Insights into Parton Structure by Direct Calculation

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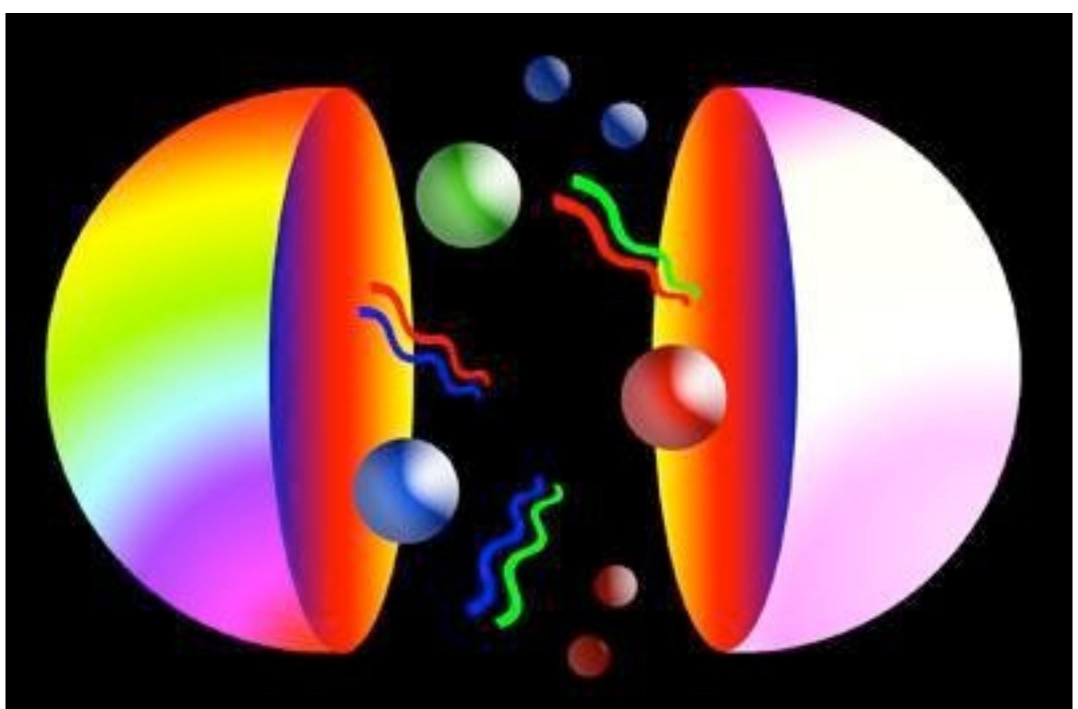

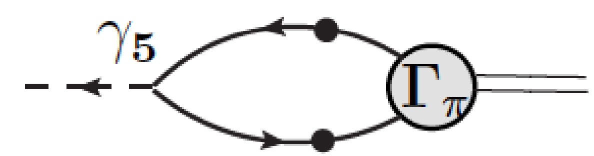

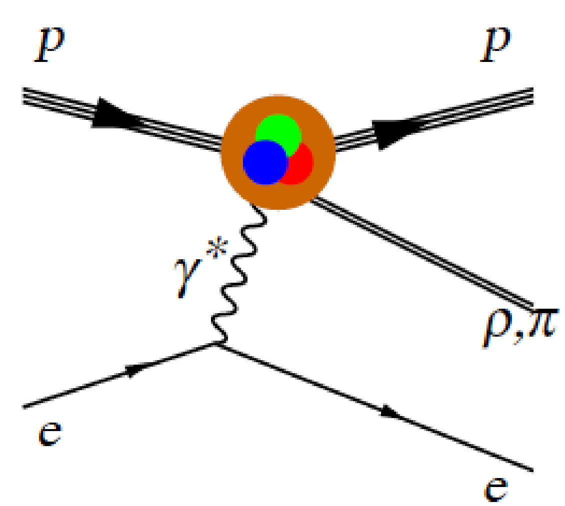

Topics

- Some things from Michael Pennington's career.
- Parton distribution amplitudes and PDFs—mainly mesons as an example. DSE-model calculations with direct connection to QCD. Comparison to LQCD.
- Some applications to uv physics
- PDFs including X. Ji's space-like correlator approximation for LQCD—a model investigation.

From the Past

Queen Mary College, U of London; Rutherford Lab., Berkeley, …Durham U, JLab….

219. **Constraints imposed on pi pi partial waves by positivity**
M.R. Pennington (Queen Mary, U. of London). 1970. 22 pp.
Published in **Nucl.Phys. B24 (1970) 317-338**

188. **What Can Asymptotic Freedom Say About e+ e- --> Hadrons?**
R.G. Moorhouse (Glasgow U.), M.R. Pennington (Durham U.), Graham G. Ross (Caltech). Dec 1976. 16 pp.
Published in **Nucl.Phys. B124 (1977) 285-300**

179. **Ambiguities in Higher Order {QCD} Predictions**
M.R. Pennington (Durham U.), Graham G. Ross (Oxford U.). Jun 1979. 6 pp.
Published in **Phys.Lett. B86 (1979) 371-376**

157. **HUNTING A HIDDEN HADRON: IS THERE A SCALAR GLUEBALL BELOW 1-GeV?**
Stephen R. Sharpe (Harvard U.), R.L. Jaffe (MIT, LNS), M.R. Pennington (Durham U.). Apr 1984. 37 pp.
Published in **Phys.Rev. D30 (1984) 1013**
HUTP-84/A017

Queen Mary College, U of London; Rutherford Lab., Berkeley, …Durham U, JLab….

141. **Preludes to Confinement: Infrared Properties of the Gluon Propagator in the Landau Gauge**
Nicholas Brown (Durham U.), M.R. Pennington (Durham U. & Brookhaven).
Nov 1987. 5 pp.
Published in **Phys.Lett. B202 (1988) 257**, Erratum: **Phys.Lett. B205 (1988) 596**

139. **Studies of Confinement: How Quarks and Gluons Propagate**
Nicholas Brown, M.R. Pennington (Brookhaven & Durham U.). Mar 1988.
43 pp.
Published in **Phys.Rev. D38 (1988) 2266**

130. **Truncating the Schwinger-Dyson equations: How multiplicative renormalizability and the Ward identity restrict the three point vertex in QED**
D.C. Curtis, M.R. Pennington (Durham U.). Apr 1990. 15 pp.
Published in **Phys.Rev. D42 (1990) 4165-4169**

127. **Masses from nothing: A Nonperturbative study of QED in three-dimensions**
M.R. Pennington, D. Walsh (Durham U.). Sep 1990. 6 pp.
Published in **Phys.Lett. B253 (1991) 246-251**

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Queen Mary College, U of London; Rutherford Lab., Berkeley, …Durham U, JLab….

120. **Is low-energy gamma gamma ---> pi0 pi0 predictable?**
D. Morgan (Rutherford), M.R. Pennington (Durham U.). Jul 1991. 5 pp.
Published in **Phys.Lett. B272 (1991) 134-138**

83. **The Nonperturbative three point vertex in massless quenched QED and perturbation theory constraints**
A. Bashir (Quaid-i-Azam U.), A. Kizilersu (Istanbul U.), M.R. Pennington (Durham U.). Jul 1997. 18 pp.
Published in **Phys.Rev. D57 (1998) 1242-1249**

47. **Sigma coupling to photons: Hidden scalar in gamma gamma ---> pi0 pi0**
M.R. Pennington (Durham U.). 2006. 4 pp.
Published in **Phys.Rev.Lett. 97 (2006) 011601**

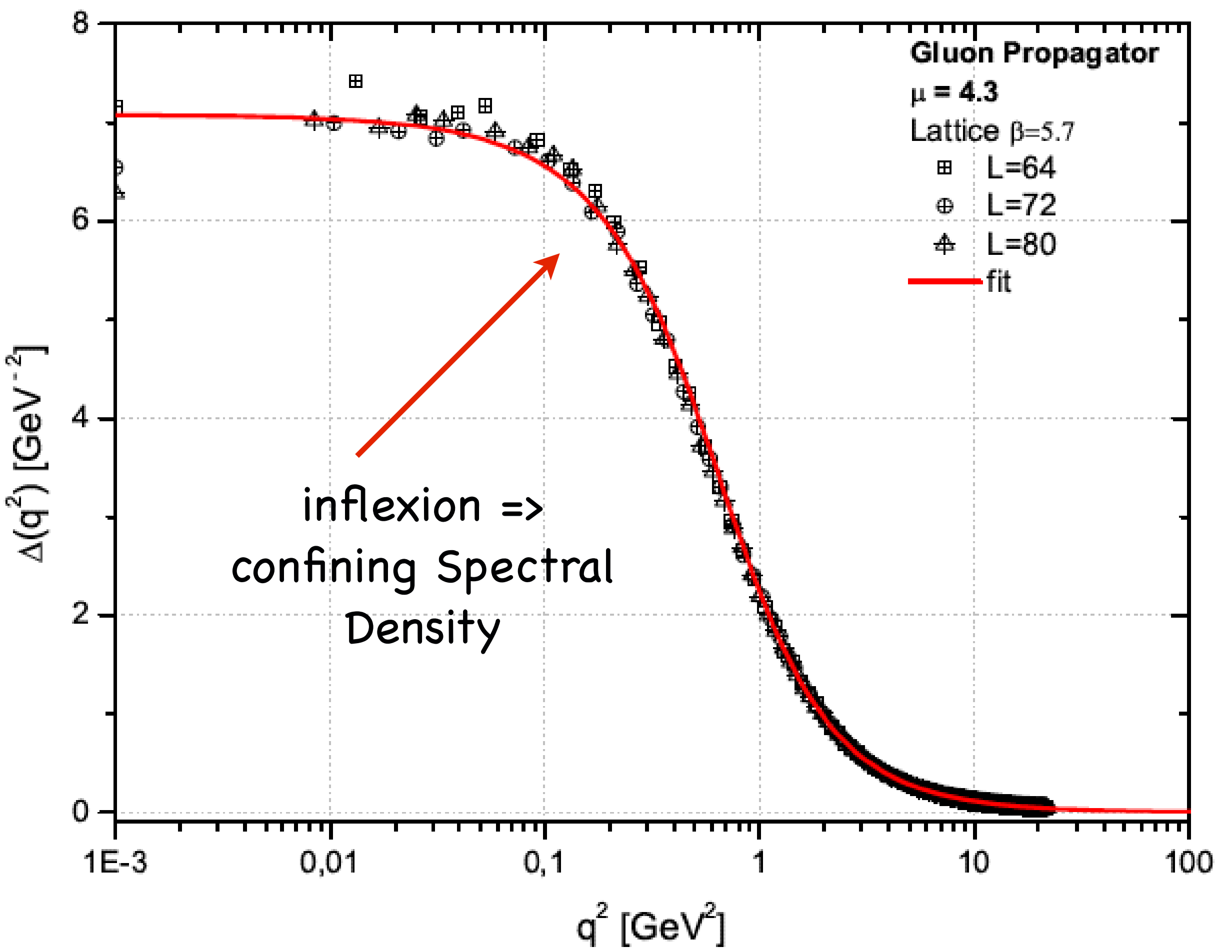
22. **Are the Dressed Gluon and Ghost Propagators in the Landau Gauge presently determined in the confinement regime of QCD?**
M.R. Pennington (Jefferson Lab), D.J. Wilson (Argonne). Sep 2011. 15 pp.
Published in **Phys.Rev. D84 (2011) 119901**
JLAB-THY-11-1426

What does one do after a career in theoretical physics?

Mike has been preparing……..


Pennington's
Barber Shop
BARBER
SHOP

Modern Context for DSE Interaction Kernel



Landau gauge, lattice – QCD gluon propagator,
I.L.Bogolubisky *et al.*, PosLAT2007, 290 (2007)

$\Rightarrow m_G(k^2)$ $\quad$ $m_G(0) \sim 0.38$ GeV

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables
Daniele Binosi (ECT, Trento & Fond. Bruno Kessler, Trento), Lei Chang (Adelaide U., Sch. Chem. Phys.), Joannis Papavassiliou (Valencia U. & Valencia U., IFIC), Craig D. Roberts (Argonne, PHY). Dec 15, 2014. 6 pp.
Published in **Phys.Lett. B742 (2015) 183-188**

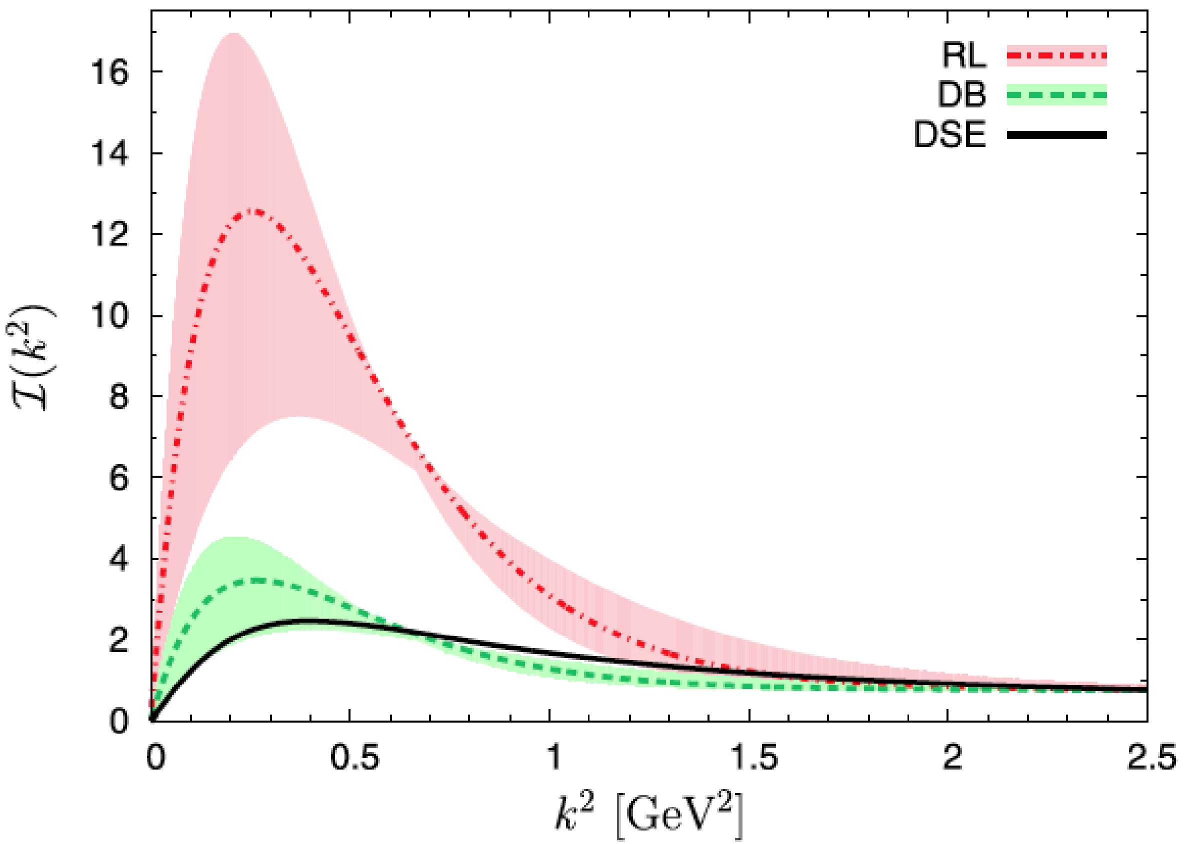
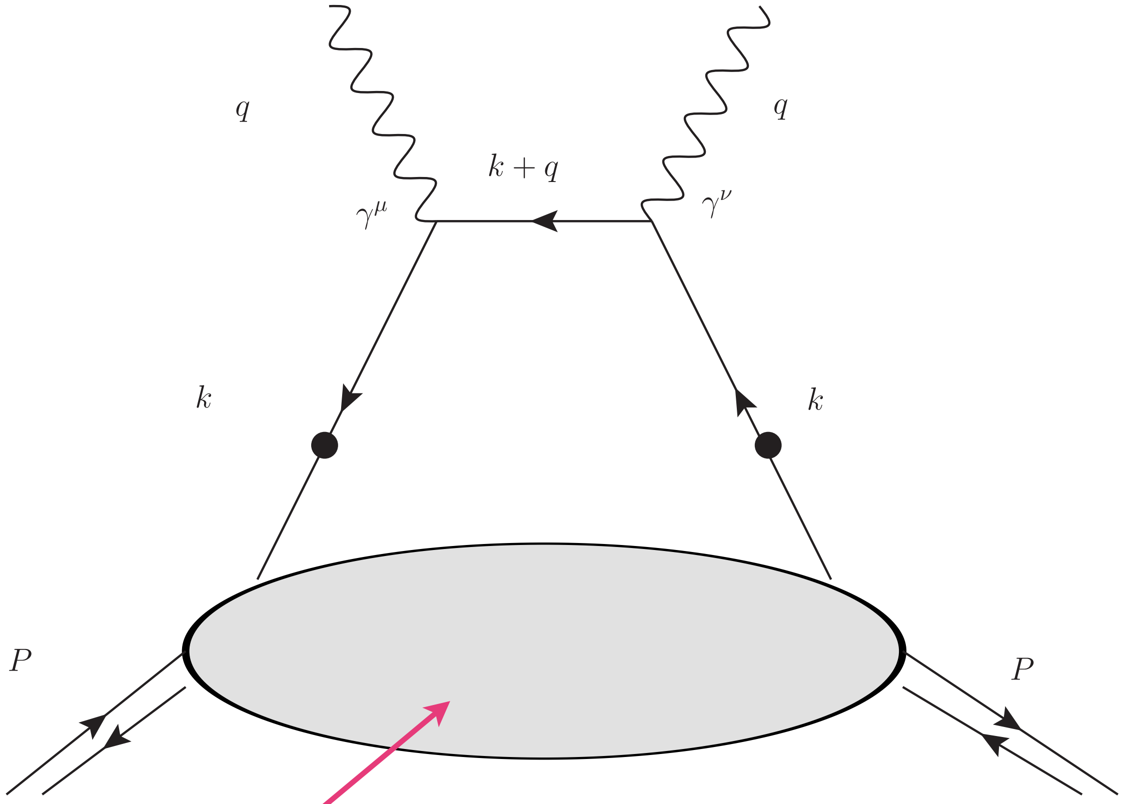


Table 1
Row 1 – Computed values determined from the interaction tension in Eq. (23), quoted in GeV; and Row 2 – the difference: $\varepsilon_\varsigma := \varsigma_{\mathcal{I}}/\varsigma_{\mathcal{I}_d} - 1$. So as to represent the domain of constant ground-state physics, described in connection with Eq. (5), we list values obtained with bottom-up interactions using $\omega = 0.5$, 0.6 GeV.

$\mathcal{I}$	$\mathcal{I}_d$	$\mathcal{I}_{DB}^{\omega=0.5}$	$\mathcal{I}_{DB}^{\omega=0.6}$	$\mathcal{I}_{RL}^{\omega=0.5}$	$\mathcal{I}_{RL}^{\omega=0.6}$
$\varsigma_{\mathcal{I}}$	1.86	1.91	1.82	3.14	2.90
ε_ς	0	2.8%	−2.4%	68.5%	55.8%

Parton Distribution Functions

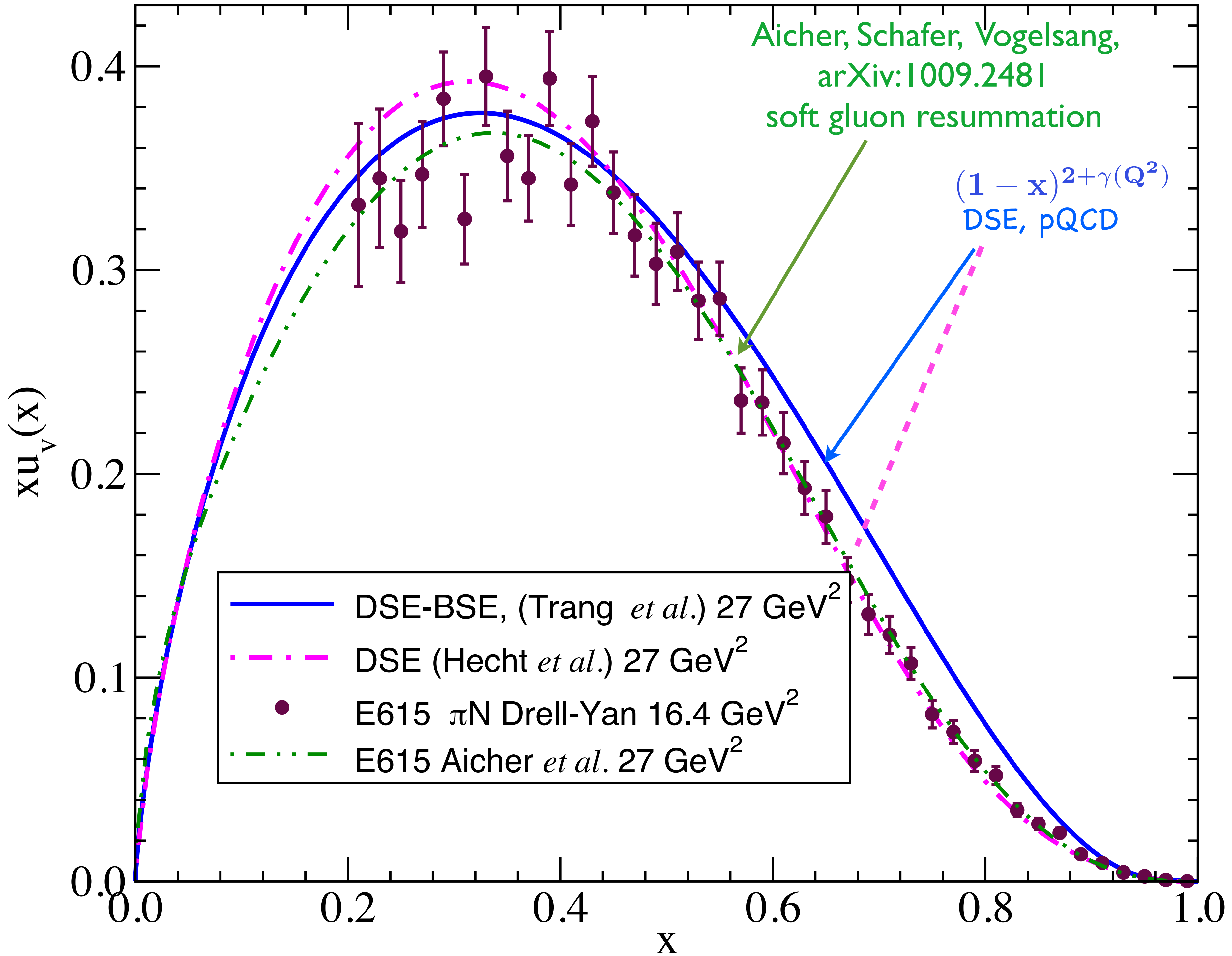


Covariant formulation
and calculation

$$\int \mathbf{d^4q\ \ F(q^2, q\cdot P, q\cdot k, k^2)}$$

Pion Valence PDF

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448



$\mathbf{q_0}$

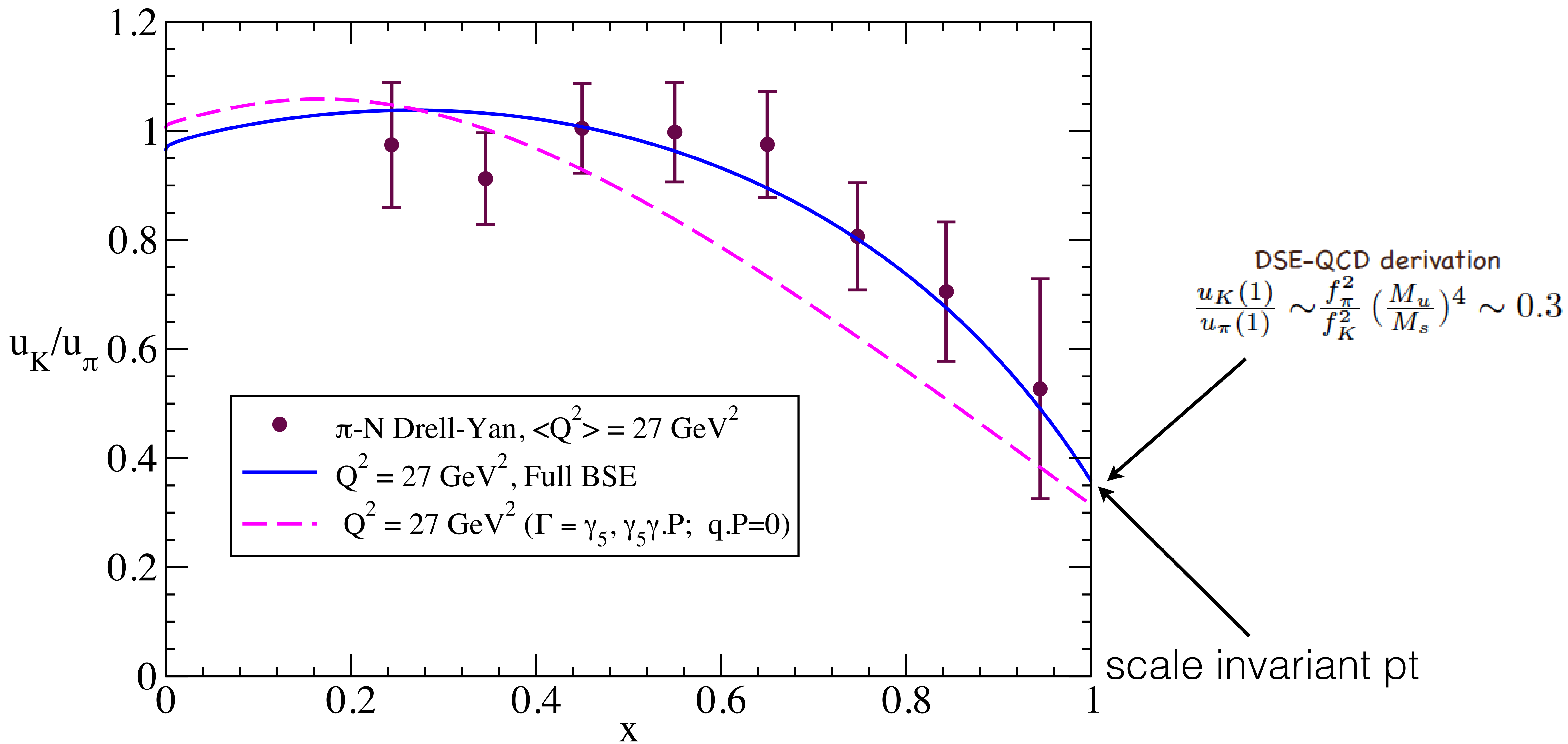
prev PDF expt parm $(1-x)^{1.5}$

CQM, duality.. $(1-x)^1$

NJL (pt π) : $(1-x)^0$

Environmental Dependence of Valence $u(x)$

Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).

