Hadron properties from nPI
Towards first principles results

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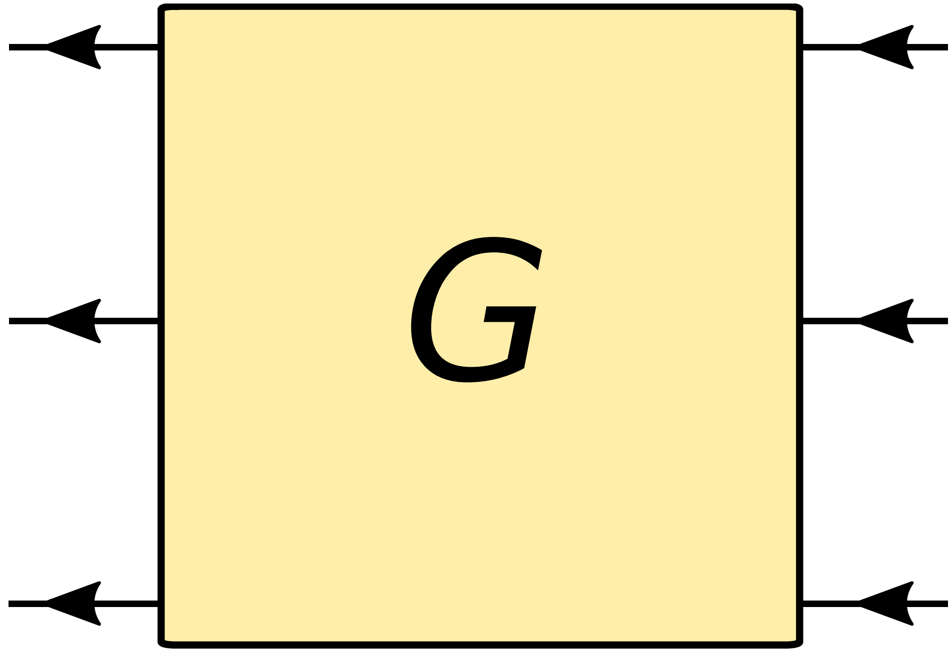
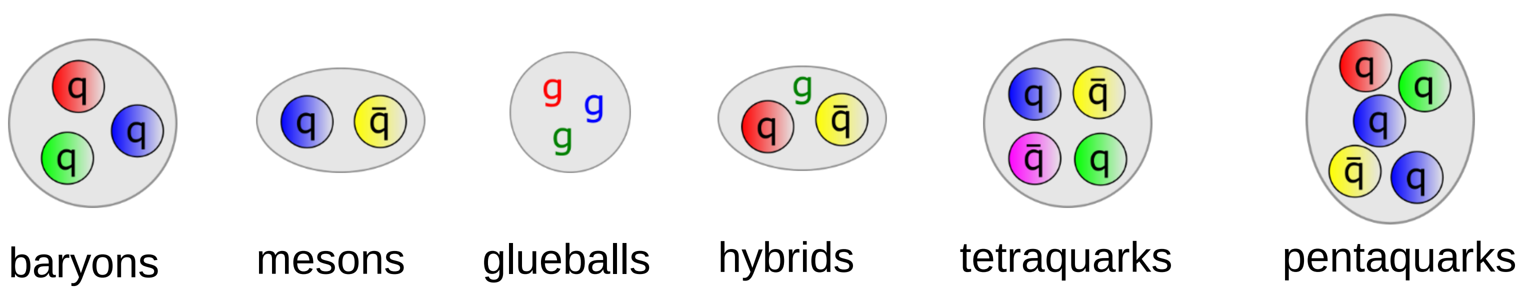

G


q q q
baryons
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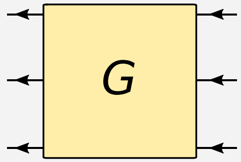
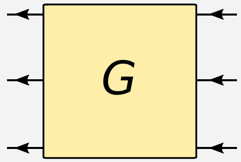
pentaquarks



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\bar{\psi}_{\alpha'}\bar{\psi}_{\beta'}\bar{\psi}_{\gamma'}\,|0\rangle$$

Extracting hadron poles from Green's functions



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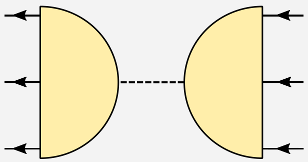
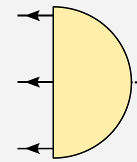


$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} \simeq \sum_\lambda \frac{\mathbf{\Psi}^\lambda_{\alpha\beta\gamma}\bar{\mathbf{\Psi}}^\lambda_{\alpha'\beta'\gamma'}}{P^2+m^2_\lambda}$$

Spectral decomposition



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Bethe-Salpeter wave function as residue

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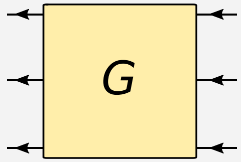

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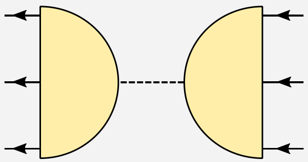

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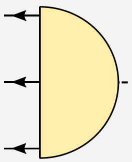

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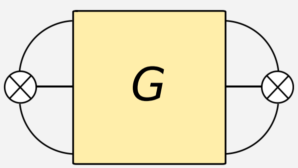
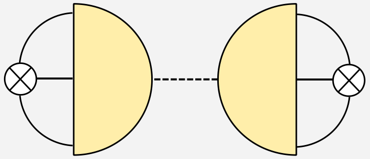


$$\mathbf{G}_{\sigma\sigma'} = \langle 0 | \, T J_\sigma \bar{J}_{\sigma'} \, | 0 \rangle \qquad J_\sigma = \Gamma_{\alpha\beta\gamma\sigma} \psi_\alpha \psi_\beta \psi_\gamma$$

On lattice, current correlators



$$\mathbf{G}_{\sigma\sigma'} = \sum_\lambda \frac{e^{-E_\lambda |\tau|}}{2 E_\lambda} [\cdots]$$

Exponential Euclidean time decay

Trade one unknown, G, for another unknown K.

Dyson equation and Bethe-Salpeter wave function

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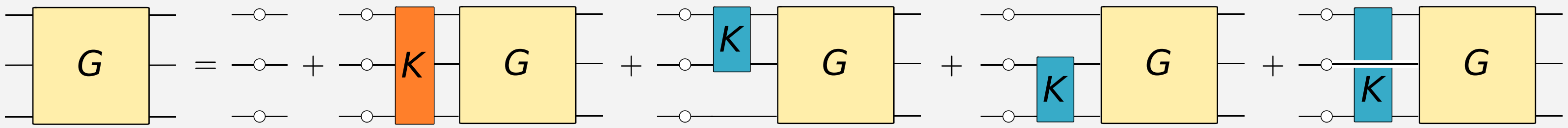
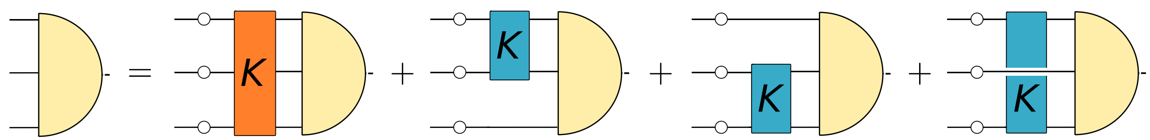


$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0\mathbf{K}\mathbf{G}$$

Solution yields on-shell particle pole and Bethe-Salpeter wave function

$$\mathbf{\Psi} = \mathbf{G}_0\mathbf{K}\mathbf{\Psi}$$



Bethe-Salpeter wave function **essential** ingredient for access to *e.g.* form-factors

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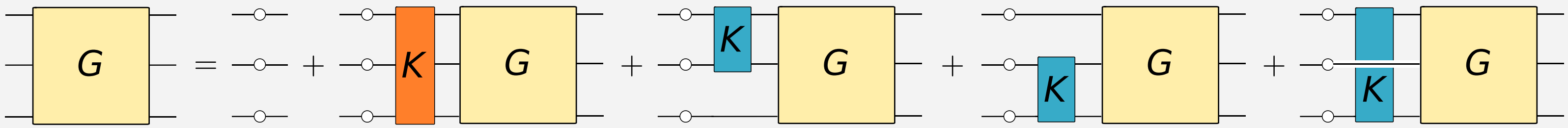

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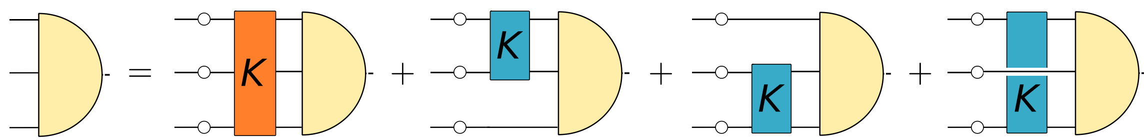

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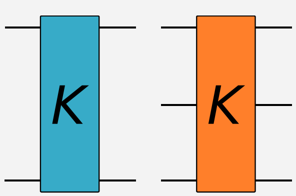

(ir)reducible two, three, four etc. body kernels the **define** equation.

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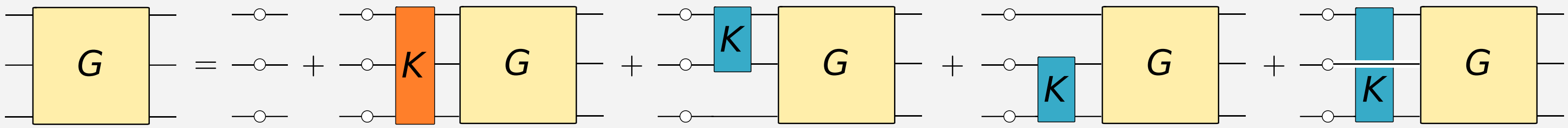

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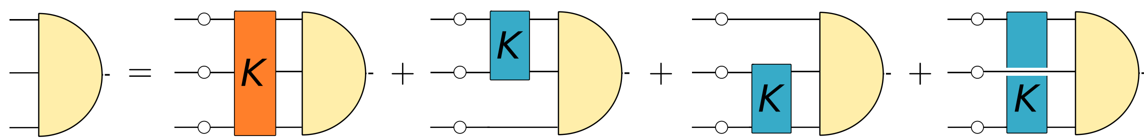

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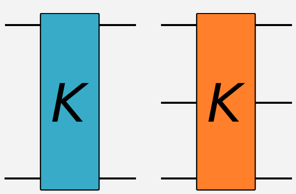

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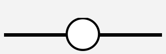
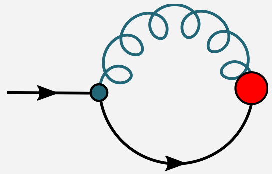
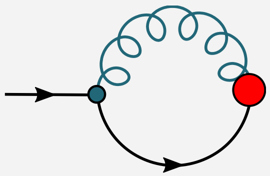


Dressed particle constituents also needed: **these are ALSO Green's functions**

$$\frac{\delta\Gamma[\phi]}{\delta\psi} = \frac{\delta S[\phi]}{\delta\psi} +$$ 

Provide access to dressed propagators and interaction vertices from which BS kernel constructed

$$\frac{\delta \Gamma[\phi]}{\delta \psi} = \frac{\delta S[\phi]}{\delta \psi} +$$ 

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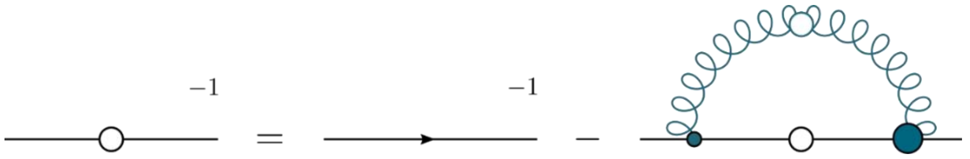
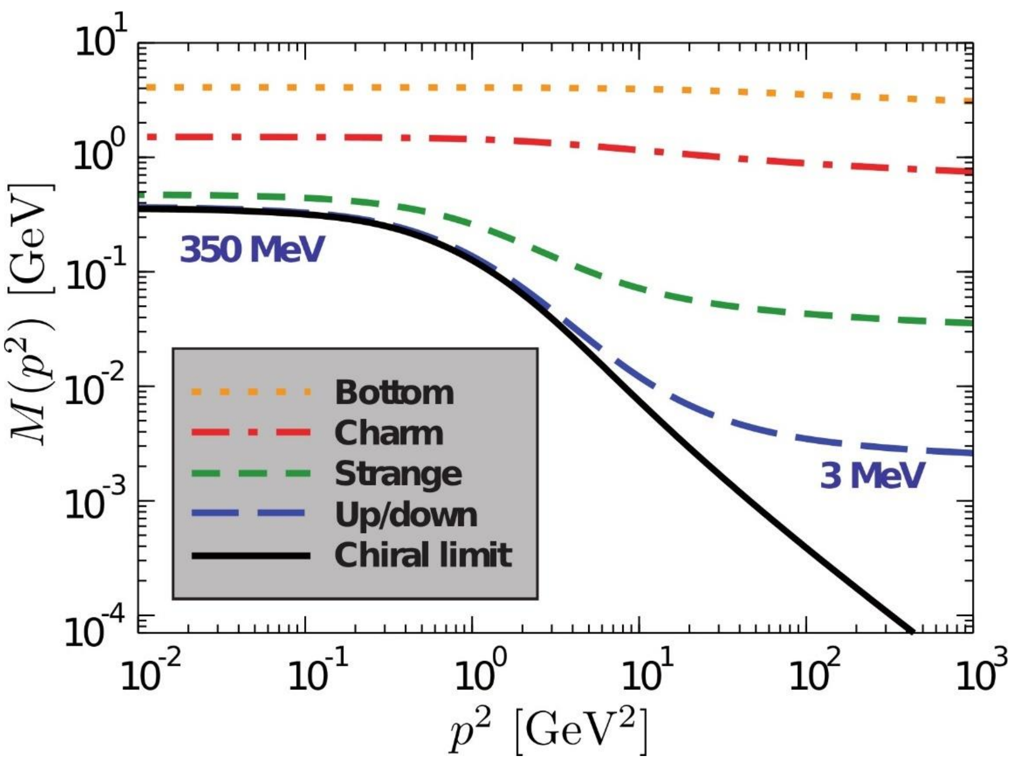


$$S^{-1}(p) = A(p^2)\left(-i\not{p} + M(p^2)\right)$$



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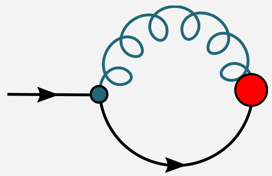

