$|V_{us}|$ FROM INCLUSIVE HADRONIC τ DECAYS

An example of the utility of combined lattice/continuum approaches

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A New Era for Hadro-Particle Physics

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OUTLINE

- *The inclusive flavor-breaking FESR V_{us} determination*
  - *The conventional implementation and the $> 3\sigma$ low $|V_{us}|$ puzzle*
  - *Continuum, lattice input: conventional implementation systematics and a new implementation strategy*
  - *Results of the new implementation and a resolution of the $> 3\sigma$ low $|V_{us}|$ puzzle*
- *A new lattice+inclusive us V+A τ data approach*

BASICS: HADRONIC τ DECAYS IN THE SM

- $\Pi^{(J)}_{ij;V/A}(Q^2)$, $\rho^{(J)}_{ij;V/A}(s)$: the flavor $ij = ud, us$, spin $J = 0, 1$ V, A current-current 2-pt function polarizations and spectral functions. Generically:

$$\begin{aligned}\Pi_{\mu\nu}(q) &= i\int d^4x\, e^{iq\cdot x}\langle 0|T\left\{J_\mu(x)J^\dagger_\nu(0)\right\}|0\rangle \\ &= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2) \\ \rho^{(J)}(s) &= \frac{1}{\pi}\mathrm{Im}\,\Pi^{(J)}(s+i\epsilon)\end{aligned}$$

- $R_{ij;V/A} \equiv \Gamma[\tau \to \nu_\tau\, \mathrm{hadrons}_{ij;V/A}\,(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$

- In SM, with $y_\tau \equiv s/m_\tau^2$, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$ [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V/A}}{ds} = \frac{12\pi^2\,|V_{ij}|^2 S_{EW}}{m_\tau^2}\left[w_\tau\,(y_\tau)\;\rho^{(0+1)}_{ij;V/A}(s) + w_L\,(y_\tau)\;\rho^{(0)}_{ij;V/A}(s)\right]$$

  - $\rho^{(1)}_{ud,us;V/A}(s)$; π, K pole contributions to $\rho^{(0)}_{ud,us;A}(s)$ all chirally unsuppressed
  - Continuum $\rho^{(0)}_{ij;V/A}(s) \propto \left(m_i \mp m_j\right)^2$

THE INCLUSIVE FB τ $|V_{us}|$ DETERMINATION

- **Context:** (conventional implementation) inclusive FB τ result c.f. K physics, 3-family unitarity

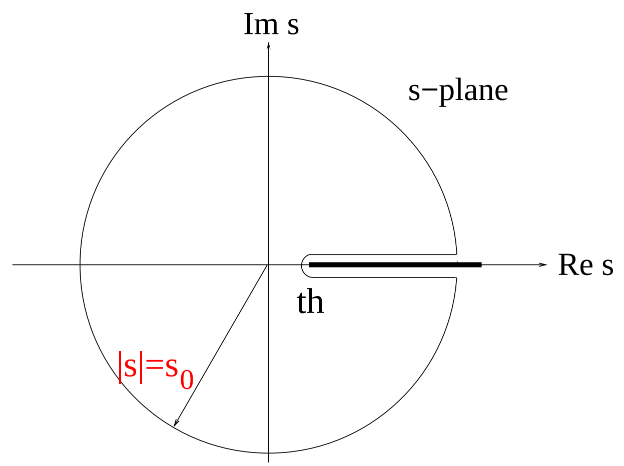
$\lvert V_{us}\rvert$	Source
0.2258(9)(?)	3-family unitarity, HT14 $\lvert V_{ud}\rvert$
$0.2231(4)_{exp}(7)_{latt}$	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
$0.2250(4)_{exp}(9)_{latt}$	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π
$0.2176(19)_{exp}(10?)_{th}$	Inclusive FB τ FESR (Passemar CKM14)

- Inclusive FB τ result $> 3\sigma$ low c.f. others: interesting **if real**, but theory systematics?

THE CONVENTIONAL IMPLEMENTATION

- Basic tool: FESRs (*Cauchy's Thm*), valid for any s_0, analytic $w(s)$, kinematic-singularity-free Π:

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$



- General inclusive FB τ sum rules for $|V_{us}|$

  * FESRs involving FB polarization, spectral function combinations $\Pi^{(J)}_{ud-us;V+A}(Q^2)$, $\rho^{(J)}_{ud-us;V+A}(s)$

  * Experimental (spectral) input: $|V_{ij}|^2\rho^{(J)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$, $R_{ij;V/A} \equiv \frac{\Gamma[\tau\rightarrow\nu_\tau\,\text{hadrons}_{ij;V/A}\,(\gamma)]}{\Gamma[\tau^-\rightarrow\nu_\tau e^-\bar{\nu}_e(\gamma)]}$

