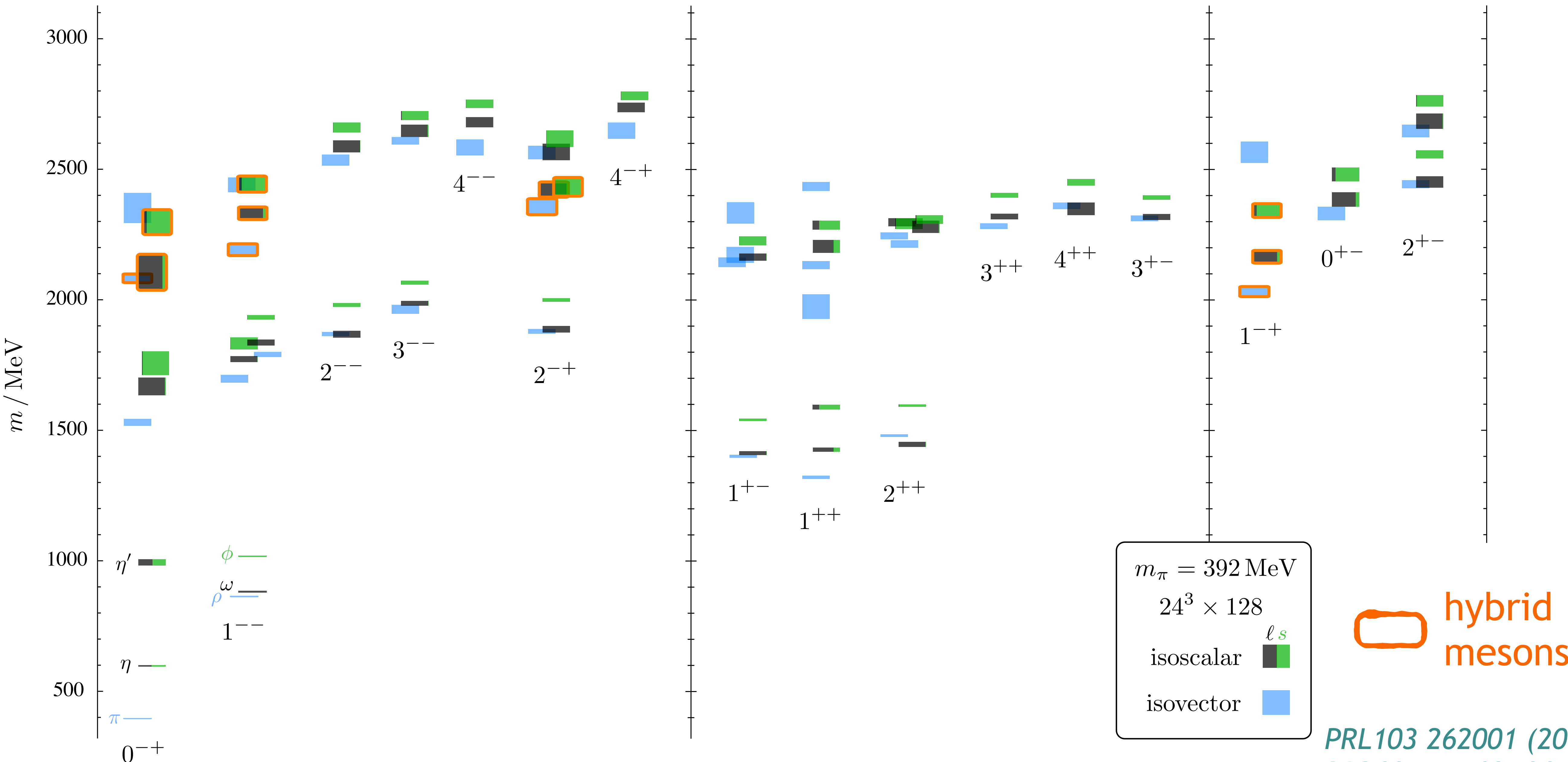
$$a \to 0 \qquad L \to \infty$$

» practical calculations often use

$$m_q^{\text{calc.}} > m_q^{\text{phys.}}$$

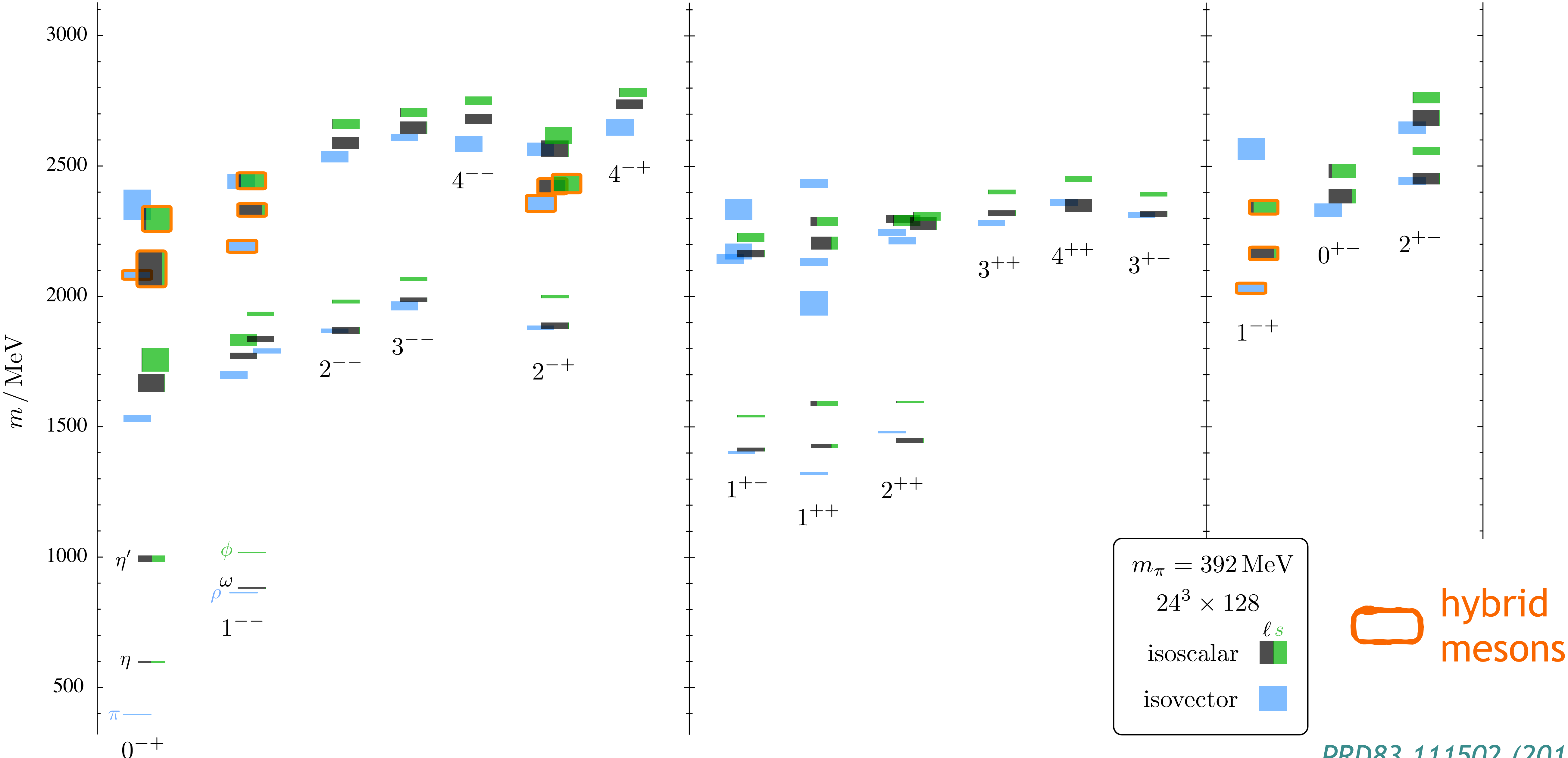
the excited meson spectrum in QCD

- build a big basis of composite QCD operators $\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$
- compute & diagonalize matrix of correlation functions



PRL103 262001 (2009)
PRD83 111502 (2011)
PRD88 094595 (2013)

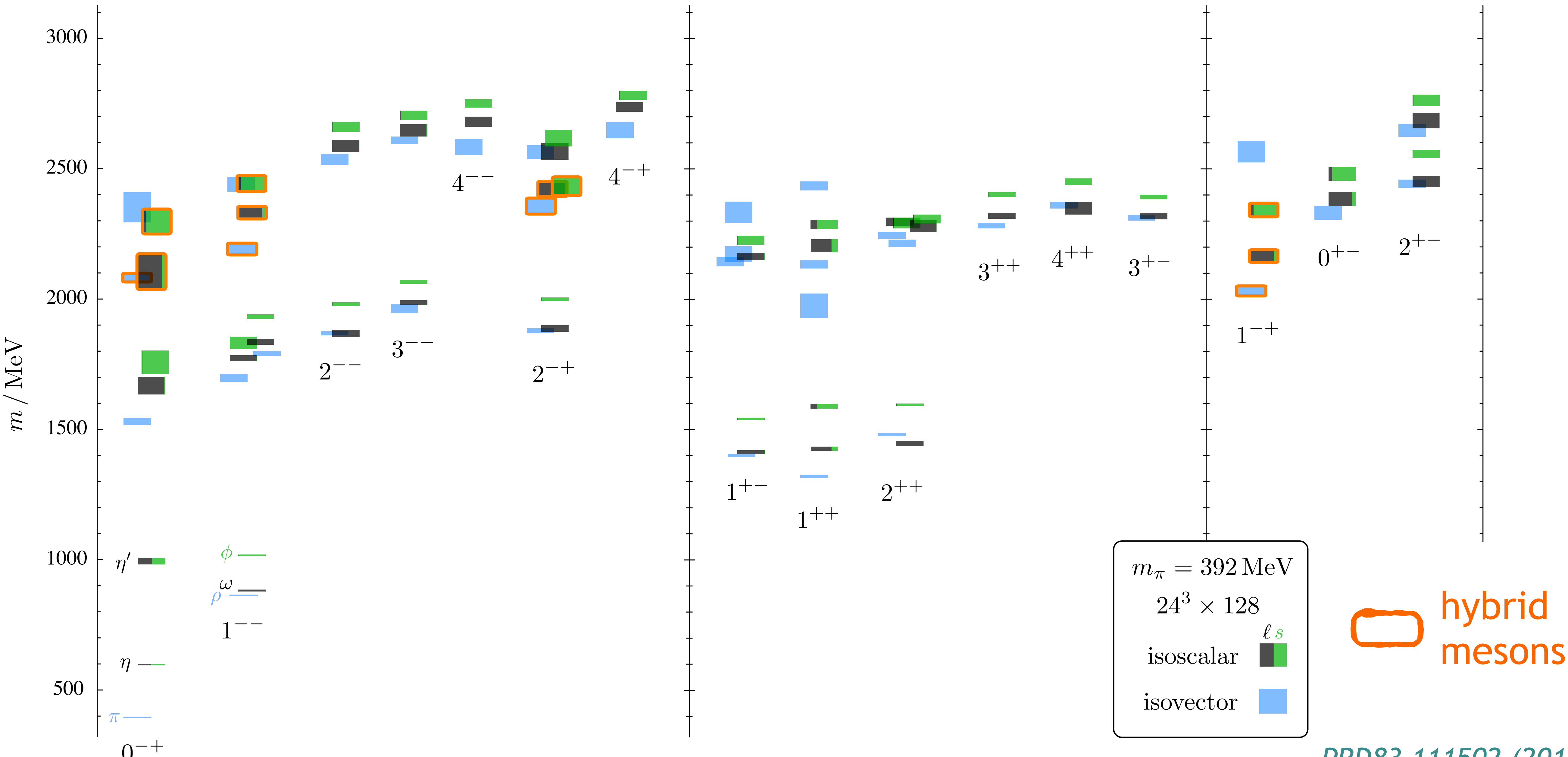
the excited meson spectrum in QCD



PRD83 111502 (2011)
PRD88 094595 (2013)

- looks a lot like the "quark model" ($q\bar{q}$ plus hybrids)

the excited meson spectrum in QCD



PRD83 111502 (2011)
PRD88 094595 (2013)

- looks a lot like the "quark model" ($q\bar{q}$ plus hybrids)
- but we know that's not all there is ...