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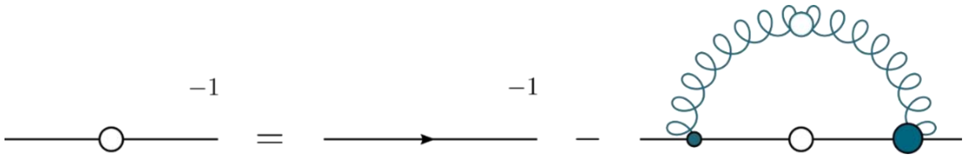
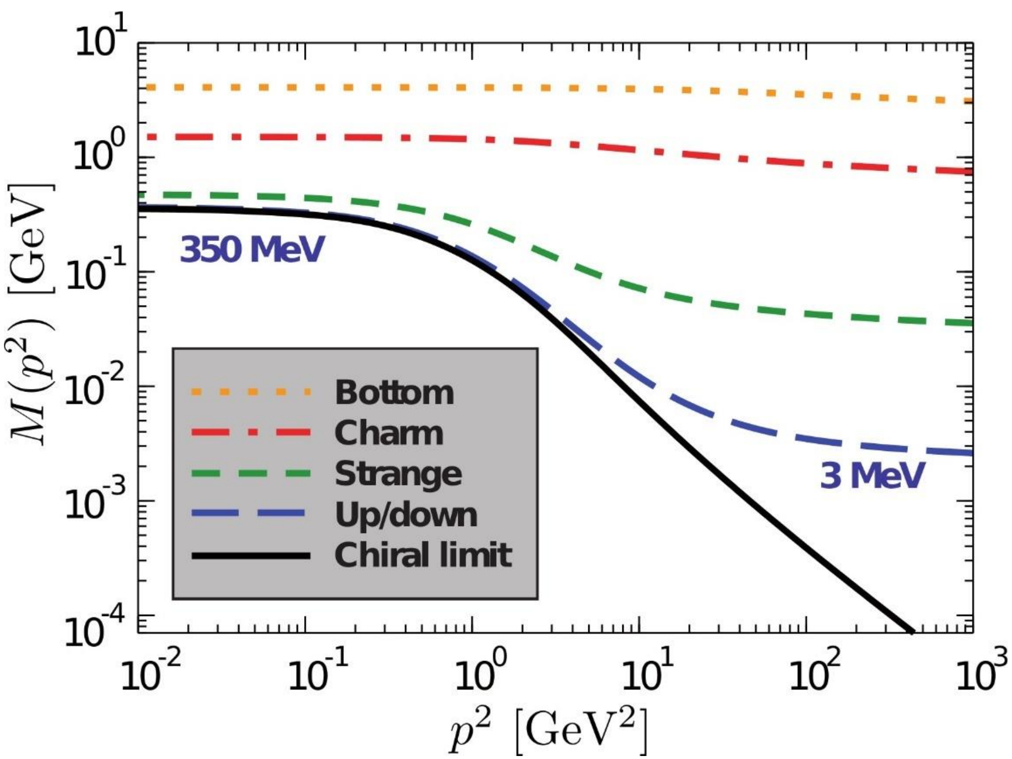




$$S^{-1}(p) = A(p^2)\left(-i\not{p} + M(p^2)\right)$$

It's QCD:

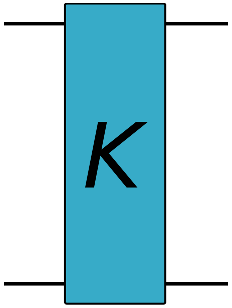
- Mass function runs
- Coupling runs
- Vertices run

See Kizilersu

Everything runs!

Very difficult to disentangle in detail

Need the Bethe-Salpeter kernel(s)



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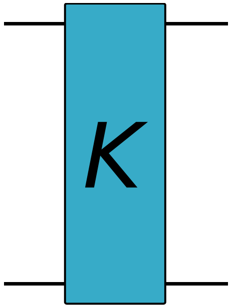
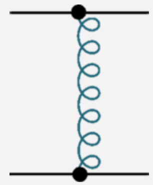


Simplest choice:
one-gluon exchange



rainbow-ladder approximation

Structure: gluon is “**dressed**”, but vertices are “**bare**”

Compensate shortcomings by replacing **dressed gluon** with **effective interaction**

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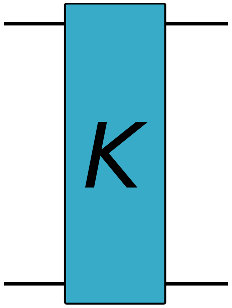

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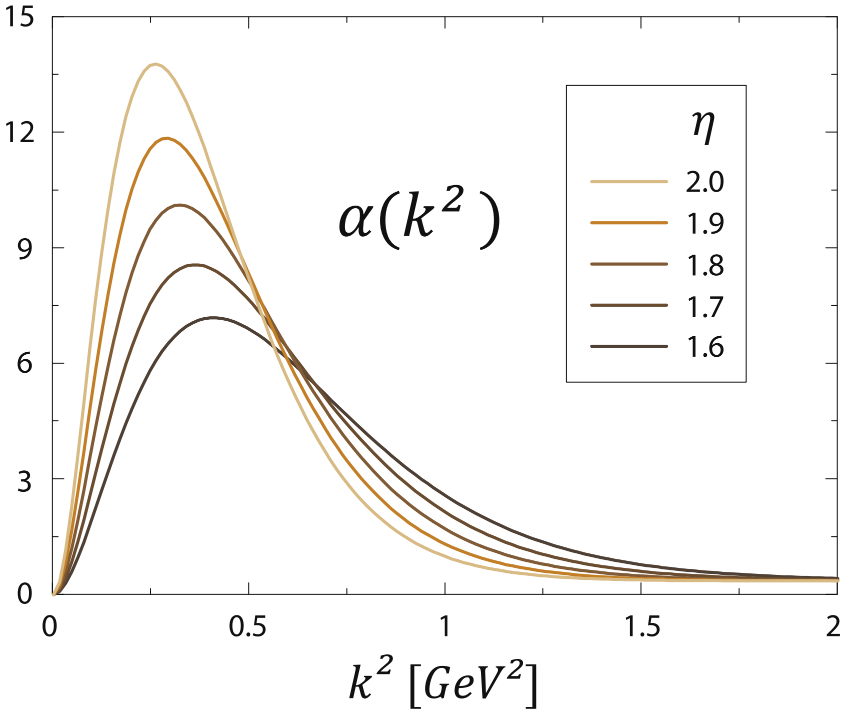
e.g. **Maris-Tandy interaction**

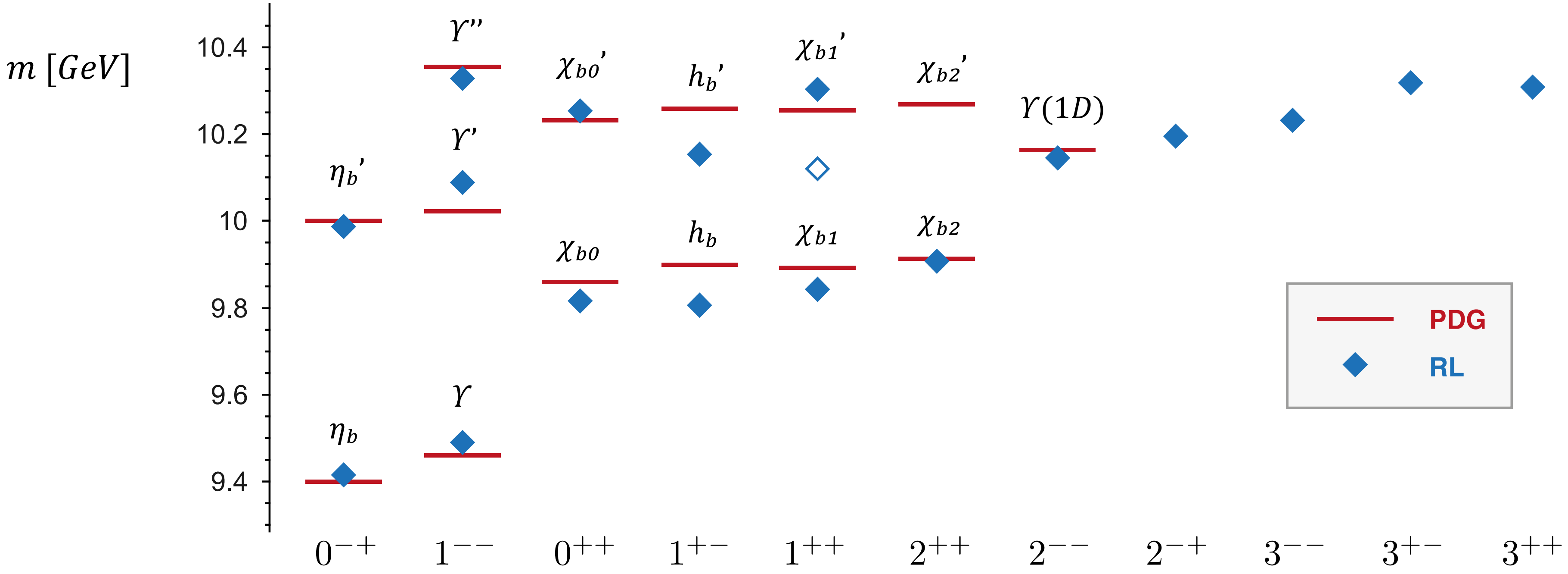


Certainly a good approximation for heavy quarks where IR effects are screened by the quark mass

Should be reliable in channels where dominated by scale of **Dynamical Chiral Symmetry Breaking**

[Maris, Tandy PRC 60 (1999) 055214]

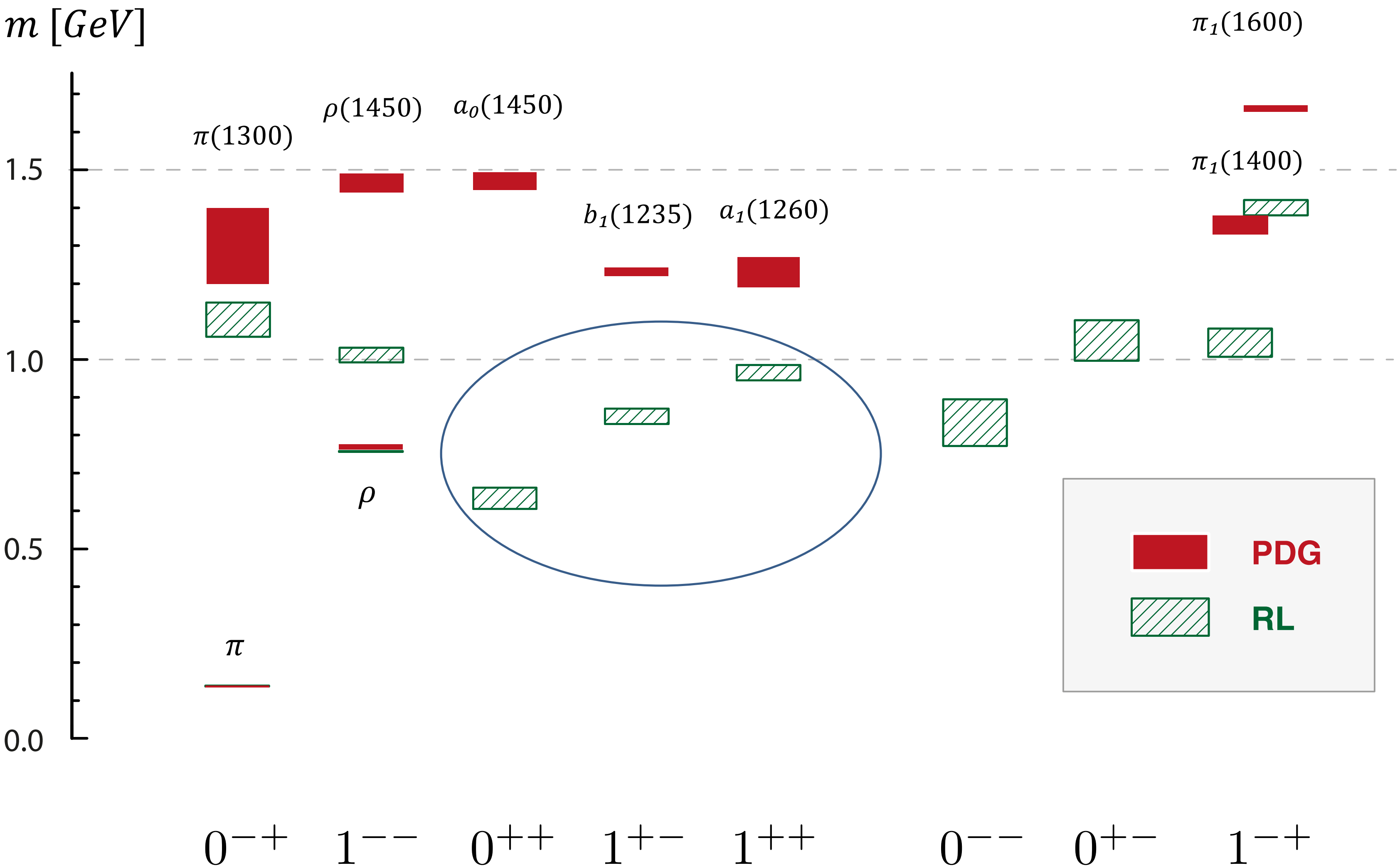
Heavy mesons (bottomonium): ground and excited states