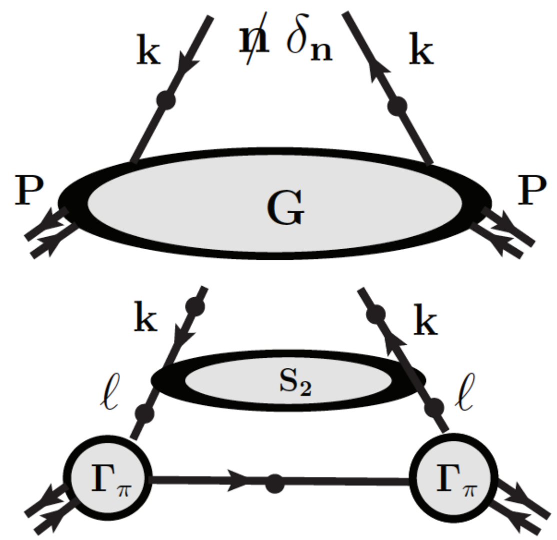


- CERN-SPS data: J. Badier et al, PLB **93**, 354 (1980)

(valence is not isolated)

The Leading Order PDF

$$\mathbf{q_f(x) = \frac{1}{4\pi} \int d\lambda \; e^{-ixP\cdot n\lambda} \, \langle \pi(P) | \, \bar{\psi}_f(\lambda n) \, \not{n} \, \psi_f(0) \, | \pi(P) \rangle_c}$$



RL DSE:

q(x) From Directly Obtained Moments

$$\langle \mathbf{x^m} \rangle^{\mathrm{RL}}_{\mathbf{v}} = \frac{-\mathbf{N_c}}{\mathbf{2P\cdot n}} \mathrm{tr} \int_\ell \mathbf{\Gamma}_\pi(\ell - \frac{\mathbf{P}}{\mathbf{2}}) \; [(\frac{\ell\cdot\mathbf{n}}{\mathbf{P\cdot n}})^{\mathbf{m}} \, \mathbf{n}\cdot\partial_\ell \mathbf{S}(\ell)] \; \mathbf{\Gamma}_\pi(\ell - \frac{\mathbf{P}}{\mathbf{2}}) \; \mathbf{S}(\ell - \mathbf{P})$$

$\mathbf{Method\ can\ easily\ exceed\ the\ Lattice-QCD\ practical\ limit: m = 3}$

Fit numerical DSE-BSE solns to PTIRs (Nakanishi)

EG: $$\Gamma_\pi(\mathbf{q^2},\mathbf{q}\cdot\mathbf{P}) = \gamma_5\left\{\mathbf{E}_\pi(\mathbf{q^2},\mathbf{q}\cdot\mathbf{P}) + \not{P}\,\mathbf{F}_\pi(..) + \not{q}\,\mathbf{q}\cdot\mathbf{P}\;\mathbf{G}_\pi(..) + \sigma:\mathbf{qP}\;\mathbf{H}_\pi(..)\right\}$$

Use Nakanishi Repn (or PTIR) (1965) :- $\mathcal{F} = \mathbf{E},\ \mathbf{F},\ \mathbf{G},\ \text{or}\ \mathbf{H}$

$$\mathcal{F}(\mathbf{q^2};\mathbf{q}\cdot\mathbf{P}) = \int_{-1}^{1}\mathbf{d}\alpha\int_0^\infty \mathbf{d\Lambda}\left\{\frac{\rho_{\mathrm{IR}}(\alpha;\mathbf{\Lambda})}{(\mathbf{q^2}+\alpha\mathbf{q}\cdot\mathbf{P}+\mathbf{\Lambda^2})^{\mathbf{m+n}}} + \frac{\rho_{\mathrm{UV}}(\alpha;\mathbf{\Lambda})}{(\mathbf{q^2}+\alpha\mathbf{q}\cdot\mathbf{P}+\mathbf{\Lambda^2})^{\mathbf{n}}}\right\}$$

npQCD info is in the variables and constants that are not momenta
---Wick rotation is trivial as in pert thy.

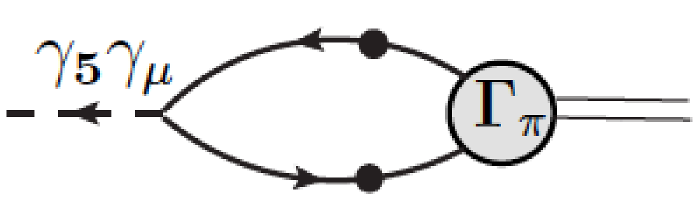
$$\rho_{\mathrm{IR}}(\alpha;\mathbf{\Lambda}) \to \rho_{\mathbf{1}}(\alpha)\,\delta(\mathbf{\Lambda}-\mathbf{\Lambda}_{\mathrm{IR}_1}) \ + \ \cdots\mathbf{3}$$

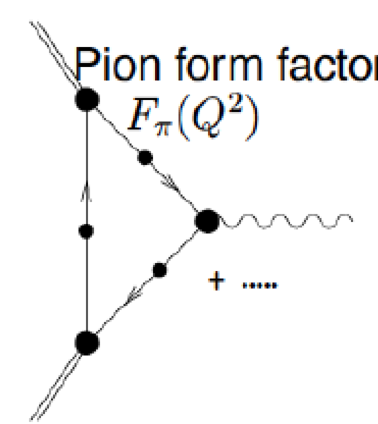
$$S(q) = \sum_{k=1}^{3}\left(\frac{z_k}{i\,\not{q}+m_k} + \frac{z_k^*}{i\,\not{q}+m_k^*}\right)$$

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.

N. Souchlas, PhD thesis KSU, (2009), J. Phys. G37, 115001 (2010)

EG: $$\mathbf{q_A}(\mathbf{x}) = \mathbf{i}\,\mathbf{N_c}\,\mathrm{tr}\int\frac{\mathbf{dk^+}\,\mathbf{dk^-}\,\mathbf{d^2k_\perp}}{(\mathbf{2}\pi)^{\mathbf{4}}}\;\delta(\mathbf{k^+}-\mathbf{x}\,\mathbf{P^+})\;\mathbf{tr}[\mathbf{\Gamma}_\pi\,\mathbf{S}\,(\mathbf{i}\gamma^+)\,\mathbf{S}\,\mathbf{\Gamma}_\pi\,\mathbf{S}]$$





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Spacelike Correlator Approximation for PDFs

To help lattice-QCD be more applicable to hadron PDFs and GPDs than just the first 3 moments ?

PRL 110, 262002 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Parton Physics on a Euclidean Lattice

Xiangdong Ji[1,2]

Standard light-cone correlator, leading twist: $\mathbf{x = k \cdot n / P \cdot n = k^+/P^+ \;\; \epsilon \;\; [0,1]}$

$$\mathbf{q_f(x) = \frac{1}{4\pi} \int d\lambda \; e^{-ix\,P\cdot n\,\lambda} \, \langle \pi(P)| \, \bar{\psi}_f(\lambda n) \; \not{n} \; \psi_f(0) \; |\pi(P)\rangle_c}$$

$$\mathbf{n^2 = 0 \;\; ; \;\; z^- = \lambda\, n \;\; ; \;\; z^+ = 0 = z_\perp}$$

Ji: Take large Pz limit of frame-dependent equal-time correlator: $\mathbf{x = kz/Pz \;\; \epsilon \;\; [-\infty, +\infty]}$

$$\mathbf{\tilde{q}_f(x; Pz) = \frac{1}{4\pi} \int dz \; e^{-ix\,P_z\,z} \, \langle \pi(P)| \, \bar{\psi}_f(z) \; \gamma_z \; \psi_f(0) \; |\pi(P)\rangle_c}$$

$$\mathbf{\rightarrow q_f(x) \text{ as } Pz \rightarrow \infty}$$

How fast?

Quark Distribution

§ Back to the continuum

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\rm QCD}^2/P_z^2\right)$$

Finite $P_z \to \infty P_z$, perturbative matching

$$\tilde{q}(x,\mu,P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu)$$

$$Z(x,\mu/P_z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang,
1310.7471 [hep-ph]; Y. Zhao, this wokshop

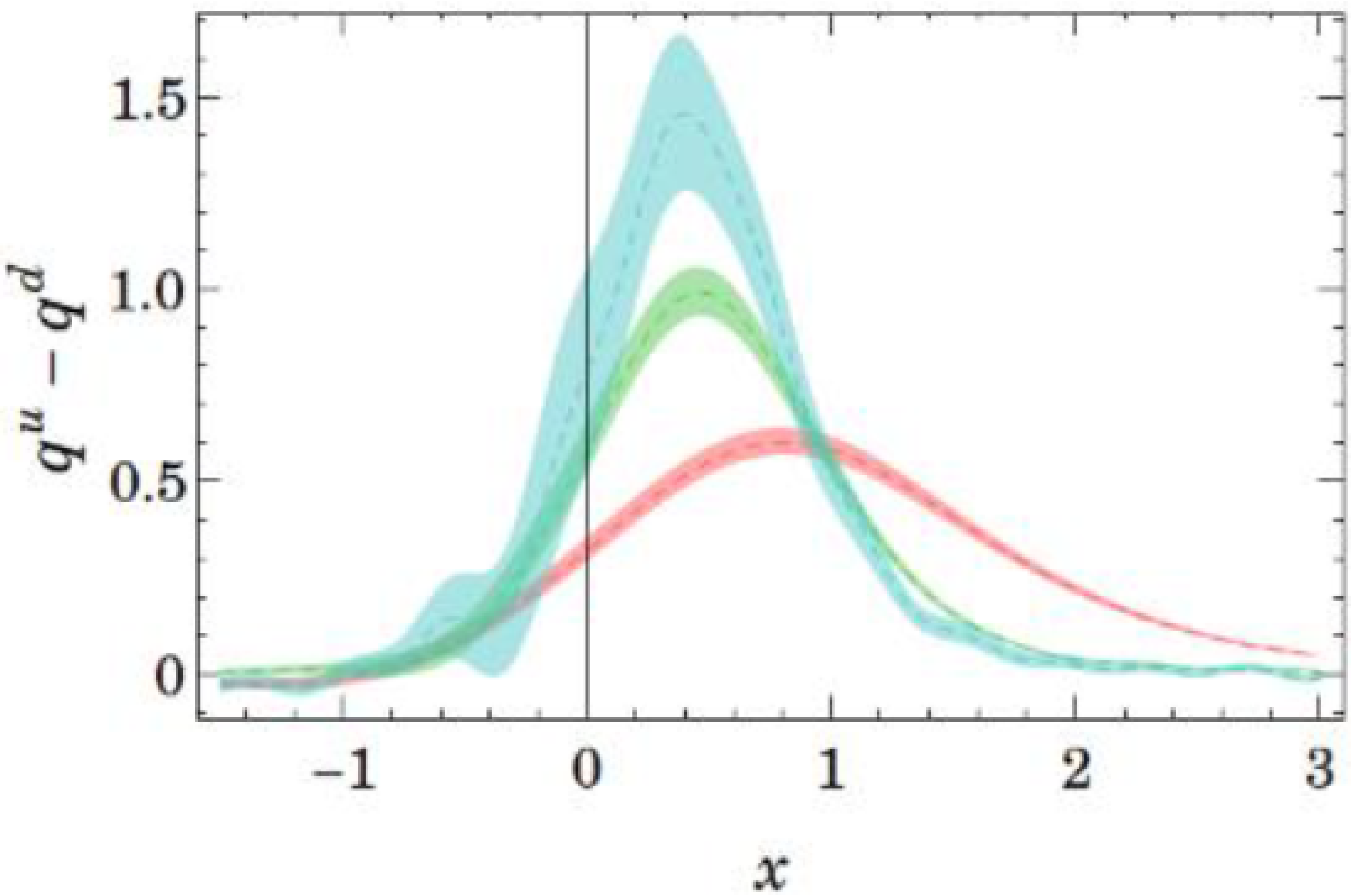
FIG. 1: One loop corrections to quasi quark distribution.

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

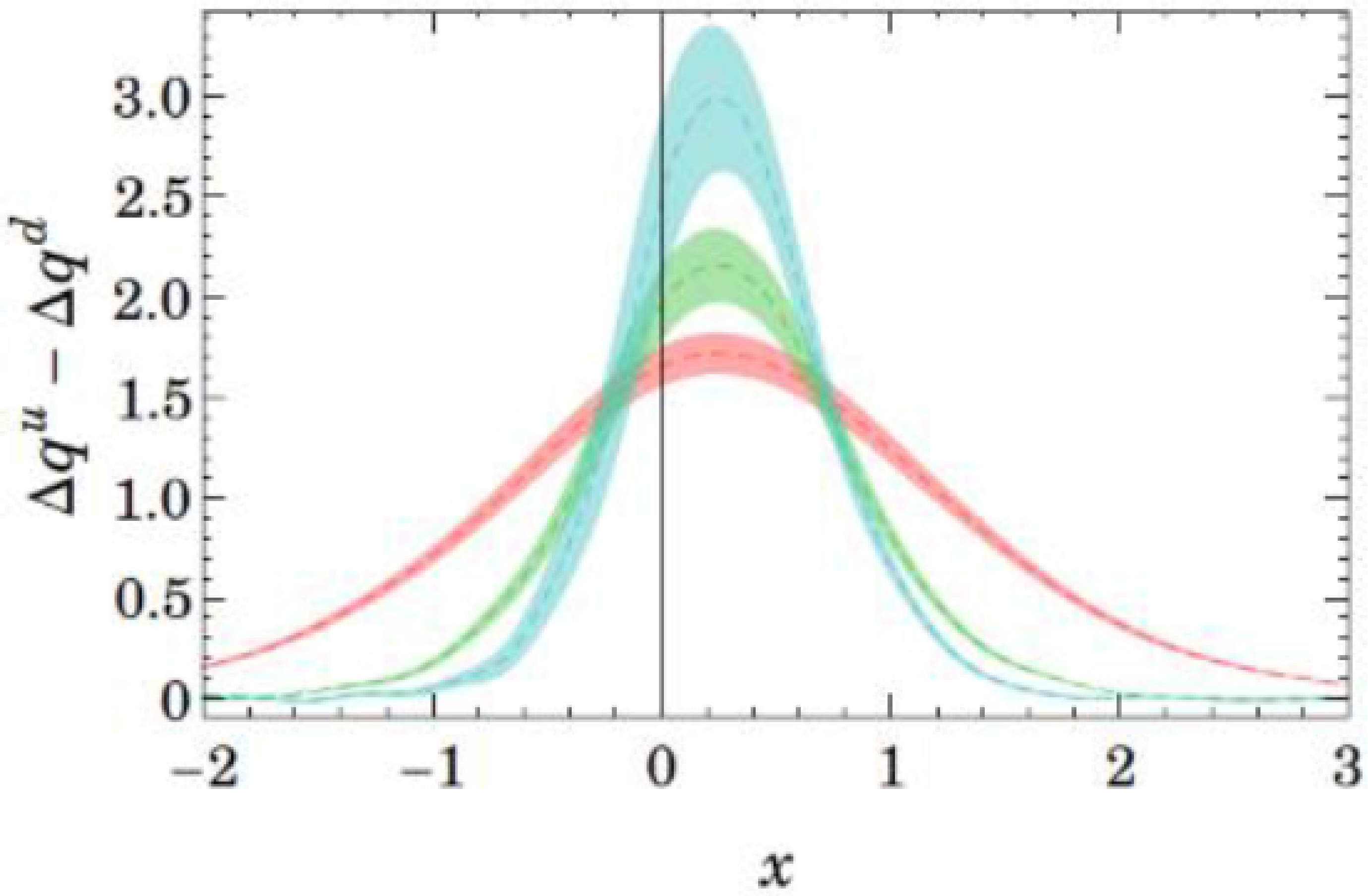
Improvement?

Work out leading-P_z corrections

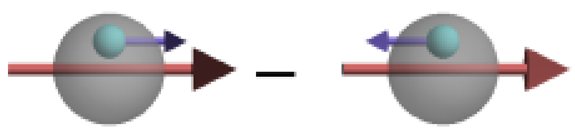
Helicity Distribution

§ Exploratory study

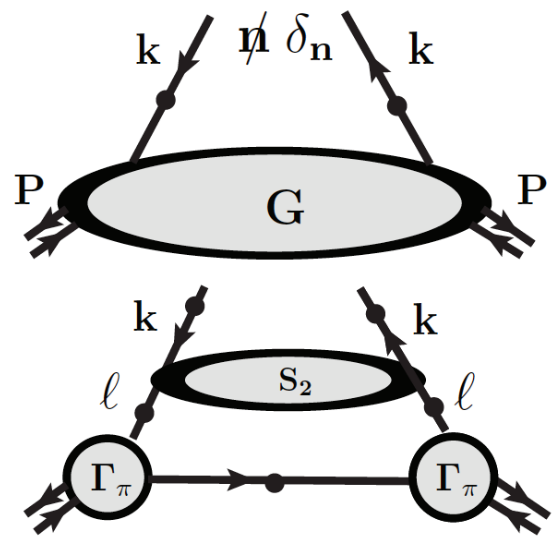
$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Uncorrected bare lattice results



Simple model for pion PDF & Quasi-PDF



$$\mathbf{S(k) = 1/(i\not k + M)\ , \quad M = 0.4\ GeV}$$

$$\mathbf{\Gamma_\pi(q, P) = \gamma_5 N_\pi \int_{-1}^{1} d\alpha\, \frac{\rho(\alpha)}{q^2 + \alpha q \cdot P + \Lambda^2}\ , \quad \rho(\alpha) = even}$$

Euclidean to Minkowski:-

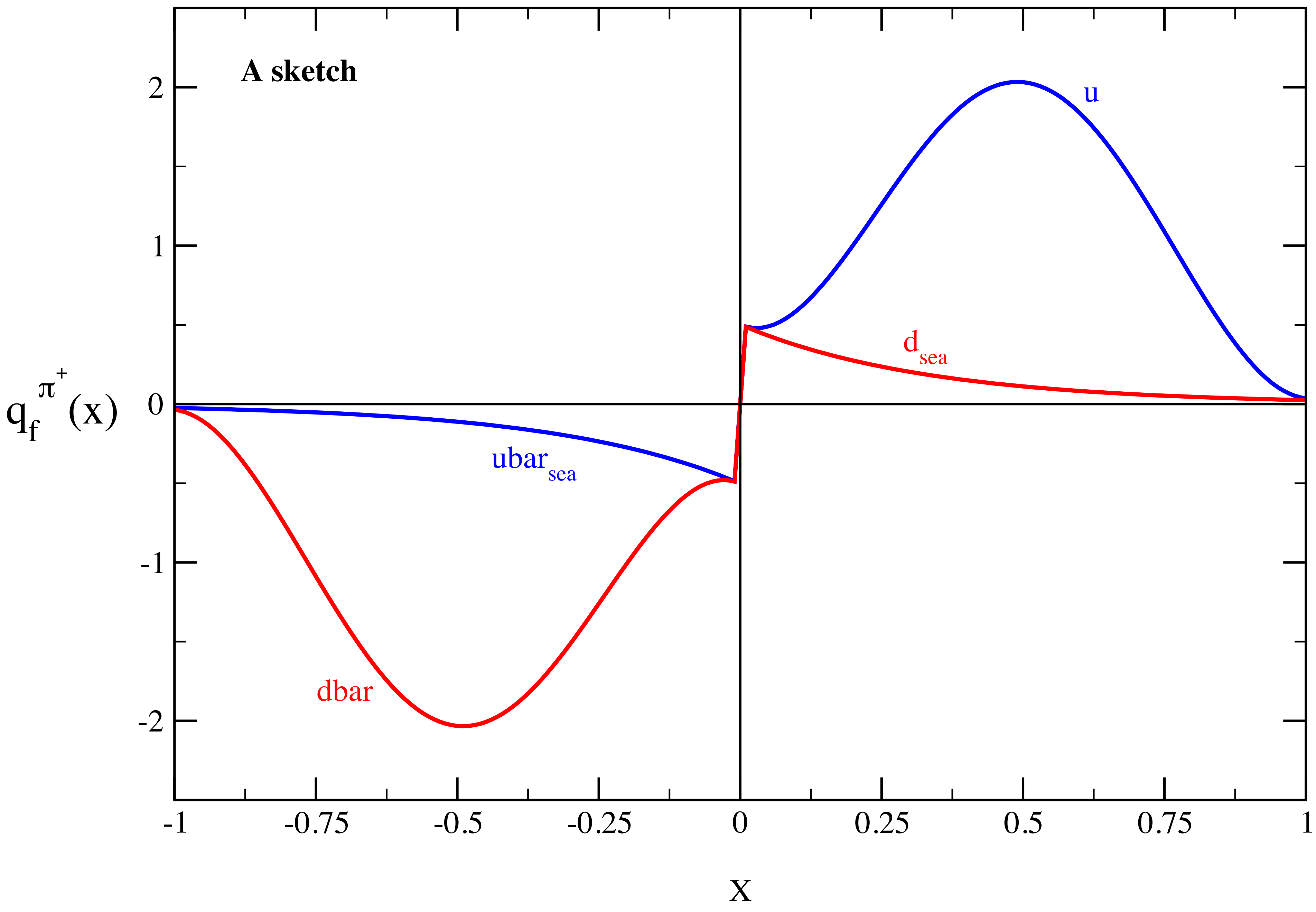
Evaluate q(x) directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} \mathbf{dk^-}$

$$\mathbf{q_A(x) = i\, N_c\, \mathrm{tr} \int \frac{dk^+\, dk^-\, d^2k_\perp}{(2\pi)^4}\ \delta(k^+ - x\, P^+)\ tr[\Gamma_\pi\ S\ (i\gamma^+)\ S\ \Gamma_\pi\ S]}$$

Evaluate $\tilde{\mathbf{q}}(\mathbf{x; P_z})$ **directly using Cauchy Residue Thm for** $\int_{-\infty}^{\infty} \mathbf{dk^0}$

$$\mathbf{\tilde{q}_A(x) = i\, N_c\, \mathrm{tr} \int \frac{dk^0\, dk_z\, d^2k_\perp}{(2\pi)^4}\ \delta(k_z - x\, P_z)\ tr[\Gamma_\pi\ S\ (i\gamma^z)\ S\ \Gamma_\pi\ S]}$$