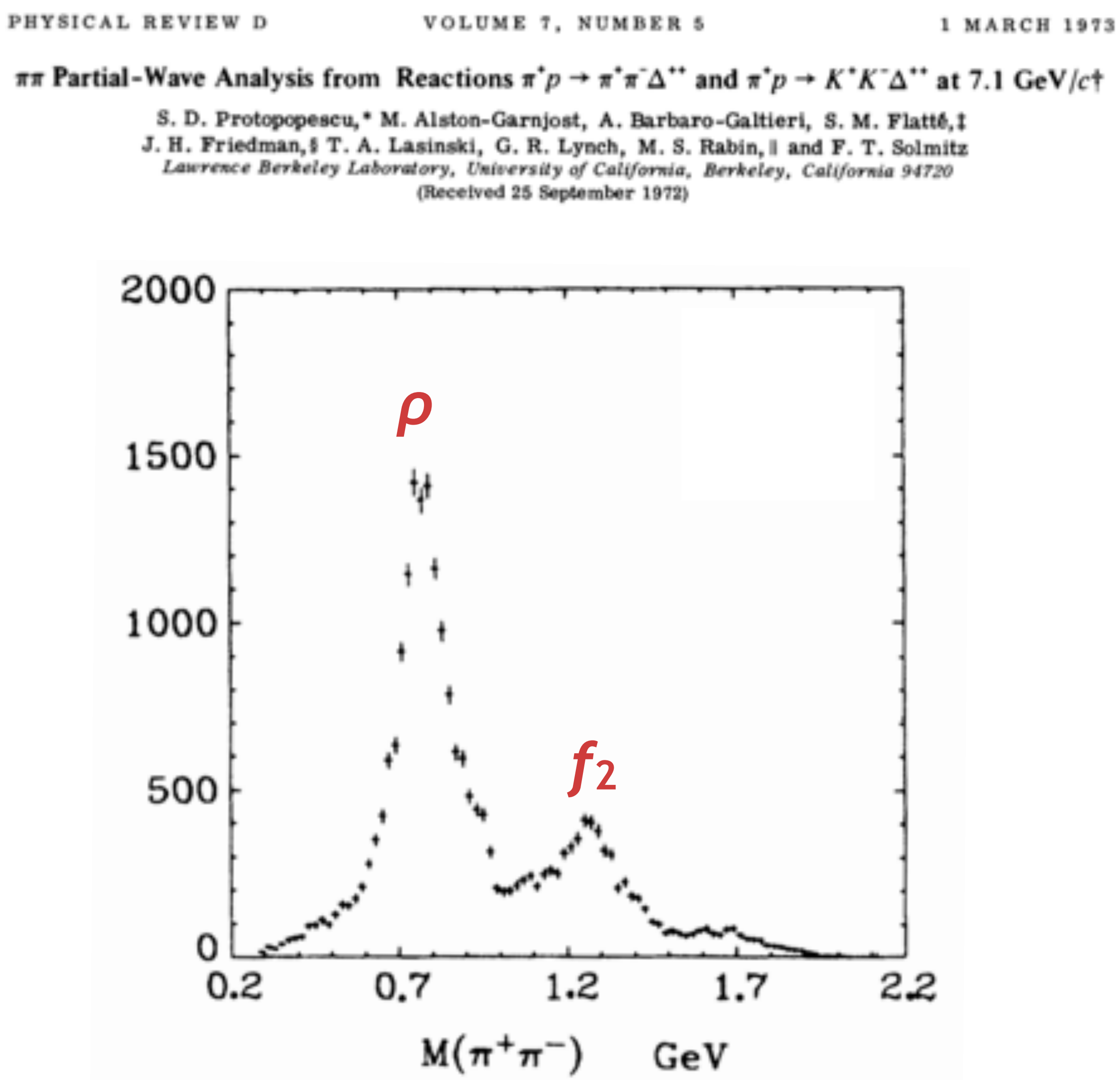
*Au, Morgan and **Pennington***

PHYSICAL REVIEW D VOLUME 35, NUMBER 5 1 MARCH 1987

Meson dynamics beyond the quark model: Study of final-state interactions

excited resonances in QCD

- excited states are really **resonances** in the scattering of lighter hadrons


PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

ππ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡ J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,∥ and F. T. Solmitz

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 25 September 1972)

continuous distribution of hadron states

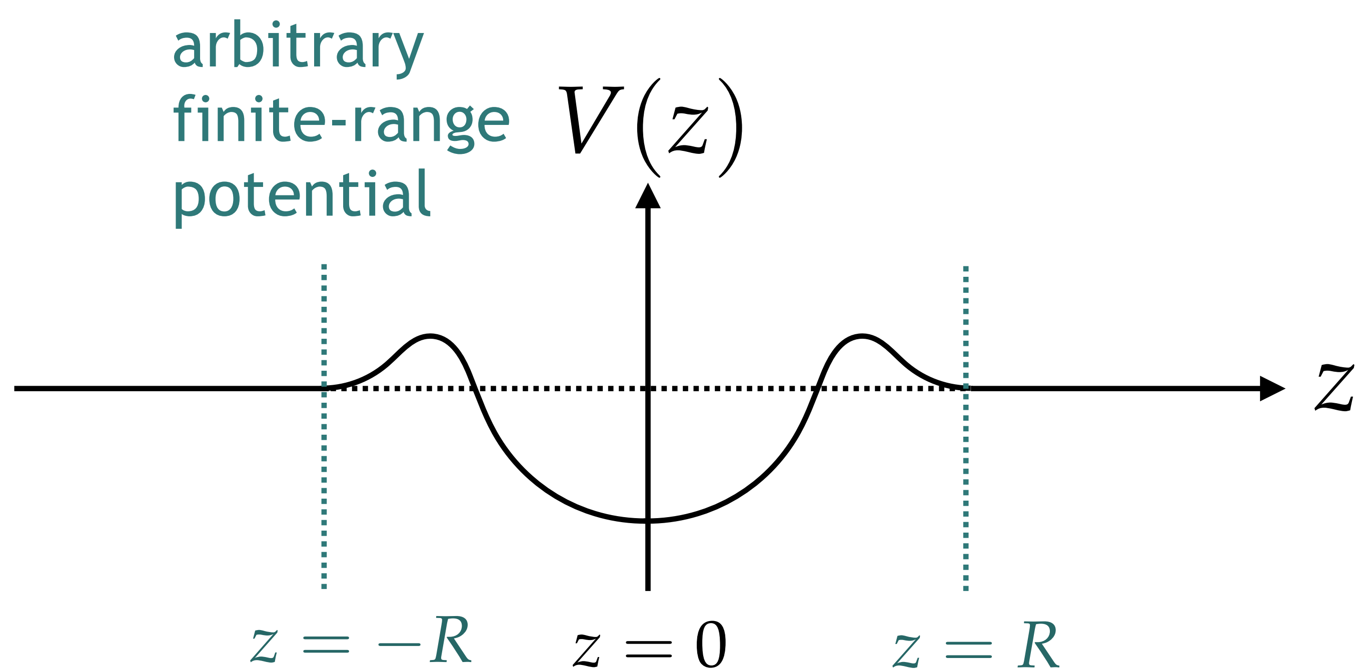
this **decay physics** should be captured in first-principles approaches to QCD

can this be achieved within lattice QCD ?

(*where the spectrum is* **discrete**)

elastic scattering in quantum mechanics

- consider scattering of two identical bosons (in one space dimension)



outside the well

$$\psi(|z|>R) \sim \cos\big(p|z| + \delta(p)\big)$$

elastic scattering in quantum mechanics

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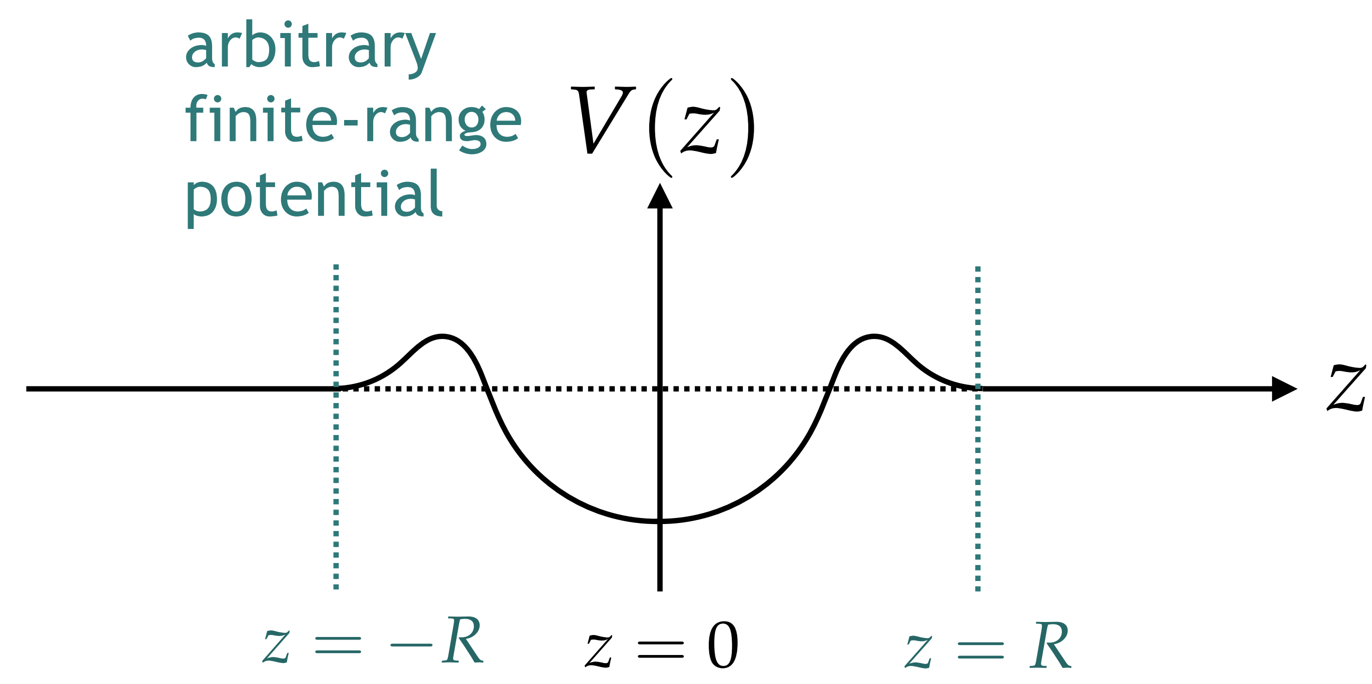


outside the well

$$\psi(|z|>R)\sim\cos\big(p|z|+\delta(p)\big)$$

elastic scattering phase-shift

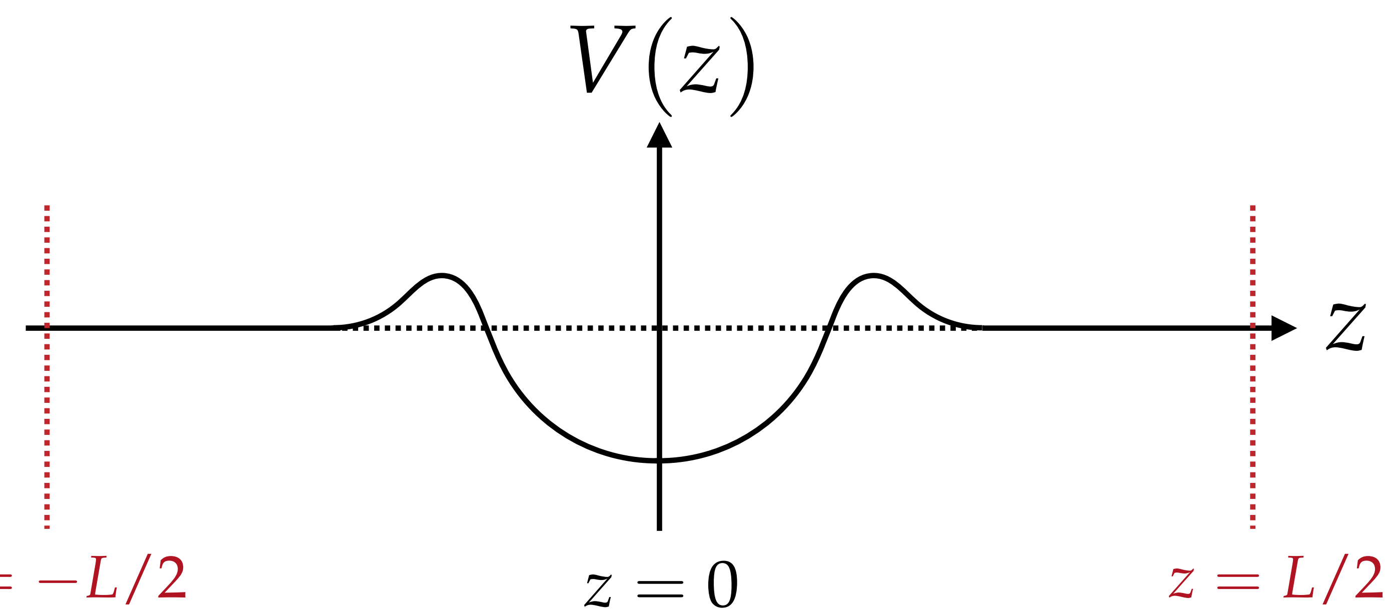
continuous momentum $\Rightarrow$ continuous spectrum of energy eigenstates think outside the box …