- Splitting between ground/excited states good
- Some deficiencies in level ordering

[Kubrak, Fischer, RW EPJA 51 (2015) 10]
[Blank, Krassnigg PRD 84 (2011) 096014]
[Hilger, Popovici, Gomez-Rocha, Krassnigg PRD 91 (2015) 034013]

Light mesons: ground and excited states



- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hilger, Gomez-Rocha, Krassnigg arXiv:1508.07183]

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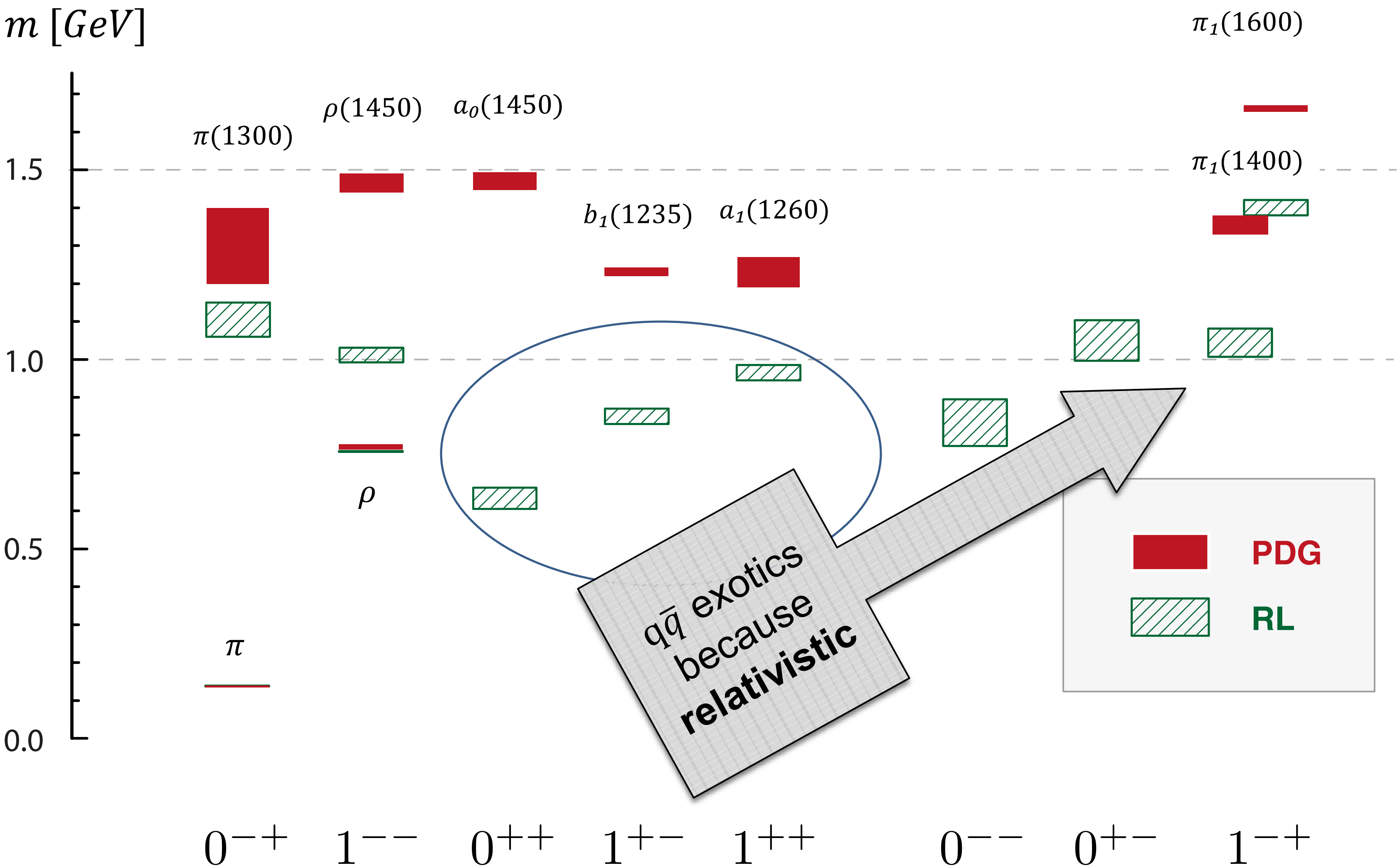

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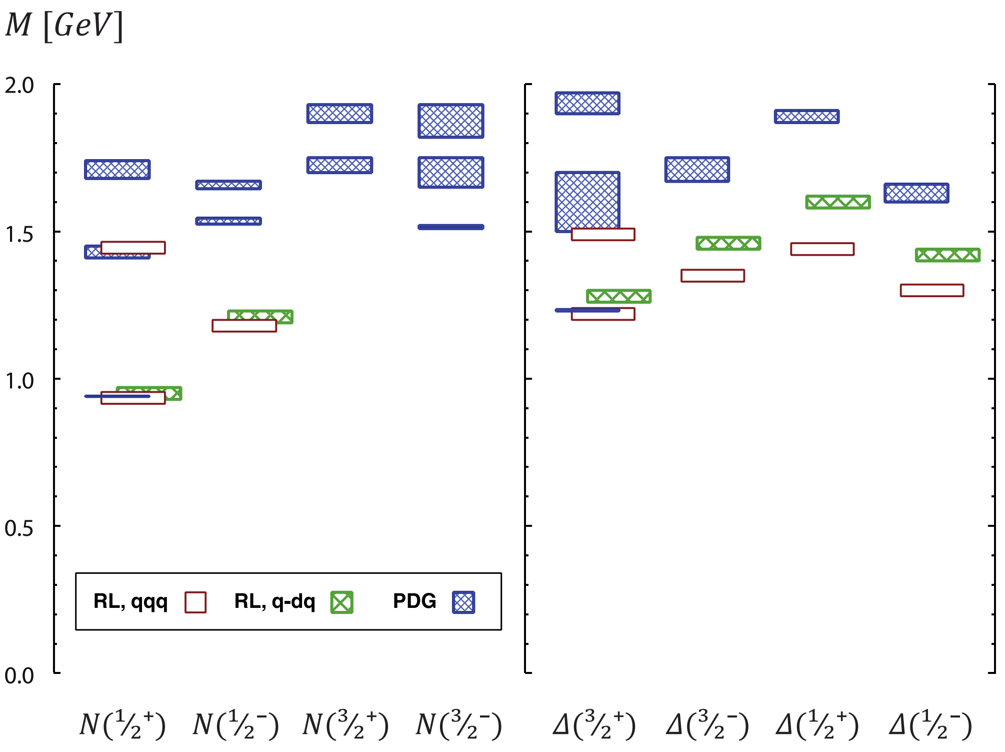
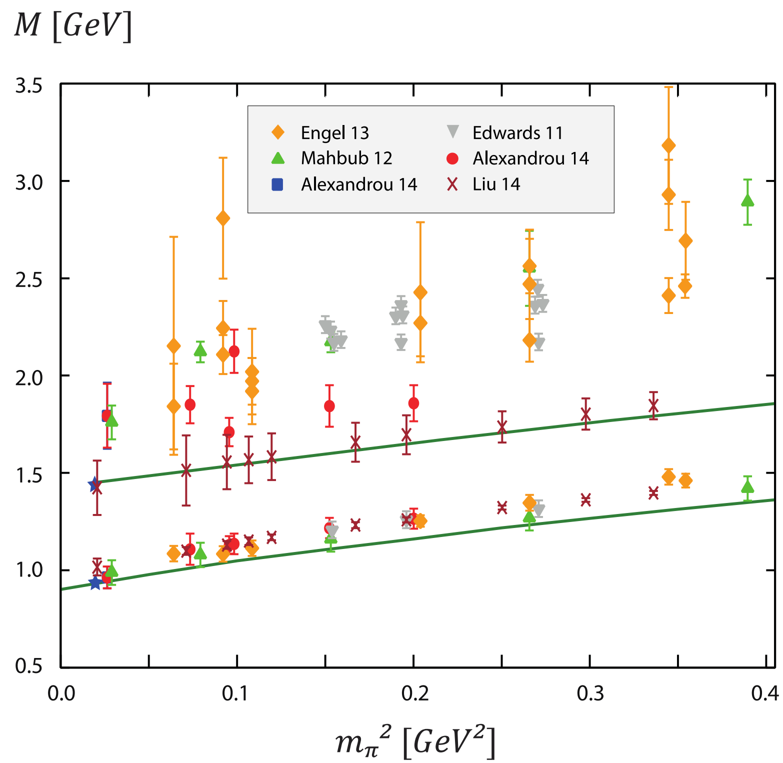
Light baryons: ground and excited states



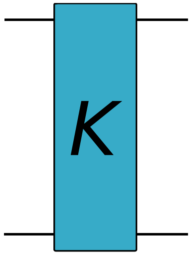


Nucleon/Delta ground + excited states good

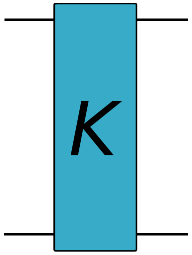
[Eichmann, Alkofer, Krassnigg, Nicmorus PRL 104 (2010) 201601]
[Sanchis-Alepuz, Eichmann, Villalba-Chavez, Alkofer, PRD 84 (2011) 096003]
[Sanchis-Alepuz, Eichmann, Fischer *in preparation*]

Expected deficiencies in diquarks/meson analogs

[Roberts, Chang, Cloet, Roberts FBS 51 (2011) 1]
[Chen, Chang, Lei, Roberts, Wan, Wilson FBS 53 (2012) 293]
[Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu, Zong PRL 115 (2015) 171801]



Expose additional corrections to the kernel



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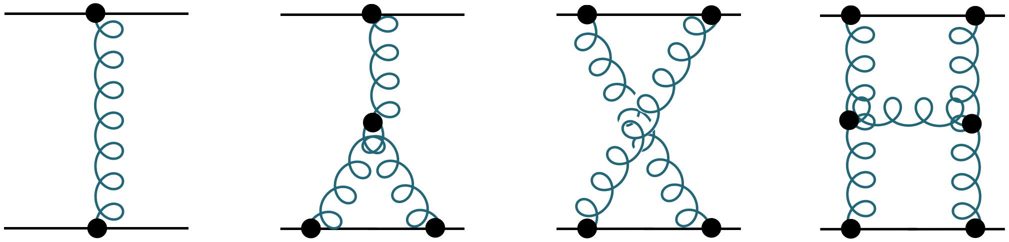
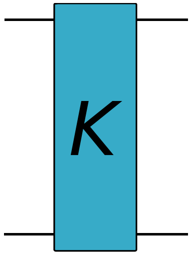
Diagrammatic



[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]



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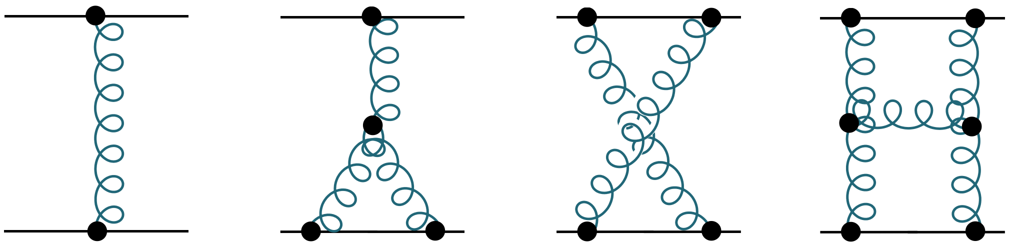

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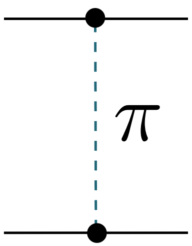
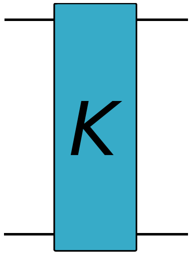
Effective/Composite



[Fischer, Nickel, Wambach ORD 76 (2007) 094009]
[Fischer, RW PRD 78 (2008) 074006]
[Sanchis-Alepuz, Fischer, Kubrak PLB 733 (2014) 151]



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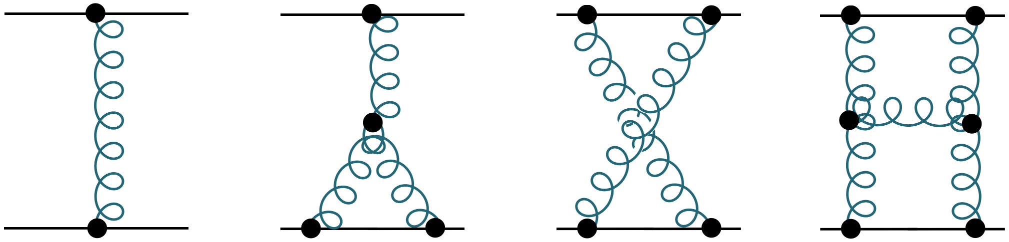

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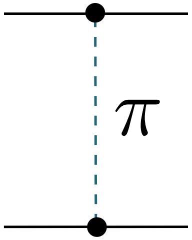

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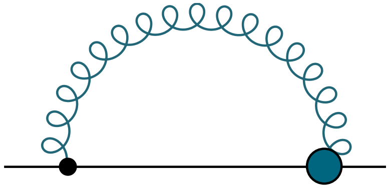
[Sanchis-Alepuz, Fischer, Kubrak PLB 733 (2014) 151]