Typical Hadron PDF q(x): a sketch for pion



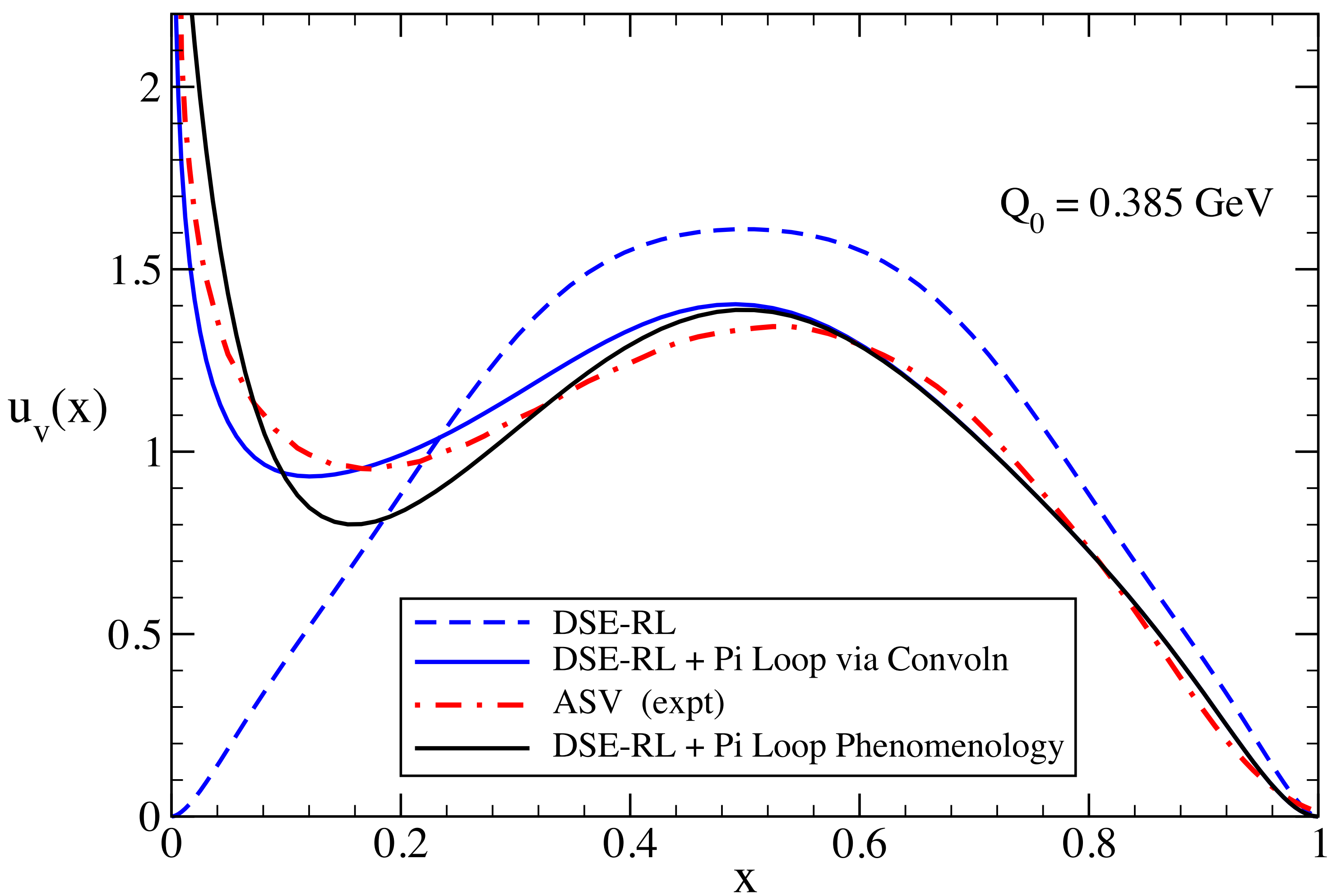
DIVERSION———A full DSE calculation of the true pion valence PDF

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

	$2\,q^{\rm RL}_{\rm val}$	$2\,q^{\rm DSE}_{\rm val}$	$4\,q^{\rm ASV}_{\rm sea}$	gluon	Total
$\langle x \rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

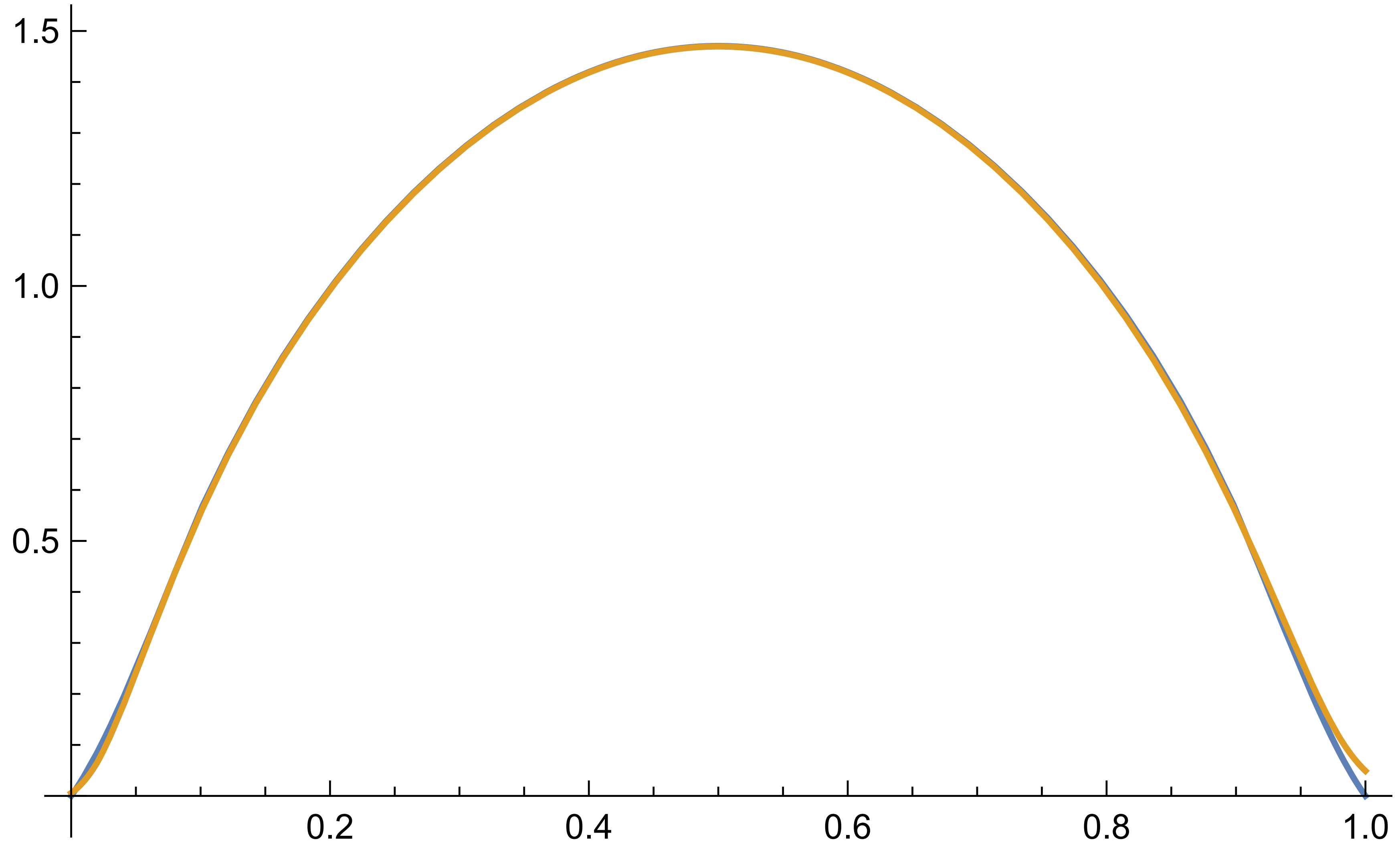
K. Khitrin, P. Tandy, in progress (2015)

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)

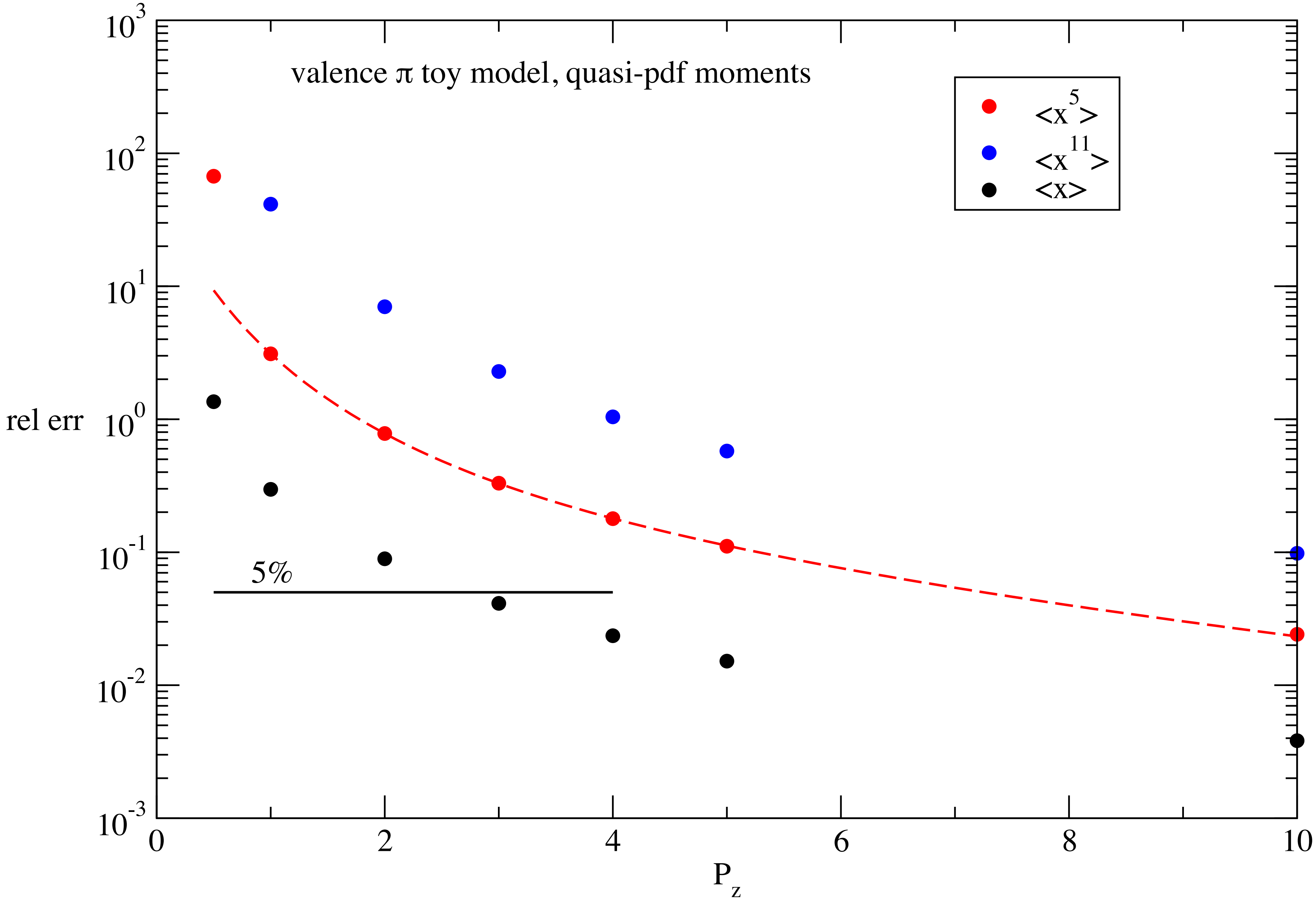


Back to: Spacelike Correlator Approximation for PDFs

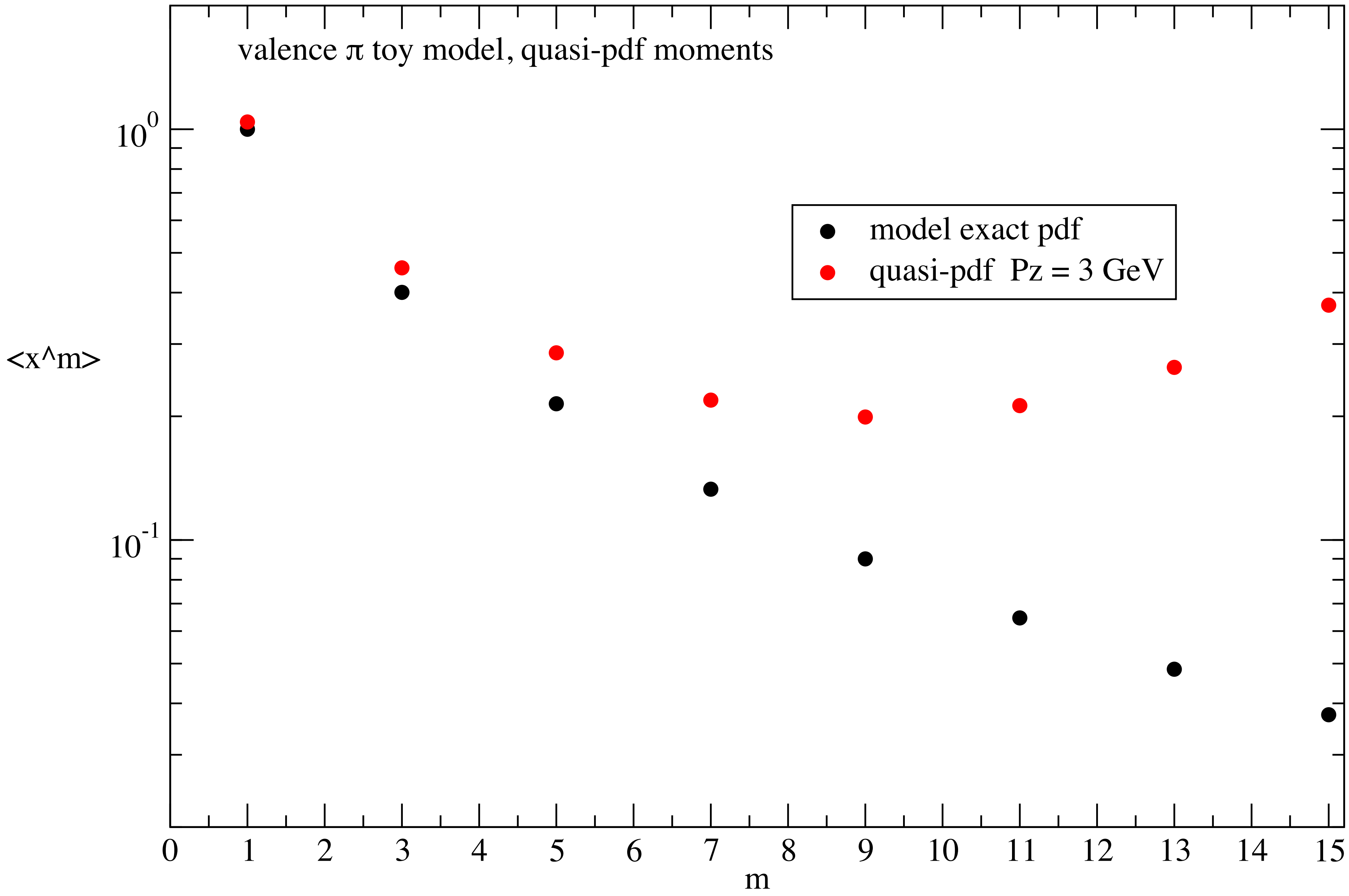
Model-exact PDF & Quasi-PDF @ Pz=10 GeV



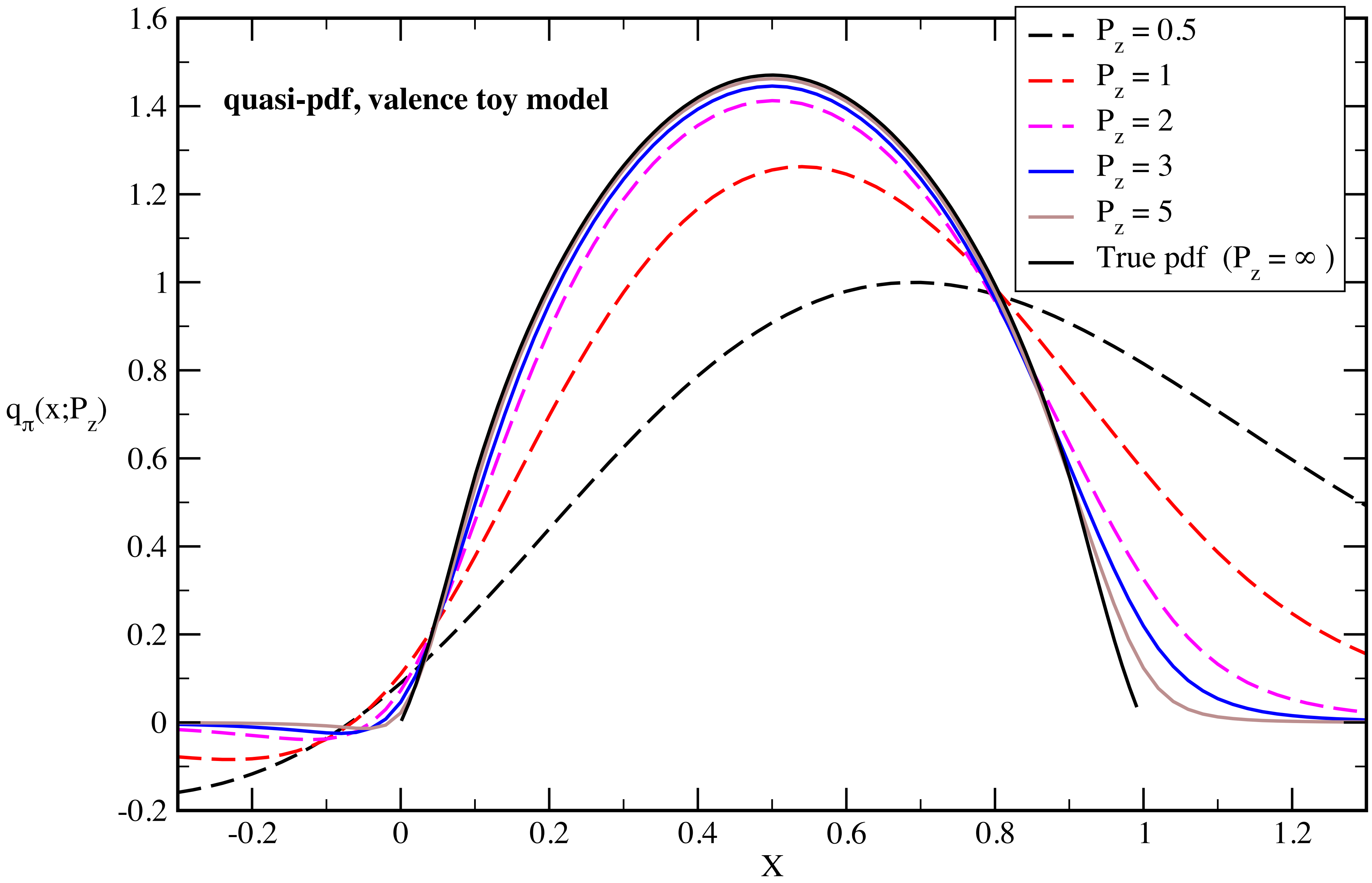
Pz Dependence of quasi-pdf of valence model pion



<x^m> for toy model pion at Pz = 3 GeV



Pz Dependence of quasi-pdf of u-ubar "pion"

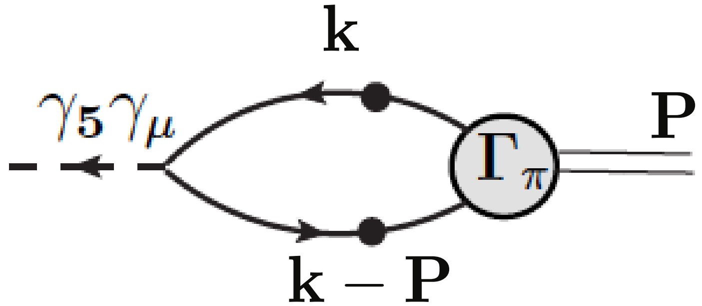


---I.Cloet, Lei Chang, PCT, in progress (2015).......

Parton Distribution Amplitudes of Mesons

Pion Distribution Amplitude (leading twist)

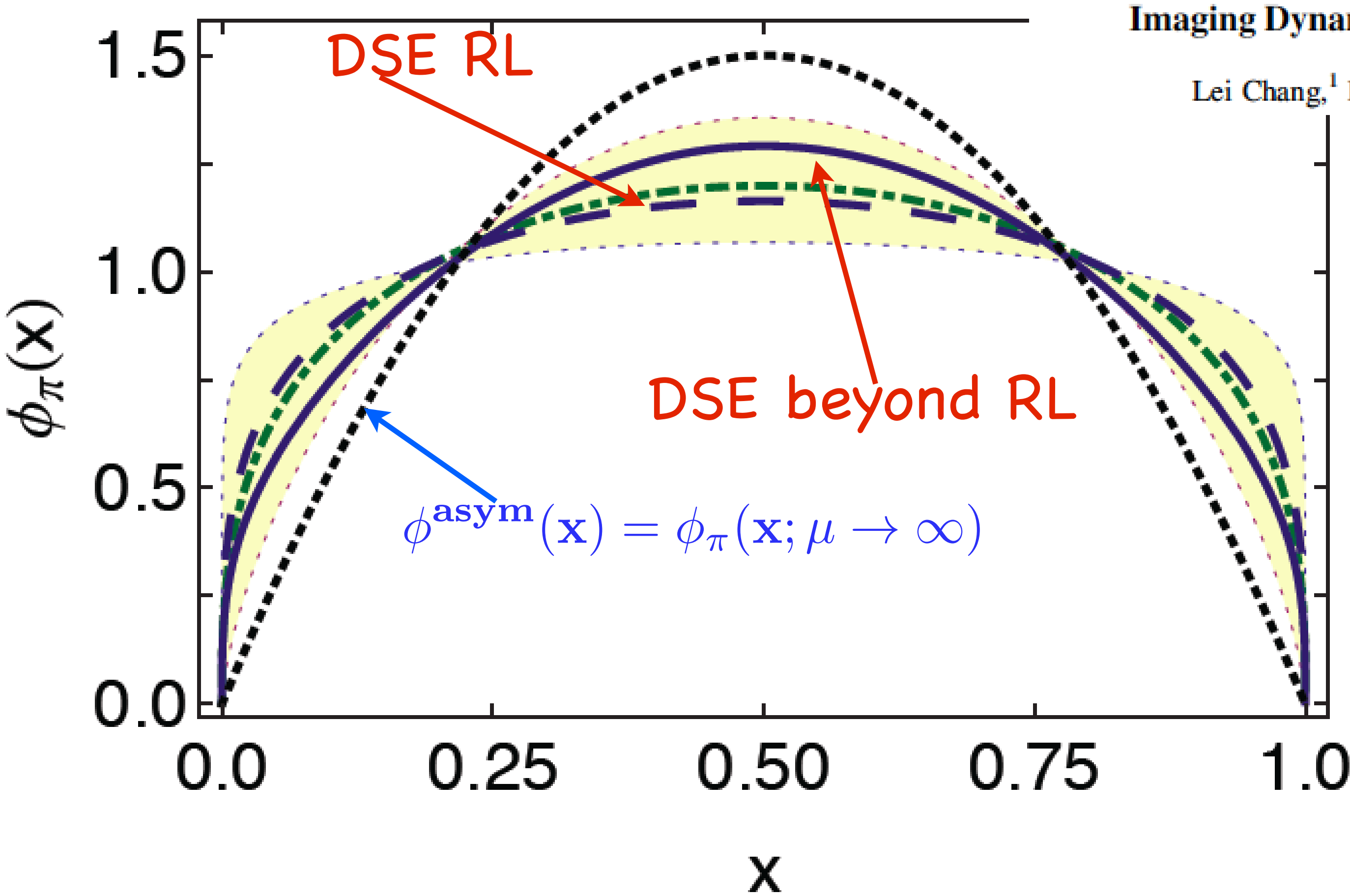
$$f_\pi \, \phi_\pi(x) = \int \frac{d\lambda}{2\pi} \, e^{-ixP.n\lambda} \, \langle 0 | \, \bar{q}(0) \, \gamma_5 \not{n} \, q(\lambda n) \, | \pi(P) \rangle$$

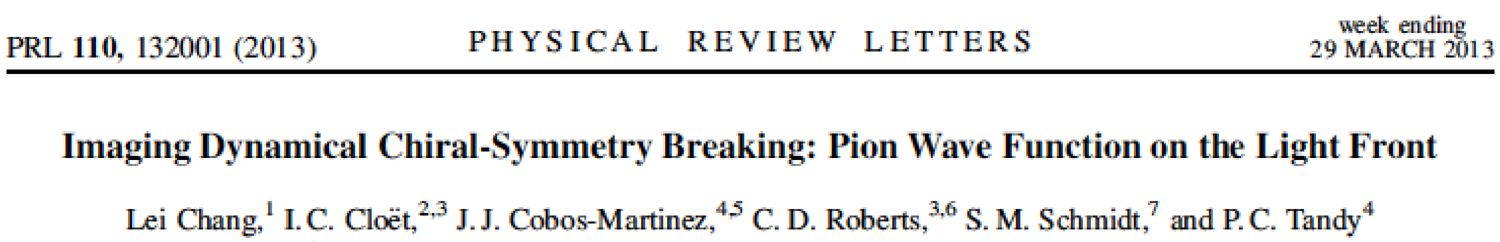


$$f_\pi \, \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \, \mathrm{tr} \int_k \left(\frac{k \cdot n}{P \cdot n}\right)^m \gamma_5 \not{n} \left[S(k) \, \Gamma_\pi(k - \frac{P}{2}; P) \, S(k - P) \right]$$