inside

think ~~outside~~ the box …

‘scattering’ in a finite-volume

- put the system in a **periodic box**



outside the well

$$\psi(|z| > R) \sim \cos\big(p|z| + \delta(p)\big)$$

- apply periodic boundary conditions

$$\left.\begin{aligned} \psi(-L/2) &= \psi(L/2) \\ \frac{d\psi}{dz}(-L/2) &= \frac{d\psi}{dz}(L/2) \end{aligned}\right\} \frac{pL}{2} + \delta(p) = n\pi$$

$$\boxed{p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)}$$

discrete energy spectrum

3+1 dim field theory version due to Lüscher

ρ resonance in $\pi\pi$ scattering

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

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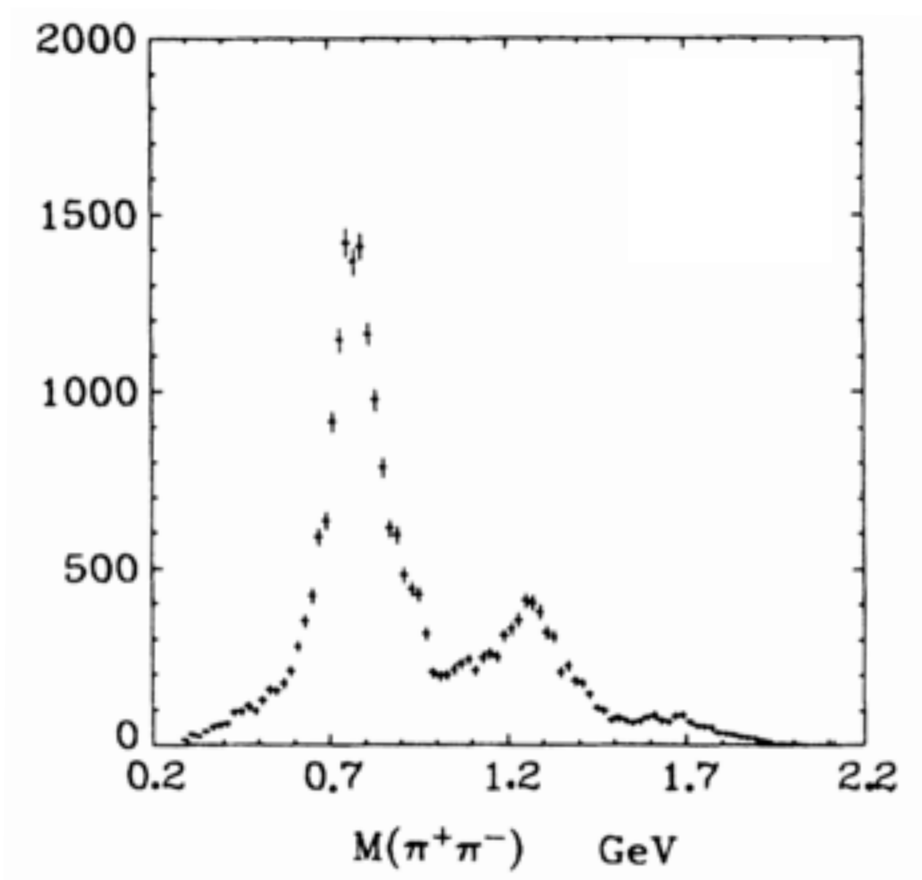
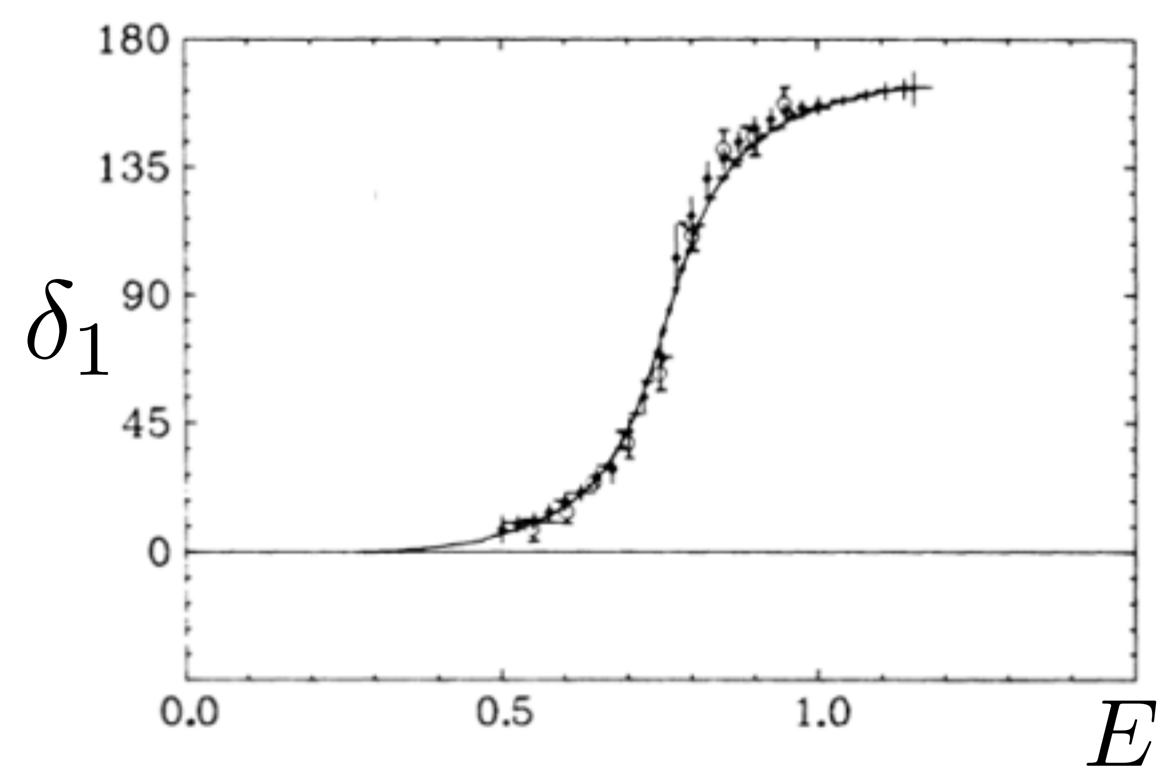


PARTIAL WAVE AMPLITUDE

$$f_\ell(E) = \frac{1}{2i}\left(e^{2i\,\delta_\ell(E)} - 1\right)$$

RESONANT PHASE SHIFT



Pennington and Protopopescu

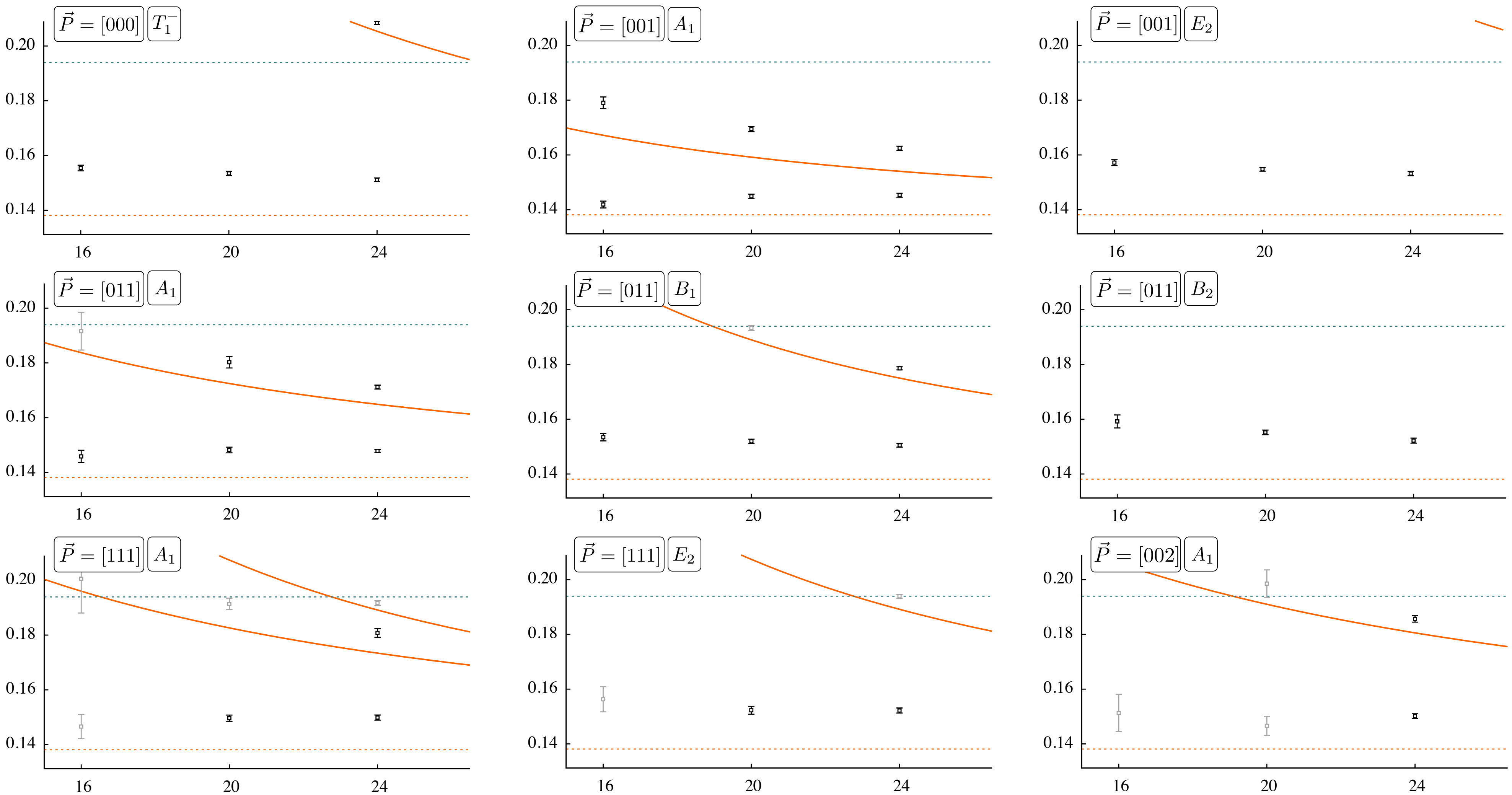
PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

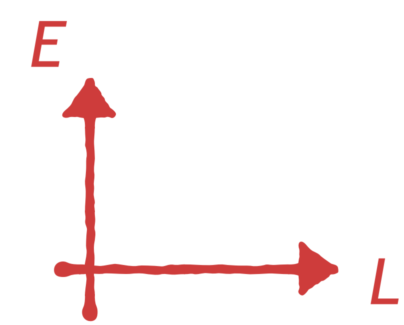
$\pi\pi$ Scattering Amplitude in the Low-Energy Region*