Technique use **n**PI effective actions expanded to **m** loops

Loop expansion of a particular resummation of dressed propagators and, perhaps, vertices

Construction of BS kernel: Munczek cutting

$$K = \frac{\delta^2 \Gamma_2[B]}{\delta B \delta B}\bigg|_{B=S} = \frac{\delta \Sigma[B]}{\delta B}\bigg|_{B=S} = \frac{\delta \Sigma[S]}{\delta S}$$

$\Sigma \quad =$ 

[Munczek, PRD 52 (1995) 4736]

- Munczek cutting assumes underlying 2PI effective action
- Dressed quark-gluon vertex is auxiliary function that defines resummation of dressed propagators / bare vertices

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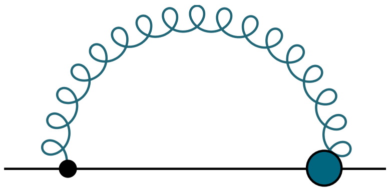

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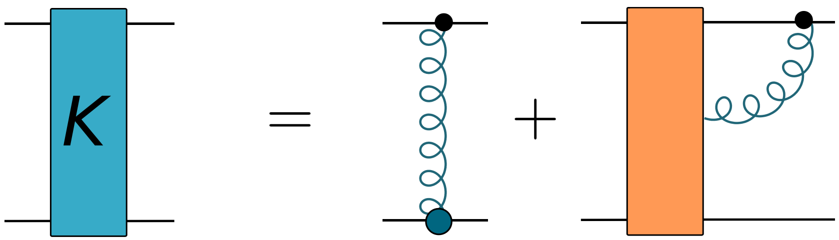


Explicit

$$K_1 \simeq [\gamma^\mu]_{\alpha\beta}[\gamma^\nu]_{\gamma\delta} D_{\mu\nu}$$

Implicit

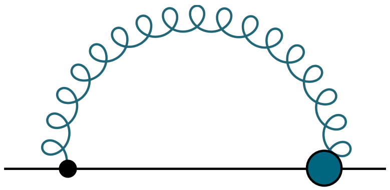
$$K_2 \simeq \left[\gamma^\mu S\right]_{\alpha\beta} \left[\frac{\delta \Gamma^\nu}{\delta S}\right]_{\gamma\delta} D_{\mu\nu}$$

Need integral representation for quark-gluon vertex to avoid ambiguity in momentum routing.

[Heupel, Goecke, Fischer EPJA 50 (2014) 85]

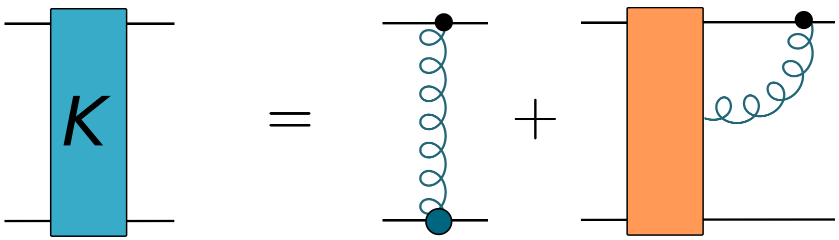
Construction of BS kernel: Munczek cutting

$$K = \left.\frac{\delta^2\Gamma_2[B]}{\delta B \delta B}\right|_{B=S} = \left.\frac{\delta\Sigma[B]}{\delta B}\right|_{B=S} = \frac{\delta\Sigma[S]}{\delta S}$$

$\Sigma =$ 

[Munczek, PRD 52 (1995) 4736]

- Munczek cutting assumes underlying 2PI effective action
- Dressed quark-gluon vertex is auxiliary function that defines resummation of dressed propagators / bare vertices



Explicit

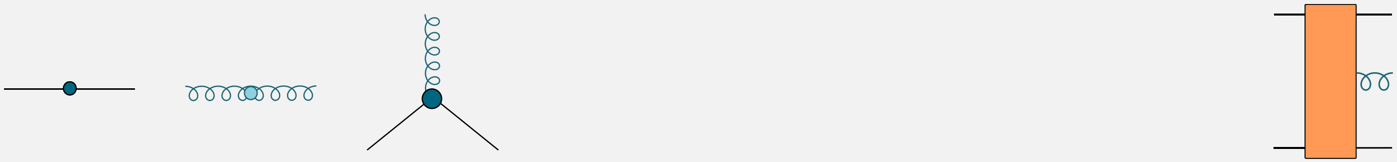
$$K_1 \simeq [\gamma^\mu]_{\alpha\beta}[\gamma^\nu]_{\gamma\delta} D_{\mu\nu}$$

Implicit

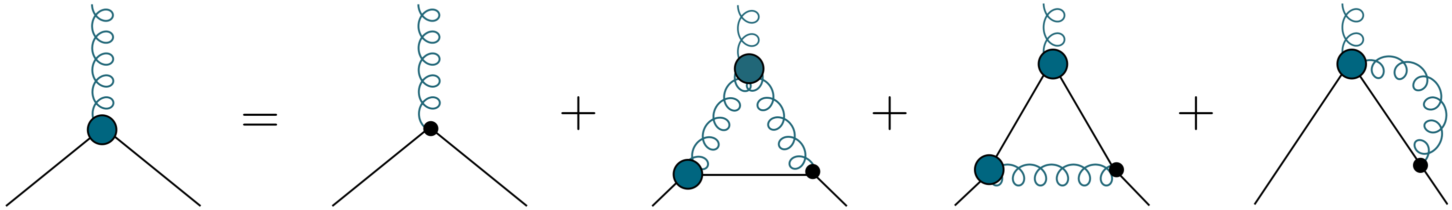
$$K_2 \simeq \left[\gamma^\mu S\right]_{\alpha\beta} \left[\frac{\delta\Gamma^\nu}{\delta S}\right]_{\gamma\delta} D_{\mu\nu}$$

Need integral representation for quark-gluon vertex to avoid ambiguity in momentum routing.

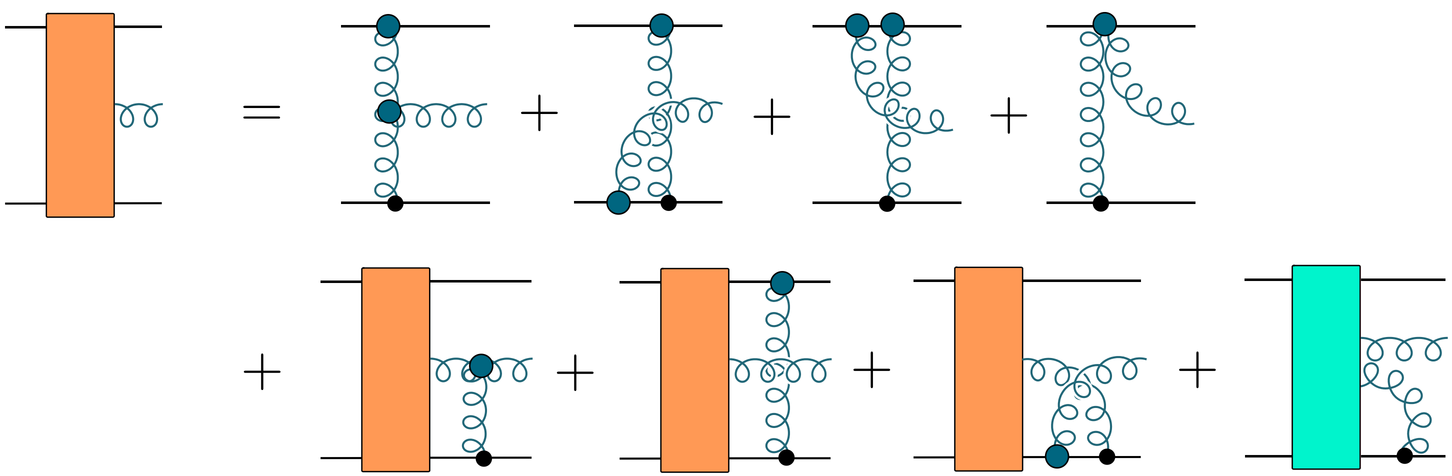
[Heupel, Goecke, Fischer EPJA 50 (2014) 85]