    (small external *us* *J=0* “subtraction” required)

    [SM “kinematic weight” w_τ]

  * $R^w_{ij;V/A}(s_0)$: Re-weighted $R_{ij;V/A}$ analogue, integrated to variable upper endpoint s_0 in spectrum

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

* FESR for $\delta R^w \equiv \frac{R^w_{ud;V+A}}{|V_{ud}|^2} - \frac{R^w_{us;V+A}}{|V_{us}|^2}$ yields

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

valid for arbitrary analytic $w(y)$, $s_0 \leq m_\tau^2$

* "Self-consistency tests":

  - $|V_{us}|$ independent of s_0, w (tests control, understanding of OPE, experimental systematics)

  - E.g., integrated $D = 2k+2$ OPE $\sim 1/s_0^k$: errors in higher D treatment $\leftrightarrow$ s_0-instability

- **Below: FESRs with variable s_0, w, $\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{ud-us;V+A}$**

- The conventional implementation [Gamiz et al. JHEP03(2003)060]
  - $s_0 = m_\tau^2$, $w = w_\tau$ only (degree 3, OPE to $D = 8$)
  - $D = 2$, $D = 4$ known, $D = 6$ OPE estimated with "VSA" (very small), $D = 8$ neglected
  - Resulting $\left[\delta R^{w_\tau}(m_\tau^2)\right]_{D\geq 2}^{OPE}$ estimate a few to several % of individual $R_{ud,us;V+A}$

    $\Rightarrow$ modest OPE accuracy enough for precision $|V_{us}|$
  - Rationale for $w = w_\tau$, $s_0 = m_\tau^2$ choice: $R_{ud,us;V+A}$ from inclusive ud, us BFs ($dR_{ij;V/A}/ds$ not needed)

    [Downside: unquantified systematic error from absence of $w(s)$-, s_0-independence tests]

- Conventional implementation tests [KM et al. arXiv:1511.08514]

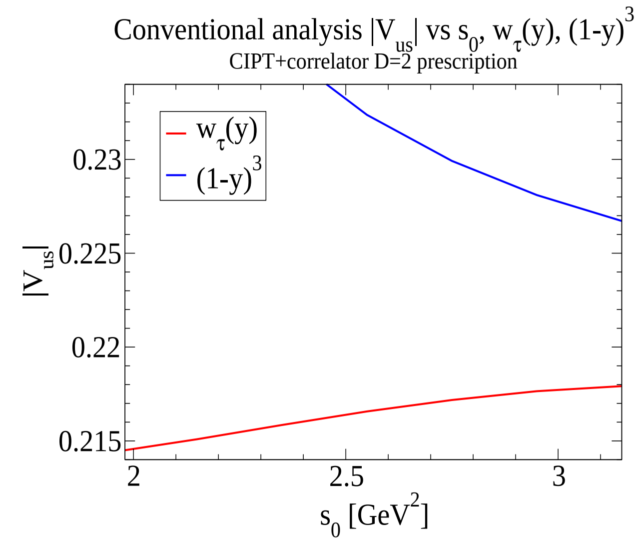
  - Variable $s_0 \leq m_\tau^2$, $|V_{us}|$ s_0-stability checks

  - Targeted $|V_{us}|$ w-independence test: $y = (s/s_0)$, $w = w_\tau(y) = 1 - 3y^2 + 2y^3$ c.f. $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$

    * $D = 6/8$ $\hat{w}$ OPE contributions $-1/-1/2$ $\times$ $D = 6/8$ w_τ OPE contributions

    * IF $D > 4$ assumptions OK for w_τ FESR $\Rightarrow$ also OK for $\hat{w}$ FESR $\Rightarrow$ $|V_{us}|$ agreement, s_0-stability

    * $D > 4$ **assumptions not OK** $\Rightarrow$ **opposite-sign** w_τ, $\hat{w}$ s_0**-instabilities, both decreasing with** s_0



- Candidate self-consistency problem sources:

  * Experiment: The less-well-known us distribution

  * Theory: Conventional $D > 4$ assumptions (w_τ, $\hat{w}$ comparison); slow $D = 2$ OPE convergence

- The $D=2$ series and slow $D=2$ convergence

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots\right]$$

with running $\overline{MS}$ $\bar{a} = \frac{\alpha_s(Q^2)}{\pi}$, $\bar{m}_s = m_s(Q^2)$

