Hadron properties from nPI
Towards first principles results

Collaborators: Alkofer, Eichmann, Fischer, Heupel, Sanchis-Alepuz
baryons     mesons     glueballs     hybrids     tetraquarks     pentaquarks

G

Richard Williams – Hadron properties from nPI: towards first principles results
Extracting hadron poles from Green’s functions

\[ G_{\alpha \beta; \alpha' \beta'} = \langle 0 | T \psi_\alpha \psi_\beta \psi_\gamma \psi_{\alpha'} \psi_{\beta'} \psi_{\gamma'} | 0 \rangle \]
Extracting hadron poles from Green’s functions

\[ G_{\alpha \beta \gamma; \alpha' \beta' \gamma'} = \langle 0 \mid T \psi_\alpha \psi_\beta \psi_\gamma \bar{\psi}_\alpha' \bar{\psi}_\beta' \bar{\psi}_{\gamma'} \mid 0 \rangle \]

\[ G_{\alpha \beta \gamma; \alpha' \beta' \gamma'} \simeq \sum_\lambda \frac{\Psi_\lambda^{\alpha \beta \gamma} \bar{\Psi}_\lambda^{\alpha' \beta' \gamma'}}{P^2 + m_\lambda^2} \]

Spectral decomposition

\[ \Psi_\lambda^{\alpha \beta \gamma} = \langle 0 \mid T \psi_\alpha \psi_\beta \psi_\gamma \mid \lambda \rangle \]

Bethe-Salpeter wave function as residue
Extracting hadron poles from Green’s functions

\[ G_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0 | T \psi_\alpha \psi_\beta \bar{\psi}_\gamma \bar{\psi}_{\alpha'} \bar{\psi}_{\beta'} \bar{\psi}_{\gamma'} | 0 \rangle \]

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Spectral decomposition

Bethe-Salpeter wave function as residue

\[ \Psi_\lambda^{\alpha\beta\gamma} = \langle 0 | T \psi_\alpha \psi_\beta \psi_\gamma | \lambda \rangle \]

On lattice, current correlators

\[ G_{\sigma\sigma'} = \langle 0 | T J_\sigma \bar{J}_{\sigma'} | 0 \rangle \quad J_\sigma = \Gamma_{\alpha\beta\gamma\sigma} \psi_\alpha \psi_\beta \psi_\gamma \]

Exponential Euclidean time decay

\[ G_{\sigma\sigma'} = \sum_\lambda \frac{e^{-E_\lambda |\tau|}}{2E_\lambda} \left[ \cdots \right] \]
Trade one unknown, G, for another unknown K.
Dyson equation and Bethe-Salpeter wave function

Trade one unknown, $G$, for another unknown $K$.

\[ G = G_0 + G_0 KG \]

Solution yields on-shell particle pole and Bethe-Salpeter wave function

\[ \Psi = G_0 K \Psi \]

Bethe-Salpeter wave function **essential** ingredient for access to *e.g.* form-factors
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Bethe-Salpeter wave function \textbf{essential} ingredient for access to \textit{e.g.} form-factors

(ir)reducible two, three, four etc. body kernels the \textbf{define} equation.
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Bethe-Salpeter wave function essential ingredient for access to e.g. form-factors

(ir)reducible two, three, four etc. body kernels the define equation.

Dressed particle constituents also needed: these are ALSO Green’s functions
Dyson-Schwinger equations

\[ \frac{\delta \Gamma[\phi]}{\delta \psi} = \frac{\delta S[\phi]}{\delta \psi} + \text{interaction vertices} \]

Provide access to dressed propagators and interaction vertices from which BS kernel constructed.
Dyson-Schwinger equations

\[ \frac{\delta \Gamma[\phi]}{\delta \psi} = \frac{\delta S[\phi]}{\delta \psi} + \quad \text{(interaction vertices)} \]

Provide access to dressed propagators and interaction vertices from which BS kernel constructed.