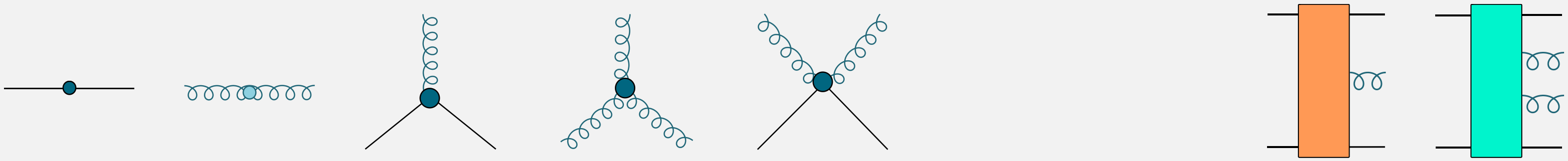
Munczek Cutting



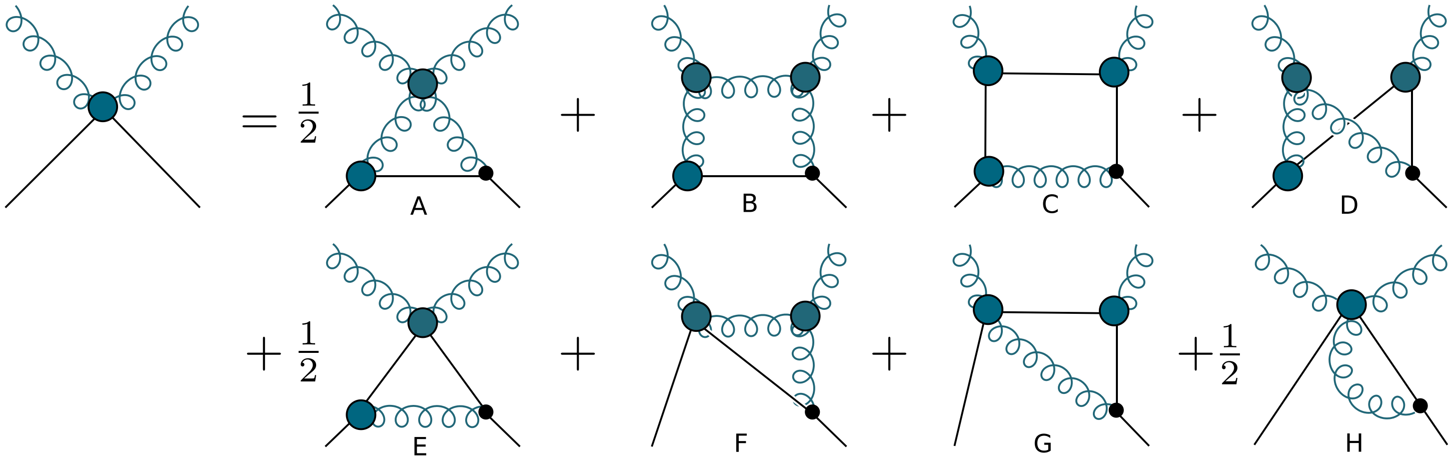
8 functions of 3 variables



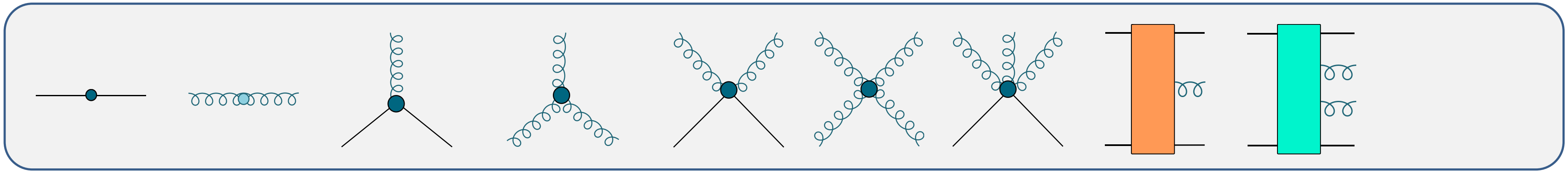
256 functions of 10 variables



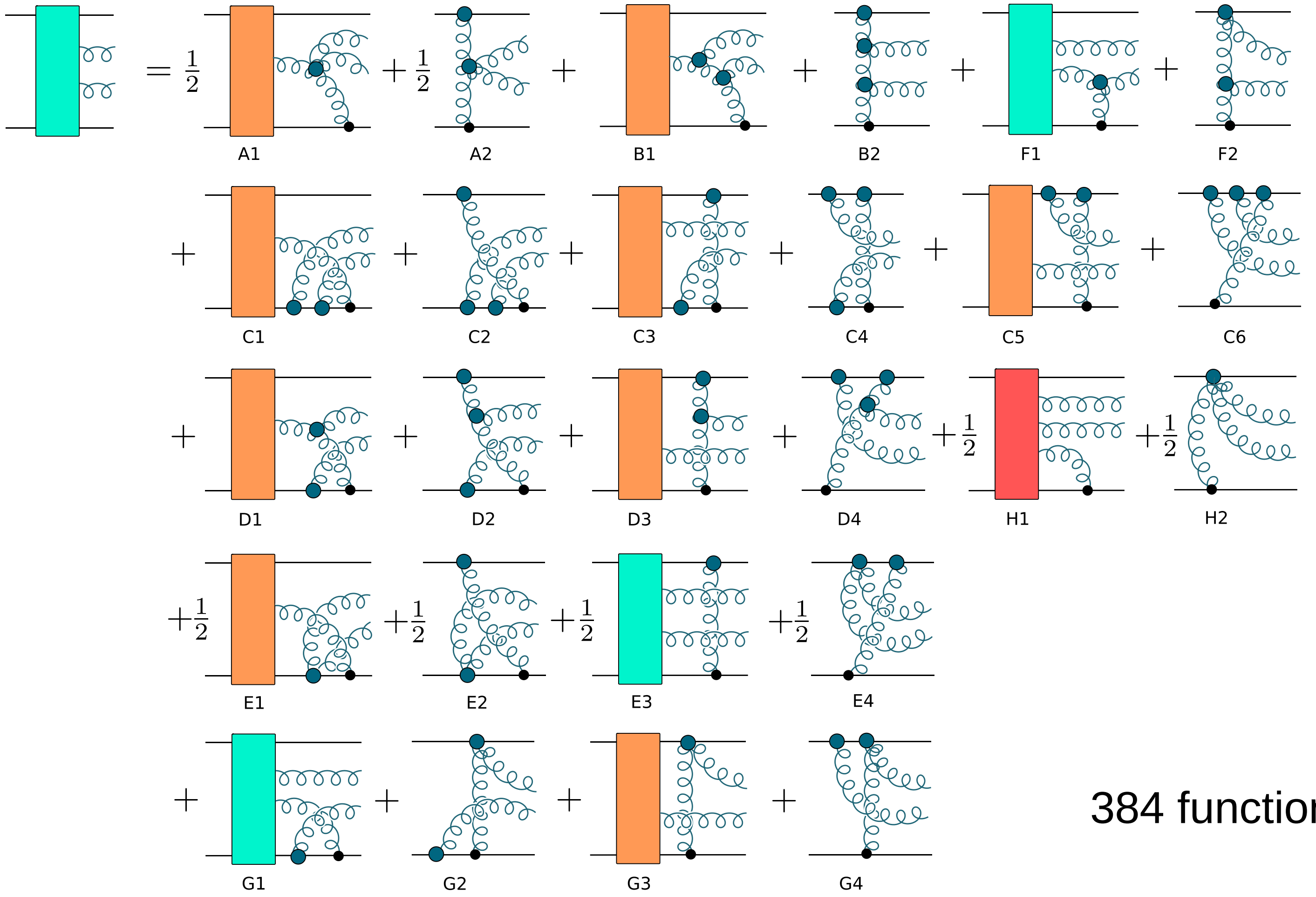
Munczek Cutting



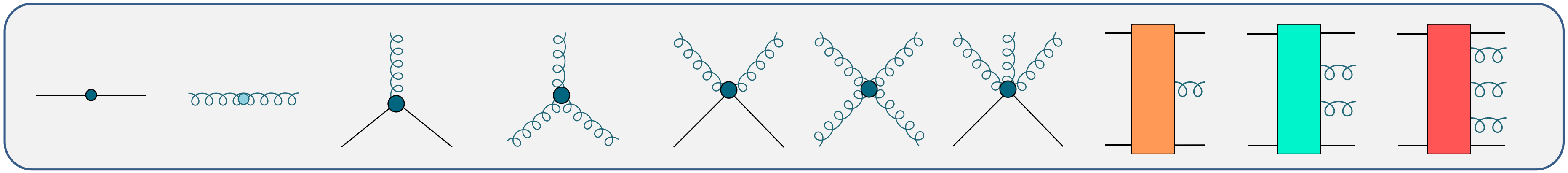
72 functions of 6 variables



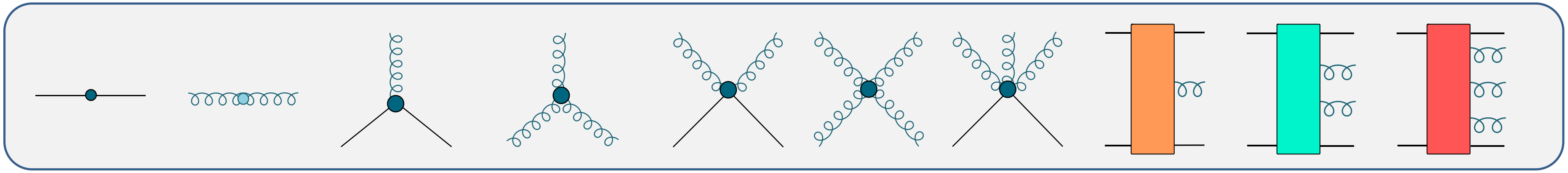
Munczek Cutting



384 functions of 15 variables

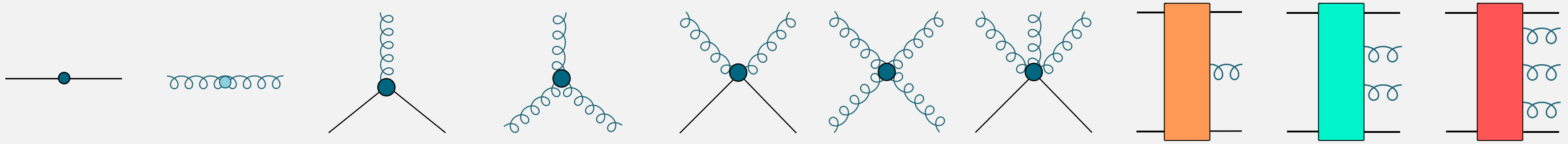


Truncations are necessary



Killer:

- multiplicity of diagrams and phase space
- Coupled system becomes **two** coupled systems

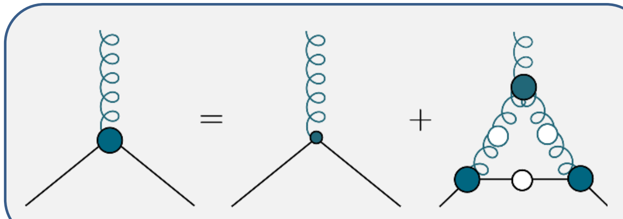


Killer:

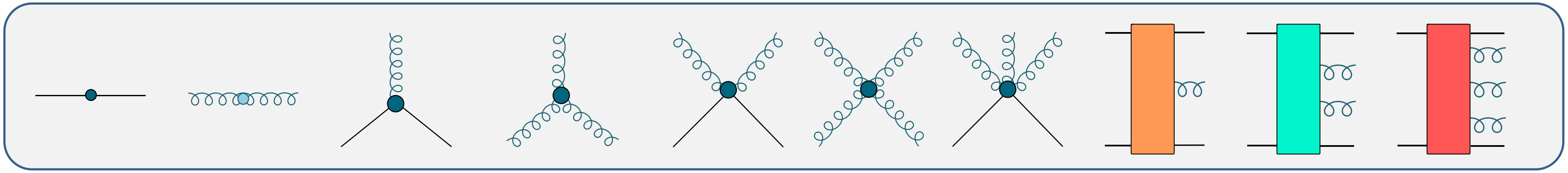
- multiplicity of diagrams and phase space
- Coupled system becomes **two** coupled systems

Because:

- Everything (relevant) functional of ***S***, as introduced by auxiliary quark-gluon vertex



- Self-coupled implies 2PI effective action to **all orders**
- Bethe-Salpeter kernel is **all orders** in loops
- Resummation expressed by 5pt functions and deps.



Killer:

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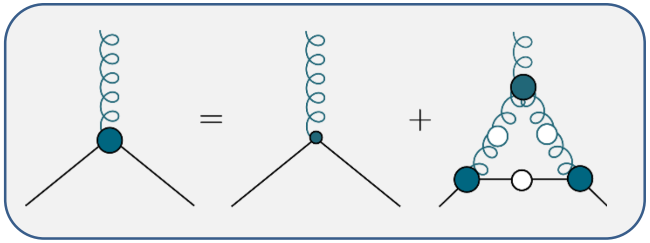


- Self-coupled implies 2PI effective action to **all orders**
- Bethe-Salpeter kernel is **all orders** in loops
- Resummation expressed by 5pt functions and deps.

Hopes:

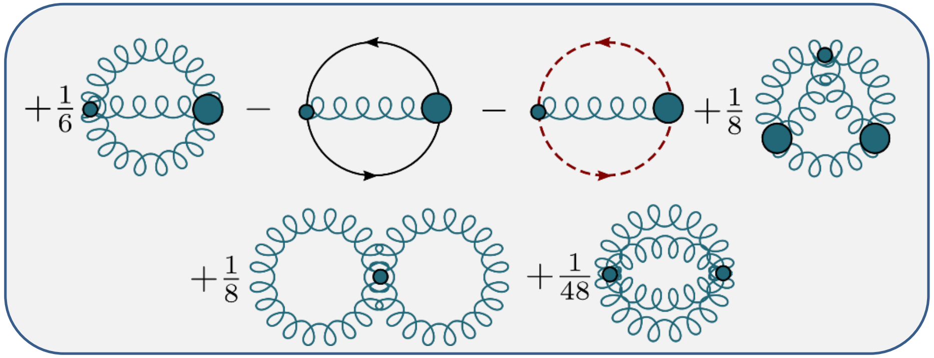
- 2PI effective action **to finite loop** order
- Simplest (coupled) 5pt function **almost** tractable

[Bhagwat et al PRC 68 (2003) 015203]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]

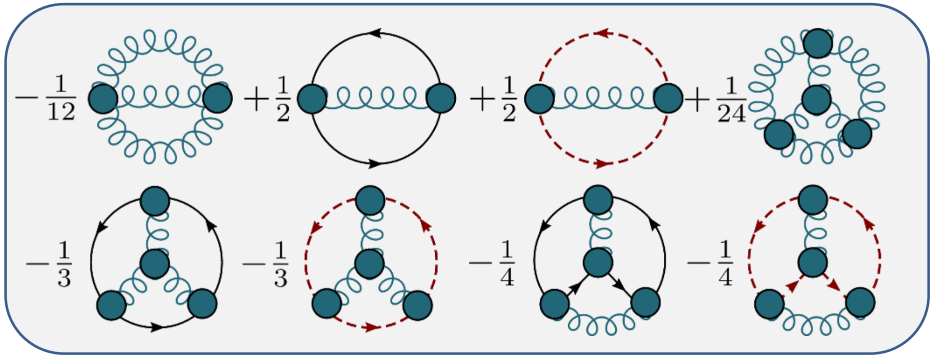
… or we switch to nPI

$$\Gamma[\phi, D, U] = S_{cl}[\phi] + \frac{i}{2} TrLnD^{-1} + \frac{i}{2} Tr\left[D_{(0)}^{-1} D\right] - i\Phi^0[\phi, D, U] - i\Phi^{int}[\phi, D, U] + const.$$

$\mathbf{\Phi^0}$: non-interacting part



$\mathbf{\Phi^{int}}$: interacting part



$$K = \left.\frac{\delta^2\Gamma_2[B,U]}{\delta B \delta B}\right|_{B=S,U=V} = \left.\frac{\delta\Sigma[B,U]}{\delta B}\right|_{B=S,U=V} \neq \frac{\delta\Sigma[S,V]}{\delta S}$$

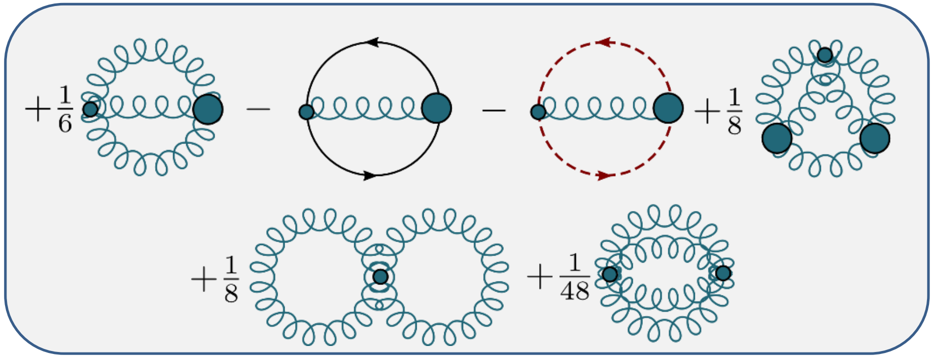
Differences:

- B, U independent. No implicit derivatives.
- Fixed order action yields fixed order equations. Vertices resummed by construction.
- No auxiliary equations or 5PI functions from implicit cutting

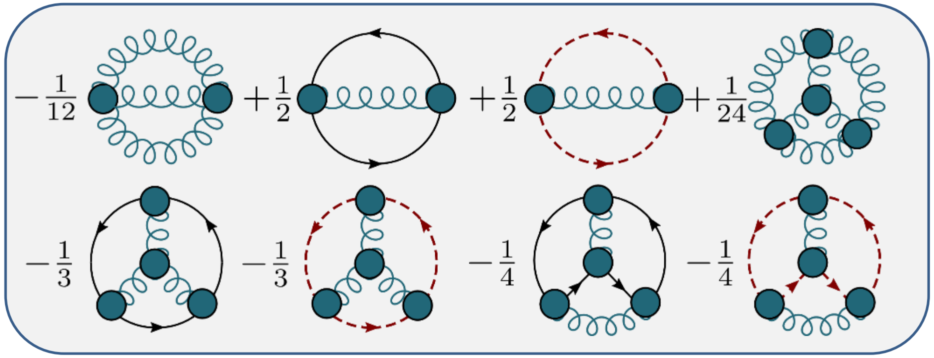
… or we switch to nPI

$$\Gamma[\phi, D, U] = S_{cl}[\phi] + \frac{i}{2} TrLnD^{-1} + \frac{i}{2} Tr\left[D_{(0)}^{-1} D\right] - i\Phi^0[\phi, D, U] - i\Phi^{int}[\phi, D, U] + const.$$