$\bar{a}(m_\tau^2) > 0.1 \Rightarrow$ slow convergence at ALL τ decay scales

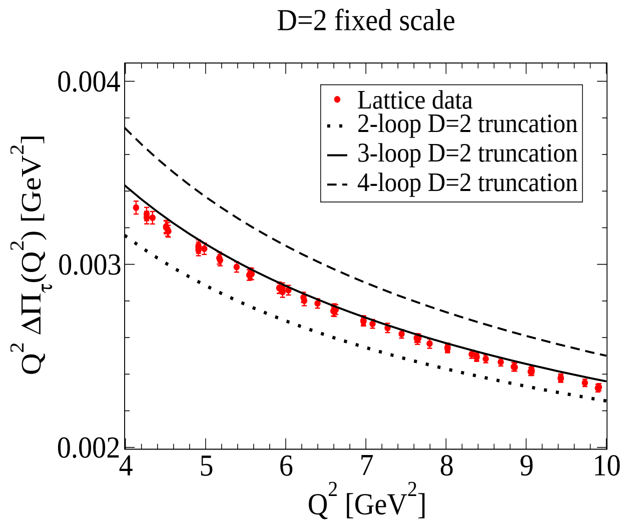
- w_τ, $\hat{w}$ comparison: $D > 4$ assumption problem(s) likely

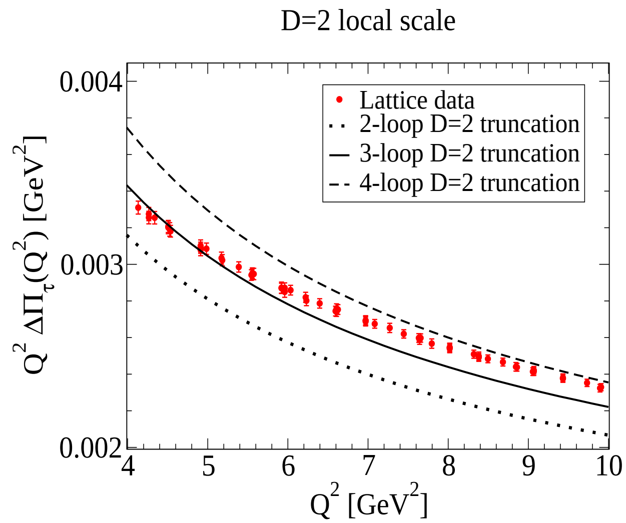
Note: VSA VERY crude; channel-dependent ud V,A breaking; caution re strong double cancellation (in ud, us V+A sums, AND resulting $ud - us$ difference)

LATTICE RE $D = 2$ SERIES, $D > 4$ OPE ISSUES

- OPE c.f. RBC/UKQCD lattice $\Delta\Pi_\tau(Q^2)$
  - KM with J. Hudspith, R. Lewis, J. Zanotti
  - Here: lightest m_π, fine ($1/a = 2.38\ GeV$) $32^3 \times 64$ $2+1$ ensemble ($m_\pi \sim 300\ MeV$, $m_\pi L \sim 4.1$)
  - Tight cylinder cut for continuum correlator behavior (width from stabilization in ud V channel α_s study)
  - Large Q^2: $D = 2+4$ OPE exploration; lower Q^2: re possible non-negligible $D > 4$ at τ decay scales

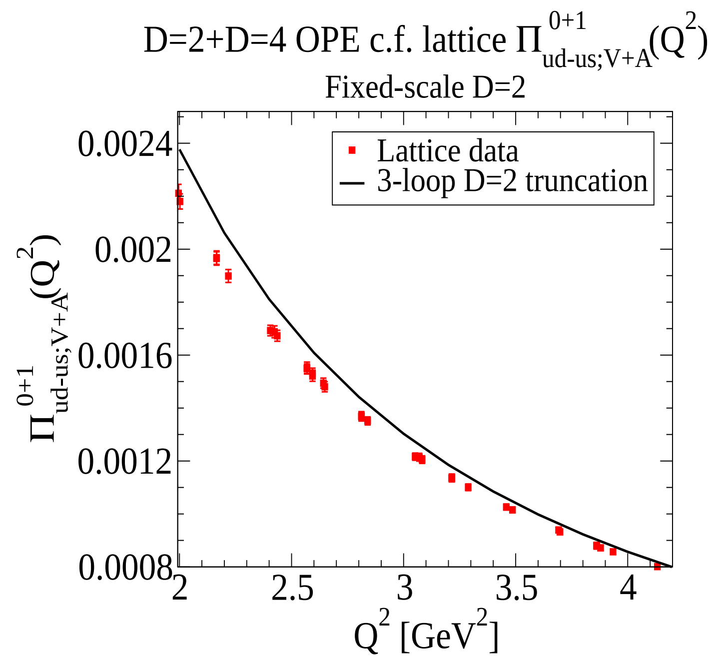
  * Fixed- or local-scale $D = 2$? [c.f. FESR FOPT/CIPT]