ρ resonance in $\pi\pi$ scattering

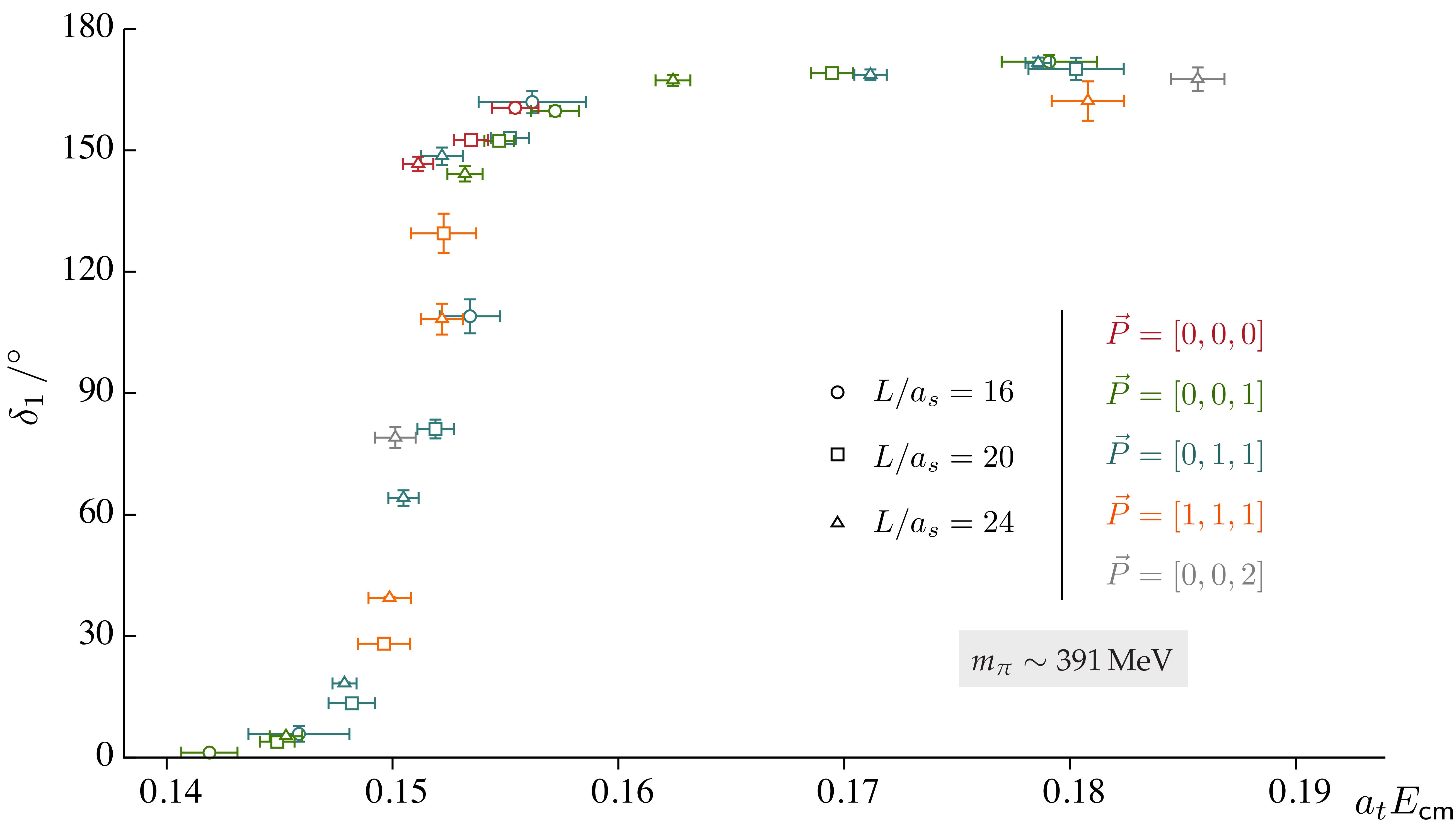
- discrete spectrum in *L*×*L*×*L* lattice QCD boxes

$m_\pi \sim 391\,\mathrm{MeV}$





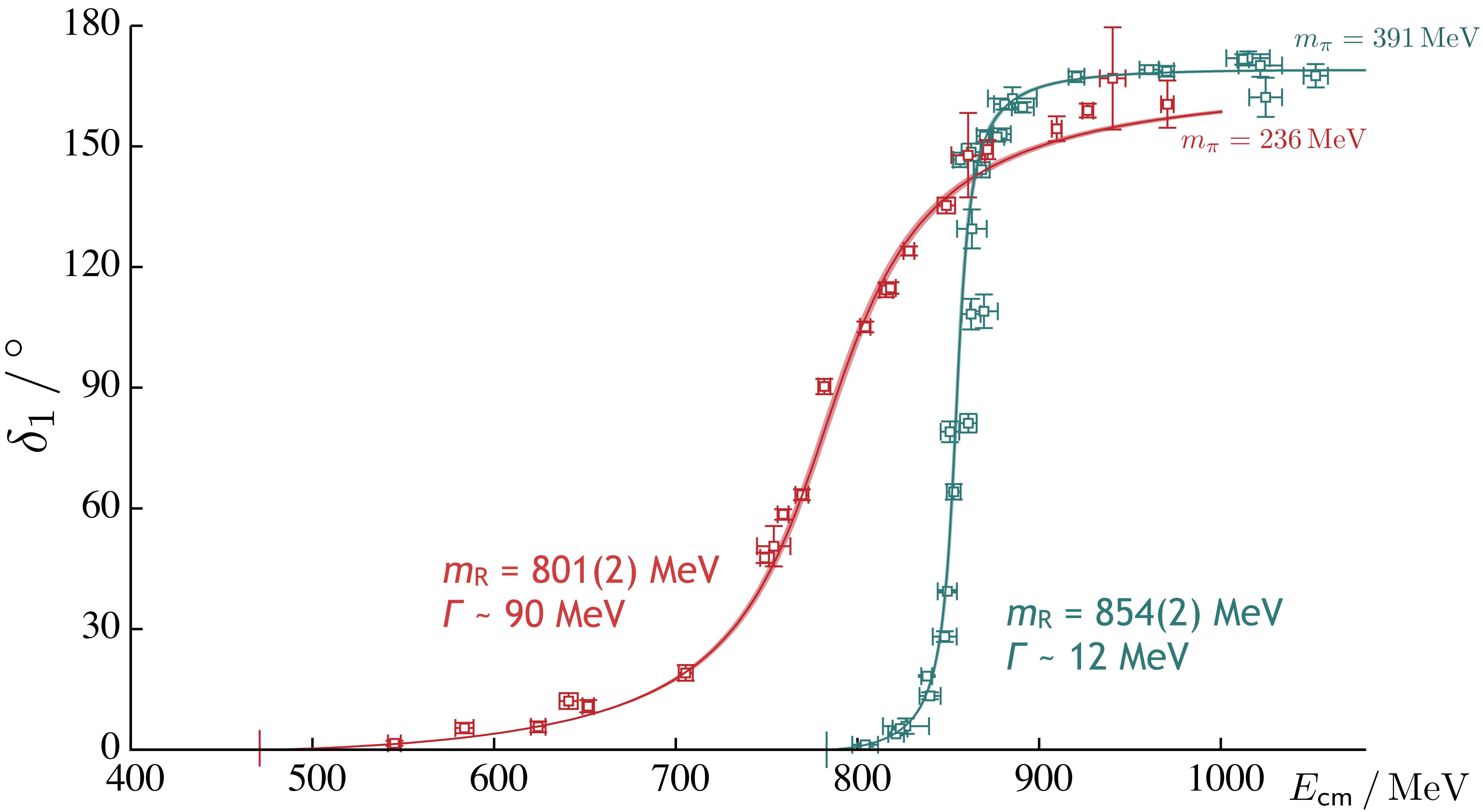
$\pi\pi$ *P*-wave phase-shift



PRD87 034505 (2013)

$\pi\pi$ *P*-wave phase-shift

- reducing the pion mass moves ρ mass, width in the right direction …



PRD87 034505 (2013)
PRD92 094502 (2015)

coupled-channel resonances in QCD

- but most excited resonances decay to more than one final state

coupled-channel resonances

Au, Morgan and ***Pennington***

PHYSICAL REVIEW D VOLUME 35, NUMBER 5 1 MARCH 1987

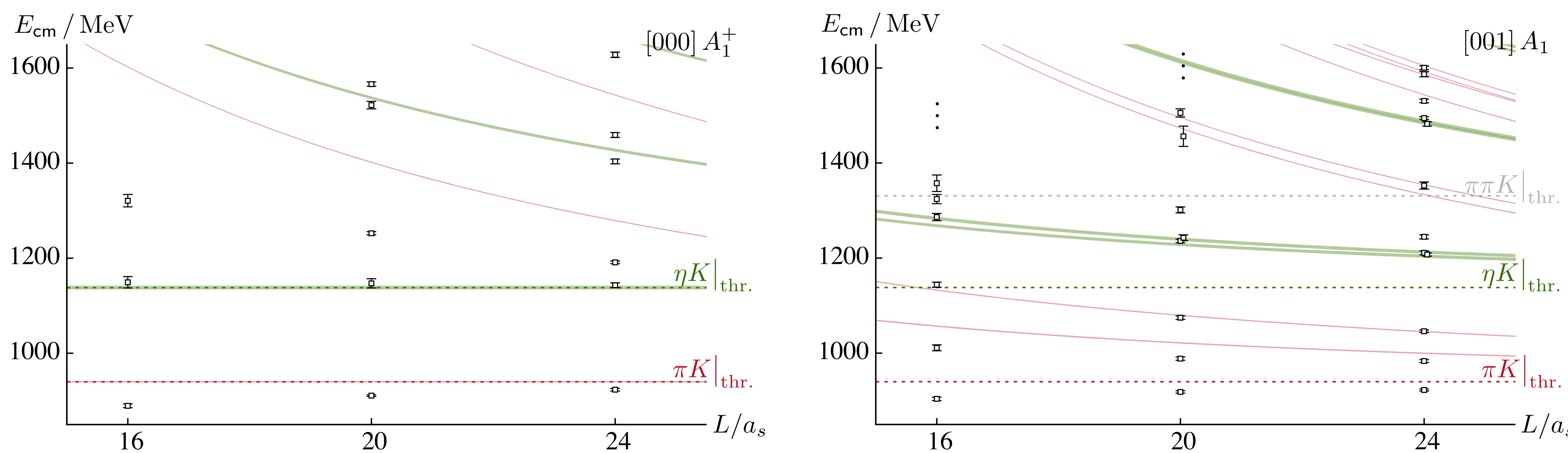
Meson dynamics beyond the quark model: Study of final-state interactions

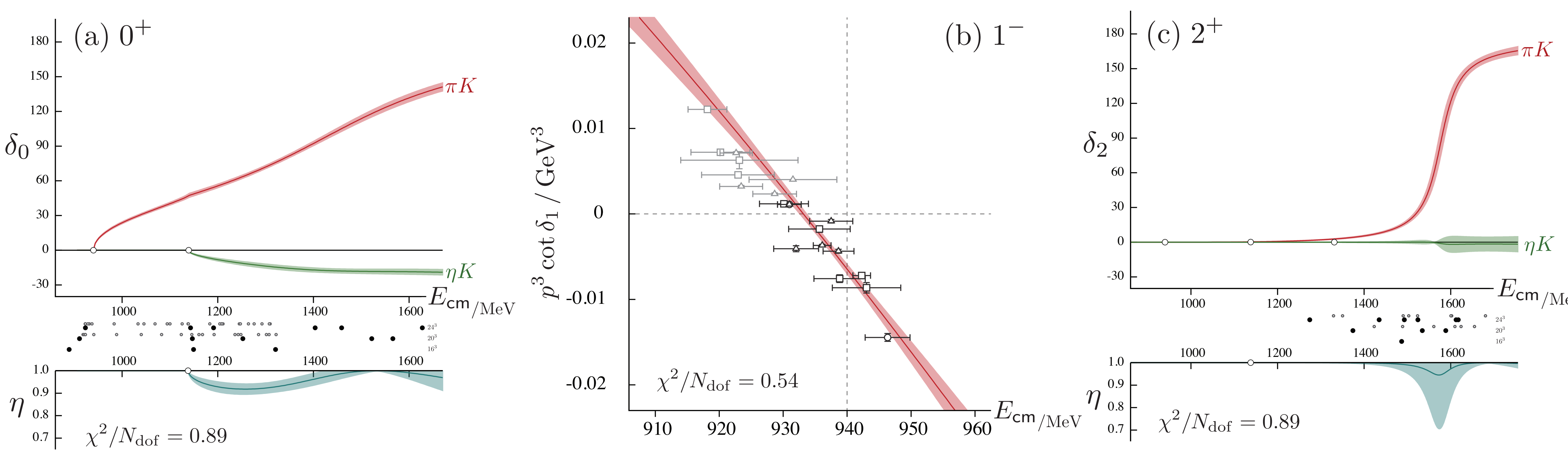
things get more interesting with strongly coupled channels ...

coupled-channel resonances in QCD

- first case calculated explicitly: *πK*/*ηK*

PRL113 182001 (2014)
PRD91 054008 (2015)