$\mathbf{\Phi^0}$: non-interacting part



$\mathbf{\Phi^{int}}$: interacting part



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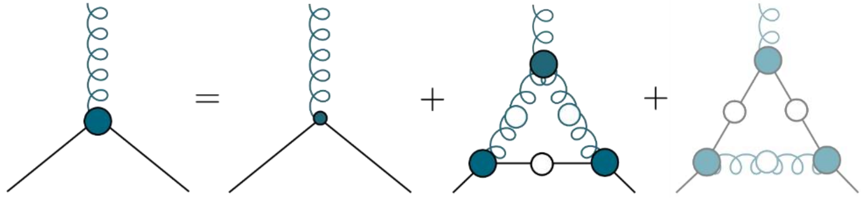
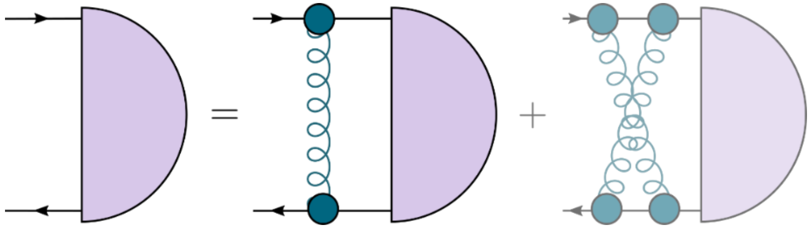
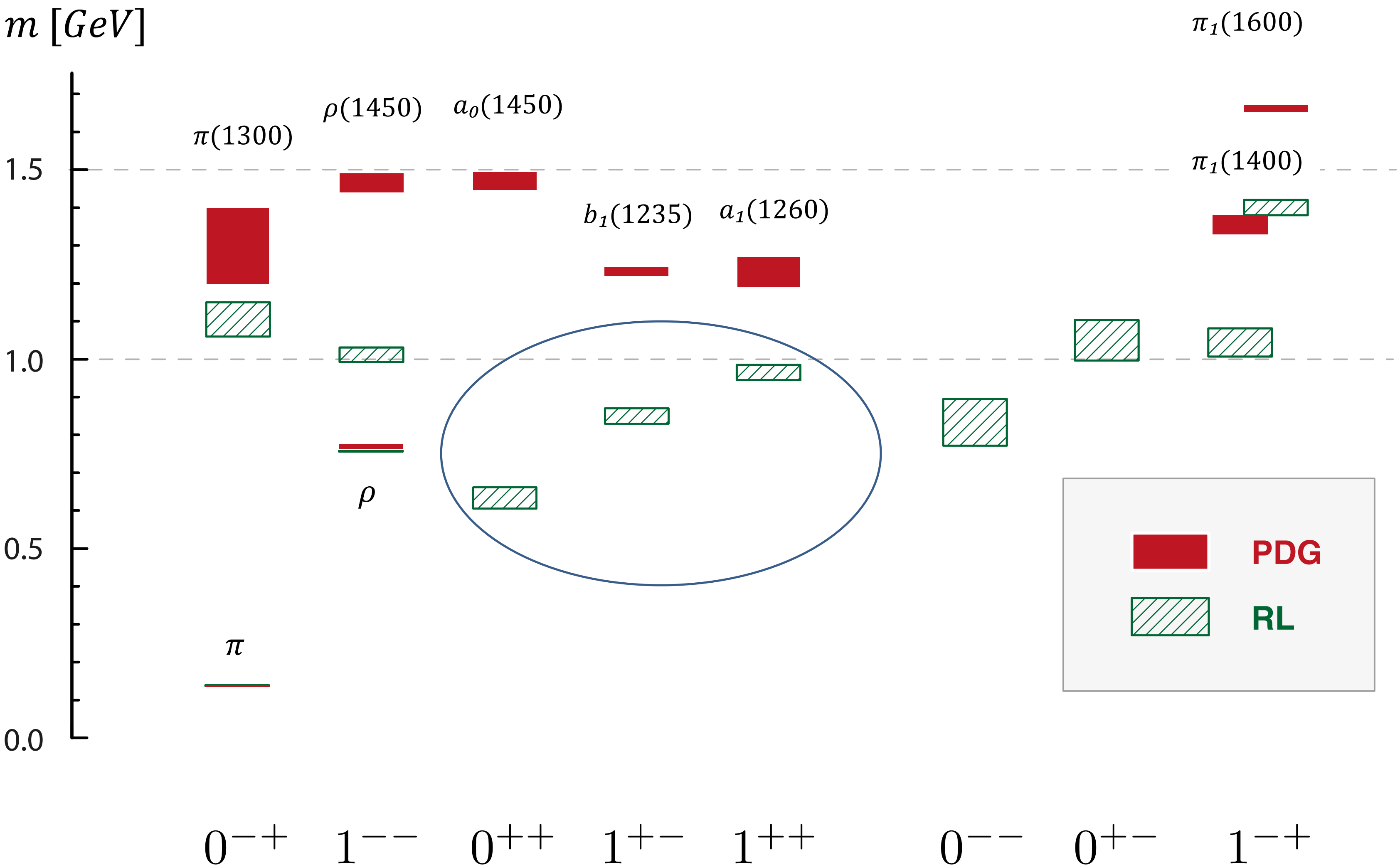




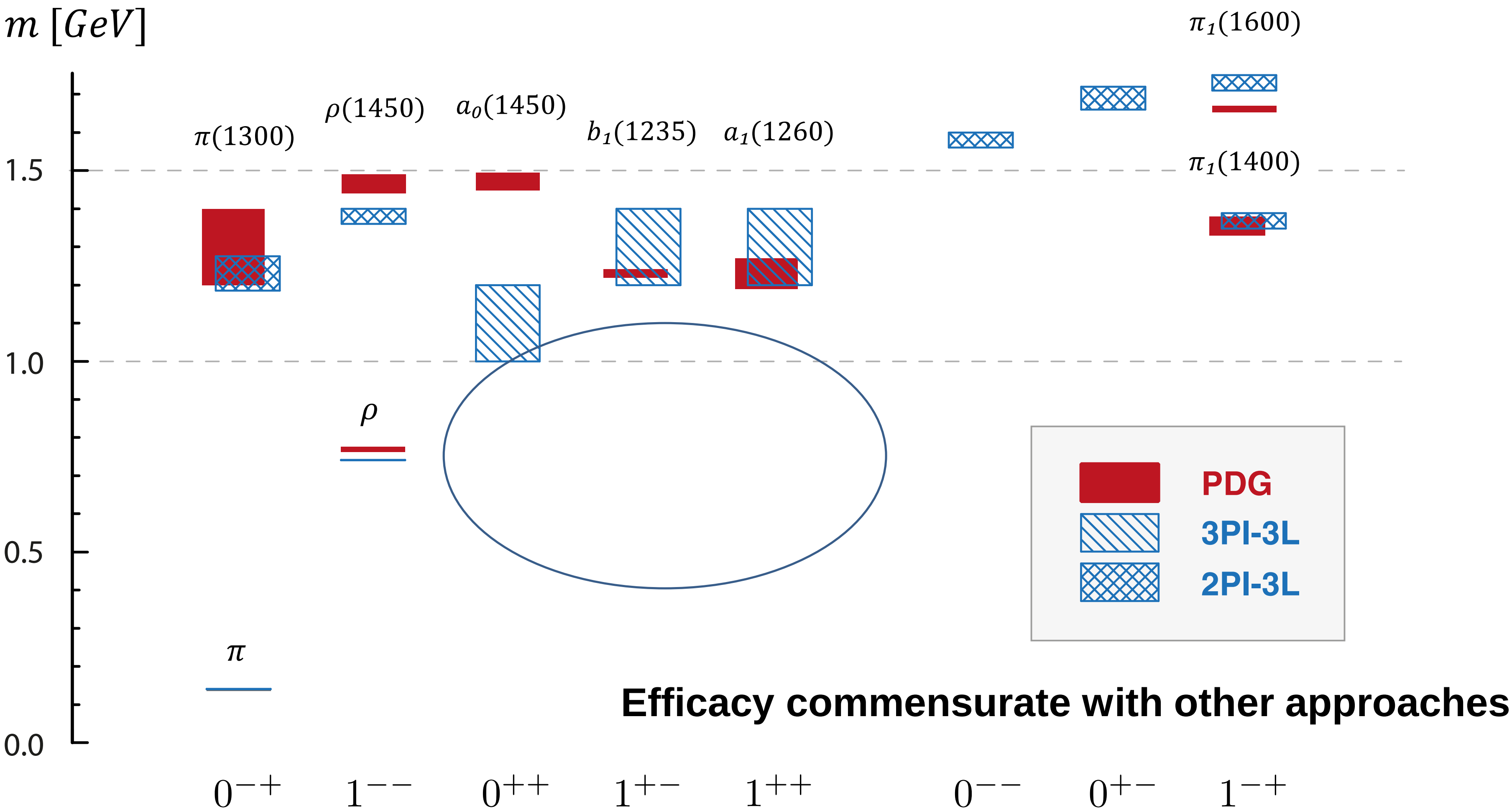
Results: rainbow-ladder



- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

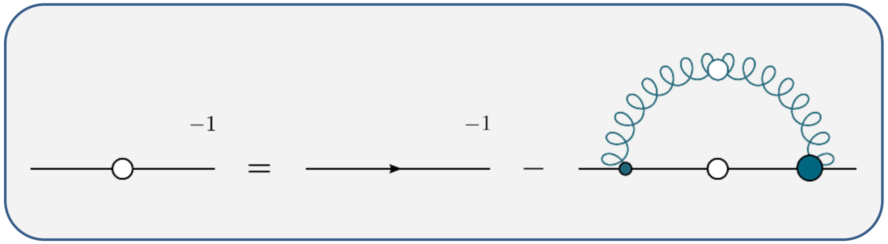
[Kubrak, Fischer, RW EPJA 50 (2014) 126]
[Hilger, Gomez-Rocha, Krassnigg arXiv:1508.07183]

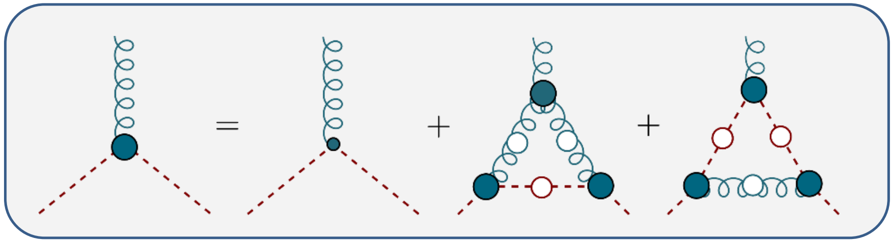
Results: beyond rainbow-ladder

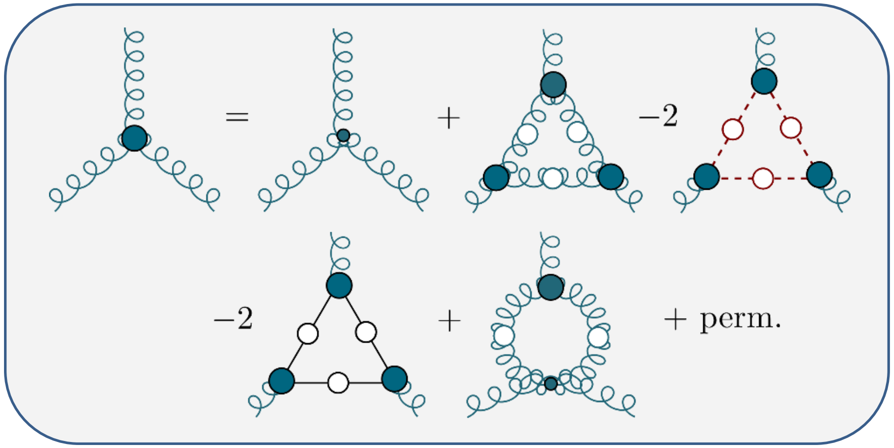


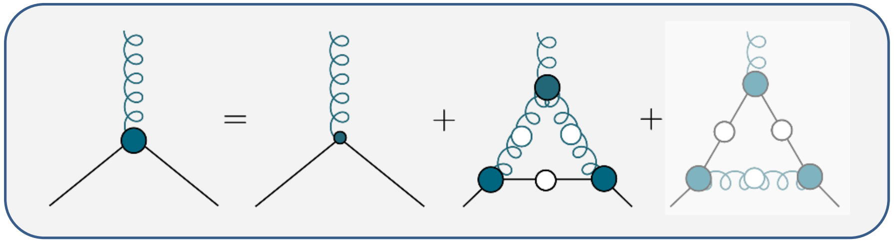
Scalar:	2PI-2L (RL) and 2PI-3L too light	[Chang, Roberts PRC 85 (2012) 052201]
$\rho - a_1$ splitting	2PI-2L (RL) and 2PI-3L too small	[Sanchis-Alepuz, RW PLB 749 (2015) 592]
$a_1 - b_1$ splitting	2PI-2L (RL) and 2PI-3L non-degenerate	[RW, Fischer, Heupel PRD 93 (2016) 034026]