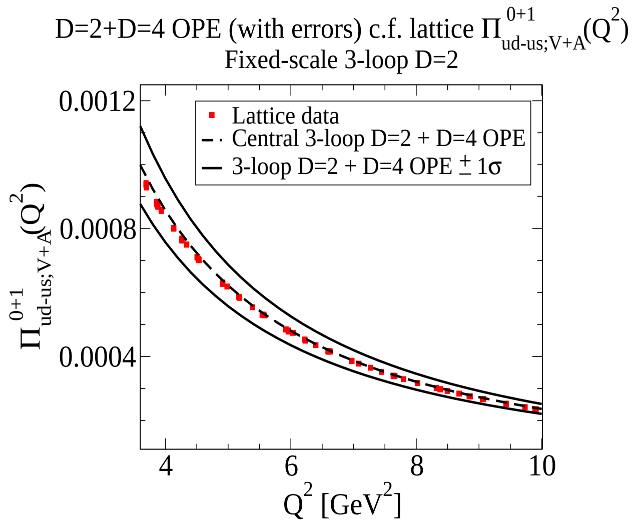


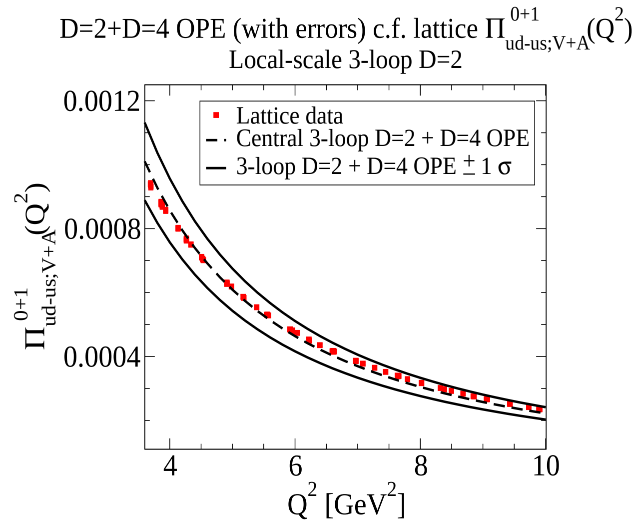
- Higher Q^2: best (excellent) lattice vs $D = 2 + 4$ OPE match for 3-loop-truncated, fixed-scale $D = 2$

- Fixed scale suggests FOPT for FESR $D = 2$



- Onset of $D > 4$ contributions below $\sim 4\ GeV^2$





- Standard $D = 2$, $D = 4$ error estimates VERY conservative, despite slow $D = 2$ series convergence

A NEW IMPLEMENTATION STRATEGY BASED ON LATTICE, CONTINUUM LESSONS

- No assumptions re $D > 4$; include contributions and fit effective condensates C_D as part of analysis

- 3-loop-truncated FOPT for $D = 2$ contribution; standard error estimates for $D = 2 + 4$

- Range of s_0 to fit both $|V_{us}|$, C_D

- Fits simplest for $w(y) = w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ (C_{2N+2}, $|V_{us}|$ as only fit parameters)

- Self-consistency check: $|V_{us}|$ from different w_N

RESULTS OF THE NEW ANALYSES

- For brevity: results for $K\pi$ normalization using preliminary BaBar $B[\tau \to K^-\pi^0\nu_\tau]$ update (Adametz thesis)

- Fitted $|V_{us}|$ (as expected) between (s_0-unstable) w_τ and $\hat{w}$ conventional implementation results

- Fitted C_D show FB cancellation in comparison to fitted ud V+A analogues (qualitative self-consistency test)

- Excellent $|V_{us}|$, C_{2N+2} stability wrt fit window variation; excellent agreement of $|V_{us}|$ from different w_N

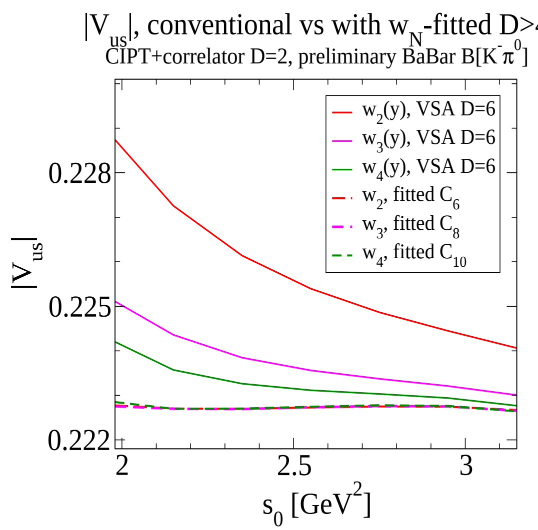
- Results using preliminary $K\pi$ BF update