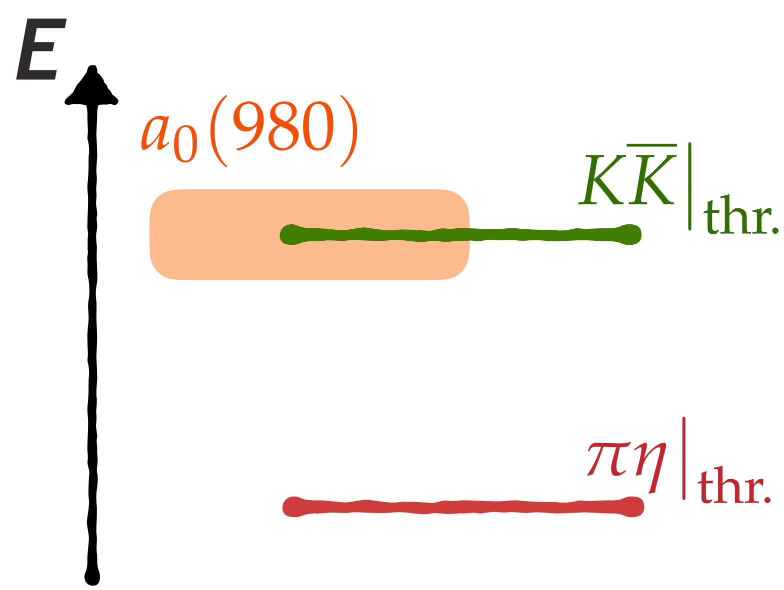


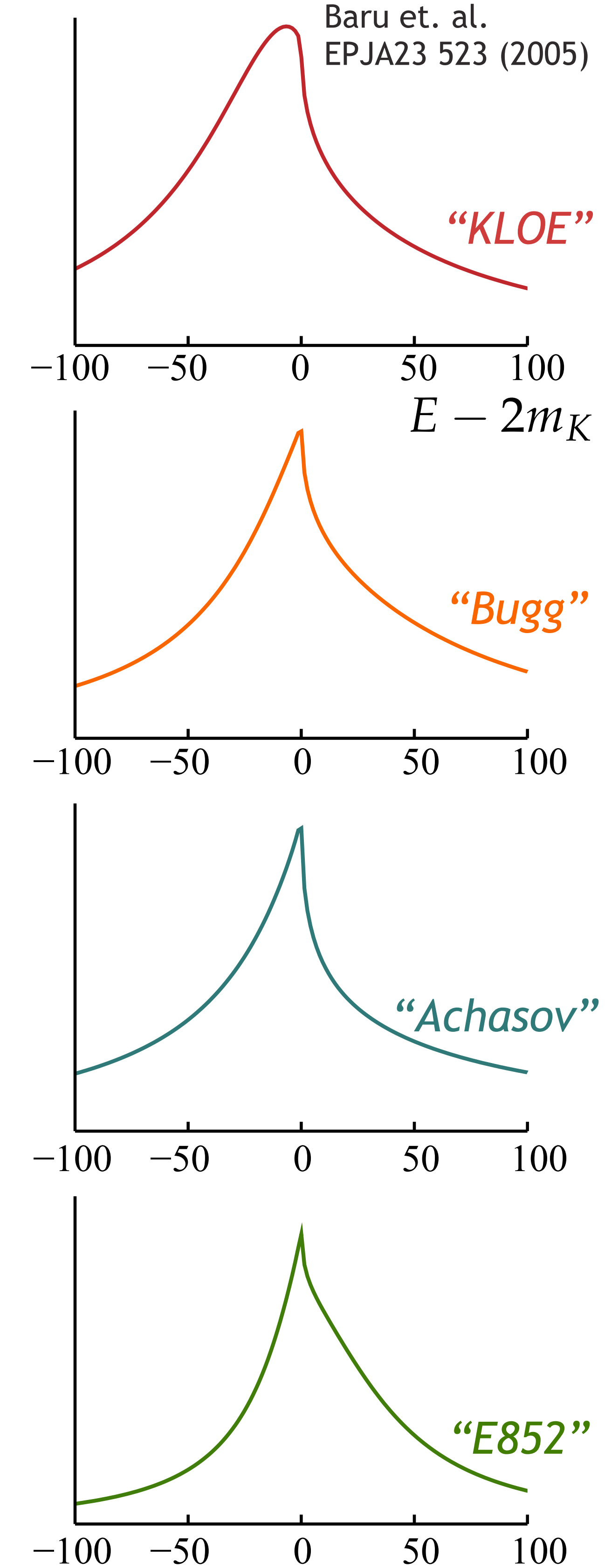


but these channels not strongly coupled …

$\pi\eta/K\bar{K}$ scattering and the $a_0(980)$

- sharp experimental enhancement at $K\bar{K}$ threshold





- usually observed in 'less-simple' production processes

e.g. $p\bar{p} \to \pi\pi\eta$

$\phi \to \gamma\pi\eta$

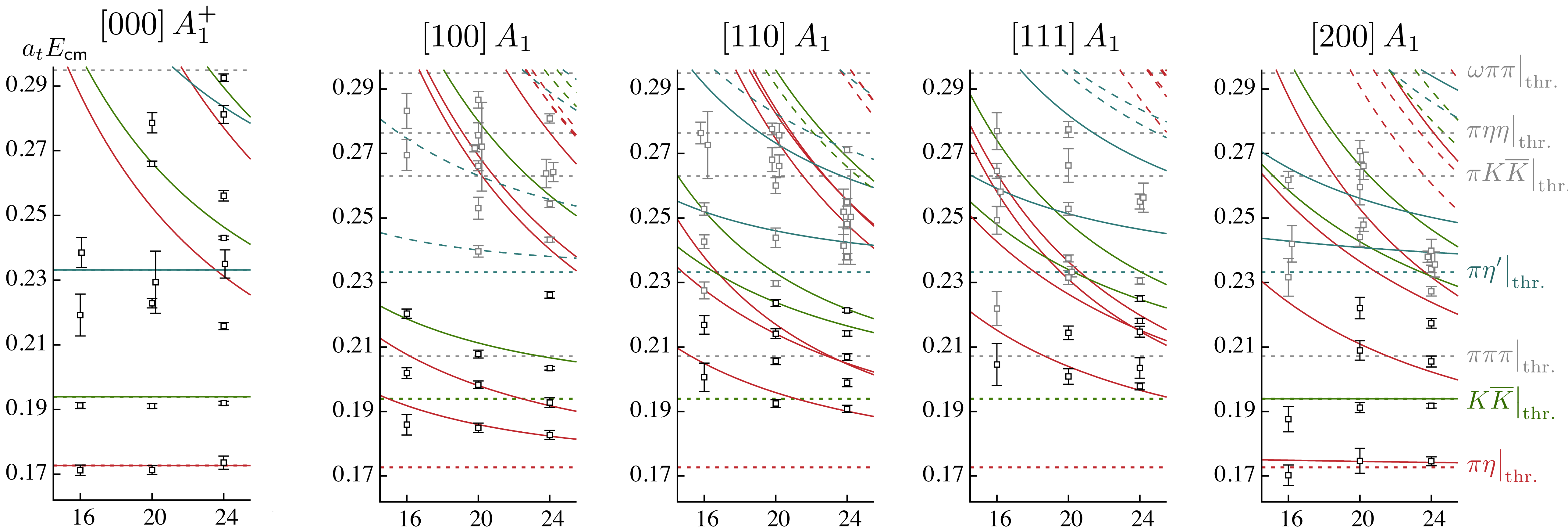
- amplitude models typically give $\dfrac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$

$\pi\eta/K\bar{K}$ scattering

- discrete spectrum in $L\times L\times L$ boxes

$m_\pi \sim 391\,\mathrm{MeV}$

PRD93 094506 (2016)



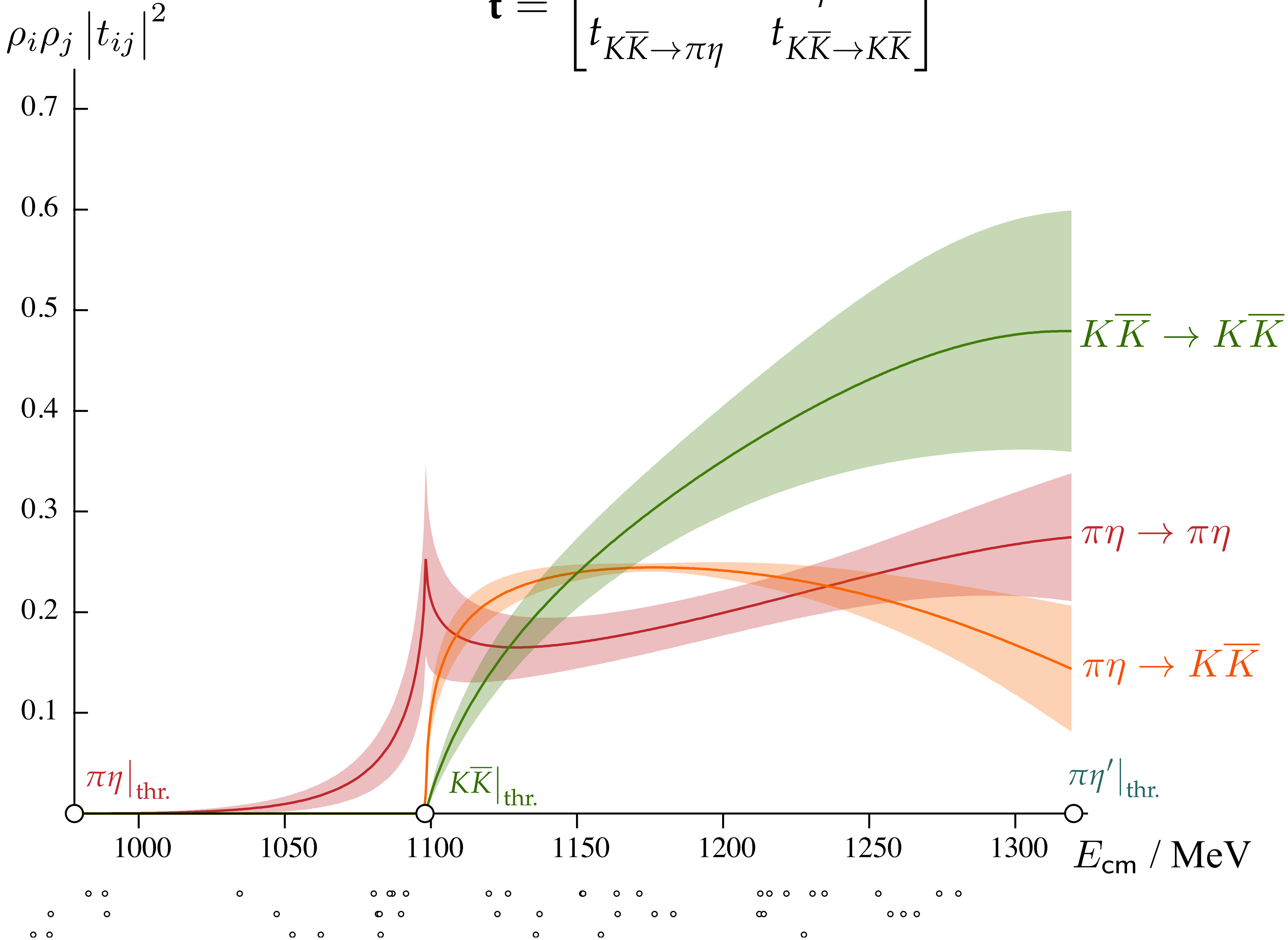
$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$

- scattering amplitudes

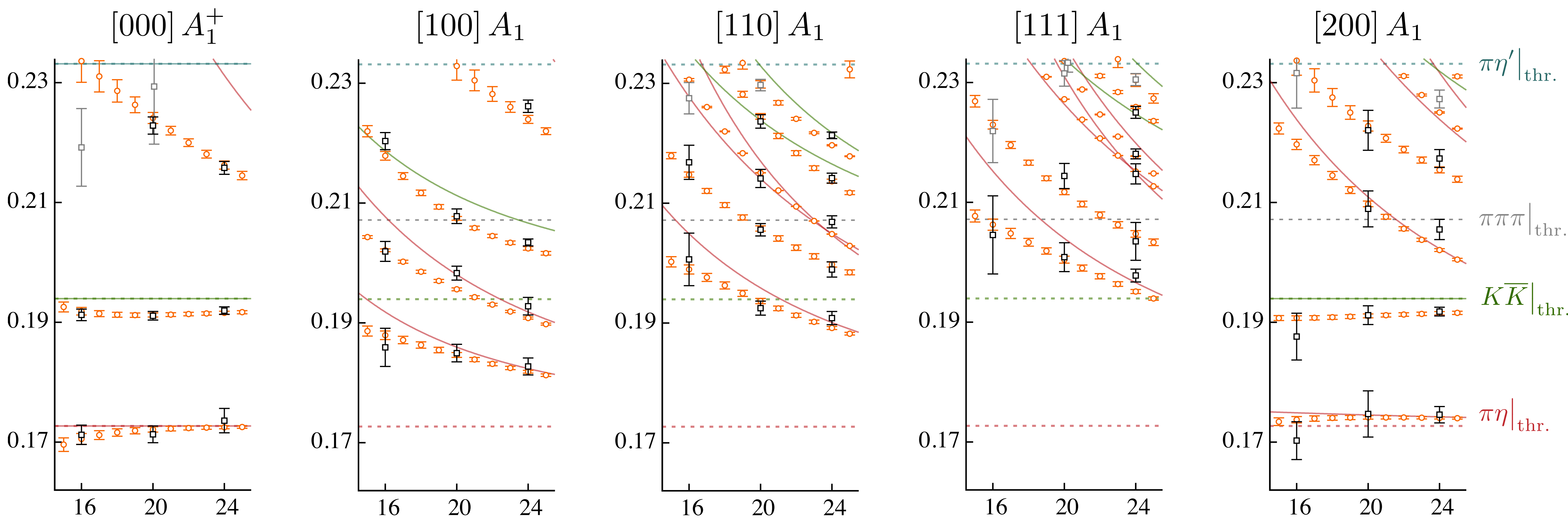
$$\mathbf{t} = \begin{bmatrix} t_{\pi\eta\to\pi\eta} & t_{\pi\eta\to K\bar{K}} \\ t_{K\bar{K}\to\pi\eta} & t_{K\bar{K}\to K\bar{K}} \end{bmatrix}$$