BS Wavefn

$\mu = 2$ GeV




PRL 110, 132001 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,[1] I. C. Cloët,[2,3] J. J. Cobos-Martinez,[4,5] C. D. Roberts,[3,6] S. M. Schmidt,[7] and P. C. Tandy[4]

Broadening of PDA is an expression of DCSB ---long sought after in LF QFT

Pion Distribution Amplitude

ERBL (~1980): $\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\;\mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$

$$\mathbf{a_n}(\mu) = \mathbf{a_n}(\mu_\mathbf{0})\left[\frac{\alpha_\mathbf{s}(\mu_\mathbf{0})}{\alpha_\mathbf{s}(\mu)}\right]^{\gamma_\mathbf{n}^{(\mathbf{0})}/\beta_\mathbf{0}}$$

Evolution to higher scales is
EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit: $\mathbf{a_n}(\mu\to\infty) = \mathbf{0}$

Efficient representation of DSE results:

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{N}_\alpha\;\mathbf{x}^\alpha(\mathbf{1-x})^\alpha\;\{\,\mathbf{1} + \Sigma_{\mathbf{n=2}}^\infty\;\tilde{\mathbf{a}}_\mathbf{n}(\mu)\,\mathbf{C_n^{\alpha+1/2}(2x-1)}\,\}$$

$$\phi_\mathbf{K}(\mathbf{x};\mu) = \mathbf{N}_\alpha\;\mathbf{x}^\alpha(\mathbf{1-x})^\alpha\;\{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\;\tilde{\mathbf{a}}_\mathbf{n}(\mu)\;\mathbf{C_n^{\alpha+1/2}(2x-1)}\,\}$$

$$+\;\mathbf{N}_\beta\;\mathbf{x}^\beta(\mathbf{1-x})^\beta\;\{\,\Sigma_{\mathbf{n=1,3\cdots}}\;\tilde{\mathbf{a}}_\mathbf{n}(\mu)\;\mathbf{C_n^{\beta+1/2}(2x-1)}\,\}$$

Low Order Truncation of ERBL-Gegenbauer Expn of PDA

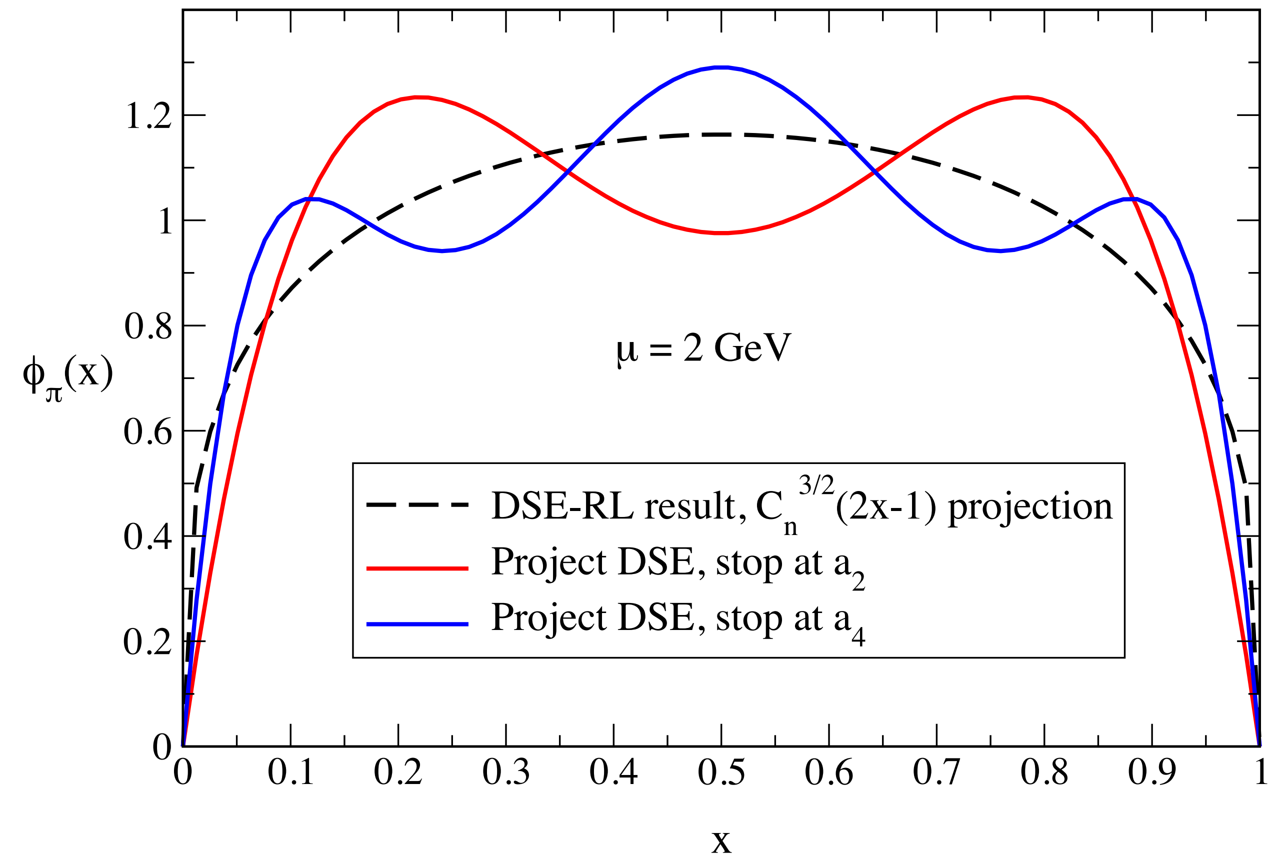
$$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\ \mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$$

DSE soln

{{0, 1.}, {**2, 0.233104**}, {4,0.112135},

{6, 0.0683202}, {8, 0.0469145},
{10, 0.0346469}, {12, 0.0268732},
{**14, 0.0215933**}, {16, 0.0178199}, **10%**
{18, 0.0150159}, {20, 0.0128672},
{22, 0.0111788}, {24, 0.00982438},
{26, 0.00871886}, {28, 0.00780296},

{30, 0.00703438}, {32, 0.0063823},
{34, 0.00582279}, {36, 0.00534272},
{38, 0.00493277}, {**40, 0.00447911**}} +.....
2%



A double-humped PDA is almost ruled out by
V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)

$$\phi_\pi^{\mathbf{QCDSR}}(\mathbf{x = 1/2};\mu = \mathbf{2}) = \mathbf{1.2 \pm 0.3}$$

One Lattice-QCD Moment Almost Determines Pion DA

PRL 111, 092001 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Pion Distribution Amplitude from Lattice QCD

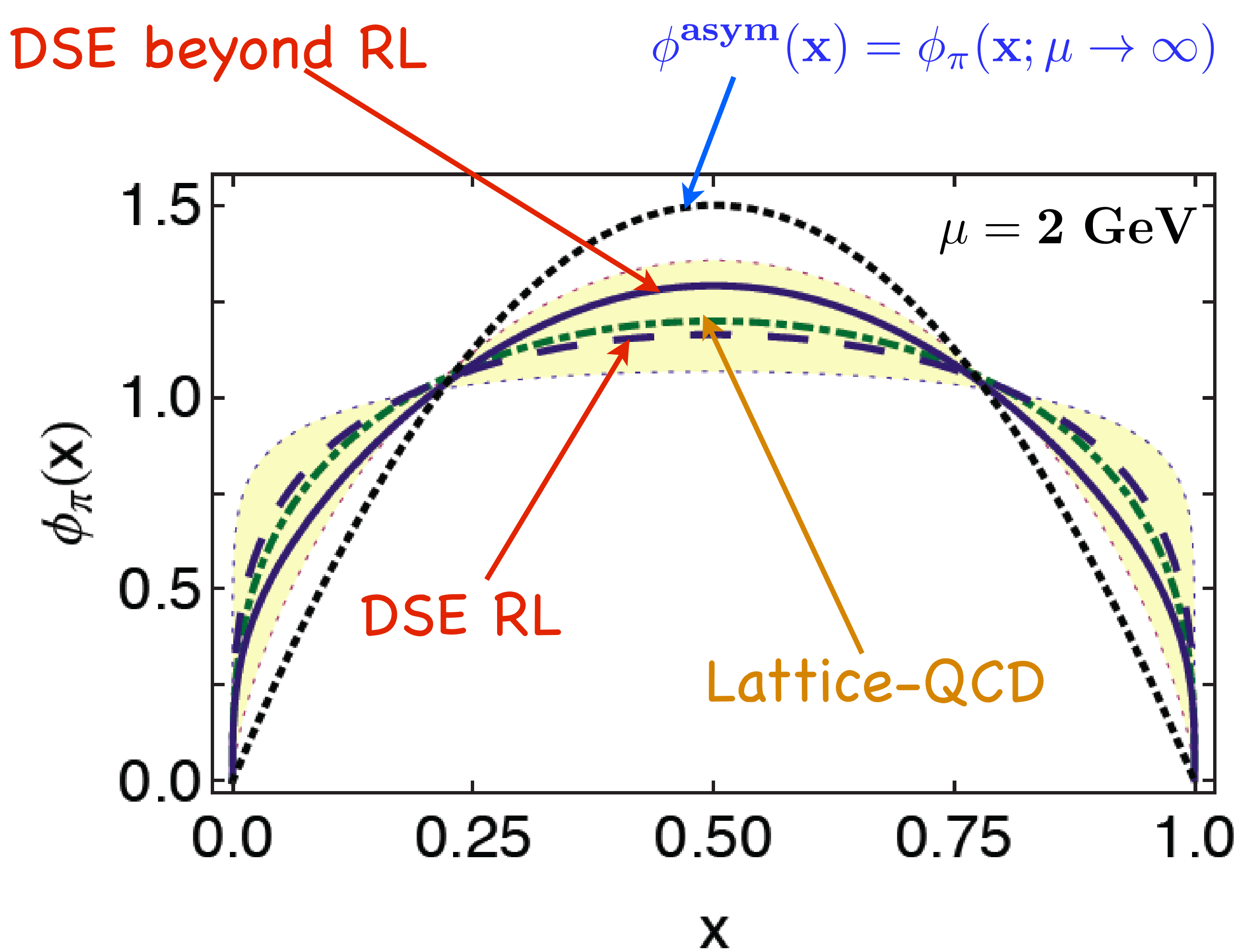
I. C. Cloët,[1] L. Chang,[2] C. D. Roberts,[1] S. M. Schmidt,[3] and P. C. Tandy[4]

$$\phi_\pi^{\mathbf{LQCD}}(\mathbf{x};\mu=\mathbf{2}) = \mathbf{N\,x}^\alpha(\mathbf{1}-\mathbf{x})^\alpha$$

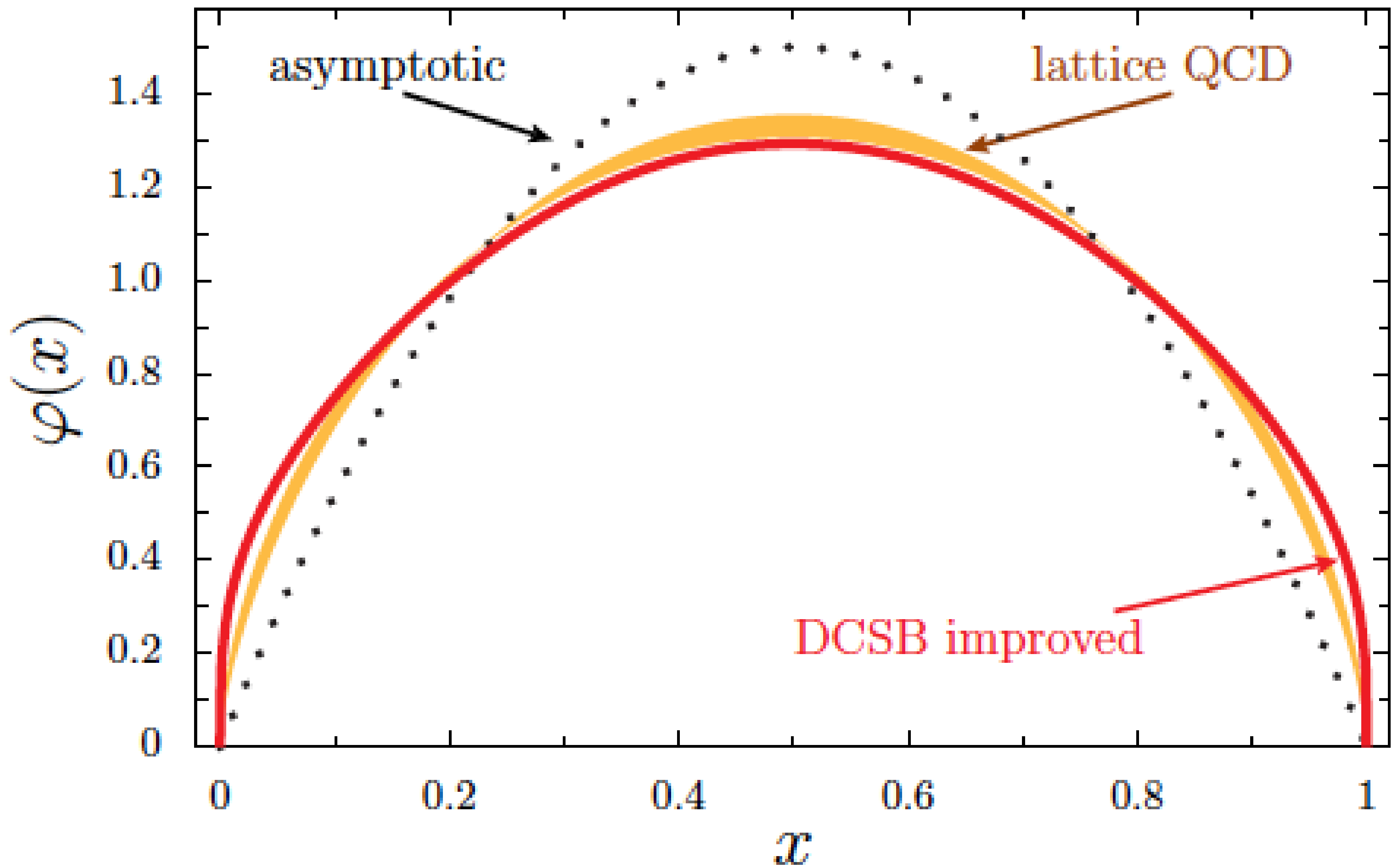
$$\alpha = \mathbf{0.35} + \mathbf{0.32} - \mathbf{0.24}$$

$$\langle(\mathbf{2x}-\mathbf{1})^2\rangle^{\mathbf{LQCD}}_{\mu=\mathbf{2}} = \mathbf{0.27} \pm \mathbf{0.04}$$

V. Braun et al., PRD74, 074501 (2006)



Pion Distribution Amplitude



$$\langle (2x-1)^2 \rangle^{LQCD}_{\mu=2\ GeV} = 0.2361\,(41)\,(39)$$

V. Braun et al., arXiv:1503.03656 [hep=lat]

DSE prediction: 0.251

Kaon Distribution Amplitude

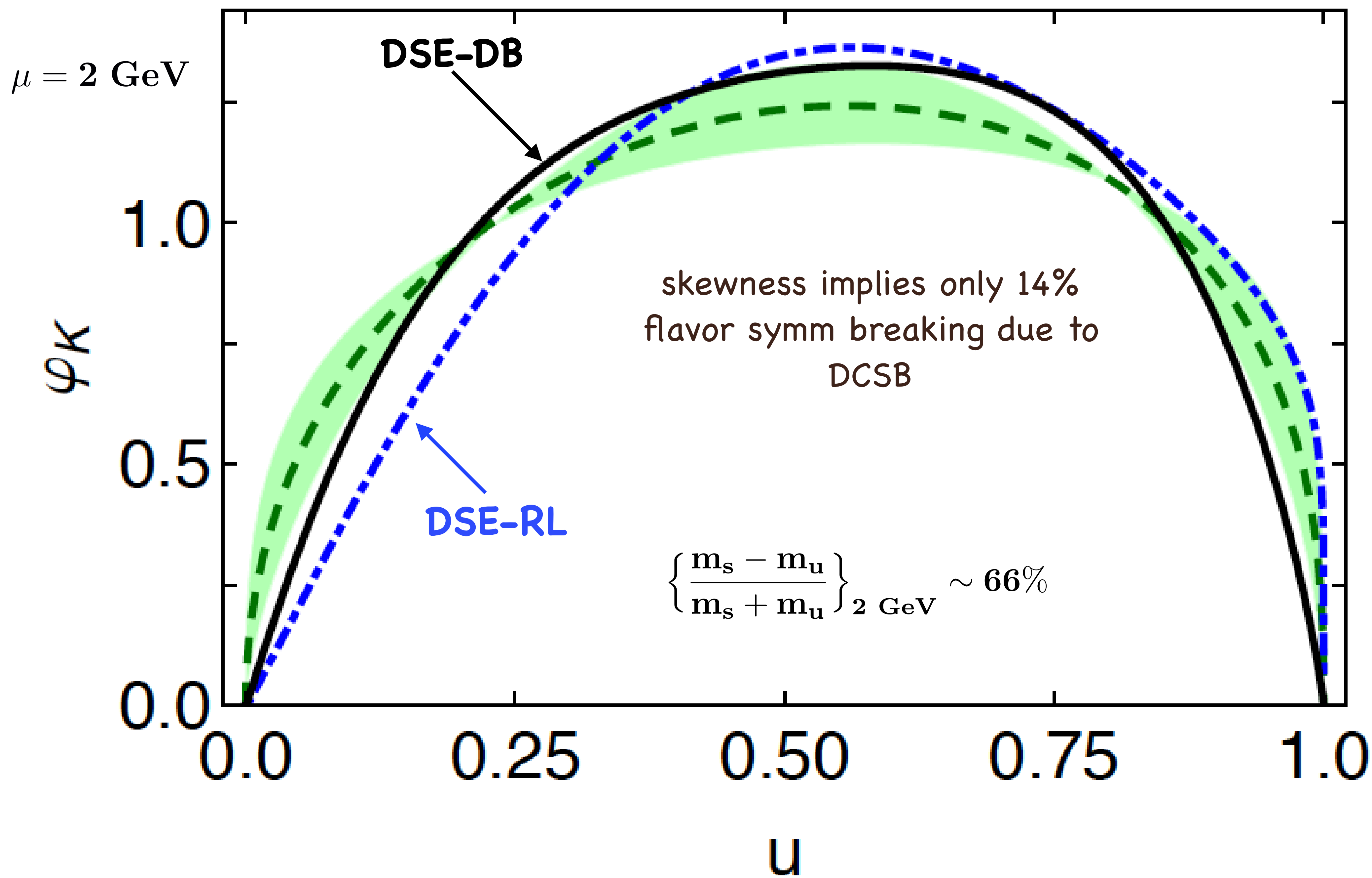
Size of SU(2)xSU(3) spin-flavor symmetry-breaking?

that, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions $B^{\pm} \to (\pi K)^{\pm}$ and $B^{\pm} \to \pi^{\pm}\pi^{0}$ provide access to the imaginary part of the CKM matrix element V_{ub}: $\gamma = \mathrm{Arg}(V^{*}_{ub})$ [4]. Factorisation theorems have been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called "non-factorisable" corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of γ is only as accurate as the evaluation of both the difference between K and π PDAs and also their respective differences from the asymptotic distribution, $\varphi^{\mathrm{asy}}(u) = 6u(1-u)$. Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)

Kaon Distribution Amplitude

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)



R. Arthur, P. Boyle, D. Brommel, M. Donnellan, J. Flynn et al, PRD83, 074505 (2011)

Kaon DA Moments

$\mu = 2$ GeV

Table 1

Moments ($u_\Delta = 2u - 1$) of the K-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall *Ansatz* for the kaon's light-front wave function. We also list values obtained with $\varphi = \varphi^{asy}$, Eq. (14), and $\varphi = \varphi_{ms}$, Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

	$\langle u_\Delta^m \rangle$	$m = 1$	2	3	4	5	6
DSE-QCD:	RL	0.11	0.24	0.064	0.12	0.045	0.076
	DB	0.040	0.23	0.021	0.11	0.013	0.063
Lattice-QCD:	[40]	0.027(2)	0.26(2)				
	[41]	0.036(2)	0.26(2)				
	[10]	0.036(2)	0.26(2)	0.020(2)	0.13(2)	0.014(2)	0.085(15)
QCD Sum Rules:	[42–46]	0.035(8)					
	[47]	0.04(2)	0.24(1)				
	$\varphi = \varphi_{ms}$	0.33	0.33	0.2	0.2	0.14	0.14
	$\varphi = \varphi^{asy}$	0	0.2	0	0.086	0	0.048

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)

Applications:-
eg: Form Factors

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$\mathbf{F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy\, \phi_\pi^\star(x;Q)\, [T_H(x,y;Q^2)]\, \phi_\pi(y;Q)}$$ + NLO/higher twist.....