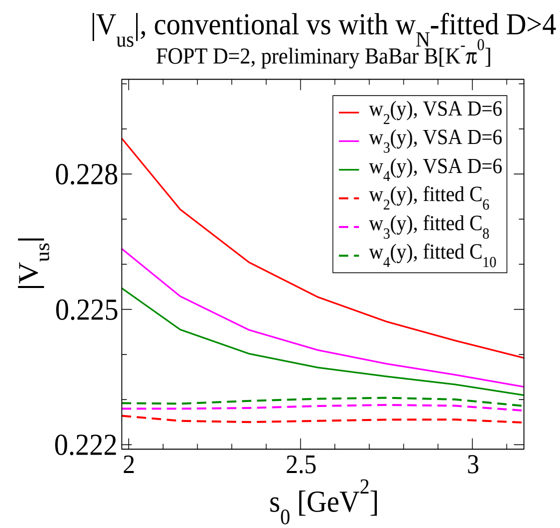
Weight	$\lvert V_{us}\rvert$ CIPT+corr $D=2$	$\lvert V_{us}\rvert$ FOPT $D=2$
w_2	0.22271(228)	0.22252(228)
w_3	0.22271(228)	0.22282(228)
w_4	0.22271(229)	0.22296(229)

Error budget, 3-loop-truncated FOPT $D=2$ fits

Source	$\delta\lvert V_{us}\rvert$ (w_2 FESR)	$\delta\lvert V_{us}\rvert$ (w_3 FESR)	$\delta\lvert V_{us}\rvert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00004	0.00004
$\delta m_s(2\ GeV)$	0.00017	0.00019	0.00019
$\delta\langle m_s\bar{s}s\rangle$	0.00035	0.00035	0.00035
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00028	0.00028
us exp	0.00226	0.00227	0.00227

Fit quality: s_0-stability for conventional implementation, except with fitted C_{2N+2}





Solid/dashed lines: with conventional implementation/new implementation (fit) treatments of $D > 4$ contributions

SUMMARY, CONVENTIONAL FB FESR ANALSIS

- Continuum, lattice $\Rightarrow$ conventional $D > 4$ assumptions untenable, must fit $D > 4$ effective condensates

  [Additional $\sim 0.0020 - 0.0030$ systematic from breakdown of $D > 4$ assumptions]

- Fitting $D > 4$ resolves s_0-, $w(y)$-dependence problems, raises $|V_{us}|$ by ~ 0.0020

- $D > 4$ fit needs variable s_0 $\Rightarrow$ **analysis using only inclusive non-strange, strange BFs NOT possible**

- Lattice $\Rightarrow$ 3-loop FOPT for $D = 2$, $D = 2 + 4$ OPE error estimate conservative

- $|V_{us}|$ errors strongly dominated by (improvable) us experimental uncertainties (currently BF dominated)

- New preliminary $K^-\pi^0$ BF normalization, 3-weight combined fit result $|V_{us}| = 0.2228(23)_{exp}(5)_{th}$ in agreement with other determinations (especially $K_{\ell 3}$)

  (c.f. $0.2200(23)_{exp}(5)_{th}$ with 2014 HFAG $B[K^-\pi^0]$)

- Theory error (conservative) ~ 0.0005 c.f. 0.0009 for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice crucial here)

- **$\Rightarrow$ FB FESR method competitive with sufficient improvement to us experimental errors**

- **However** sub-0.5% $|V_{us}|$ needs sub-% $R^w_{us;V+A}$ error

Relative exclusive mode $R^w_{us:V+A}$ contributions

Wt	s_0 $[GeV^2]$	K	$K\pi$	$K\pi\pi$ (B-factory)	Other
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

- **"Other":** 1999 ALEPH data/MC, $\sim$ 25% error

$\Rightarrow$ **"sufficient improvement" includes experimentally (much) more challenging higher-multiplicity modes**

A PROMISING τ-BASED ALTERNATIVE

- Work with J. Hudspith, T. Izubuchi, R. Lewis, H. Ohki, C. Lehner + RBC/UKQCD

- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us V+A polarizations with weights having poles at Euclidean Q^2
  - $\tilde{\Pi}(Q^2)$ such that inclusive $dR_{us;V+A}/ds$ distribution directly determines related spectral function $\tilde{\rho}(s)$
  - Theory: Lattice us 2-point function data (no OPE)
  - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- Experimental $dR_{us;V+A}/ds$ distribution yields $|V_{us}|^2\,\tilde{\rho}(s)$,