$m_\pi \sim 391\,\text{MeV}$

PRD93 094506 (2016)

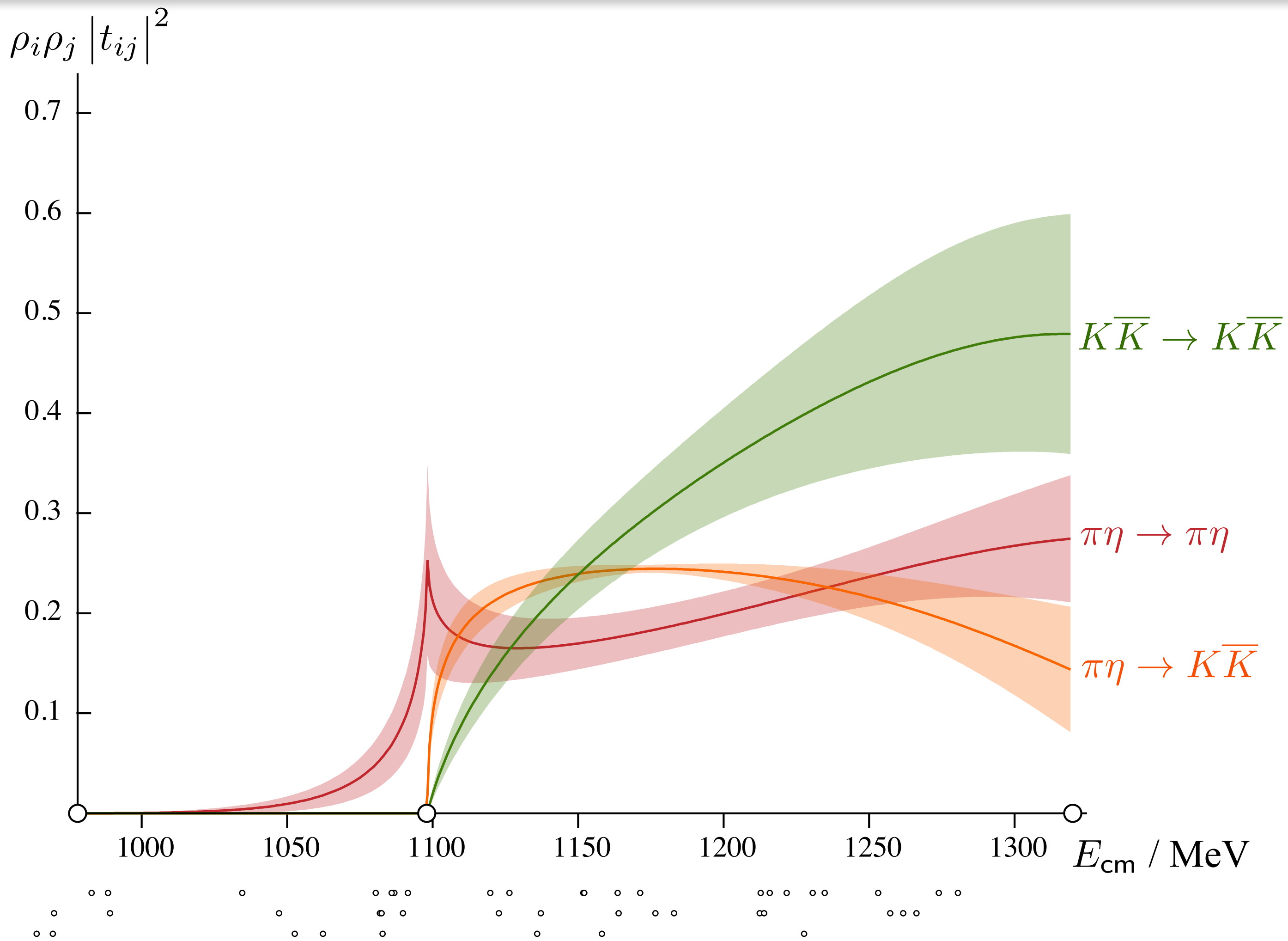


$\pi\eta/K\bar{K}$ scattering



- these amplitudes describe the calculated spectra

$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$



$m_\pi \sim 391\,\mathrm{MeV}$

PRD93 094506 (2016)

strong cusp in $\pi\eta$ at $K\bar{K}$ threshold

rapid turn-on of $K\bar{K}$ amplitudes

indicative of a nearby **resonance** ?

can you fit poles in your box ?

*Morgan and **Pennington***

VOLUME 59, NUMBER 24 PHYSICAL REVIEW LETTERS 14 DECEMBER 1987

Pole Structures in Nuclear and Particle Physics

Resonances correspond to poles on unphysical sheets of the energy plane. Experiment usually reflects only that on the nearest of these and the more distant ones are shadow poles. However, in special circumstances, near to strongly opening thresholds, more than one pole can become physically significant.

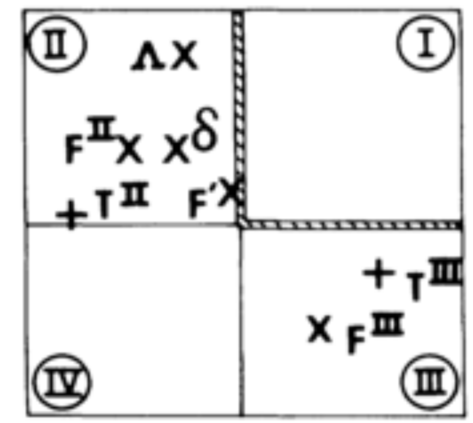


FIG. 1. Complex k_2-plane display of various resonance-pole locations (cf. text).

resonance

= a **pole** at complex $s = s_0$

$$t_{ij}(s) \sim \frac{g_i\, g_j}{s_0 - s}$$

Re[$\sqrt{s_0}$] ~ 'mass'

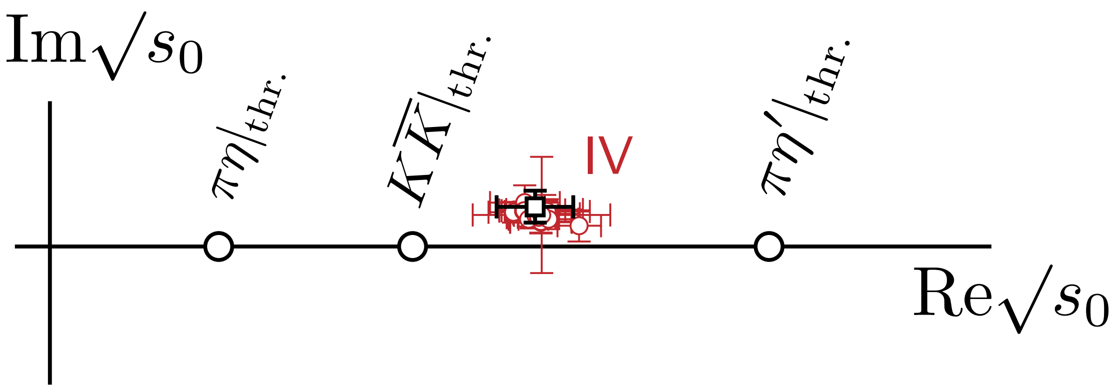
2·Im[$\sqrt{s_0}$] ~ 'width'

$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$

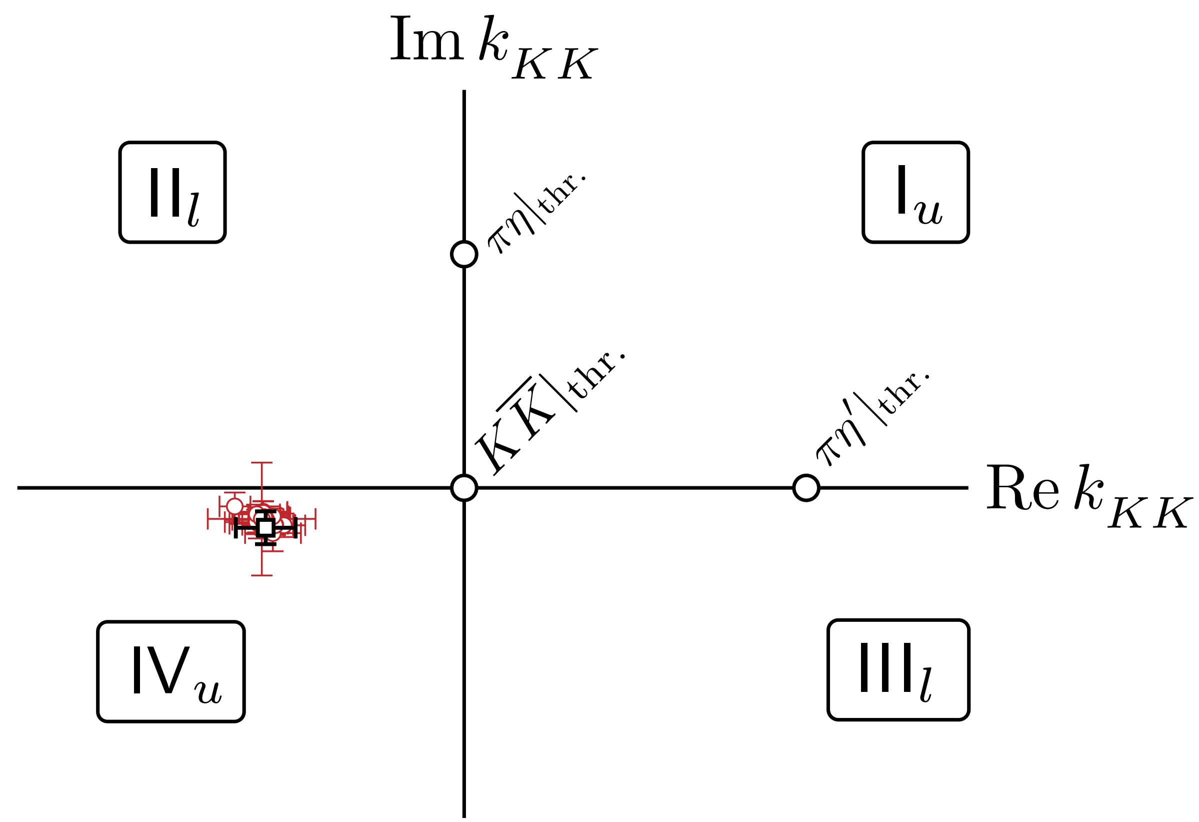
- our amplitudes have a single dominant pole

$m_\pi \sim 391\,\text{MeV}$

PRD93 094506 (2016)



Sheet	$\text{Im}k_{\pi\eta}$	$\text{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−



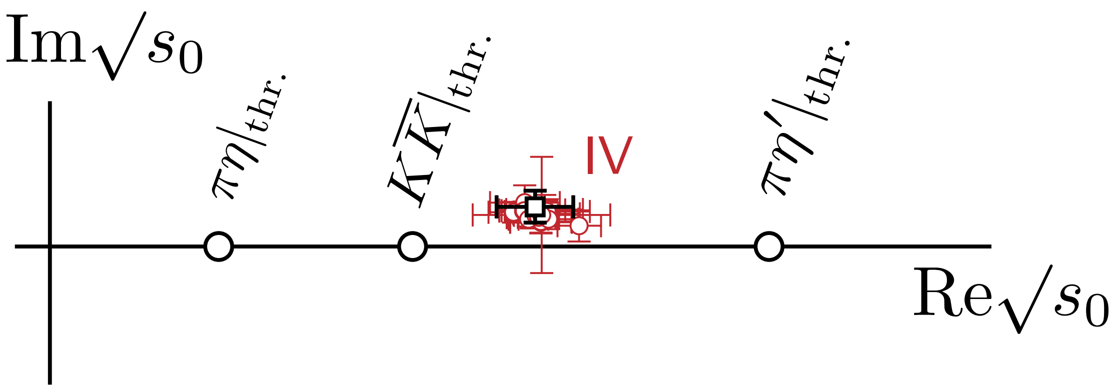
a single pole on sheet IV ⇒ a **molecular interpretation** ?