---LFQCD, Brodsky, LePage PRD (1980)

$$\mathbf{Q^2 >> \Lambda^2_{QCD} :\ Q^2 F_\pi(Q^2) \to 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, \omega_\phi^2(Q^2) \ +\ \mathcal{O}(1/Q^2)}$$

$$\mathbf{\omega_\phi(Q^2) = \frac{1}{3}\int_0^1 dx\, \frac{\phi_\pi(x;Q)}{x}}$$

$$\mathbf{\to 1\ ,\quad Q^2 \to \infty}$$

$\mathbf{at\ Q^2 \sim 3 - 4\ GeV^2,\ \Rightarrow 0.1}$

$\mathbf{JLab\ expt,\ \ Theory \Rightarrow 0.45}$

$$\mathbf{But, recent\ DSE\ theory \Rightarrow \phi_\pi(x; \mu = 2\ GeV) \Rightarrow \omega_\phi^2 = 3.3}$$

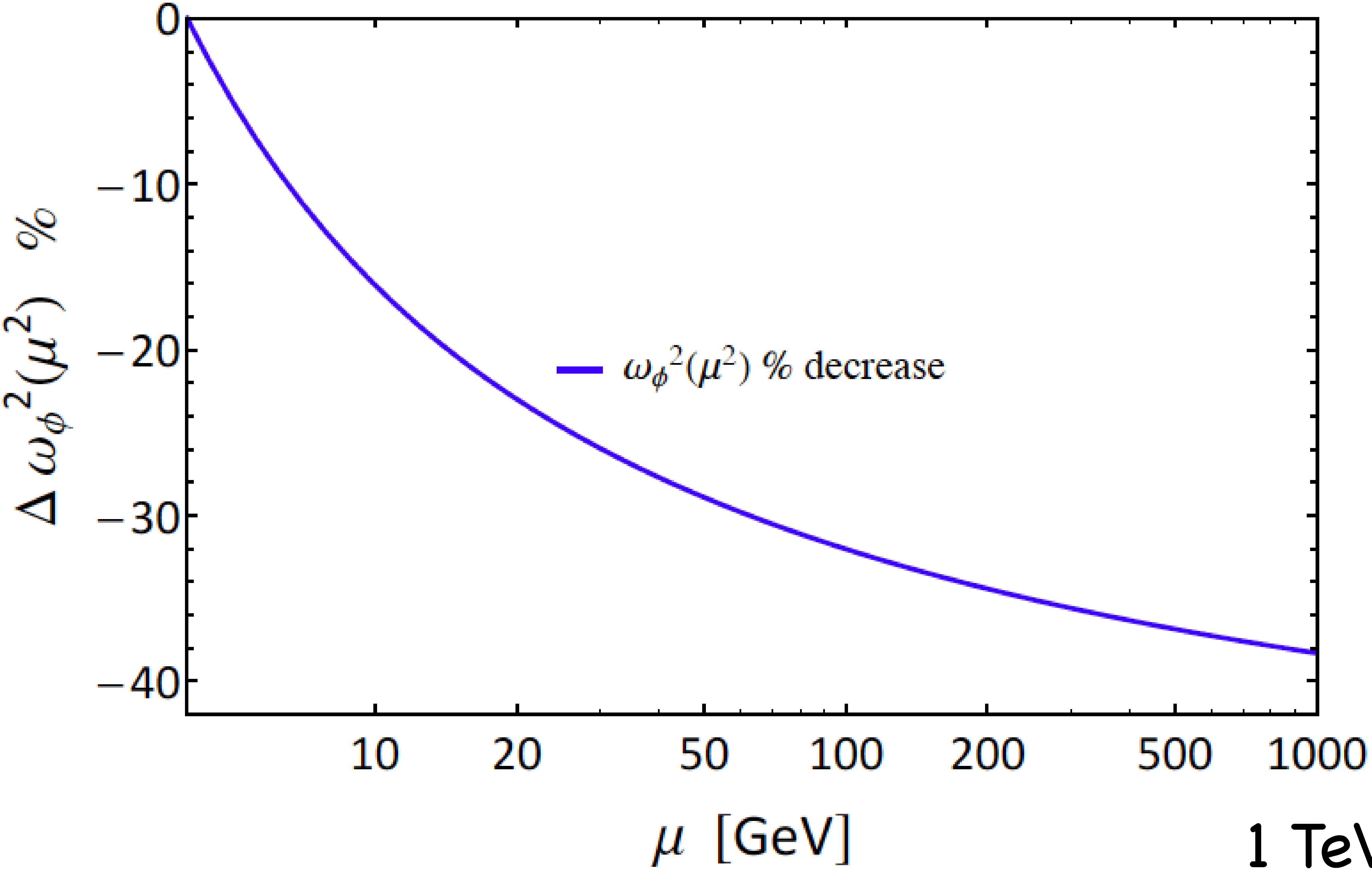
PRL **111,** 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I. C. Cloët,[2] C. D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

UV-QCD is not Asymptotic QCD

$$\mathbf{Q^2} >> \mathbf{\Lambda^2_{\rm QCD}} : \ \mathbf{Q^2 F_\pi(Q^2)} \to \mathbf{16}\,\pi\,\mathbf{f_\pi^2}\,\alpha_\mathbf{s}(\mathbf{Q^2})\ \omega_\phi^\mathbf{2}(\mathbf{Q^2})\ +\ \mathcal{O}(\mathbf{1/Q^2})$$

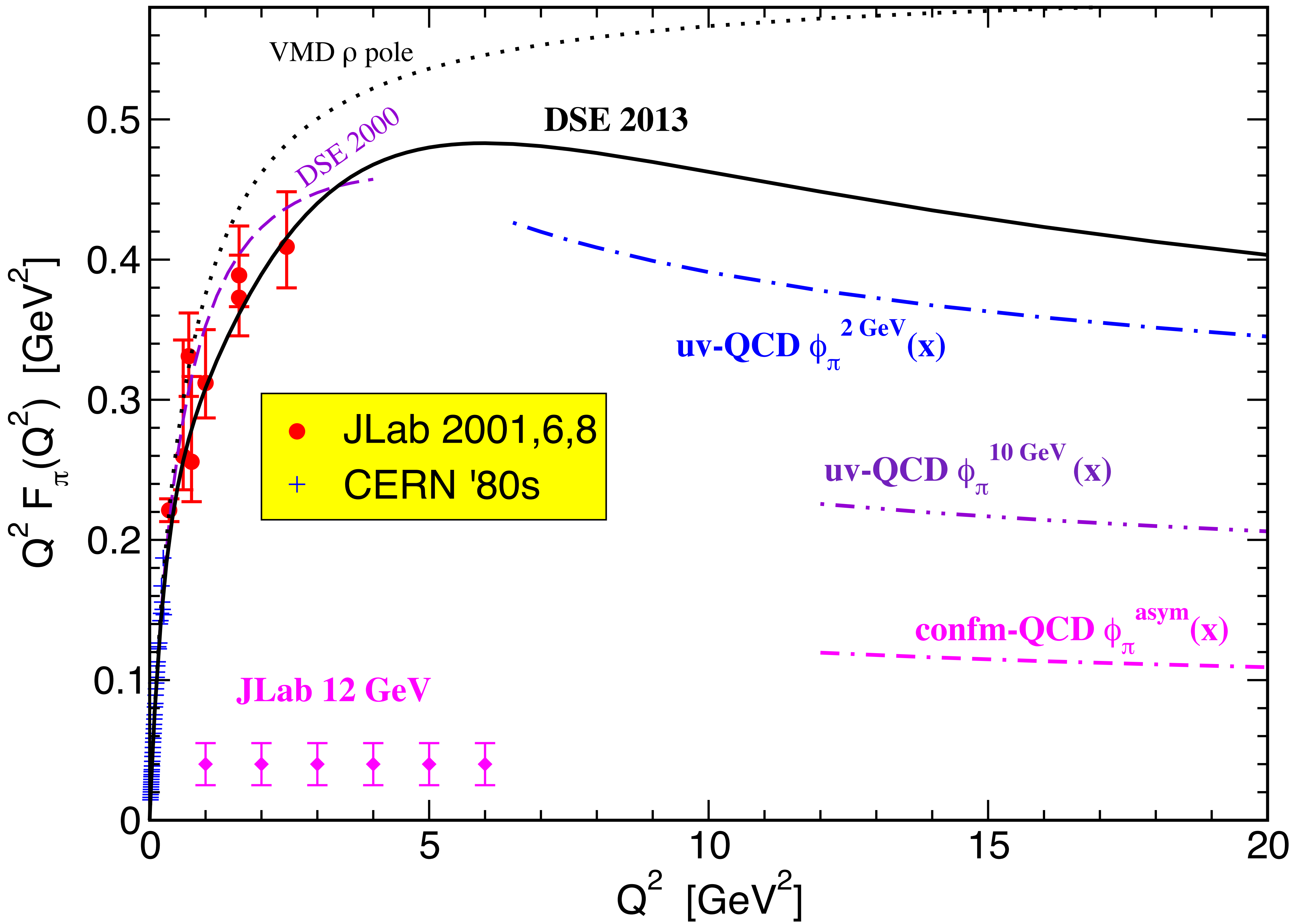


1 TeV

PRL 111, 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I.C. Cloët,[2] C.D. Roberts,[2] S.M. Schmidt,[3] and P.C. Tandy[4]



Jab data: G. Huber et al., PRC78, 045203 (2008)

Pion Transition Form Factor

K. Raya, L. Chang, A. Bashir, J.J.Cobos-Martinez, L.X. Gutierez-Guerrero, C.D.Roberts, P.C.Tandy, PRD93, 074017 (2016)

This afternoon…

Summary

- **X. Ji's space-like correlator approach to PDFs—a model investigation. Spurious anti-quark contributions seem unavoidable if Pz < 2 GeV. For x > 0.8, need Pz > 4 GeV for confidence in the qualitative shape. Further work in progress.**

- **Parton Distribution Amplitudes (pion, kaon). DSE approach shows good contact with available lattice-QCD moments. Flavor symmetry breaking & dynamical chiral symmetry breaking evident and quantitative in the shapes.**

- **Pion Transition & Elastic Form Factors DSE TFF calculation for all Q^2—-agrees with Belle not BaBar. DSE elFF——Connection with ultraviolet QCD reconciled. Identify that the ultraviolet partonic behavior is within reach of proposed JLab pion FF experiments.**

- **Parton Distribution Functions (pion). Qualitative behavior of empirical data fits reproduced by DSE q-qbar + pion loop analysis.**

- **Time to declare we understand the pion and kaon in QCD ?**

The End

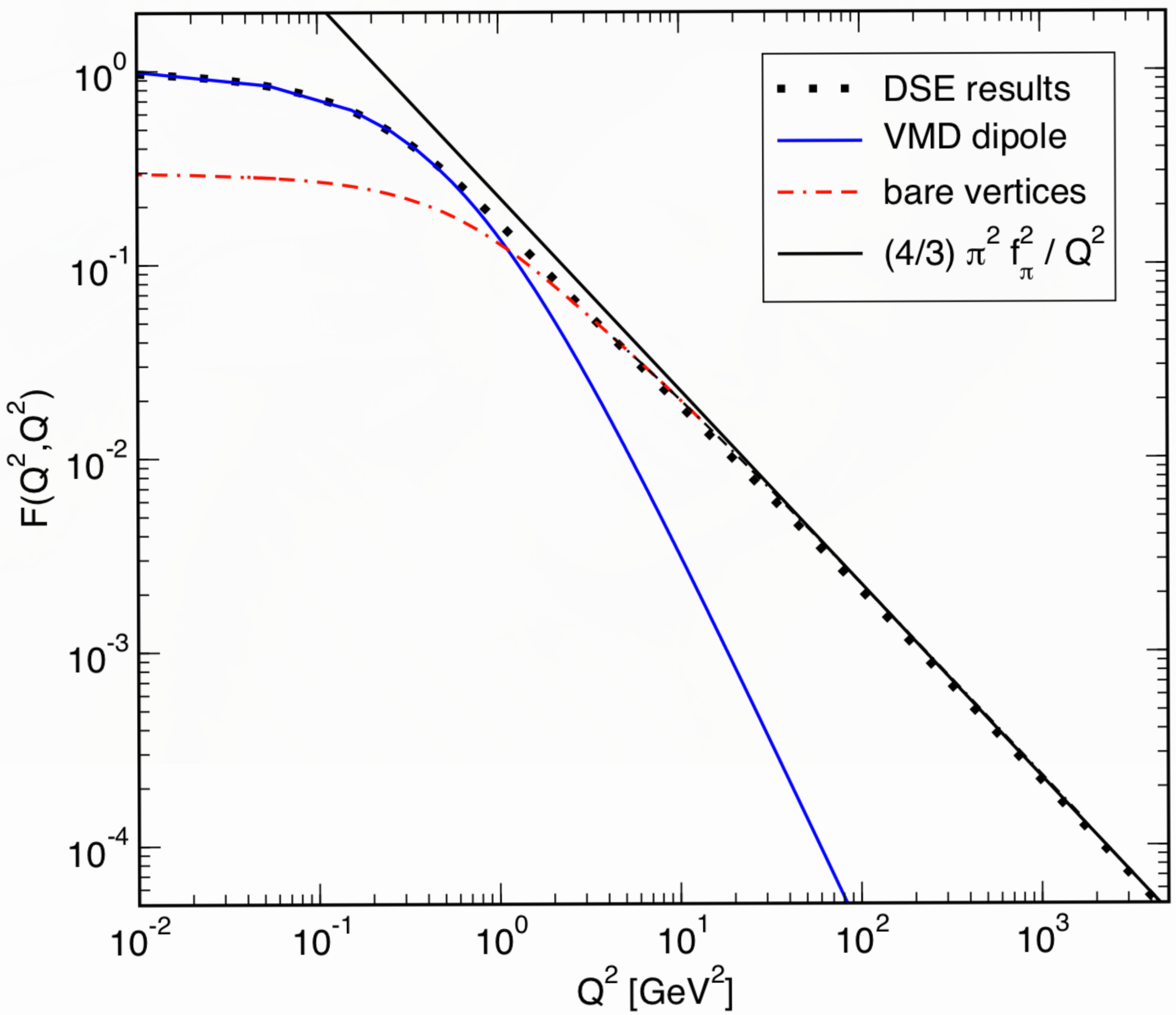
Collaborators

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Sixue Qin, Argonne National Lab, USA
- Hong-shi Zong, Nanjing Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico

Where Asym FF Could be Calculated, its Power Law was Correct:-

$\gamma^\star\pi\gamma^\star$ *Asymptotic Limit*

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒



Mazatlan Nov09 – p. 20/5

Estimate 1-Pion Loop Contribution to Pion PDF

$$\pi^+ \; : \; \langle x^1 \rangle_\mu = \int_0^1 dx\, x\, \{u + \bar{u}_{sea} + \bar{d} + d_{sea} + g(x)\} \approx 2\langle x\, q_v(x)\rangle + 4\langle x\, q_{sea}(x)\rangle + \langle x\, g(x)\rangle = 1$$

$u = u_v + u_{sea}\,,\quad \bar{d} = \bar{d}_v + \bar{d}_{sea}$ Empirical GRS/ASV $\Rightarrow$ universal $q_v(x),\ q_{sea}(x)$ at $\mu = 0.630$ GeV

$$\Gamma_\pi = \sqrt{1-\alpha^2}\,\Gamma^{RL}_{q\bar{q}} + \alpha\,\Gamma_{\pi q\bar{q}}$$

CPT: 18% effect

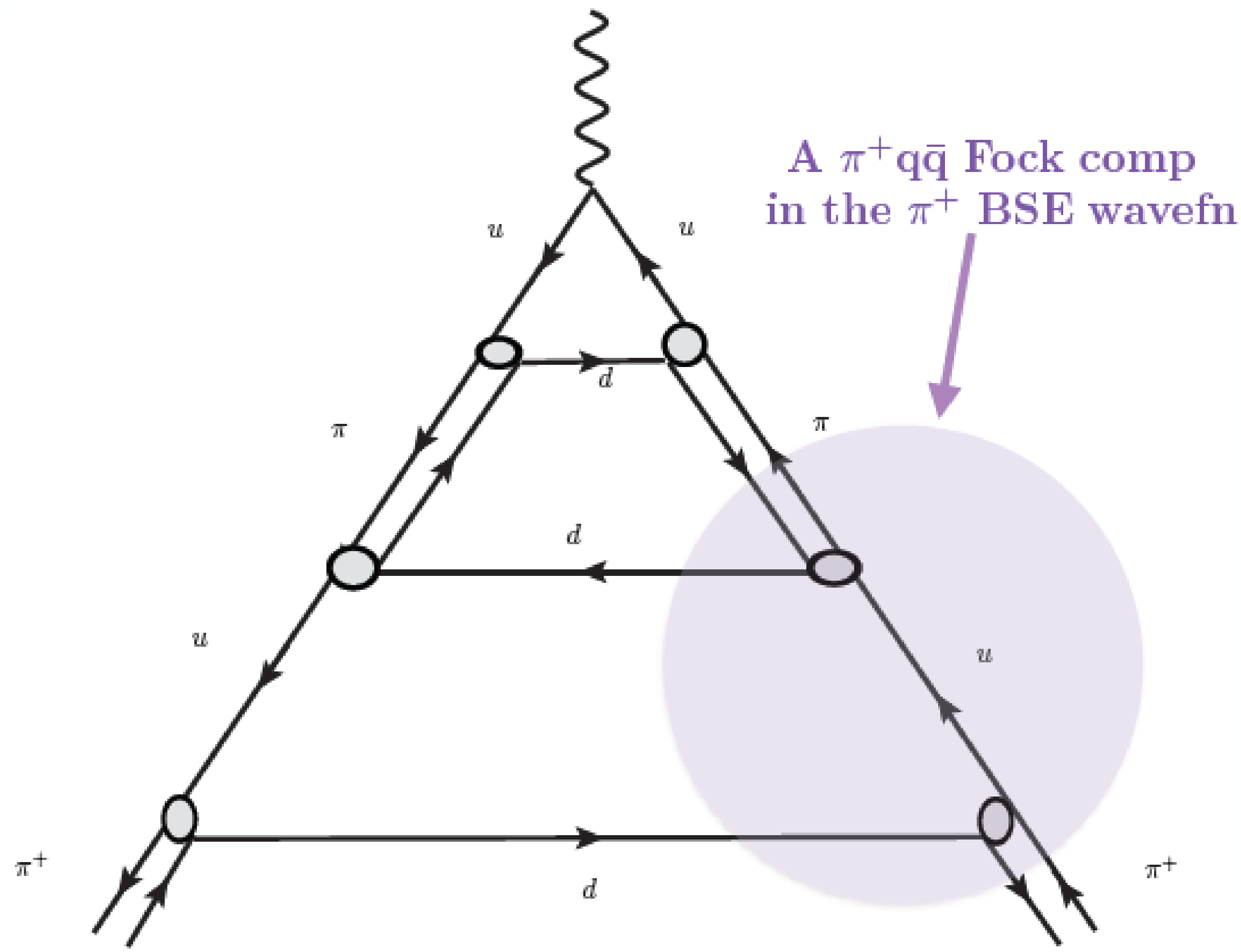
$$r^2_{ch} = (1-\alpha^2)\, r^2_{RL} + \alpha^2\, r^2_{\pi-lp}$$

DSE-RL: $r^2_{RL} = r^2_{ch} \Rightarrow \alpha^2 = 18\%$

PDF Consequence:

$$q_v(x) = (1-\alpha^2)\, q^{RL}(x) + q_v^{\pi-lp}(x)$$

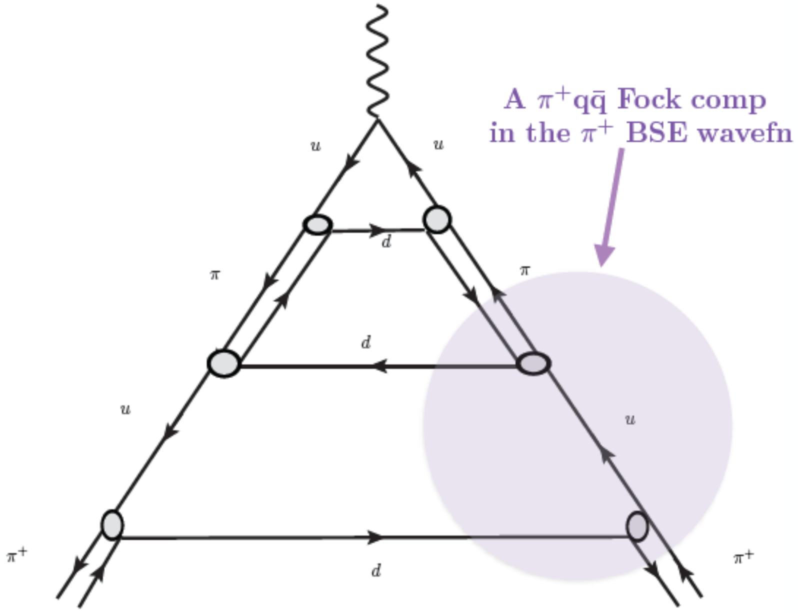
with $\langle q_v^{\pi-lp}(x)\rangle = \alpha^2 = 0.18$



Convolution Model for q(x) from virtual pi loop

$$\mathbf{q_v^{\pi-lp}(x)} \sim \mathcal{P}_{\mathbf{q/T}}(\mathbf{x}) = \int_{\mathbf{x}}^{\mathbf{1}} \mathbf{dy}\ \mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})\ \mathcal{P}_{\mathbf{q}/\pi}(\tfrac{\mathbf{x}}{\mathbf{y}})\ ,$$

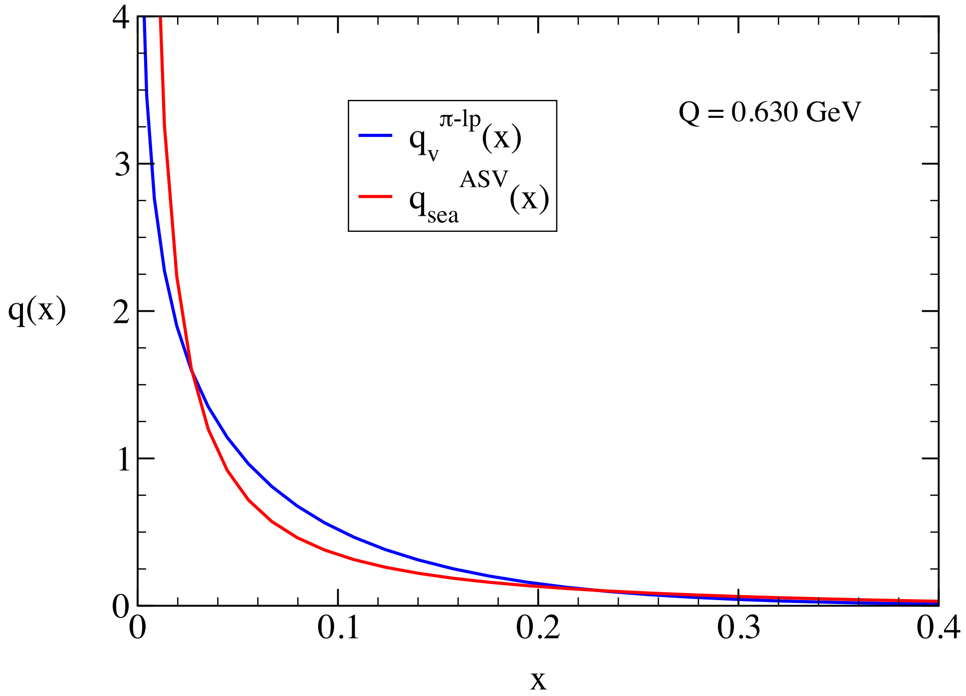
$$\mathbf{T} = \ \mathbf{target} = \pi\ \mathbf{here}$$



$\mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})$ **should strongly favor** $\mathbf{y} \leq \frac{\mathbf{m}_\pi}{\mathbf{2M_q + m_\pi}} \approx \mathbf{0.2}$,

$\mathcal{P}_{\mathbf{q}/\pi}(\frac{\mathbf{x}}{\mathbf{y}})$ **is self – consistently determined**

Result is strongly constrained



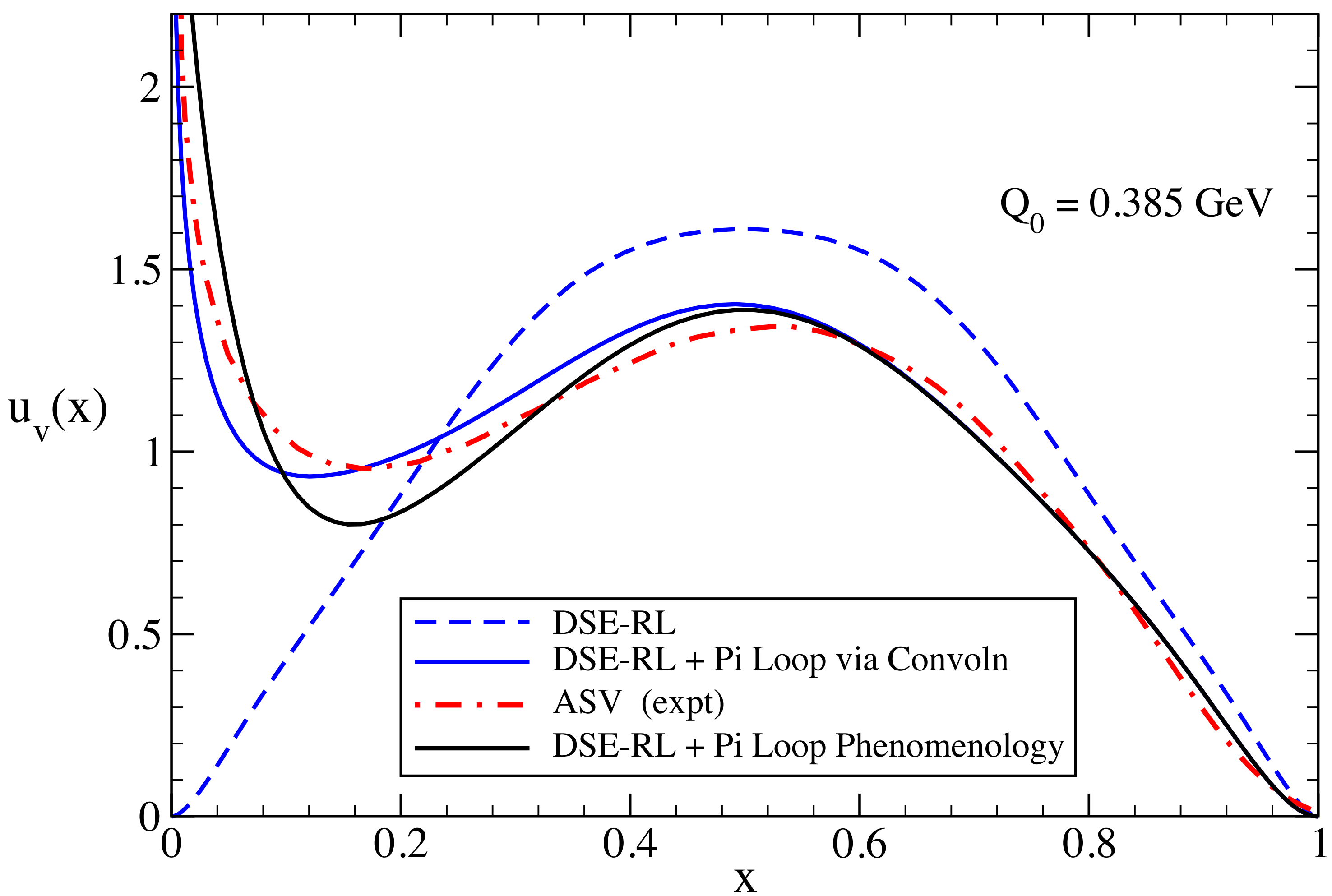
Analysis of Pion Parton Momentum Sum Rule

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

	$2\,q_{\rm val}^{\rm RL}$	$2\,q_{\rm val}^{\rm DSE}$	$4\,q_{\rm sea}^{\rm ASV}$	gluon	Total
$\langle x \rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

K. Khitrin, P. Tandy, in progress (2015)

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)