3PI results: Recipe

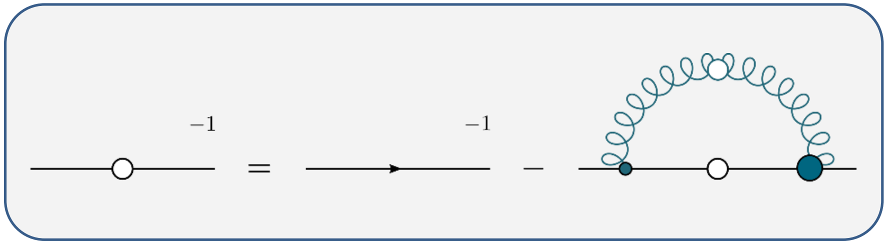


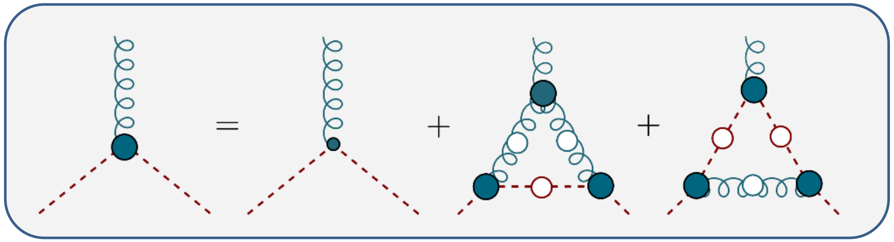


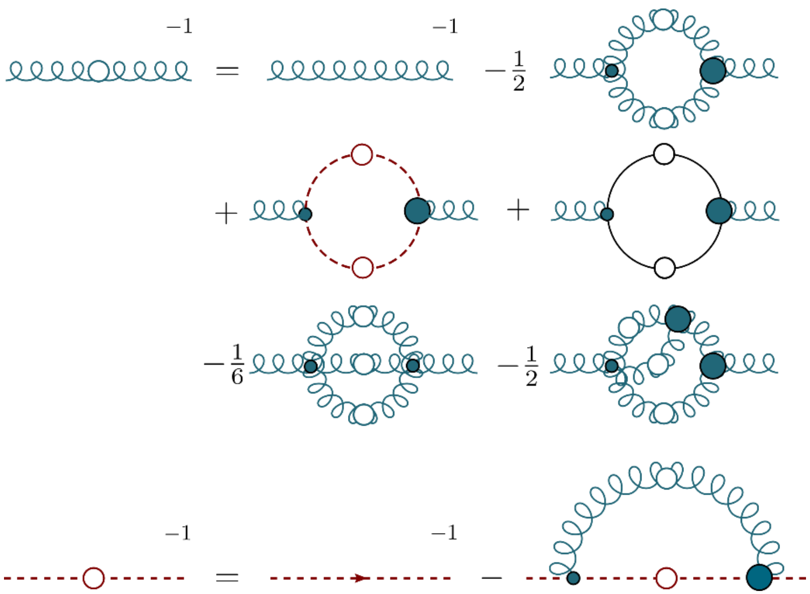


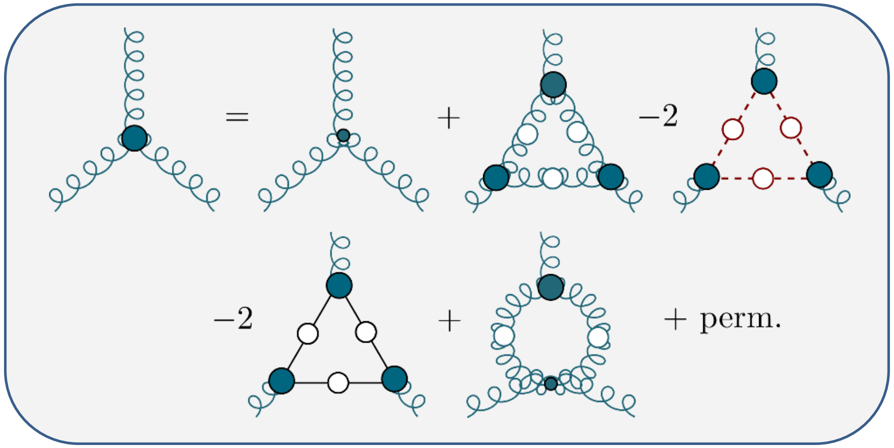


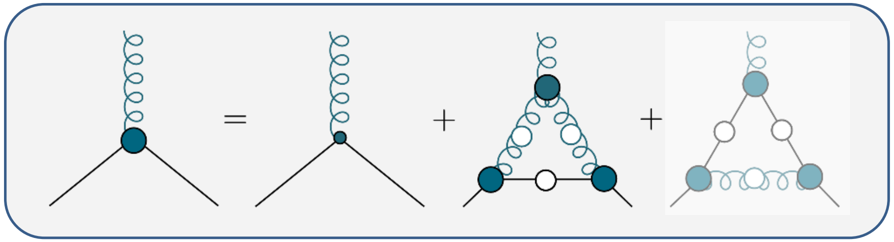
3PI results: Recipe



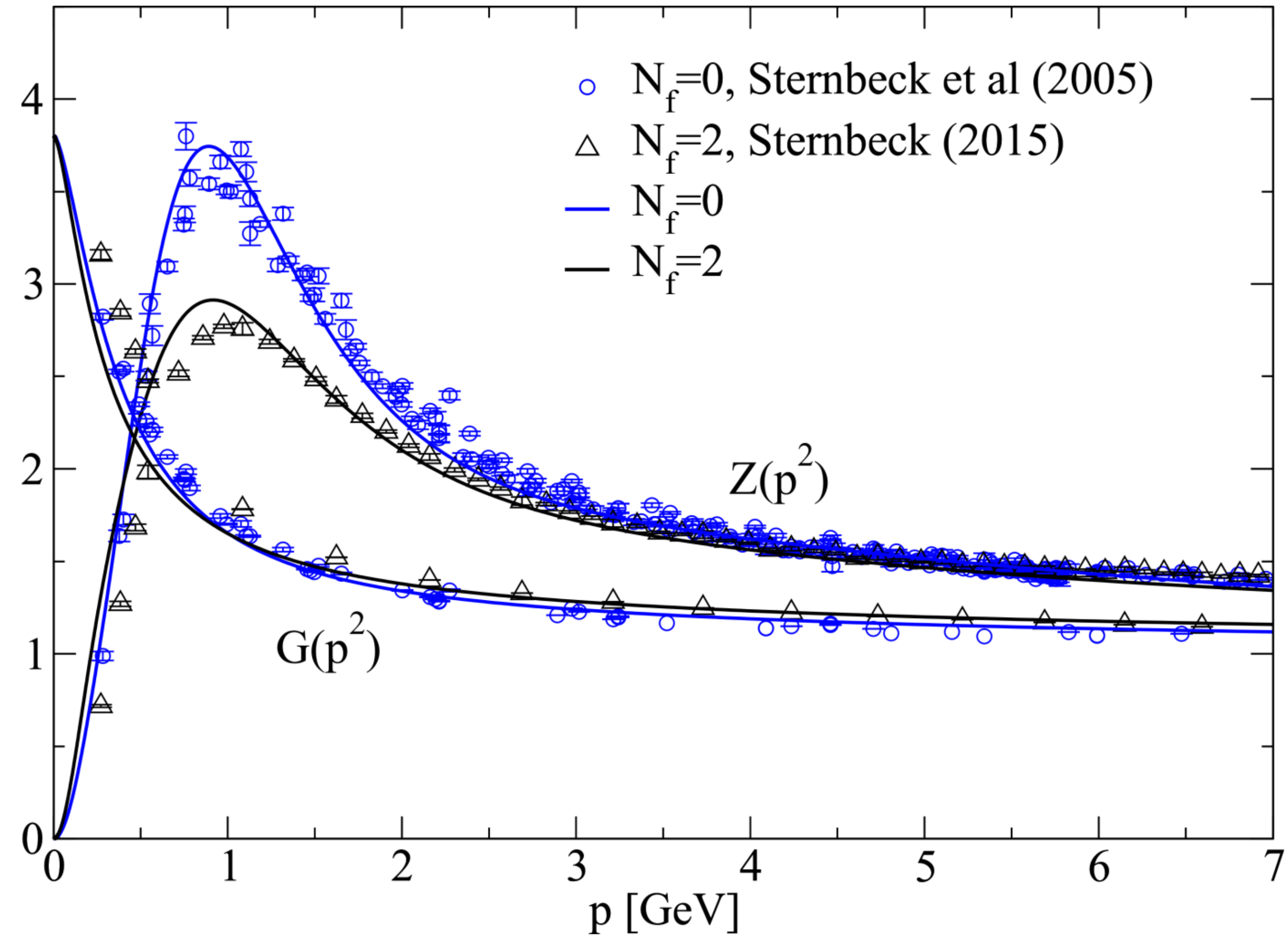








Ghost/gluon solved independently of 3PI



Employ lattice data for ghost/gluon and scale setting

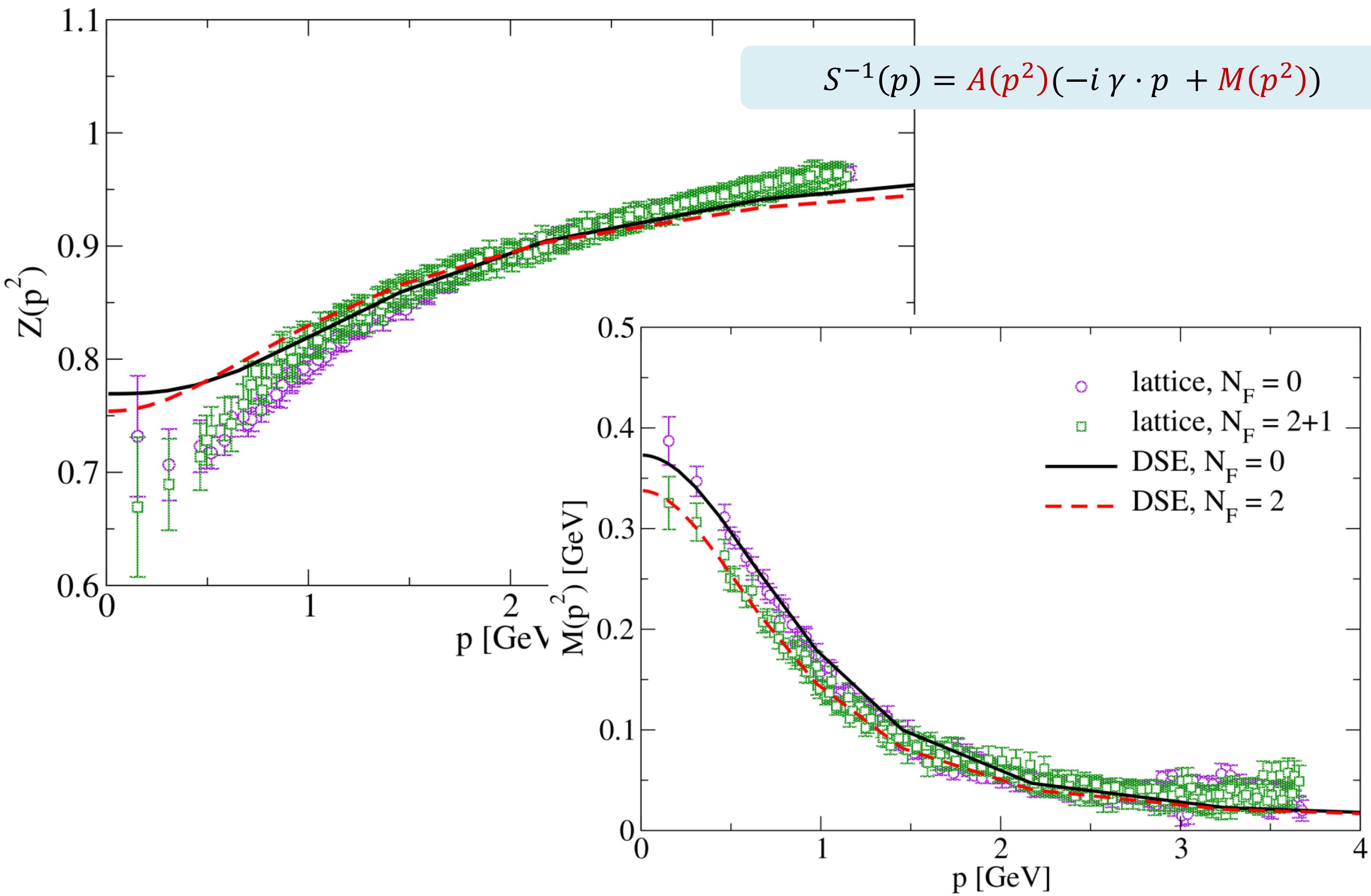
Provides input parameters of calculation (e.g. g_s)

$$D_G(p) = -\frac{G(p^2)}{p^2}$$

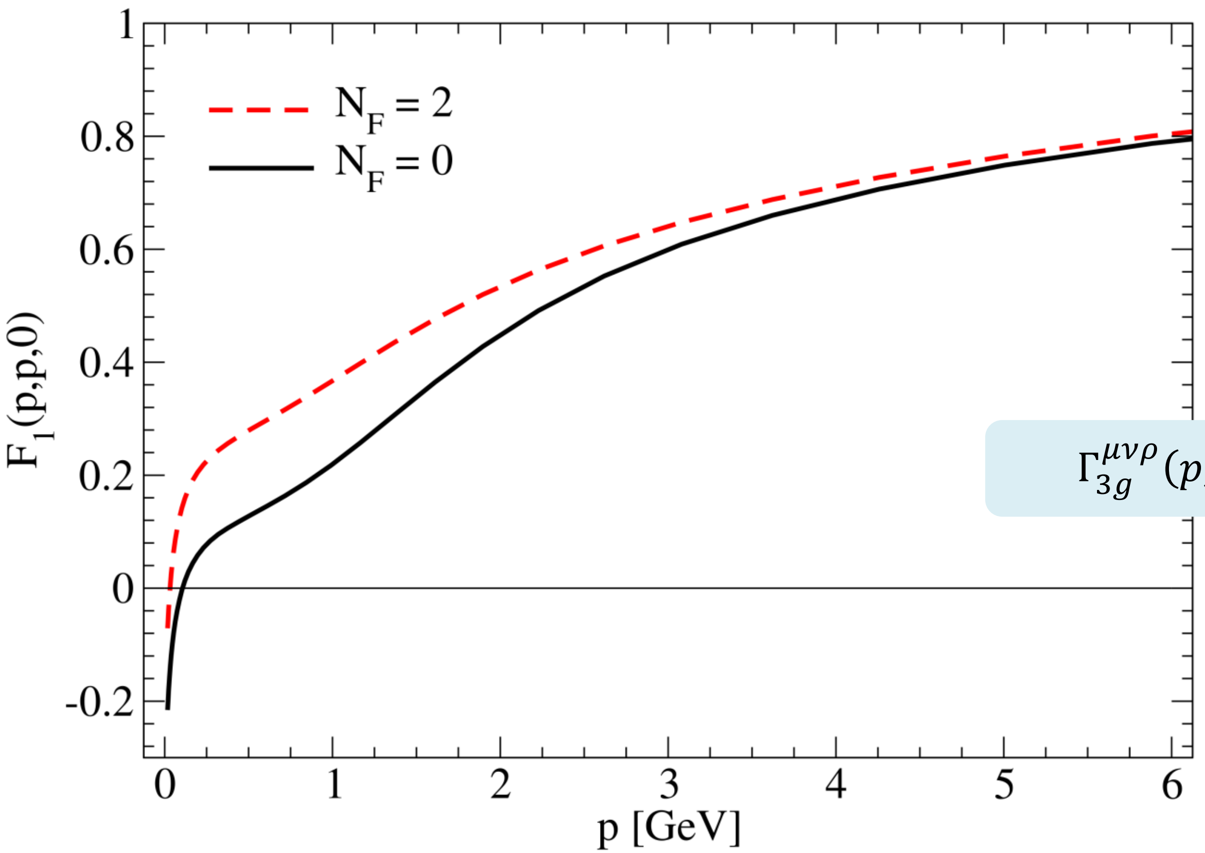
$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