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1 + 2\frac{s}{m_\tau^2}\right)\rho^{(J=1)}_{us;V+A}(s) + \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J=0$ subtraction needed)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right)\Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

Reminder: strange hadronic τ decays in the SM

With $y_\tau \equiv s/m_\tau^2$ [Tsai PRD4 (1971) 2821]

$$\frac{dR_{us;V+A}}{ds} = \frac{12\pi^2\,|V_{us}|^2 S_{EW}}{m_\tau^2}\,[1-y_\tau]^2 \cdot\left[(1+2y_\tau)\,\rho^{(J=1)}_{us;V+A}(s) + \rho^{(J=0)}_{us;V+A}(s)\right]$$

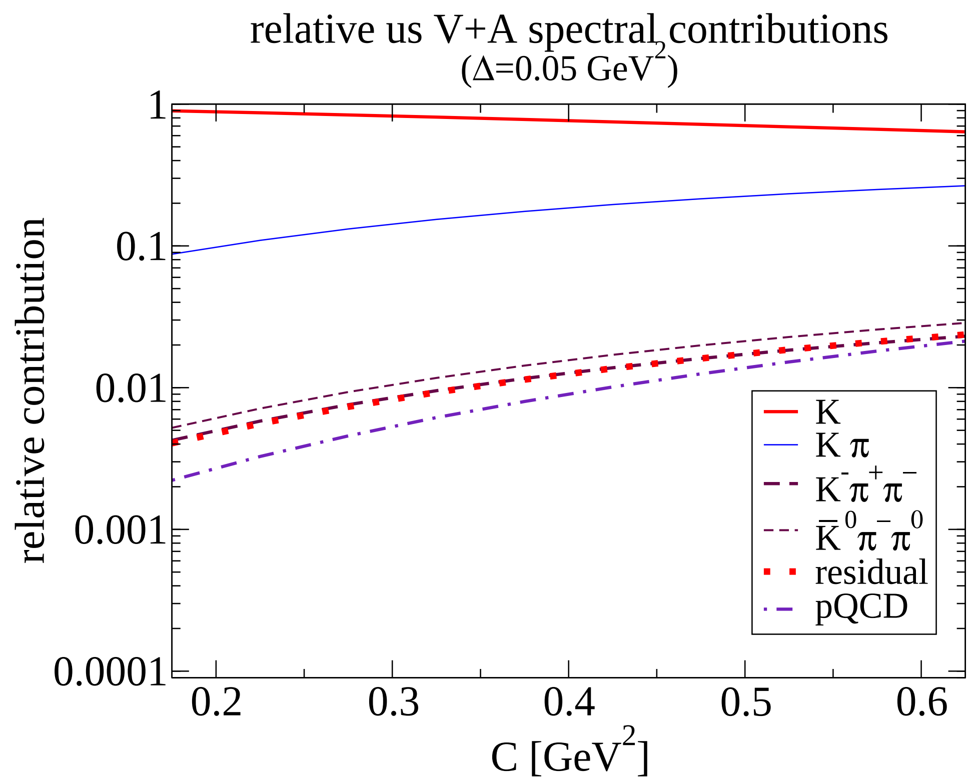
$$= \frac{12\pi^2\,|V_{us}|^2 S_{EW}}{m_\tau^2}\,[1-y_\tau]^2\,\tilde\rho_{us;V+A}(s)$$

$\Rightarrow$Division by known factors yields $\tilde\rho_{us;V+A}(s)$ with no need for continuum $\rho^{(0)}_{us;V/A}(s)$ subtraction

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

$$\int_{th}^{\infty} ds\, w_N(s)\, \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)}$$

  - Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS
  - LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$
  - $w_N(s)$: rapid fall-off if all $Q_k^2 < 1\ GeV^2$ $\Rightarrow K$, $K\pi$ **dominate LHS, near-endpoint multi-particle, $s > m_\tau^2$ contributions strongly suppressed [FIG]**
  - Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error



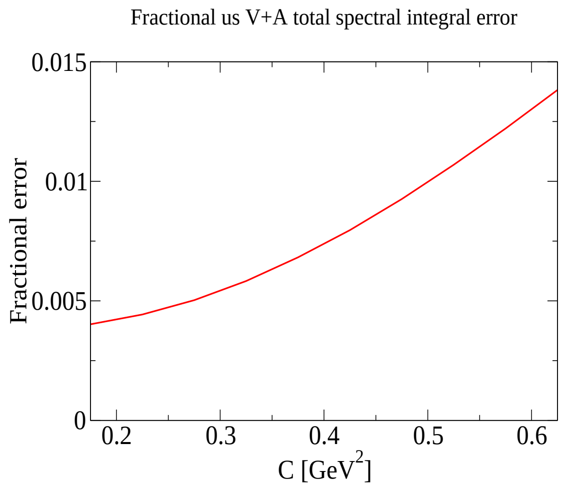
4-pole weights, pole spacing Δ, pole centroid C