Pion PDFs—Expt "Data" Parameterizations

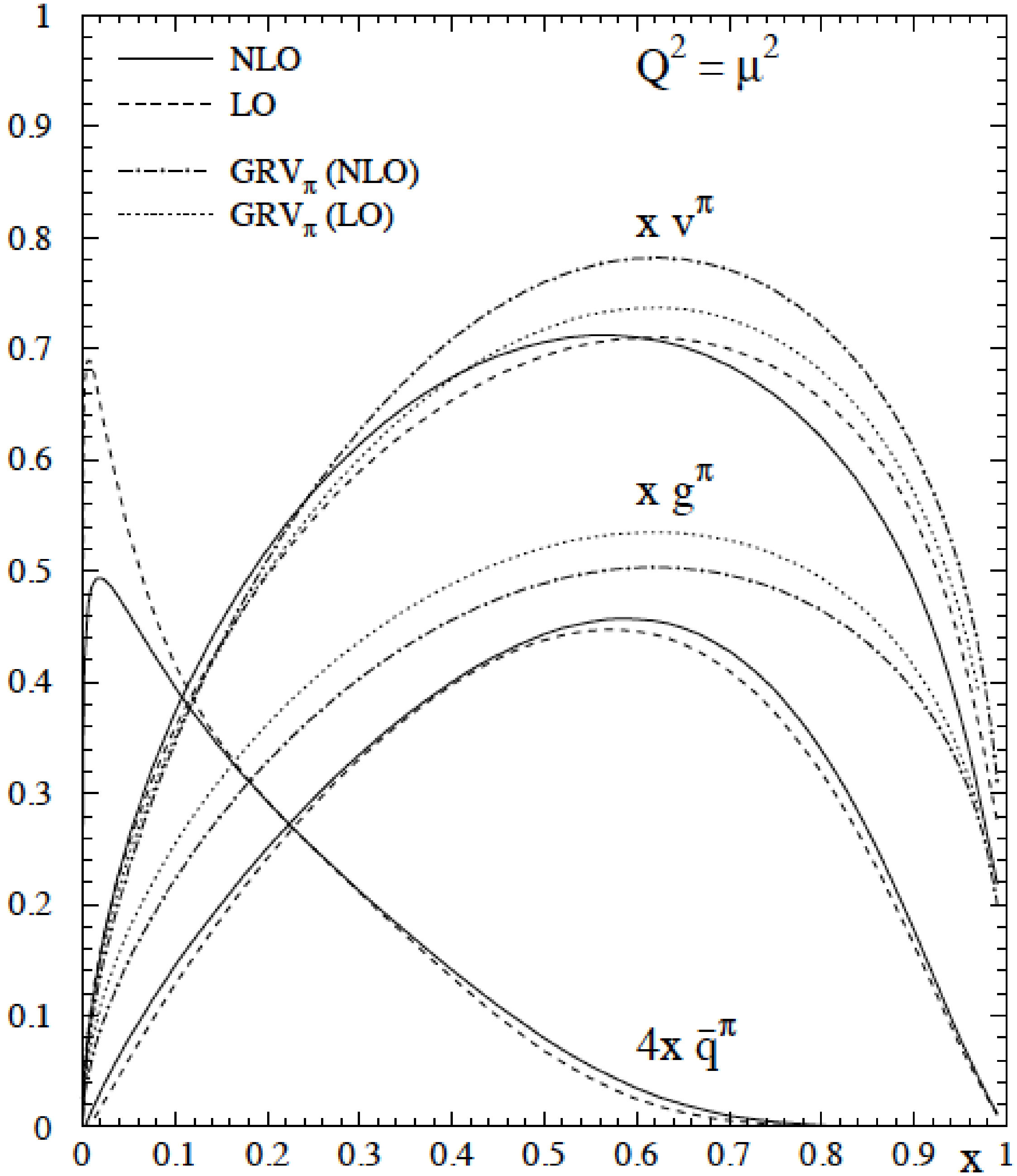


Fig. 1. The valence and valence–like input distributions $xf^\pi(x, Q^2 = \mu^2)$ with $f = v, \bar{q}, g$ as compared to those of GRV$_\pi$ [5]. Notice that GRV$_\pi$ employs a vanishing SU(3)$_{\text{flavor}}$ symmetric $\bar{q}^\pi$ input at $\mu^2_{\text{LO}} = 0.25$ GeV^2 and $\mu^2_{\text{NLO}} = 0.3$ GeV^2 [5]. Our present SU(3)$_{\text{flavor}}$ broken sea densities refer to a vanishing s^π input in (3), as for GRV$_\pi$ [5]

Eur. Phys. J. C 10, 313–317 (1999)
Digital Object Identifier (DOI) 10.1007/s100529900124

Pionic parton distributions revisited

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Aicher, Schafer, Vogelsang,
arXiv:1009.2481
soft gluon resummation

Excited Pion (1300) Distribution Amplitude

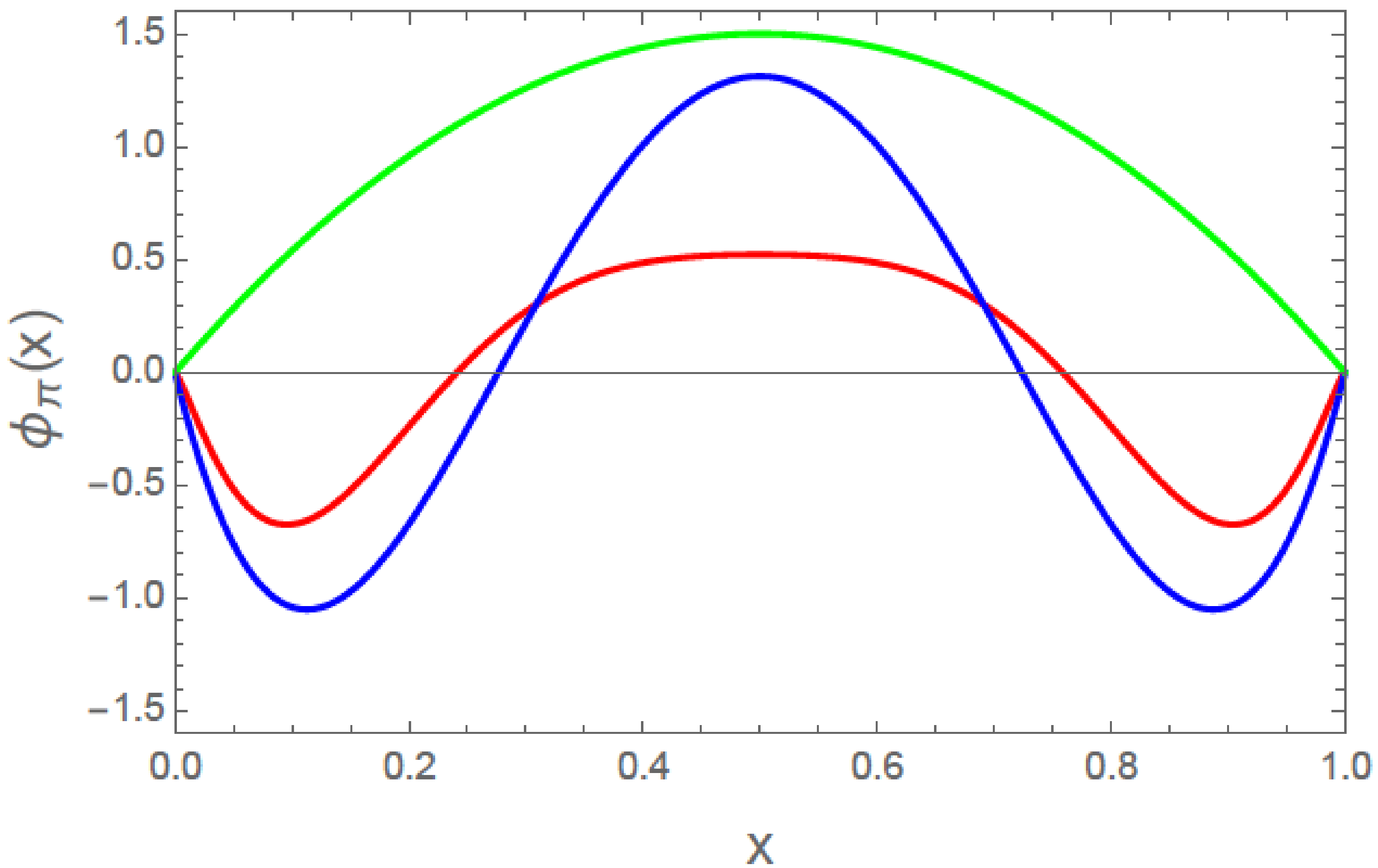


Figure 2: The red line is the calculated radial excited pion distribution amplitude, while the blue line is its conformal limit. The green line is the conformal limit of the ground pion distribution amplitude.

Bo-Lin, L. Chang, C.D.Roberts, H-S., Zong, in progress 2016, Nanjing U.

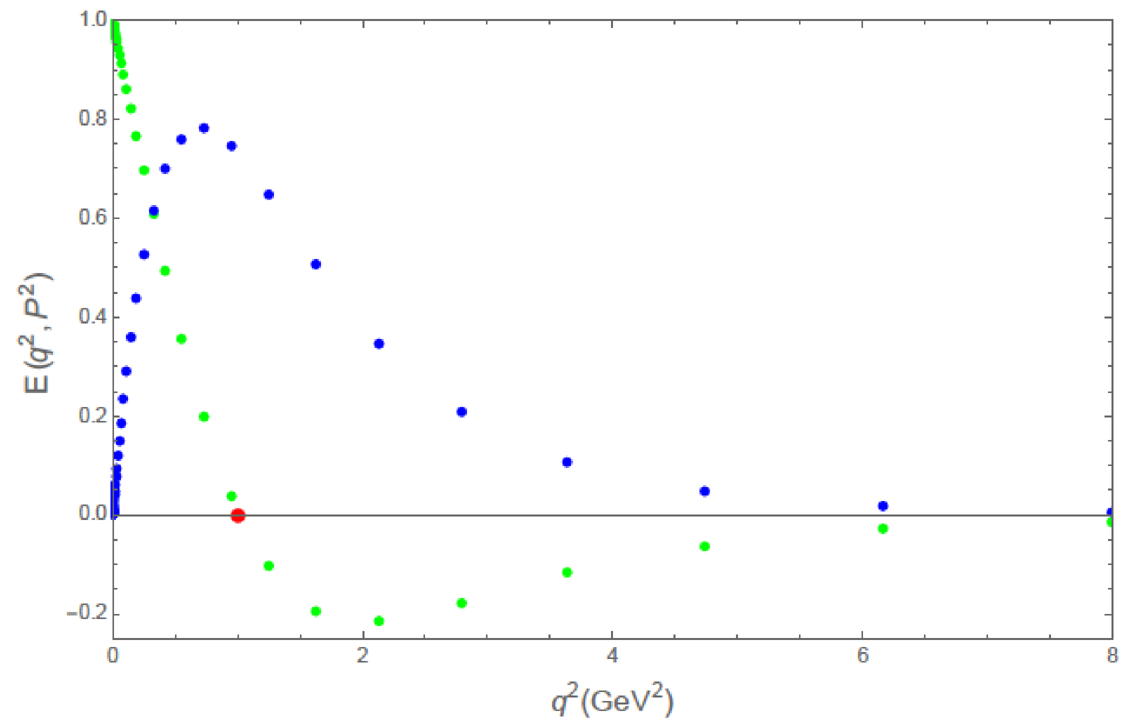


Figure 3: The green dotted line is the zeroth Chebyshev moment of E(q ; P), while the blue dotted one is the second Chebyshev moment.

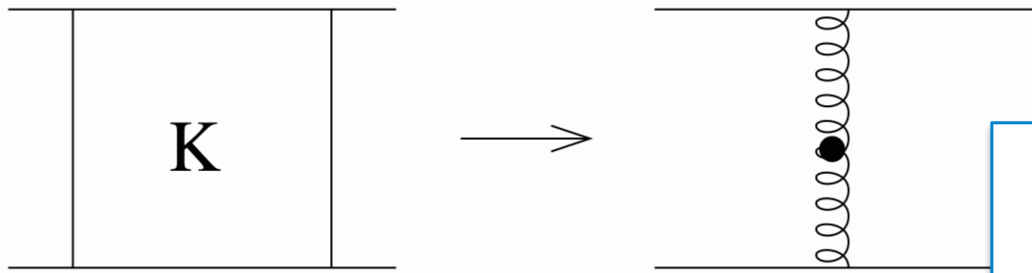
DSE Modeling of Hadron Physics

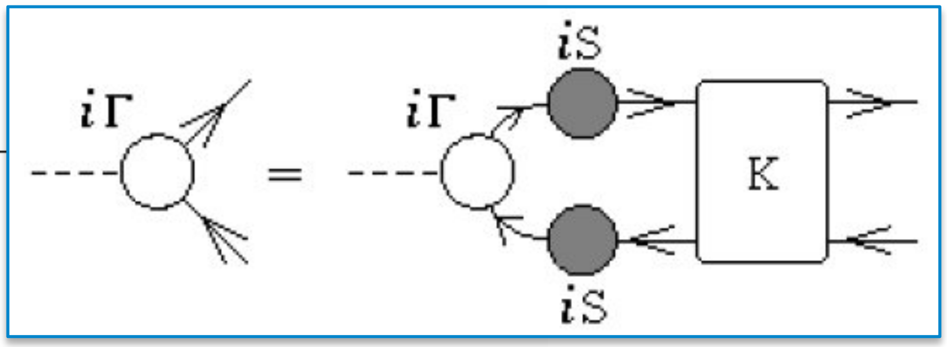
- Most common: Rainbow-ladder truncation of QCD's eqns of motion. Approximation to full BSE kernel now being utilized.
- Constrain modeling by preserving AV-Ward-Takahashi Id, V-WTI. [Color singlet]
- Naturally implements DCSB, conserved vector current, Goldstone Thm, PCAC…
- RL truncation only good for ground state vector & pseudoscalar mesons, q-qq descriptions of baryons with AV and S diquarks.
- At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant. Applicable to all scales, high Q^2 form factors, etc
- Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- Pion & kaon q-qbar Bethe-Salpeter wavefn is very well known

$$\mathbf{AV-WTI: m_q \to 0, P \to 0 \Rightarrow} \ \Gamma_{\pi q\bar{q}}(k^2) = i\gamma_5 \frac{\frac{1}{4}\mathrm{tr}S_0^{-1}(k)}{f_\pi^0} + \mathcal{O}(P)$$

Ladder-Rainbow Model

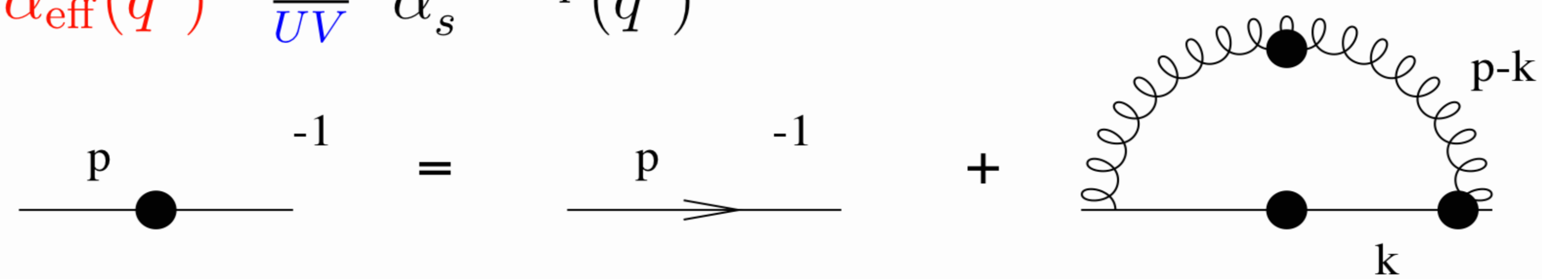
Landau gauge only





- $K_{\rm BSE} \rightarrow -\gamma_\mu \frac{\lambda^a}{2}\; 4\pi\alpha_{\rm eff}(q^2)\; D^{\rm free}_{\mu\nu}(q)\; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \;\underset{IR}{\rightarrow}\; \langle \bar{q}q\rangle_{\mu=1\ {\rm GeV}} = -(240\text{MeV})^3$, incl vertex dressing
- $\alpha_{\rm eff}(q^2) \;\underset{UV}{\rightarrow}\; \alpha_s^{1-\rm loop}(q^2)$

1 phen parameter

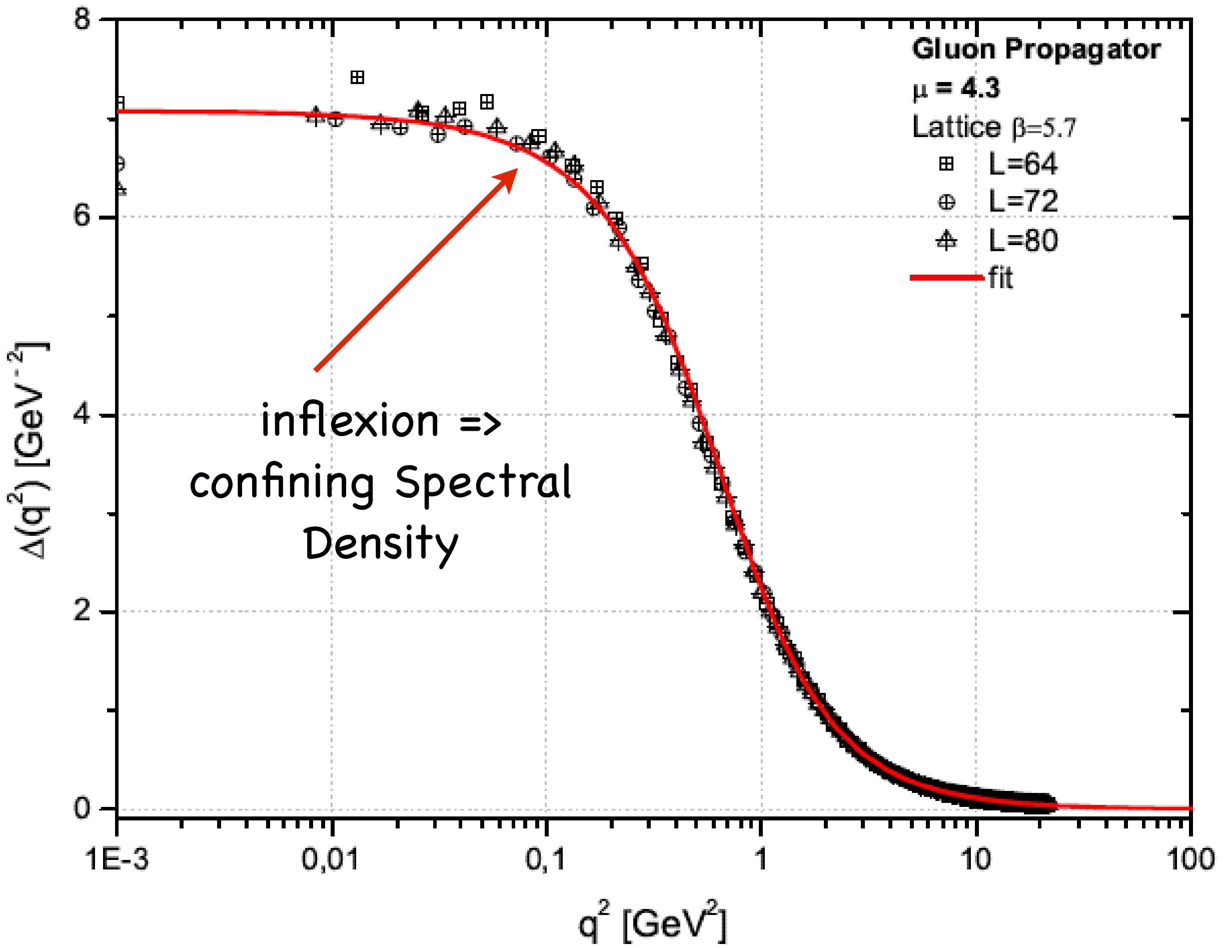


- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10% $[\mathbf{fit: m_\pi, m_K, f_\pi}], \mathbf{f_K (2\%)}$

An Ansatz for the FULL QCD kernel: L. Chang, C.D. Roberts, PRL103, 081601 (2009), + S. Qin (2015).

A more modern RL kernel: S. Qin, L. Chang, C.D. Roberts, D.J. Wilson, PRC84, 042202 (2011).

Modern Context for Rainbow-Ladder Kernel



Landau gauge, lattice – QCD gluon propagator, I.L.Bogolubisky *et al.*, PosLAT2007, 290 (2007)

Identified enough stength for physical DCSB

$\Rightarrow m_G(k^2)$ $\quad m_G(0) \sim 0.38$ GeV

$$K^{RL}_{BSE} = \frac{4\pi\hat{\alpha}_{eff}(q^2)}{m_G^2(q^2)+q^2} \Rightarrow \frac{\alpha^{RL}_{eff}(0)}{\pi} \approx 3$$

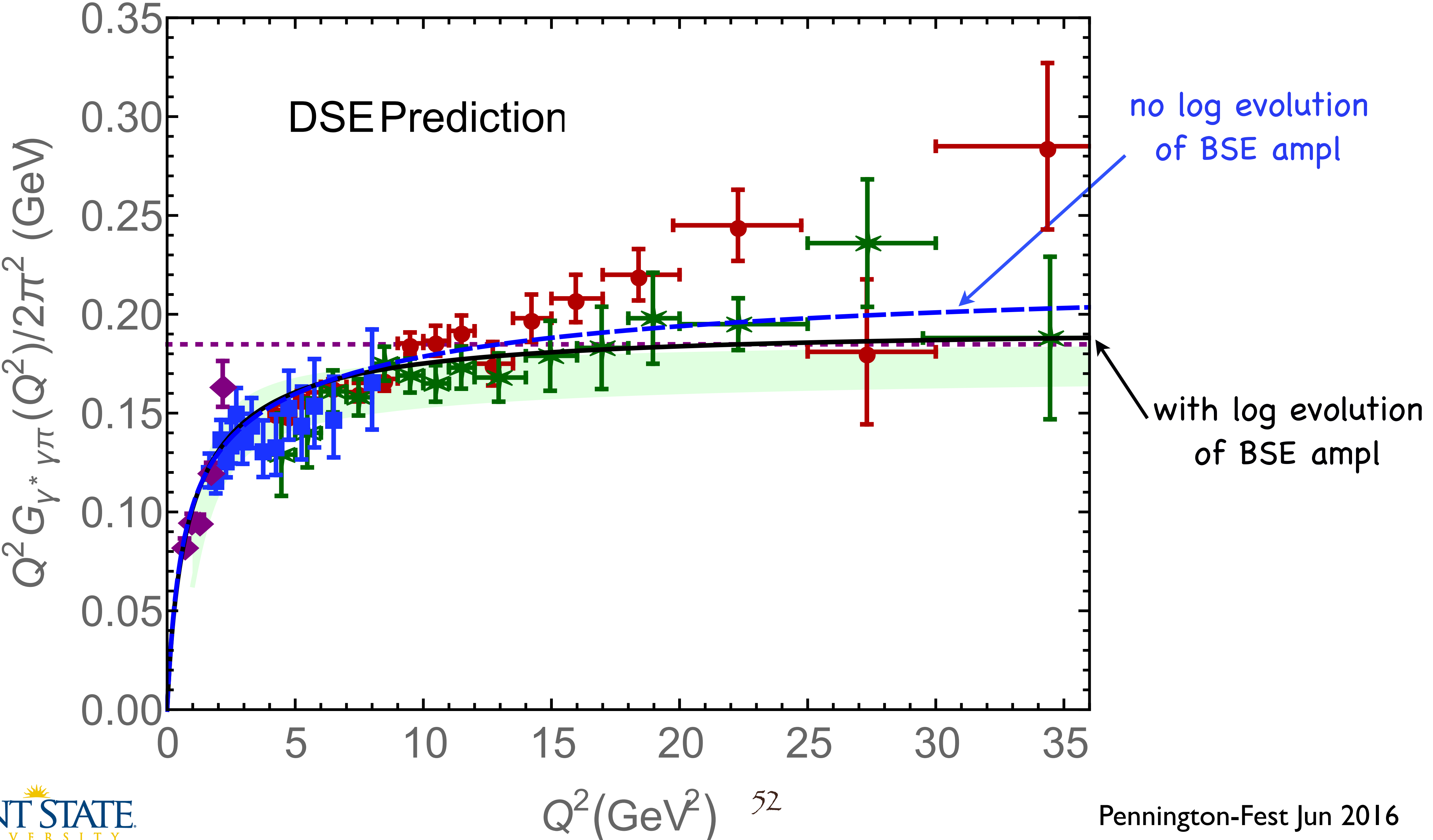
$$\Rightarrow \frac{\alpha^{DB}_{eff}(0)}{\pi} \approx 1 \;, \quad \text{[with dressed vertex effects]}$$

BSE kernel from ab initio gauge sector DSE work now agrees satisfactorily with the kernel from fitting data: Binosi, Chang, Papavassiliou, Roberts, PLB742, 183 (2015)