\[ S^{-1}(p) = A(p^2) \left(-i\not{\psi} + M(p^2)\right) \]
Dyson-Schwinger equations

\[ \frac{\delta \Gamma[\phi]}{\delta \psi} = \frac{\delta S[\phi]}{\delta \psi} + \quad \text{Provide access to dressed propagators and interaction vertices from which BS kernel constructed} \]

\[ S^{-1}(p) = A(p^2) \left( -i\not{\psi} + M(p^2) \right) \]

It’s QCD:
- Mass function runs
- Coupling runs
- Vertices run

Everything runs!

Very difficult to disentangle in detail

See Kizilersu
Need the Bethe-Salpeter kernel(s)
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Simplest choice: one-gluon exchange

Structure: gluon is “dressed”, but vertices are “bare”

Compensate shortcomings by replacing dressed gluon with effective interaction
Need the Bethe-Salpeter kernel(s)

Simplest choice: one-gluon exchange

Structure: gluon is “dressed”, but vertices are “bare”

Compensate shortcomings by replacing dressed gluon with effective interaction

e.g. Maris-Tandy interaction

Certainly a good approximation for heavy quarks where IR effects are screened by the quark mass

Should be reliable in channels where dominated by scale of Dynamical Chiral Symmetry Breaking

[Maris, Tandy PRC 60 (1999) 055214]
Heavy mesons (bottomonium): ground and excited states

- Splitting between ground/excited states good
- Some deficiencies in level ordering

[Kubrak, Fischer, RW EPJA 51 (2015) 10]
[Blank, Krassnigg PRD 84 (2011) 096014]
[Hilger, Popovici, Gomez-Rocha, Krassnigg PRD 91 (2015) 034013]
Light mesons: ground and excited states

- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hilger, Gomez-Rocha, Krassnigg arXiv:1508.07183]
Light mesons: ground and excited states

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- Deficiencies in many channels

[Sensitivities to interaction exasperated in light sector, deficiencies in many channels.]

[Diagram showing the mass spectrum of light mesons, with peaks at various masses and labels such as $\pi(1300)$, $\rho(1450)$, $a_0(1450)$, $b_1(1235)$, $a_1(1260)$, $\pi_1(1600)$, and $\pi_1(1400)$.]

Light baryons: ground and excited states

Nucleon/Delta ground + excited states good

Expected deficiencies in diquarks/meson analogs

[Roberts, Chang, Cloet, Roberts FBS 51 (2011) 1]

Expanding the discussion on hadron properties from nPI towards first principles results, the expected deficiencies in diquarks/meson analogs are highlighted. The combination of various studies, including Eichmann, Alkofer, Krassnigg, Nicmorus PRL 104 (2010) 201601, Sanchis-Alepuz, Eichmann, Villalba-Chavez, Alkofer, PRD 84 (2011) 096003, and Sanchis-Alepuz, Eichmann, Fischer in preparation, provides a comprehensive view on the topic.
Beyond rainbow-ladder?

Expose additional corrections to the kernel
Exposé additional corrections to the kernel

Diagrammatic

[Expositions: Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]
Beyond rainbow-ladder?

Exposé additional corrections to the kernel

Diagrammatic

Effective/Composite

[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]

[Fischer, Nickel, Wambach ORD 76 (2007) 094009]
[Fischer, RW PRD 78 (2008) 074006]
[Sanchis-Alepuz, Fischer, Kubrak PLB 733 (2014) 151]
Beyond rainbow-ladder?