$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$

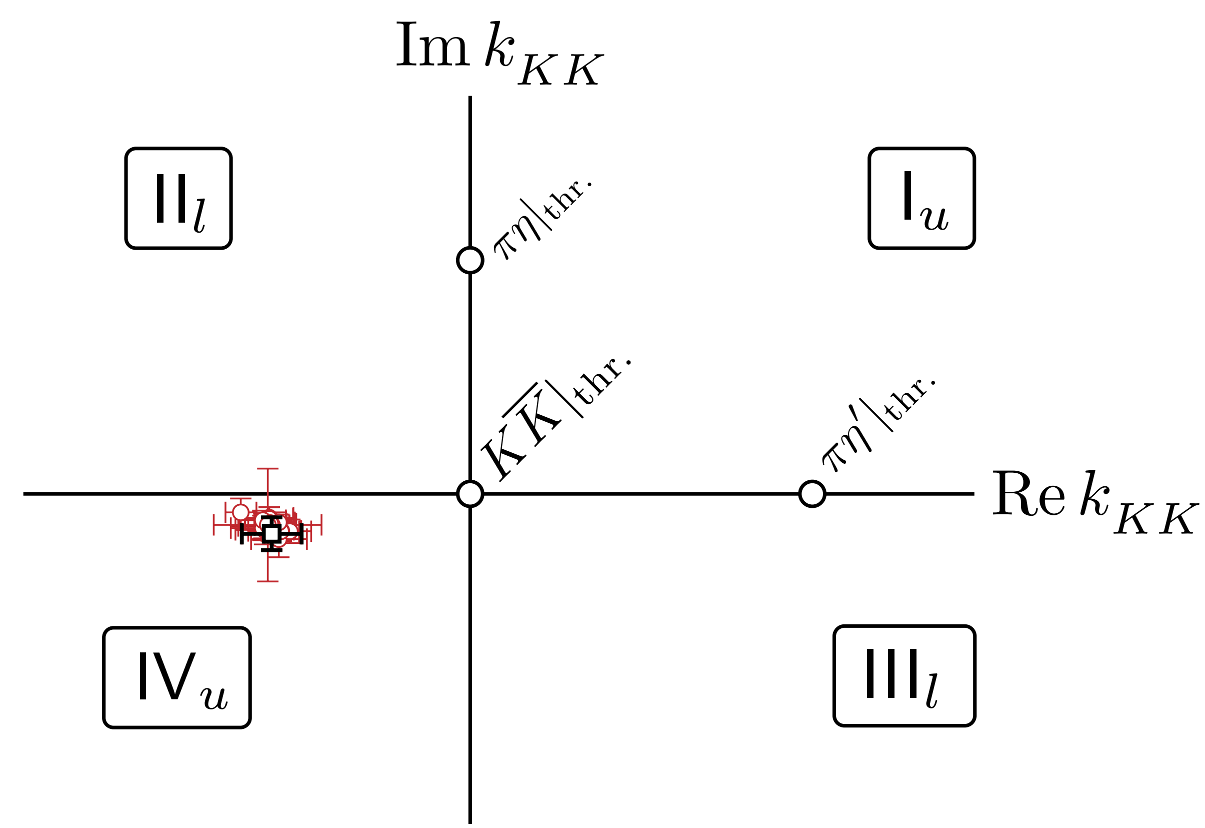
- our amplitudes have a single dominant pole

$m_\pi \sim 391\,\mathrm{MeV}$

PRD93 094506 (2016)



Sheet	$\mathrm{Im}k_{\pi\eta}$	$\mathrm{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−



a single pole on sheet IV ⇒ a **molecular interpretation** ?

Morgan and Pennington

Volume 258, number 3,4 PHYSICS LETTERS B 11 April 1991

$f_0(S^*)$: molecule or quark state?

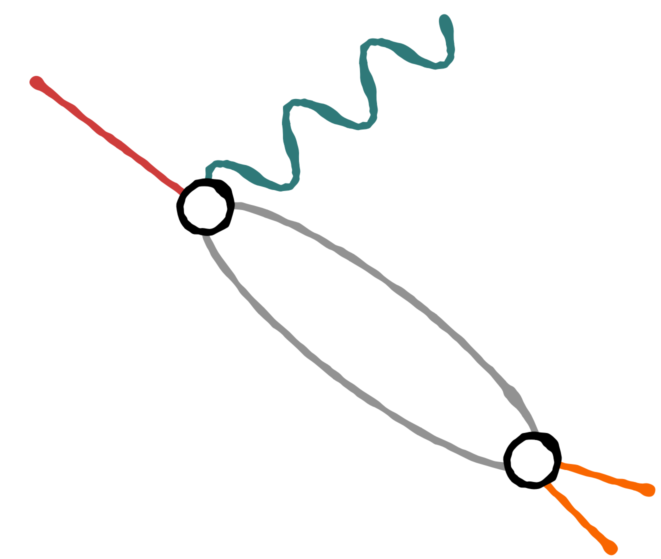
resonances and currents

- more to learn from couplings to external currents …

*de Fazio and **Pennington***

Physics Letters B 521 (2001) 15–21

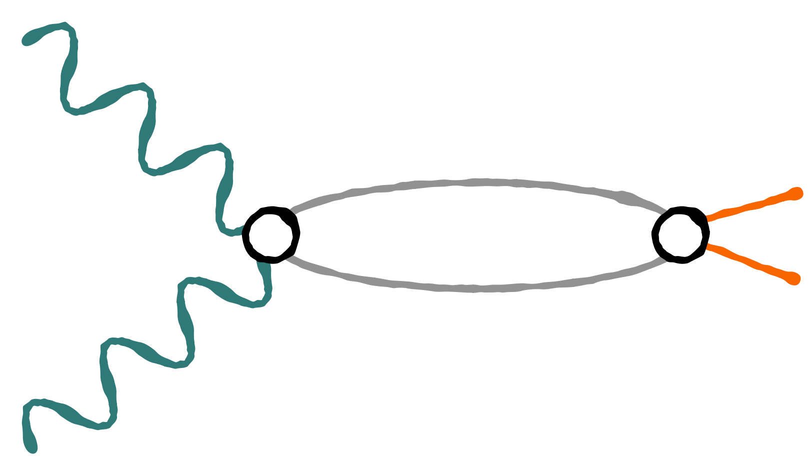
Probing the structure of $f_0(980)$ through radiative ϕ decays



*Morgan and **Pennington***

Zeitschrift für Physik C Particles and Fields
September 1988, Volume 37, Issue 3, pp 431-447

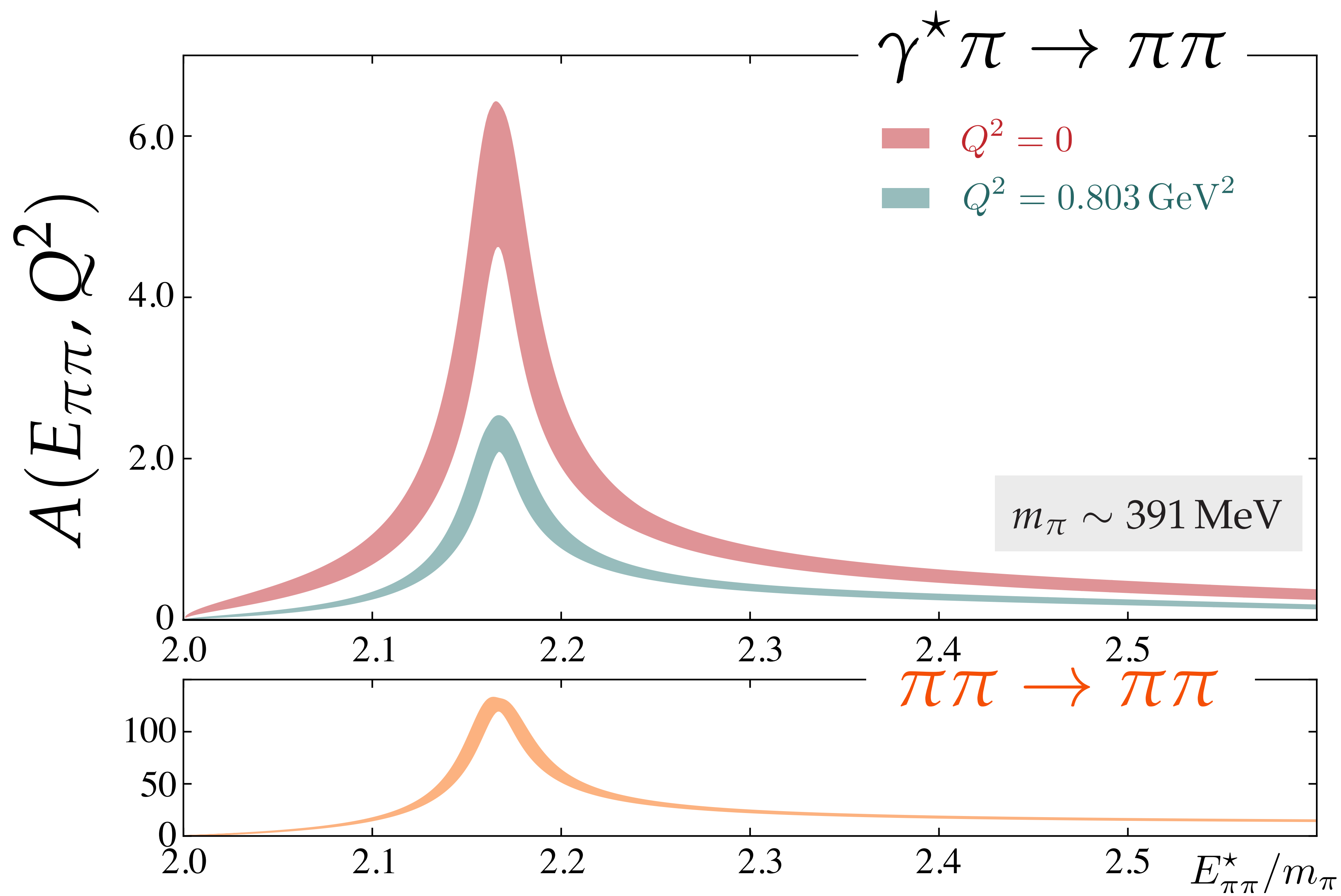
What we can learn from $\gamma\gamma\to\pi\pi, K\bar{K}$ in the resonance region



resonances and currents : e.g. $\gamma\pi \to \pi\pi$

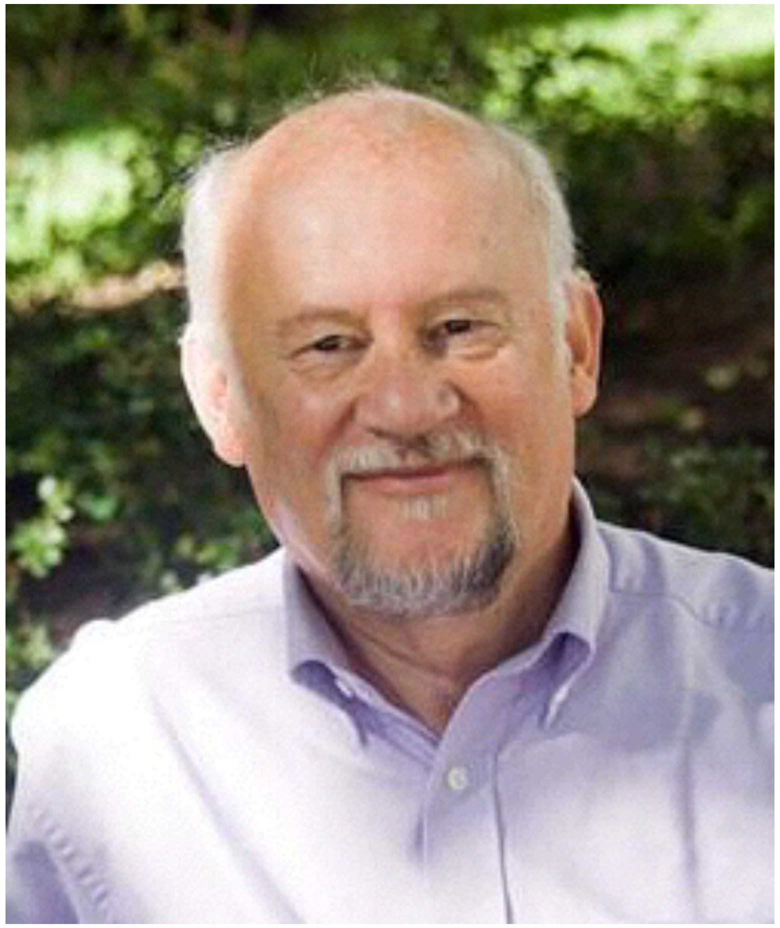
- first such calculation (of a simpler case) has recently appeared

Raul Briceno
JLab Isgur Fellow



PRL 115 242001 (2015)