Pion Transition Form Factor

K. Raya, L. Chang, A. Bashir, J.J.Cobos-Martinez, L.X. Gutierez-Guerrero, C.D.Roberts, P.C.Tandy, arXiv:1510.02799

From unified treatment of DA, elastic FF, and transition FF



Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int \; D\bar{q}qG \;\; \mathcal{O}(\bar{q}, q, G) \;\; e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence m_q, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole
- EOMs (DSEs): $0 = \int \; D\bar{q}qG \;\; \frac{\delta}{\delta q(x)} \; e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, $\cdots$

Expect: qualitatively new insight where other methods can't, eg high Q^2

Do not expect: final, precision-QCD results, except in special cases

Other Meson Distribution Amplitudes

Table 1: Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with $N_f = 2+1$ domain-wall fermions and nonperturbative renormalisation of lattice operators [29]: linear extrapolation to physical pion mass, $\overline{\text{MS}}$-scheme at $\zeta = 2\,\text{GeV}$, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the s-quark mass, discretisation and renormalisation.

meson	$\langle (x-\bar{x})^n \rangle$	$16^3 \times 32$	$24^3 \times 64$
π	n=2	0.25(1)(2)	0.28(1)(2)
$\rho_\parallel$	n=2	0.25(2)(2)	0.27(1)(2)
ϕ	n=2	0.25(2)(2)	0.25(2)(1)
K	n=1	0.035(2)(2)	0.036(1)(2)
$K^*_\parallel$	n=1	0.037(1)(2)	0.043(2)(3)
K	n=2	0.25(1)(2)	0.26(1)(2)
$K^*_\parallel$	n=2	0.25(1)(2)	0.25(2)(2)

$$\varphi(x) = x^\alpha (1-x)^\beta / B(\alpha,\beta).$$

$$16^3 \times 32: \quad \alpha_{us} = 0.56^{+0.21}_{-0.18}\,,\ \beta_{us} = 0.45^{+0.19}_{-0.16}\,,$$
$$24^3 \times 64: \quad \alpha_{us} = 0.48^{+0.19}_{-0.16}\,,\ \beta_{us} = 0.38^{+0.17}_{-0.15}\,.$$

DAs of light quark mesons look much the same--with small flavor breaking

DSE analysis of LQCD moments:
Segovia, Chang, Cloet, Roberts, Schmidt, Zong
PLB731, 13, (2014)

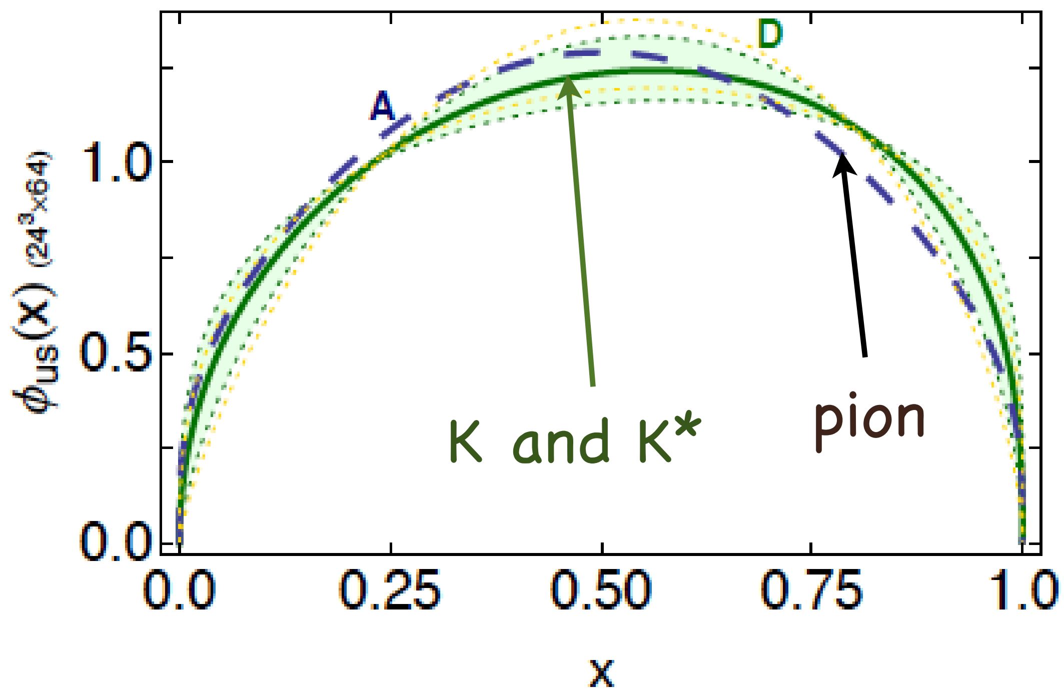
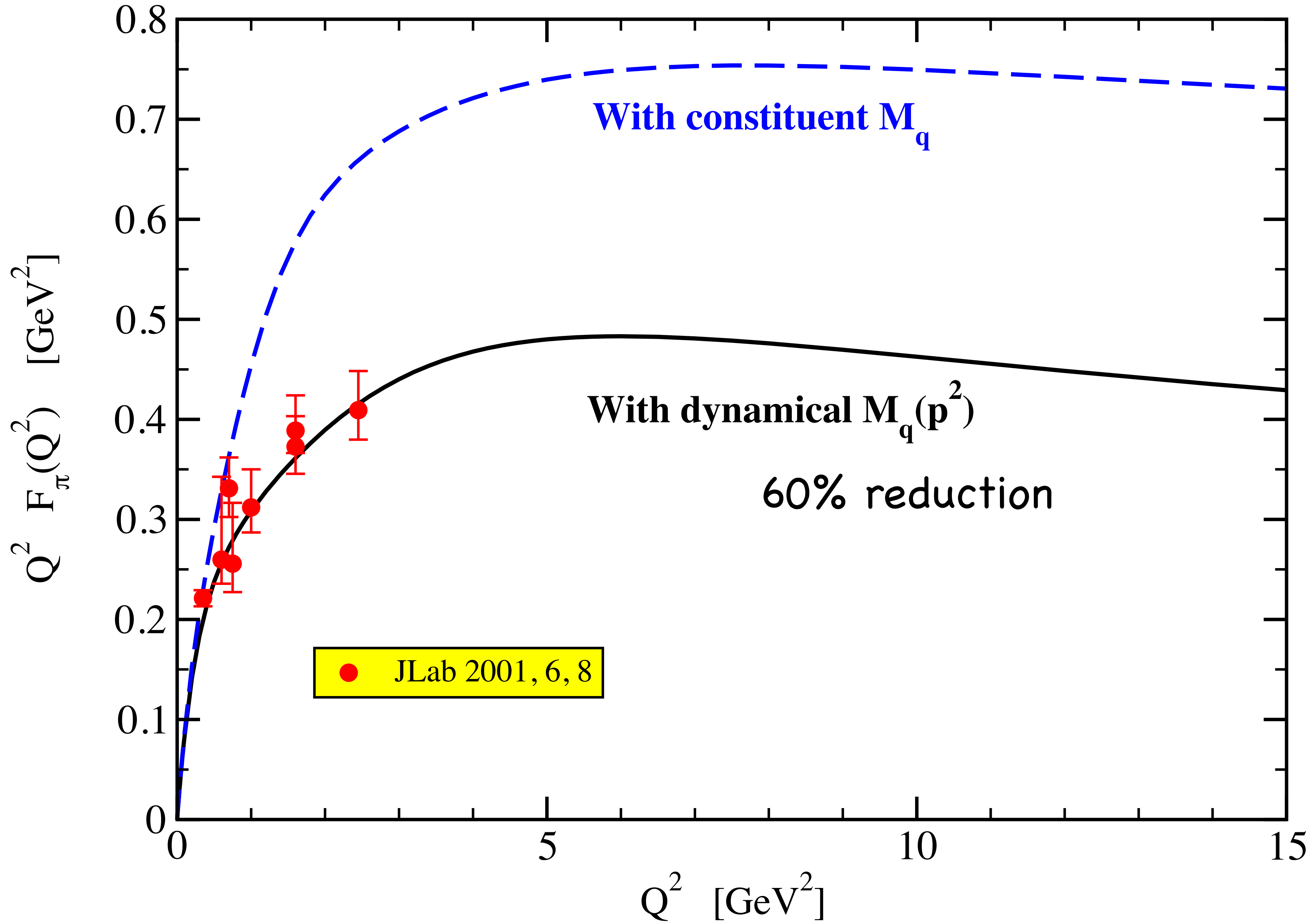


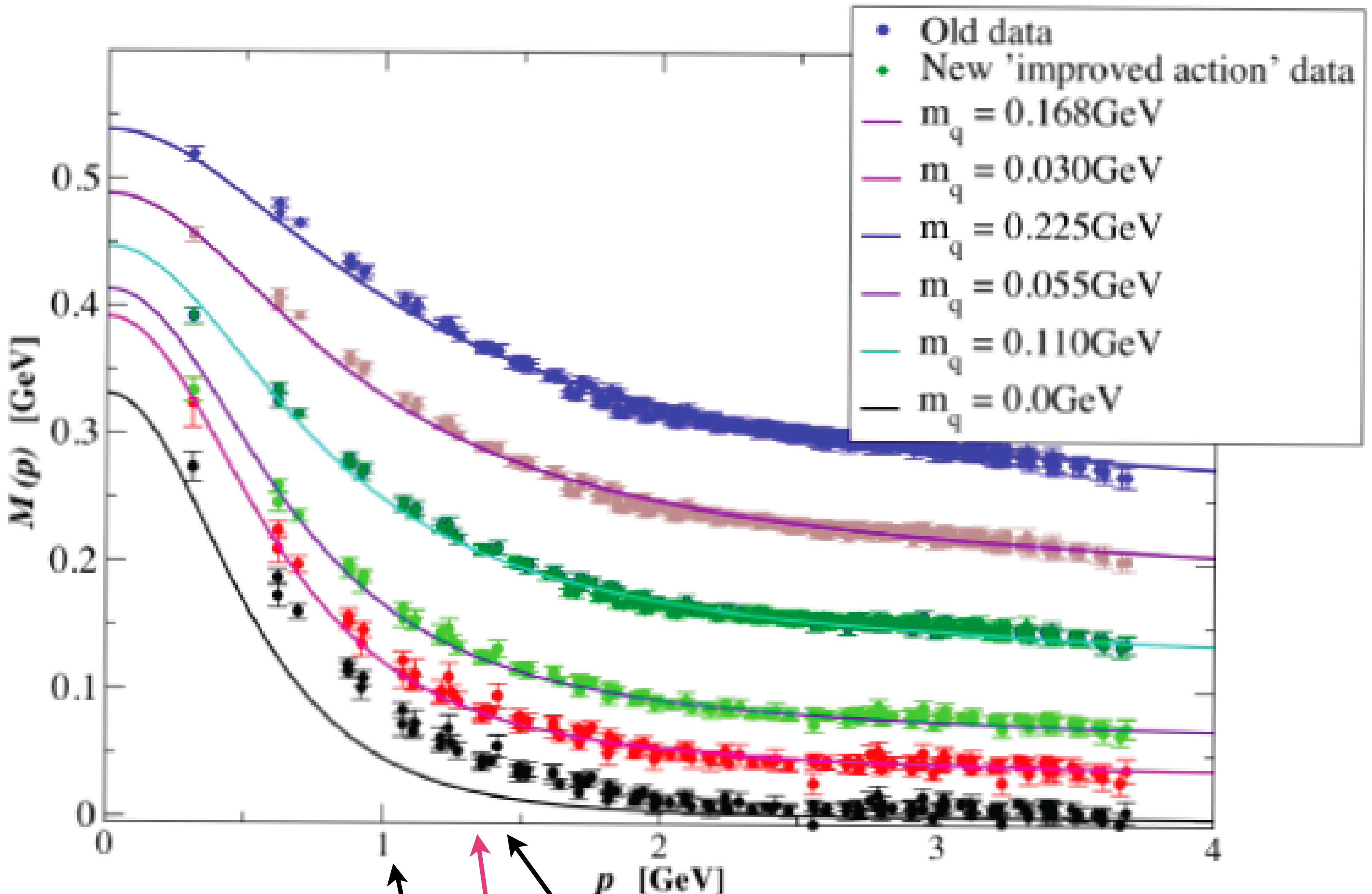
Figure 2: Solid curve and associated error band (shaded region labelled "D"): PDA in Eq. (15), describing $u\bar{s}$ pseudoscalar and vector mesons, reconstructed using Eq. (8) and obtained from the $24^3 \times 64$-lattice configurations. The result obtained from the $16^3{\times}32$-lattice moments in Table 1 is not materially different. The dashed curve "A" is the DSE prediction for the pion's PDA in Eq. (13).

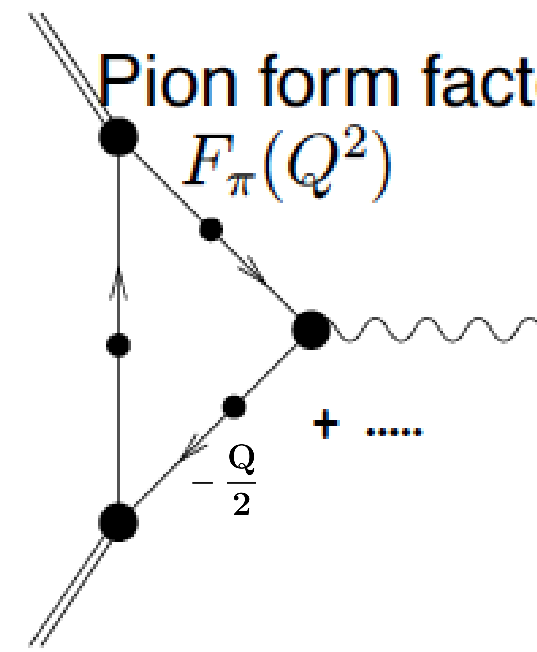
Pion Form Factor: Running q Mass Fn Effect



Jab data: G. Huber et al., PRC78, 045203 (2008)

Transition from constituent to parton quark



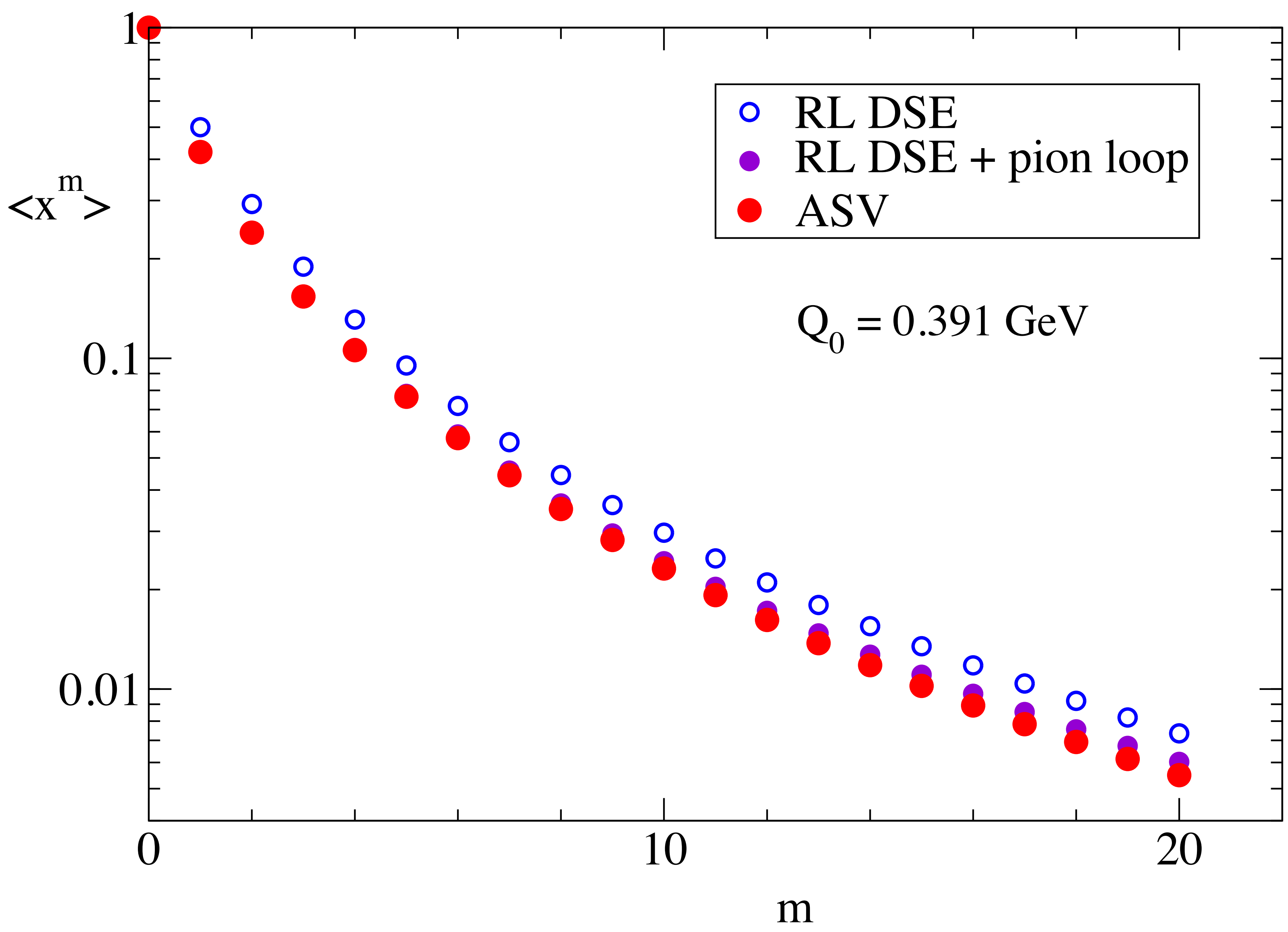


$$4\ \mathbf{GeV}^2 = \mathbf{Q}^2 \Leftarrow \frac{\mathbf{Q}}{\mathbf{2}}$$

$$\frac{\mathbf{Q}}{\mathbf{2}} \Rightarrow \mathbf{Q}^2 = 8\ \mathbf{GeV}^2$$

JLAb 12 GeV

Many Moments via Feyn PTIR--Easy

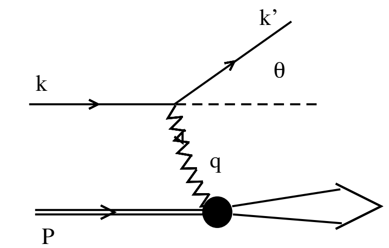


Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang,
(ASV) PRL 105, 252003 (2010)

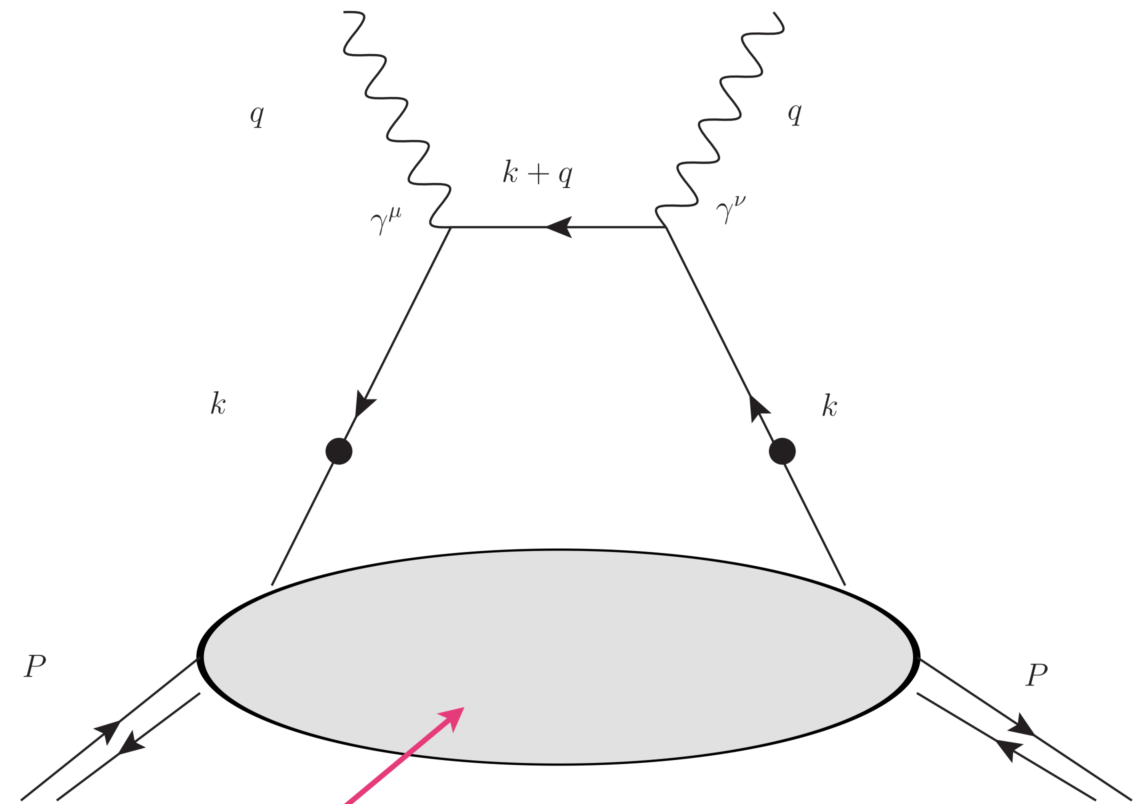
Summary

- DCSB: A large u/d quark constituent mass is generated from almost nothing for the same reason & and by the same mechanism that makes the pion almost massless!
- DCSB causes the shape of the pion DA to be significantly broader than the asymptotic-QCD DA at accessible scales for hadron physics, and a new analysis technique shows that lattice-QCD moments say the same thing.   [DCSB identified in a LF-defined quantity.]
- The scale running of distribution amplitudes is exceedingly SLOW---even at LHC scales asymptotic-QCD for DAs and form factors they influence there are persistent sizeable npQCD effects and DCSB in the hadron states.
- The elastic form factor of the pion makes a transition from non-perturbative/constituent quark behavior to partonic perturbative behavior for Q^2 at 6-8 GeV^2 and the relevant extension of the Brodsky-LePage uv-QCD leading formula is just 15% below the recent DSE calculation there.
- The new DSE approach is applicable to form factors for all spacelike Q^2.
- DSE-QCD can now be applied to light-front-defined bound state properties as a fn of momentum fraction x. Meson DAs and PDFs work out well, nucleon PDFs and GPDs await...