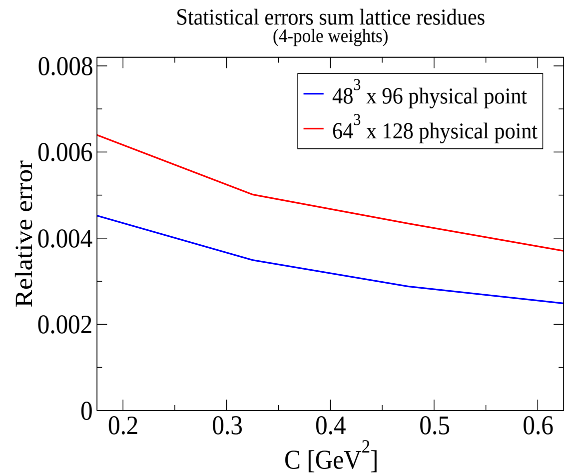
- An illustration of expected precision: $N = 4$, $C = 0.325\ GeV^2$ $[\{Q_k^2\} = \{0.25, 0.30, 0.35, 0.40\ GeV^2\}]$
  - Spectral integral error $\sim 0.4\%$
  - Statistical error, sum of residues, near-physical-point $n_f = 2+1$ RBC/UKQCD ensembles:

    $\sim 0.4\,\%$ for $48^3 \times 96$, $1/a = 1.73\ GeV$

    $\sim 0.5\,\%$ for $64^3 \times 128$, $1/a = 2.36\ GeV$
  - Quadrature sum $\Rightarrow \delta|V_{us}| \sim 0.0007$, ~ 0.0008 for near-physical-point 48^3, 64^3 ensembles, respectively

- Optimization/systematics (FV, continuum limit, correcting for small m_ℓ, m_s mis-tunings) under study





Relative errors as a function of C, 4-pole weights, pole spacing $\Delta = 0.05\ GeV^2$, pole interval centroid C