Expose additional corrections to the kernel

Diagrammatic

Effective/Composite

Technique use nPI effective actions expanded to m loops

Loop expansion of a particular resummation of dressed propagators and, perhaps, vertices

[Fischer, RW PRL 103 (2009) 122001]
[Fischer, Nickel, Wambach ORD 76 (2007) 094009]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]

[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Sanchis-Alepuz, Fischer, Kebkak PLB 733 (2014) 151]

[Fischer, RW PRD 78 (2008) 074006]
Construction of BS kernel: Munczek cutting

\[ K = \frac{\delta^2 \Gamma_2[B]}{\delta B \delta B} \bigg|_{B=S} = \frac{\delta \Sigma[B]}{\delta B} \bigg|_{B=S} = \frac{\delta \Sigma[S]}{\delta S} \sum \]

- Munczek cutting assumes underlying 2PI effective action
- Dressed quark-gluon vertex is auxiliary function that defines resummation of dressed propagators / bare vertices

[Munczek, PRD 52 (1995) 4736]
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\[ \Sigma = \sum \]

- Munczek cutting assumes underlying 2PI effective action
- Dressed quark-gluon vertex is auxiliary function that defines resummation of dressed propagators / bare vertices

\[ K_1 \simeq [\gamma^\mu]_{\alpha \beta} [\gamma^\nu]_{\gamma \delta} D_{\mu \nu} \]

\[ K_2 \simeq \left[ \gamma^\mu S \right]_{\alpha \beta} \left[ \frac{\delta \Gamma^\nu}{\delta S} \right]_{\gamma \delta} D_{\mu \nu} \]

Explicit

Implicit

Need integral representation for quark-gluon vertex to avoid ambiguity in momentum routing.

[Heupel, Goecke, Fischer EPJA 50 (2014) 85]
Construction of BS kernel: Munczek cutting

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\[ \Sigma = \]

\[ [\text{Munczek, PRD 52 (1995) 4736}] \]

- Munczek cutting assumes underlying 2PI effective action
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\[ \text{Explicit} \quad K_1 \simeq [\gamma^\mu]_{\alpha\beta} [\gamma^\nu] \gamma \delta D_{\mu\nu} \]

\[ \text{Implicit} \quad K_2 \simeq \left[ \gamma^\mu S \right]_{\alpha\beta} \left[ \frac{\delta \Gamma^\nu}{\delta S} \right]_{\gamma\delta} D_{\mu\nu} \]

Need integral representation for quark-gluon vertex to avoid ambiguity in momentum routing.

[Heupel, Goecke, Fischer EPJA 50 (2014) 85]
Munczek Cutting

\[
\begin{align*}
\text{8 functions of 3 variables} & \\
\text{256 functions of 10 variables}
\end{align*}
\]
\[
\frac{1}{2} A + \frac{1}{2} E + \frac{1}{2} B + \frac{1}{2} F + \frac{1}{2} C + \frac{1}{2} D + \frac{1}{2} G + \frac{1}{2} H
\]

72 functions of 6 variables
Munczek Cutting

\[
\begin{align*}
& = \frac{1}{2} A1 + \frac{1}{2} A2 + \frac{1}{2} B1 + B2 + \frac{1}{2} F1 + \frac{1}{2} F2 + C1 + C2 + C3 + C4 + \frac{1}{2} C5 + \frac{1}{2} C6 + D1 + D2 + D3 + D4 + \frac{1}{2} H1 + \frac{1}{2} H2 + \frac{1}{2} E1 + \frac{1}{2} E2 + \frac{1}{2} E3 + \frac{1}{2} E4 + G1 + G2 + G3 + G4
\end{align*}
\]

384 functions of 15 variables
Truncations are necessary

Killer:

- multiplicity of diagrams and phase space
- Coupled system becomes two coupled systems
Truncations are necessary

Killer:
- multiplicity of diagrams and phase space
- Coupled system becomes two coupled systems

Because:
- Everything (relevant) functional of $S$, as introduced by auxiliary quark-gluon vertex
- Self-coupled implies 2PI effective action to all orders
- Bethe-Salpeter kernel is all orders in loops
- Resummation expressed by 5pt functions and deps.
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Because:
- Everything (relevant) functional of $S$, as introduced by auxiliary quark-gluon vertex
  - Self-coupled implies 2PI effective action to all orders
  - Bethe-Salpeter kernel is all orders in loops
  - Resummation expressed by 5pt functions and deps.