Deep Inelastic Lepton Scattering



- PDFs: $u_\pi(x), \quad u_K(x), \quad s_K(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit fixes quark $\mathbf{k}^+$
- Covariant formulation: $\int \mathbf{d^4q \;\; F(q^2, q\cdot P, q\cdot k, k^2)}$
- Evolve from model scale via LO DGLAP

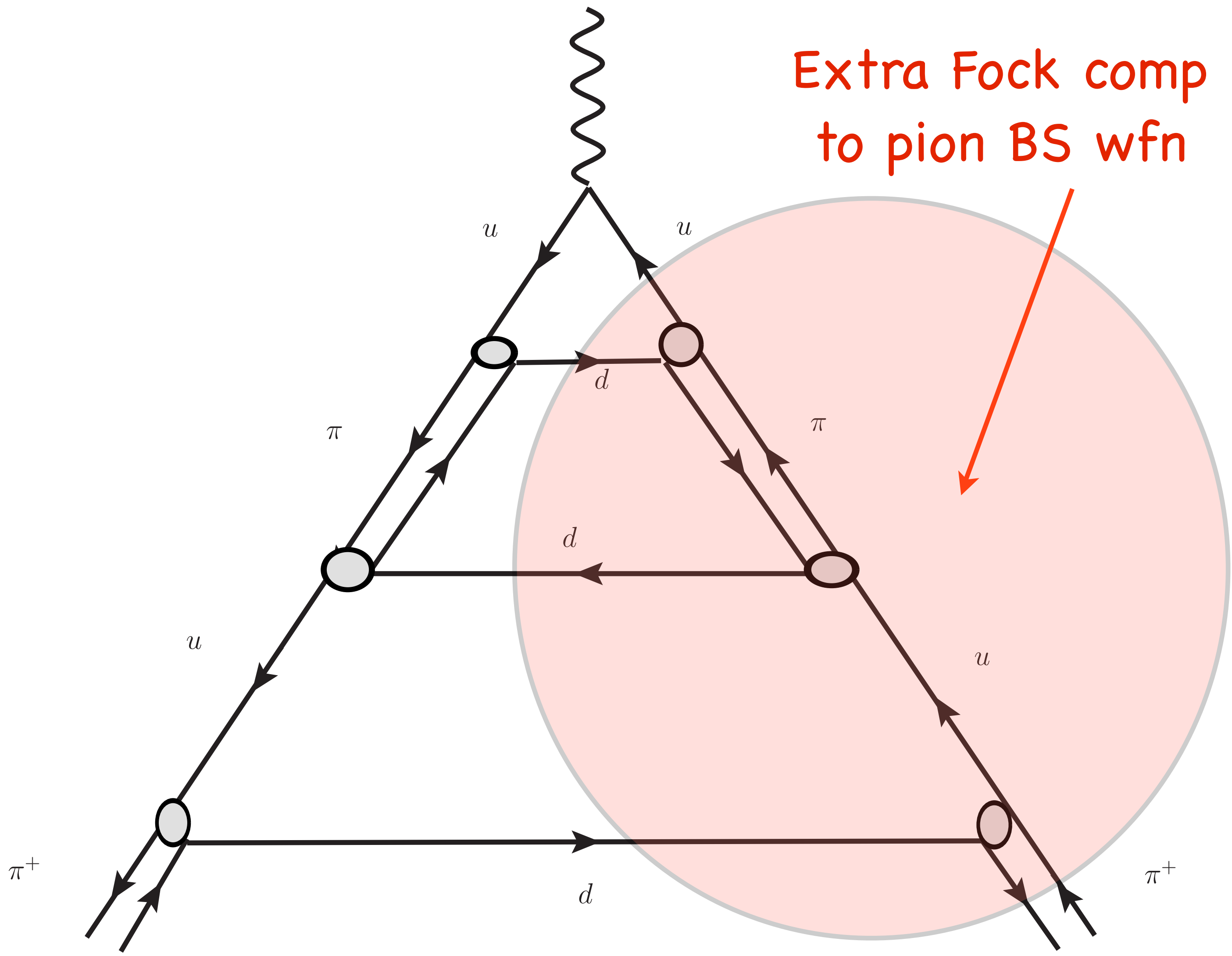


Pion Loop in Pion Charge Form Factor

$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1} - \alpha^\mathbf{2})\mathbf{F}_\pi^{\mathrm{RL}}(\mathbf{Q^2}) + \alpha^\mathbf{2}\,\mathbf{F}_\pi^{\pi-\mathbf{lp}}(\mathbf{Q^2})$$

$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1} - \alpha^\mathbf{2})(\mathbf{1} - \frac{\mathbf{Q^2}\,\mathbf{r}^\mathbf{2}_{\mathrm{RL}}}{\mathbf{6}} + \cdots) + \alpha^\mathbf{2}(\mathbf{1} - \frac{\mathbf{Q^2}\,\mathbf{r}^\mathbf{2}_{\pi-\mathbf{lp}}}{\mathbf{6}} + \cdots)$$

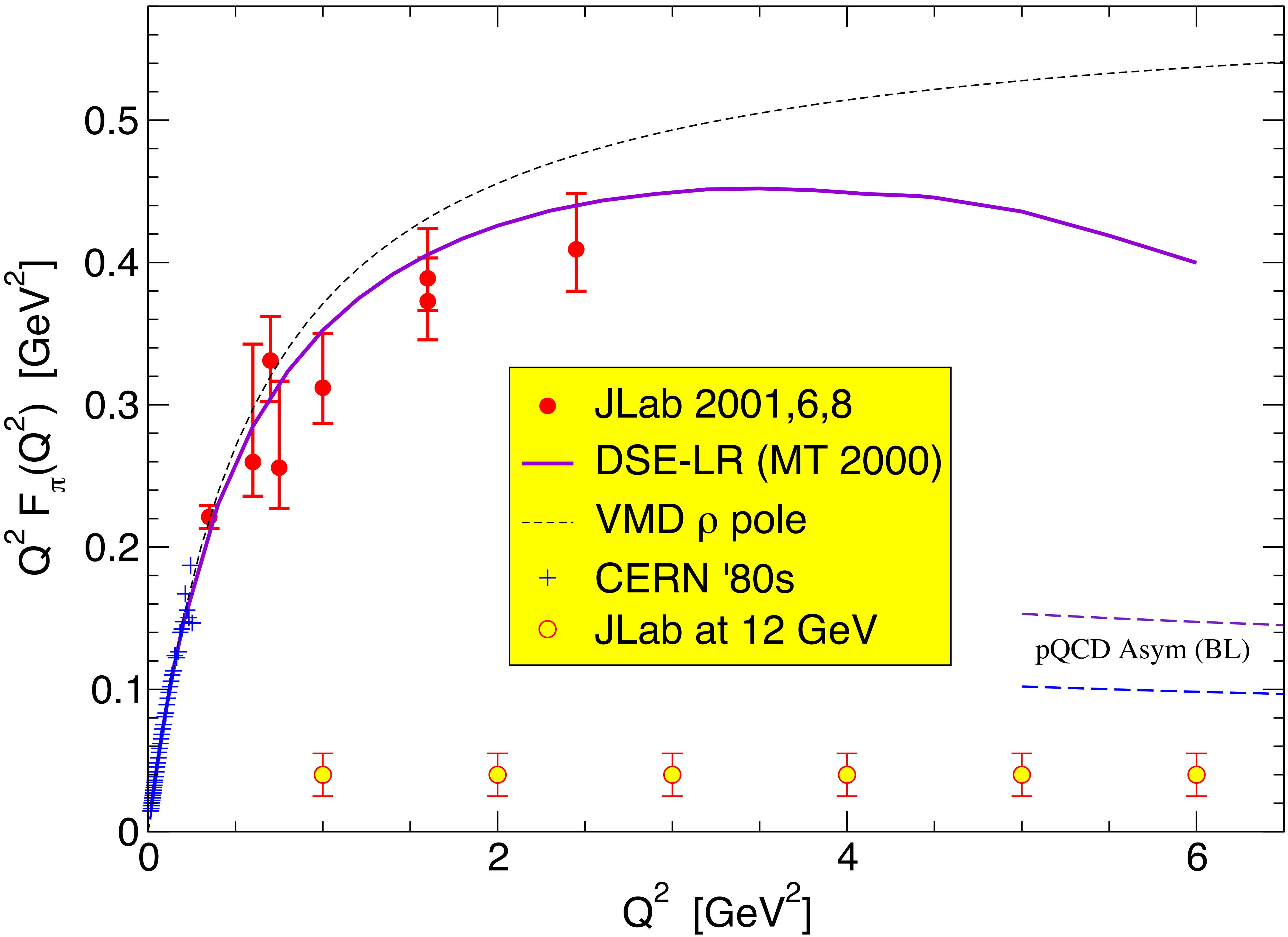
$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1} - \frac{\mathbf{Q^2}\,\mathbf{r}^\mathbf{2}_{\mathrm{TOT}}}{\mathbf{6}} + \cdots), \qquad \mathbf{r}^\mathbf{2}_\mathbf{ch} = (\mathbf{1} - \alpha^\mathbf{2})\mathbf{r}^\mathbf{2}_{\mathrm{RL}} + [\alpha^\mathbf{2}\,\mathbf{r}^\mathbf{2}_{\pi-\mathbf{lp}}]$$



The Future?

- Excited meson & baryons states, especially exotics & hybrids
- PDFs and GPDS for nucleons and pions
- Continue to enhance understanding of EM form factors of baryons
- Focus on observables where LQCD has difficulty, FFS, GPDs, chem potl > 0
- Parton DAs for nucleons
- Will LQCD be able to obtain the x-dependence of PDFs, GPDs, rather than 2-3 moments?
- Direct solution of BSE and Faddeev eqn for excited mesons and baryons? J/Psi tower of states? It looks possible to directly solve the meson BSE to obtain the essential features of the Nakanishi "spectral function" $\rho(\alpha; \mathbf{\Lambda})$.

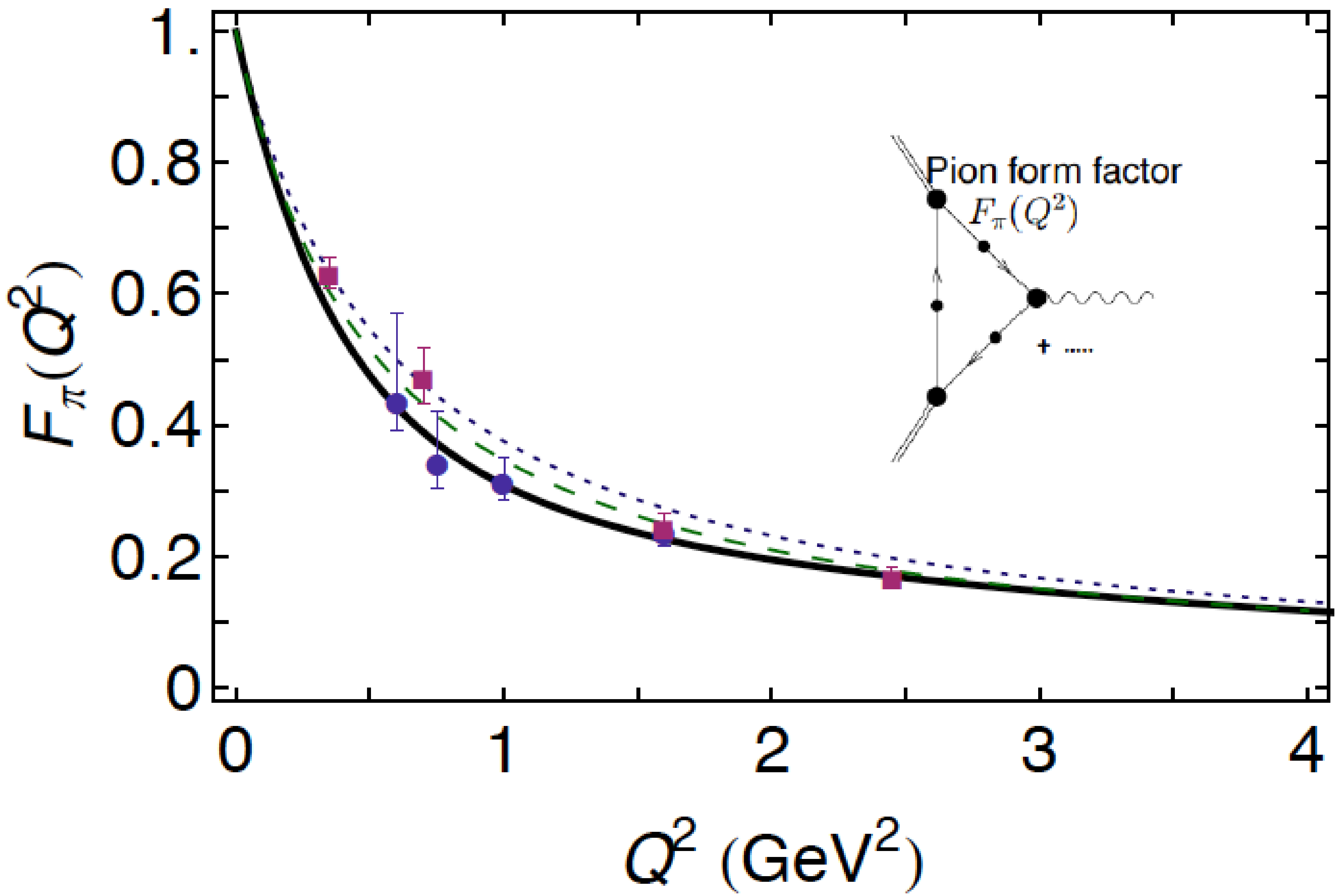
P. Maris and P.C. Tandy, PRC62, 055204, (2000)



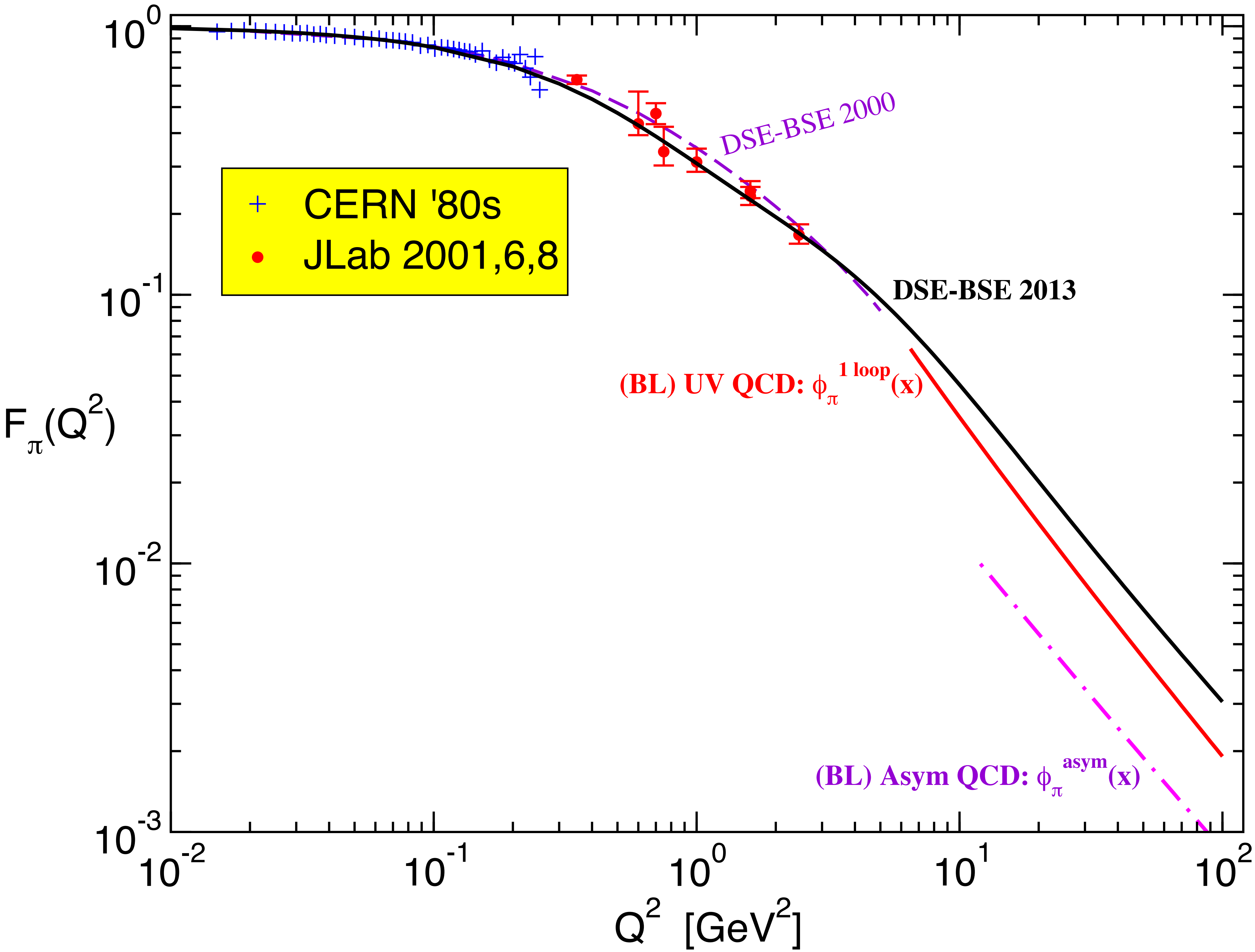
Jab data: G. Huber et al., PRC78, 045203 (2008)

Previous DSE Limited Result 2000

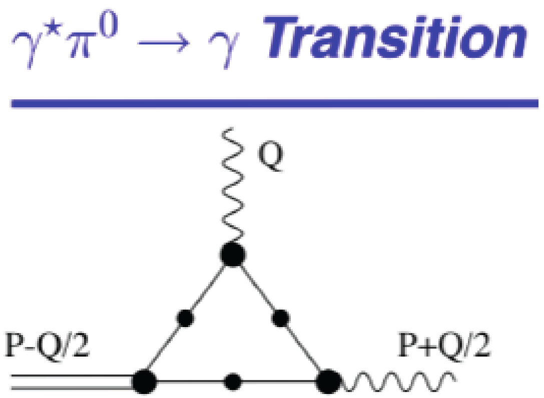
P. Maris and P.C. Tandy, PRC62, 055204, (2000)

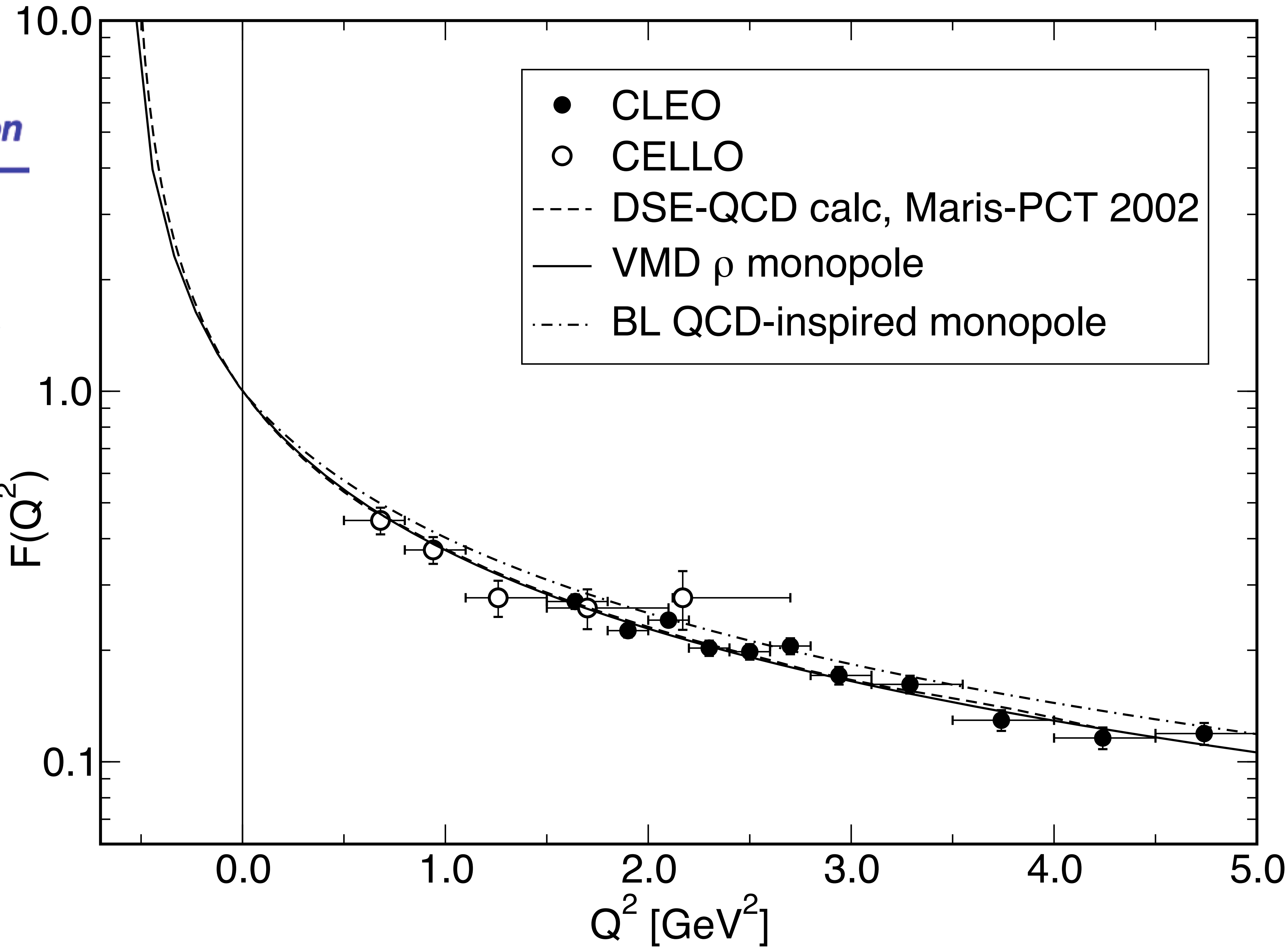


Pion Form Factor: Broad Picture



Pion Transition Form Factor





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Dyson–Schwinger Equations and their Application to Hadronic Physics

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Analysis of a quenched lattice QCD dressed quark propagator
M.S. Bhagwat, M.A. Pichowsky (Kent State U.),
C.D. Roberts (Argonne, PHY), P.C. Tandy (Kent State U.). Apr 2003. 9 pp.
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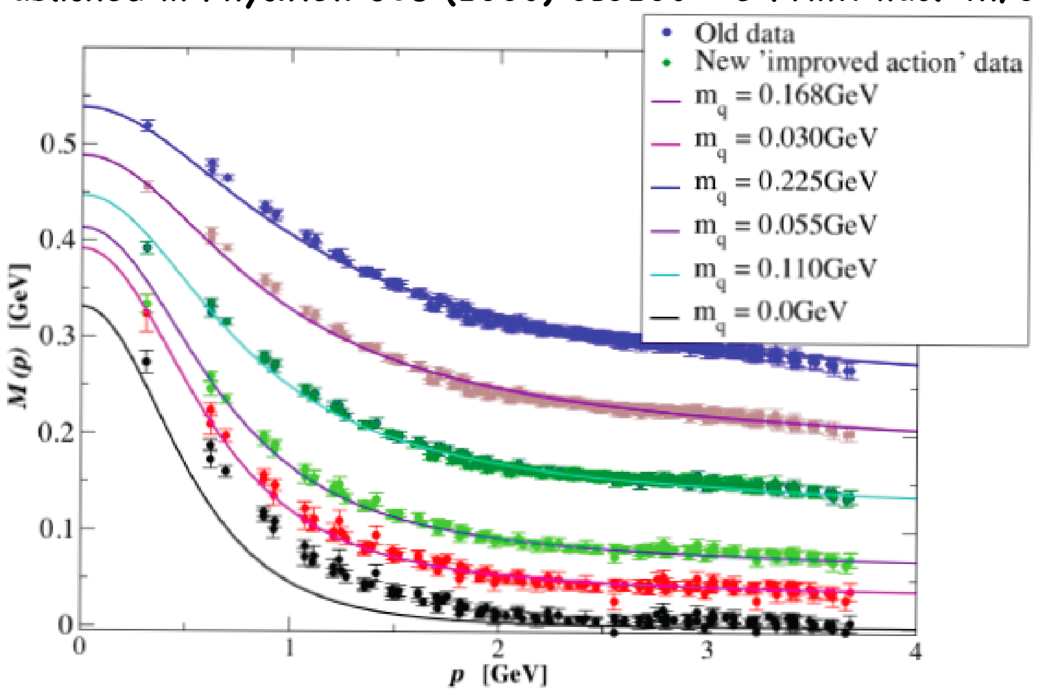

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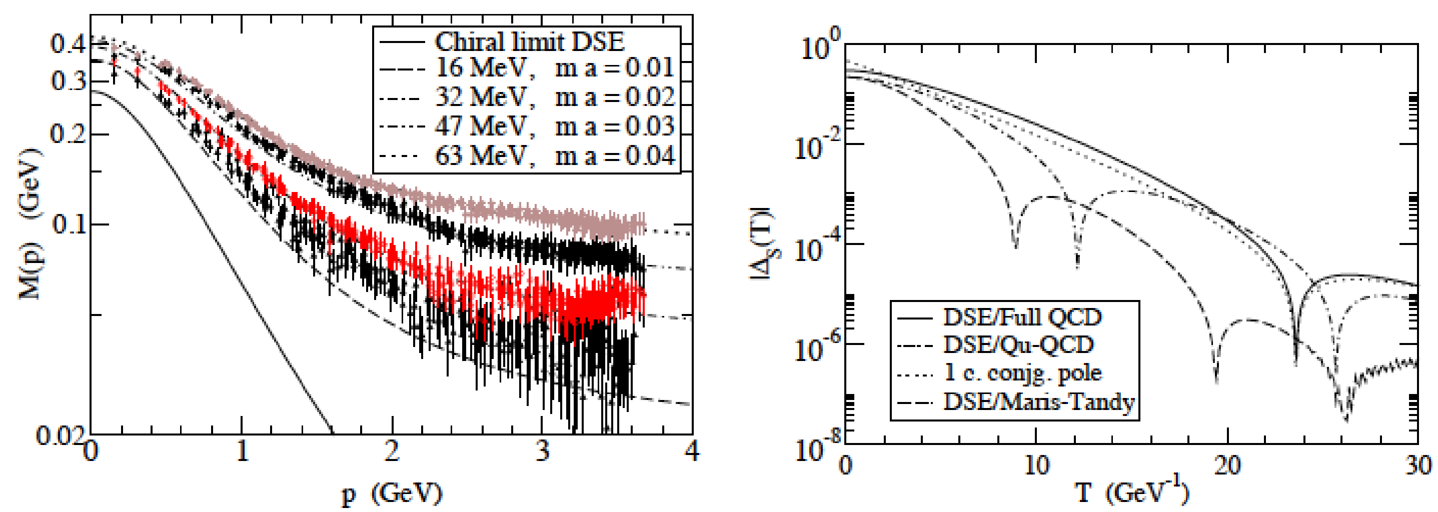
[14] P.O. Bowman, U.M. Heller, D.B. Leinweber, and A.G. Williams, hep-lat/0209129.



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