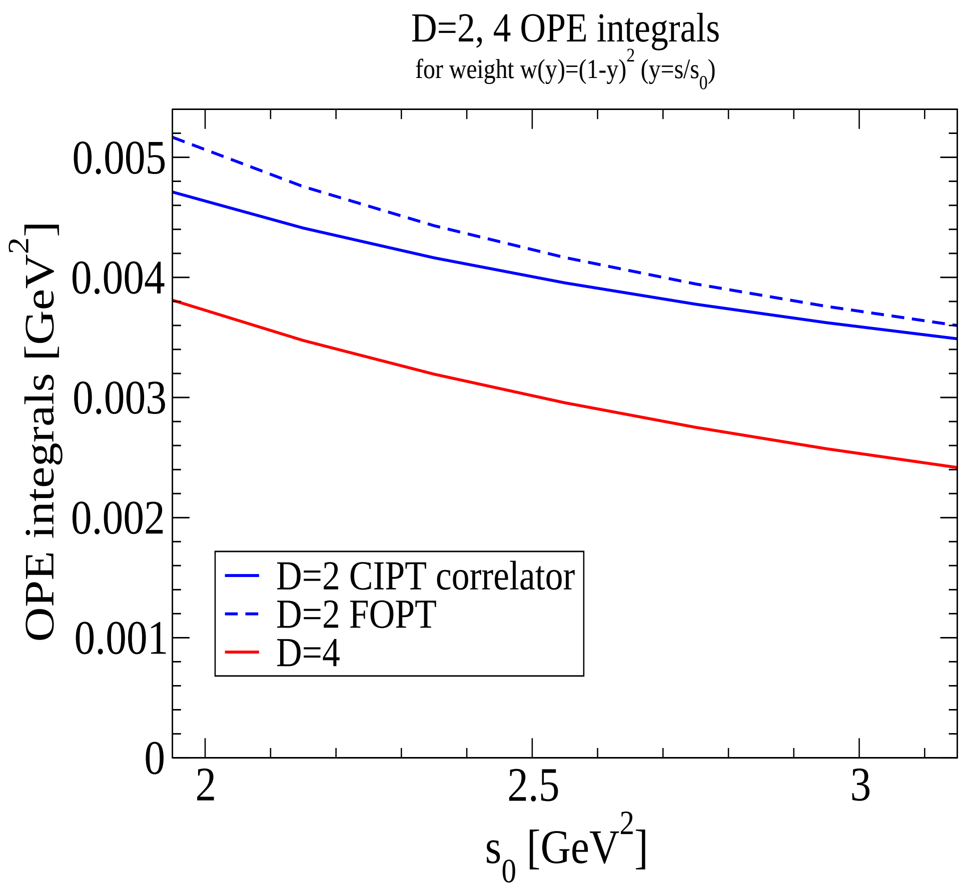
OVERALL SUMMARY

- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
  - Alternate, no-assumptions implementation: $|V_{us}|$ higher by ~ 0.0020, compatible with other determinations
  - Near-term improvements feasible through improvements in us exclusive mode BFs
  - Highly favorable theoretical error situation
  - However, for competitive $|V_{us}|$ need improvements to old ALEPH higher-multiplicity, low-statistics data [unlikely in the near-term]

- Advantage of new lattice-inclusive us $V+A$ τ approach
  - Theory:
    * Lattice in place of OPE; no (albeit mildly model-dependent) us $J=0$ continuum subtraction; improvement through increased statistics
    * Easily parasitic on lattice a_μ effort (major effort being pursued by many lattice collaborations)
  - Spectral integrals:
    * Theory errors still small for weights strongly suppressing higher multiplicity contributions
    * Strong K, $K\pi$ dominance of spectral integral
    * Significant experimental improvements possible through just improved $K\pi$ BFs, distributions

BACKUP SLIDES

- Are $D = 6, 8$ OPE contributions likely to be small for the conventional inclusive FB FESRs?
  - $D = 4 << D = 2$ for $w_T(y) = 1 - 3y^2 + 2y^3$, $y = s/s_0$, "accidental" [$O(\alpha_s^2)$ suppression due to absence of term linear in y in $w_T(y)$]
  - Comparison of $D = 2, 4$ OPE contributions for $w(y) = (1-y)^2$ (a case without this suppression) to see natural relative sizes



D=2, 4 OPE integrals
for weight w(y)=(1-y)^2 (y=s/s_0)
OPE integrals [GeV^2]
0.005
0.004
0.003
0.002
0.001
0
2
2.5
3
s_0 [GeV^2]
D=2 CIPT correlator
D=2 FOPT
D=4

OPE, SPECTRAL INPUT

- PDG, FLAG, HPQCD input for $D = 2, 4$ OPE
- ud V+A spectral data from ALEPH 2013
- us V+A spectral data from sum over exclusive modes [$>$ 90% of B_{us}^{TOT} from $K_{\ell 2}$, Belle, BaBar $K\pi$, $K\pi\pi$, 3K results; residual: 1999 ALEPH]
- Here, for brevity, $K\pi$ normalization including preliminary BaBar $B[\tau \to K^- \pi^0 \nu_\tau]$ update (Adametz thesis)

MORE ON THE us DATA

- K pole via $f_K|V_{us}|$ from $K_{\ell 2}$
- Rather precise unit-normalized $K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $3K$ distributions from Belle, BaBar (main uncertainties from BFs)
- K, B-factory modes over 90% of B_{us}^{TOT}
- Residual us exclusive mode contributions from 1999 ALEPH data, covariances

THE EXPERIMENTAL $K\pi$ BF SITUATION

- HFAG 2014 $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0126$

- HFAG 2014 $B[K^-\pi^0\nu_\tau] = 0.00431(15)$ value $\rightarrow$ preliminary BaBar (Adametz thesis) result 0.00500(15) yields $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0133$

- Central $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau]$ from $K_{\ell 3}$, dispersive analysis expectations [ACLP13] also 0.0133

- 0.07% difference "small" but represents $\sim 2.4\%$ of B_{us}^{TOT}, hence $\sim 1.2\%$ increase in $|V_{us}|$

Results for $|V_{us}|$ for current HFAG 2014 $K\pi$ BFs

Weight	$\lvert V_{us}\rvert$ CIPT+corr $D=2$	$\lvert V_{us}\rvert$ FOPT $D=2$
w_2	0.21985(230)	0.21966(230)
w_3	0.21985(231)	0.21966(231)
w_4	0.21985(231)	0.22009(231)

Error budget, existing $K\pi$ BFs

Source	$\delta\lvert V_{us}\rvert$ (w_2 FESR)	$\delta\lvert V_{us}\rvert$ (w_3 FESR)	$\delta\lvert V_{us}\rvert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00003	0.00005
$\delta m_s(2\ GeV)$	0.00017	0.00018	0.00020
$\delta\langle m_s\bar{s}s\rangle$	0.00034	0.00034	0.00034
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00027	0.00027
us exp	0.00229	0.00229	0.00230

Stability of $|V_{us}|$ with fitted C_{2N+2} input, existing $K\pi$ BF normalization