Hopes:
- 2PI effective action to finite loop order
- Simplest (coupled) 5pt function almost tractable

[Bhagwat et al PRC 68 (2003) 015203]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
... or we switch to nPI

\[ \Gamma[\phi, D, U] = S_{cl}[\phi] + \frac{i}{2} Tr \ln D^{-1} + \frac{i}{2} Tr \left[ D_{(0)}^{-1} D \right] - i \Phi^0[\phi, D, U] - i \Phi^{\text{int}}[\phi, D, U] + \text{const.} \]

**\( \Phi^0 \): non-interacting part

\[ \begin{align*}
\frac{1}{6} & \quad - \quad \frac{1}{8} \\
\frac{1}{8} & \quad + \quad \frac{1}{48}
\end{align*} \]

**\( \Phi^{\text{int}} \): interacting part

\[ \begin{align*}
- \frac{1}{12} & \quad + \quad \frac{1}{2} \\
+ \frac{1}{2} & \quad + \quad \frac{1}{2} \\
+ \frac{1}{24} & \quad + \quad \frac{1}{24}
\end{align*} \]

\[ K = \left. \frac{\delta^2 \Gamma_2[B, U]}{\delta B \delta B} \right|_{B=S, U=V} = \left. \frac{\delta \Sigma[B, U]}{\delta B} \right|_{B=S, U=V} \neq \left. \frac{\delta \Sigma[S, V]}{\delta S} \right|_{B=S, U=V} \]

**Differences:**

- B, U independent. No implicit derivatives.
- Fixed order action yields fixed order equations. Vertices resummed by construction.
- No auxiliary equations or 5PI functions from implicit cutting
... or we switch to nPI

\[ \Gamma[\phi, D, U] = S_{cl}[\phi] + \frac{i}{2} Tr \ln D^{-1} + \frac{i}{2} Tr \left[ D^{-1}(0) \right] - i \Phi^0[\phi, D, U] - i \Phi^{int}[\phi, D, U] + \text{const.} \]

\( \Phi^0 \): non-interacting part

\( \Phi^{int} \): interacting part

\[ K = \frac{\delta^2 \Gamma_2[B, U]}{\delta B \delta B} \bigg|_{B=S, U=V} = \frac{\delta \Sigma[B, U]}{\delta B} \bigg|_{B=S, U=V} \neq \frac{\delta \Sigma[S, V]}{\delta S} \]
Richard Williams – Hadron properties from nPI: towards first principles results

23.06.2016

Results: rainbow-ladder

- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

\[ m \text{ [GeV]} \]

\[ \pi(1300) \quad \rho(1450) \quad a_0(1450) \quad \pi_1(1400) \quad \pi_1(1600) \]

\[ \rho \]

\[ \pi \]

\[ 0^{-+} \quad 1^{--} \quad 0^{++} \quad 1^{+-} \quad 1^{++} \quad 0^{--} \quad 0^{+-} \quad 1^{-+} \]

[ Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hilger, Gomez-Rocha, Krassnigg arXiv:1508.07183]
Results: beyond rainbow-ladder

$m \text{ [GeV]}$

$\pi(1300)$  $\rho(1450)$  $a_0(1450)$  $b_1(1235)$  $a_1(1260)$  $\pi_1(1400)$  $\pi_1(1600)$

Efficacy commensurate with other approaches

Scalar: 2PI-2L (RL) and 2PI-3L too light
$\rho - a_1$ splitting: 2PI-2L (RL) and 2PI-3L too small
$a_1 - b_1$ splitting: 2PI-2L (RL) and 2PI-3L non-degenerate