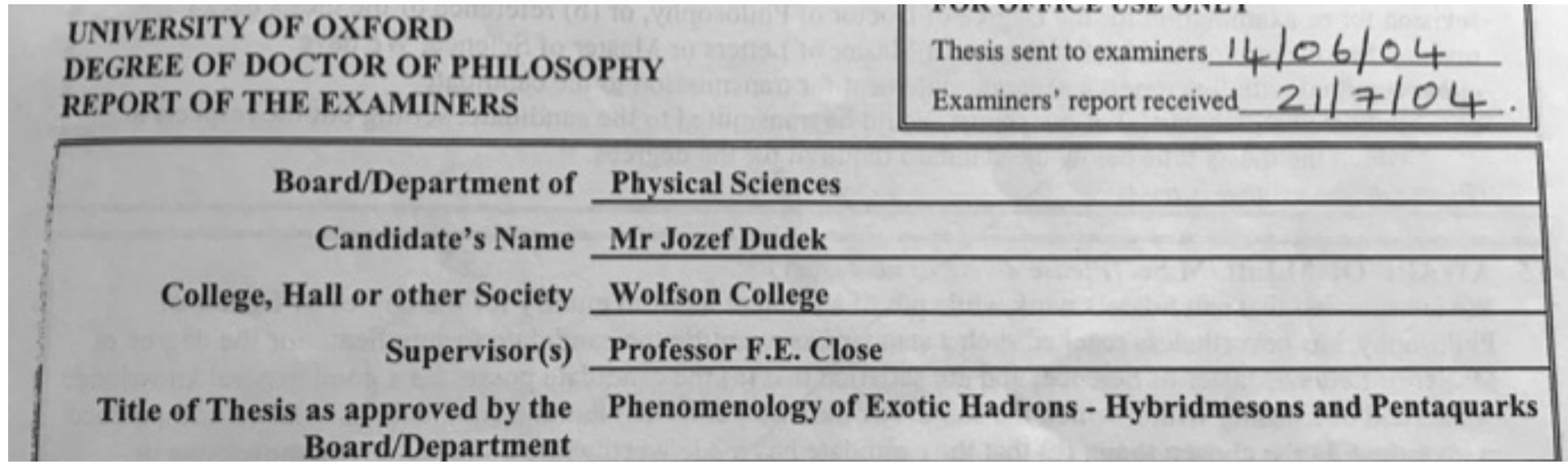
convinced the skeptics ?



- well, a Lancastrian and a Yorkshireman agree …

Mike Pennington

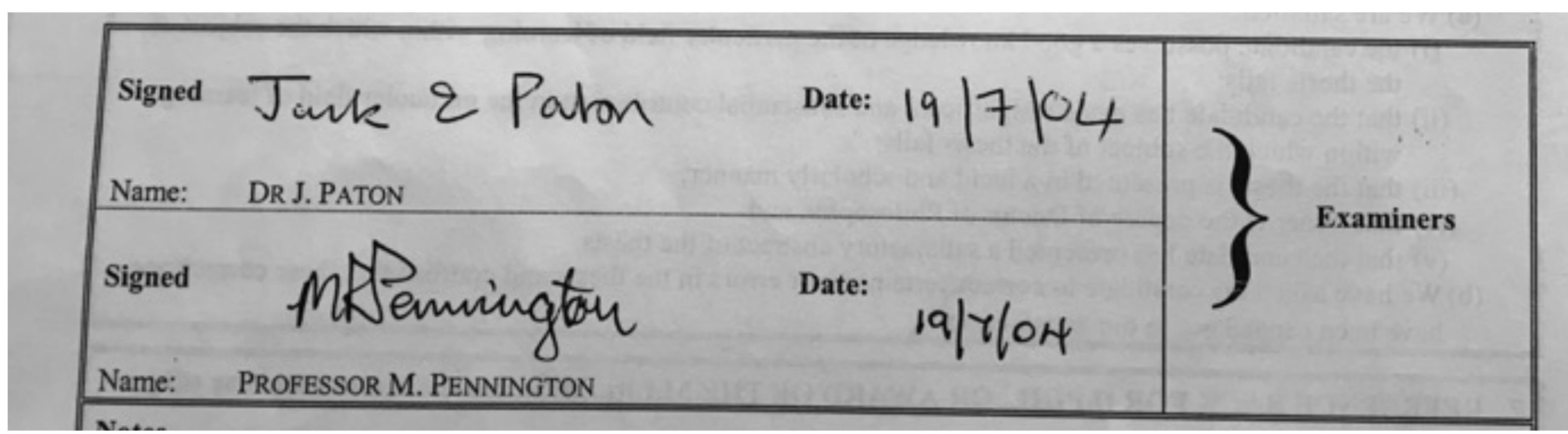
- like many people here, I have too many things to thank Mike for, but here's the first:



UNIVERSITY OF OXFORD
DEGREE OF DOCTOR OF PHILOSOPHY
REPORT OF THE EXAMINERS

FOR OFFICE USE ONLY
Thesis sent to examiners 14/06/04
Examiners' report received 21/7/04.

Board/Department of	Physical Sciences
Candidate's Name	Mr Jozef Dudek
College, Hall or other Society	Wolfson College
Supervisor(s)	Professor F.E. Close
Title of Thesis as approved by the Board/Department	Phenomenology of Exotic Hadrons - Hybridmesons and Pentaquarks



Signed Jack E Paton Date: 19/7/04
Name: DR J. PATON

Signed M R Pennington Date: 19/7/04
Name: PROFESSOR M. PENNINGTON

} Examiners

Mike Pennington

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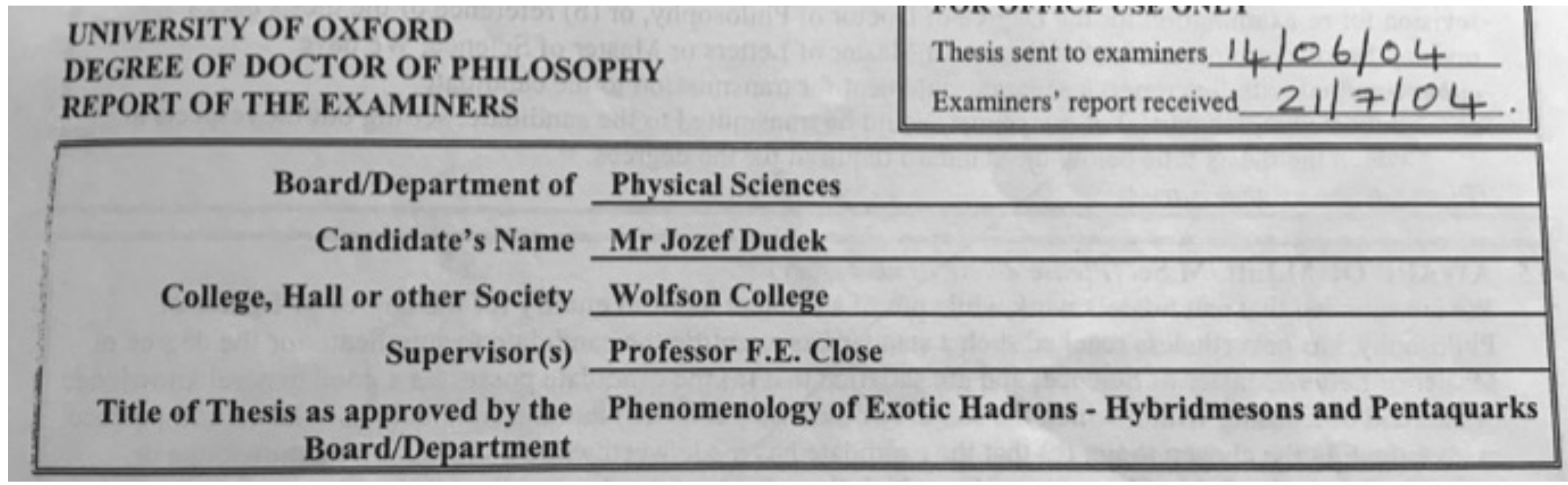

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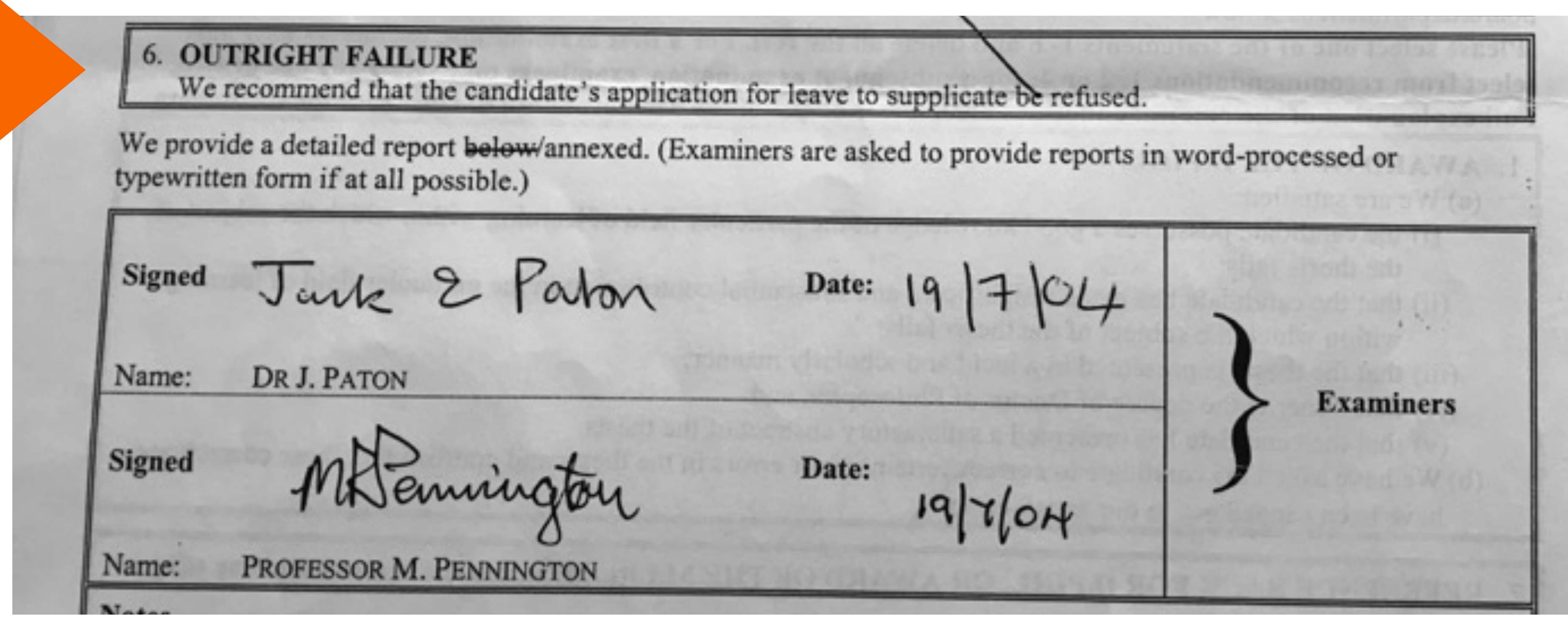
thanks for not thinking inside this box !



6. OUTRIGHT FAILURE
We recommend that the candidate's application for leave to supplicate be refused.

We provide a detailed report ~~below~~/annexed. (Examiners are asked to provide reports in word-processed or typewritten form if at all possible.)

Signed Jack E Paton Date: 19/7/04
Name: DR J. PATON

Signed M R Pennington Date: 19/7/04
Name: PROFESSOR M. PENNINGTON

Examiners

hadron spectrum collaboration

JEFFERSON LAB

Jozef Dudek
Robert Edwards
Balint Joo
David Richards
Raul Briceno

TRINITY, DUBLIN

Michael Peardon
Sinead Ryan

CAMBRIDGE

Christopher Thomas
Graham Moir
David Wilson

MESON SPECTRUM

***PRL**103 262001 (2009)*	$I=1$
PRD82 034508 (2010)	$I=1,\ K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	$D,\ D_s$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1,\ \rho$
***PRL**113 182001 (2014)*	$\pi K, \eta K : K^\star$
PRD91 054008 (2015)	$\pi K, \eta K : K^\star$
PRD92 094502 (2015)	$\pi\pi, K\bar{K} : \rho$
PRD93 094506 (2016)	$\pi\eta, K\bar{K} : a_0$

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\rm hyb}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
PRD91 094502 (2015)	$\Xi^\star_{cc}$

MATRIX ELEMENTS

PRD90 014511 (2014)	$f_{\pi^\star}$
PRD91 114501 (2015)	$M' \to \gamma M$
***PRL**115 242001 (2015)*	$\gamma^\star\pi \to \pi\pi$
PRD93 114508 (2016)	$\gamma^\star\pi \to \pi\pi$

LATTICE TECH.

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p} > 0$