[Chang, Roberts PRC 85 (2012) 052201]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[RW, Fischer, Heupel PRD 93 (2016) 034026]
3PI results: Recipe

\[ -1 = \text{Diagram 1} \]

\[ -1 = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \]

\[ = \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} - 2 \]

\[ -2 = \text{Diagram 8} + \text{Diagram 9} + \text{perm.} \]

\[ = \text{Diagram 10} + \text{Diagram 11} + \text{Diagram 12} \]
3PI results: Recipe

\[ \begin{align*}
-1 &= -1 - \\
\text{Ghost/gluon solved independently of 3PI}
\end{align*} \]
3PI results: Ghost/gluon propagator

Employ lattice data for ghost/gluon and scale setting

Provides input parameters of calculation (e.g. $g_s$)

\[ D_G(p) = \frac{-G(p^2)}{p^2} \]

\[ D_{\mu\nu}(p) = \left( \delta_{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \frac{Z(p^2)}{p^2} \]
3PI results: quark propagator

\[ S^{-1}(p) = A(p^2)(-i \gamma \cdot p + M(p^2)) \]
3PI results: three-gluon vertex

- Tree-level structure dominant
- Single phase-space slice sufficient with S3 (e.g. soft-gluon)

• Quark-loop enhances three-gluon vertex
• Zero crossing pushed to deep IR

\[ Γ^{\mu\nu\rho}_{3g} (p, k, q) = F_1 (p, k, q) Γ^{\mu\nu\rho}_{3g,0} (p, k, q) \]

[Blum, Huber, Mitter, von Smekal PRD 89 (2014) 061703]
[Aguilar, Binosi, Ibanez, Papavassiliou PRD 89 (2014) 085008]
[Eichmann, RW, Alkofer, Vujinovic PRD 89 (2014) 105014]
3PI results: ghost-gluon vertex

- Unquenching effects negligible.
- Lattice data needs improvement

One tensor structure

\[ \Gamma_{gh}^{\mu}(l, q) = f(l, q) T_{(q)}^{\mu\nu} l^\nu \]
3PI results: (quenched) quark-gluon vertex

1\textsuperscript{st} and 3\textsuperscript{rd} structures comparable.

Difficult systematics (lattice) in 2\textsuperscript{nd}

[Skullerud, Bowman, Kizilersu, Leinweber, Williams JHEP 04 (2003) 047]
[RW, Fischer, Heupel PRD 93 (2016) 034026]
3PI results: (unquenched) quark-gluon vertex

- Strong dynamical enhancement in running of vertex.
- DCSB plays a large role

See Kizilersu

[Chang, Roberts PRC 85 (2012) 052201]
[RW, Fischer, Heupel PRD 93 (2016) 034026]
Other Applications: Tetraquarks

Permuted two body kernel

Tensor decomposition
512 components
9 kinematic variables

Time-like constituents
- Analytic continuation
- Resonance structure

[Eichmann, Fischer, Heupel PLB 753 (2016) 282]
Other Applications: Glueballs

Tensor decomposition

(Derive using Helicity formalism)

\[ J = 0^+ : 4 \text{ covariants (2 Landau gauge)} \]

\[ J = 0^- : 1 \text{ covariants} \]

Time-like constituents

- Analytic continuation

[Meyers, Swanson PRD 87 (2013) 036009]
[Fukamarchi, Kondo, Nishino, Shinohara arXiv:1605.01841]
Mesons $q\bar{q}$
- Only now exploring details of quark-gluon interaction on spectrum
- No longer disconnected from gauge sector. Implicit flavor dependence.

Developing framework
- Unified description of mesons and baryons consistent with symmetries
- Calculation of higher spin and/or excited mesons and baryons

Extensible to other bound-states via nPI
- Baryons
- Tetraquarks
- Glueballs and Hybrid mesons

A functional derivative (or two) away …
- Calculation of form-factors, EM transitions and decays
Conclusions

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see G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. Fischer for a review
Conclusions

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see G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. Fischer for a review