3PI results: quark propagator

$$S^{-1}(p) = A(p^2)(-i\,\gamma \cdot p \,+ M(p^2))$$



3PI results: three-gluon vertex



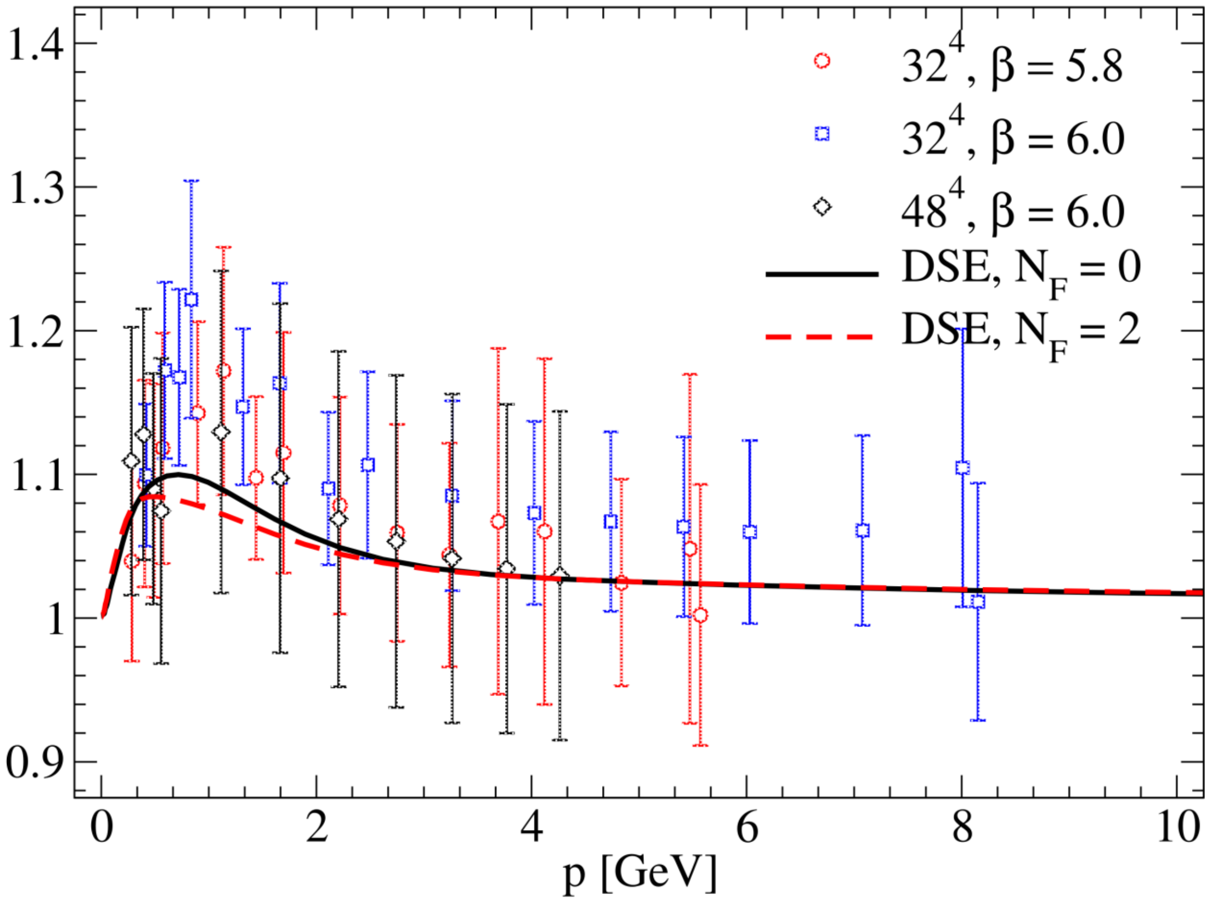
- Quark-loop enhances three-gluon vertex
- Zero crossing pushed to deep IR

$$\Gamma_{3g}^{\mu\nu\rho}(p,k,q) = F_1(p,k,q)\Gamma_{3g,0}^{\mu\nu\rho}(p,k,q)$$

- Tree-level structure dominant
- Single phase-space slice sufficient with S3 (e.g. soft-gluon)

[Blum, Huber, Mitter, von Smekal PRD 89 (2014) 061703]
[Aguilar, Binosi, Ibanez, Papavassiliou PRD 89 (2014) 085008]
[Eichmann, RW, Alkofer, Vujinovic PRD 89 (2014) 105014]

3PI results: ghost-gluon vertex

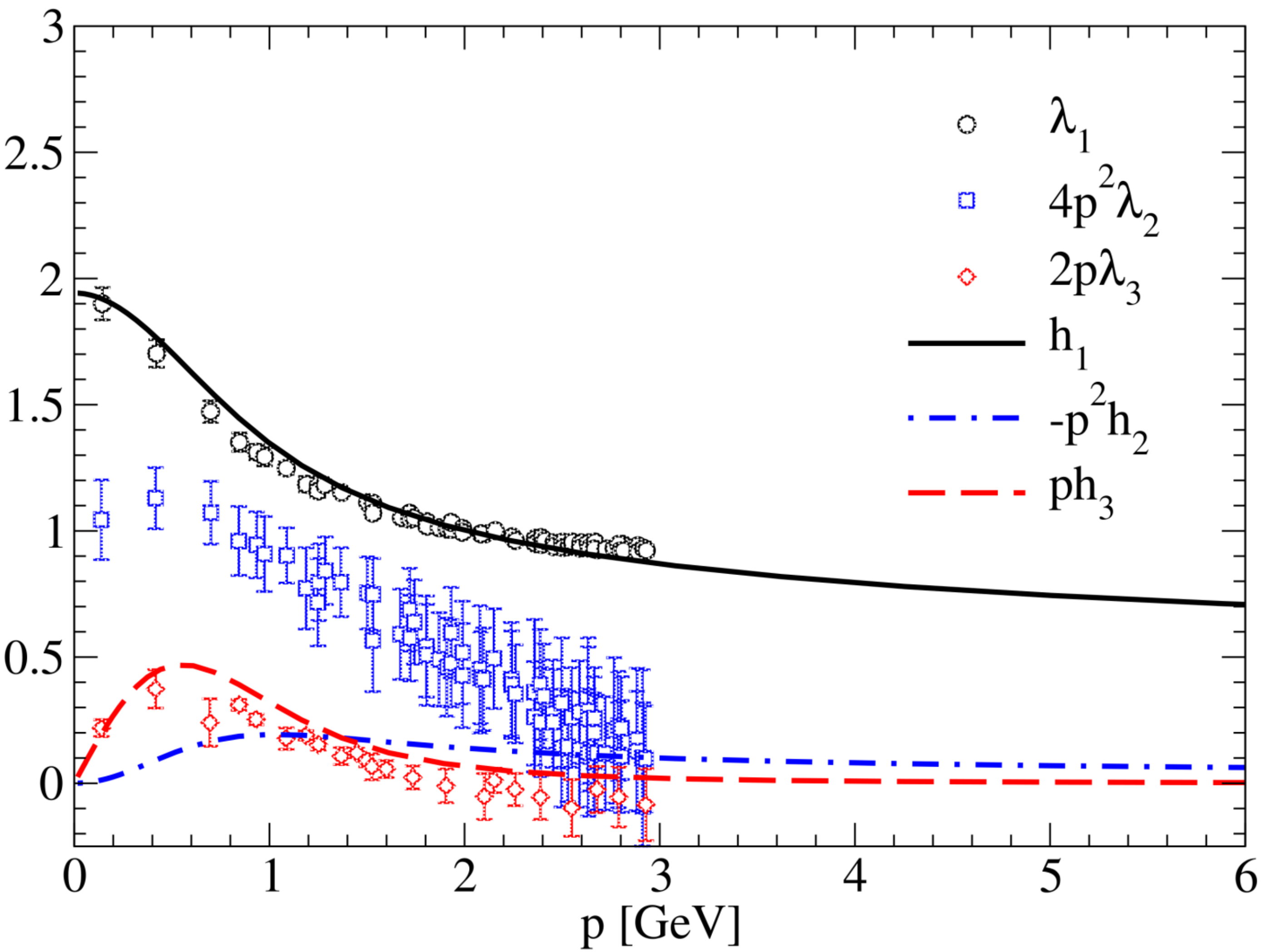


One tensor structure

$$\Gamma^{\mu}_{gh}(l,q) = f(l,q)T^{\mu\nu}_{(q)}l^{\nu}$$

- Unquenching effects negligible.
- Lattice data needs improvement

3PI results: (quenched) quark-gluon vertex

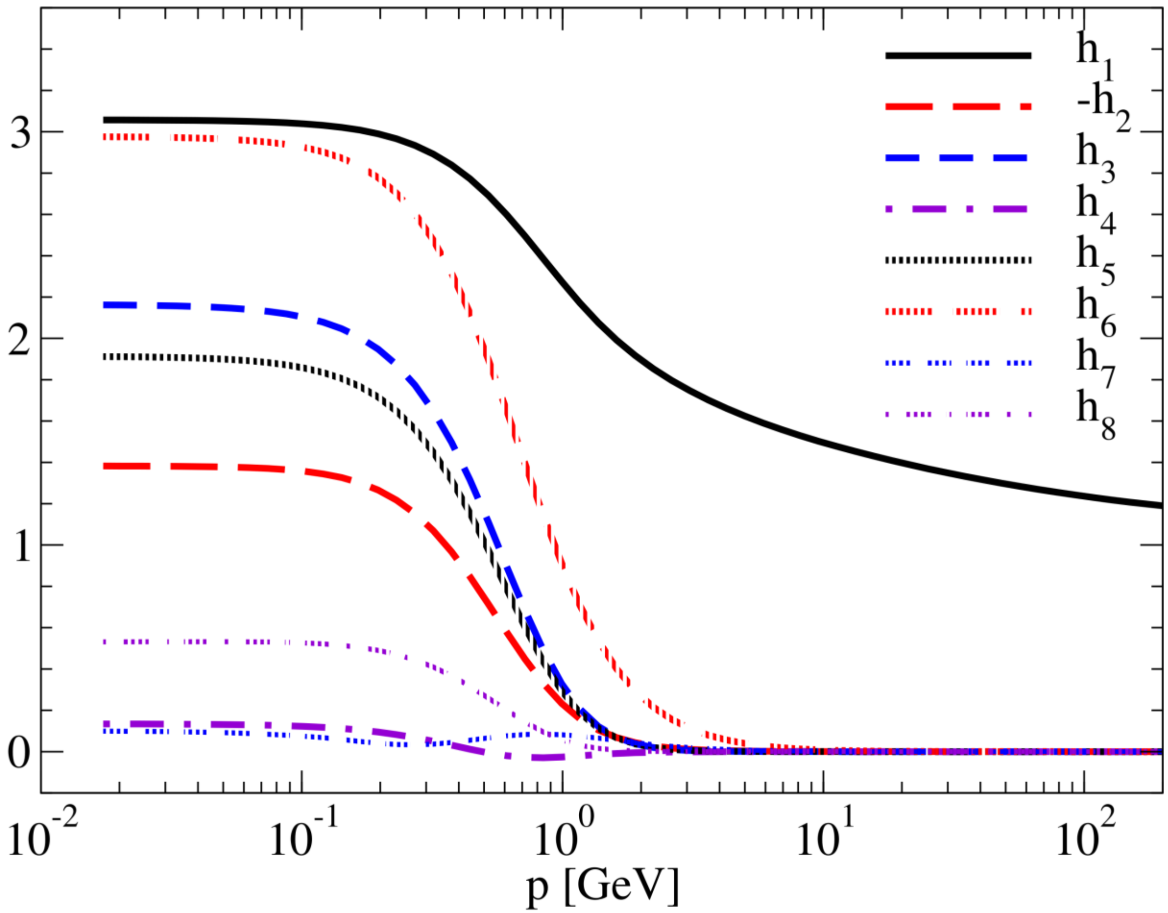


1st and 3rd structures comparable.

Difficult systematics (lattice) in 2nd

[Skullerud, Bowman, Kizilersu, Leinweber, Williams JHEP 04 (2003) 047]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

3PI results: (unquenched) quark-gluon vertex

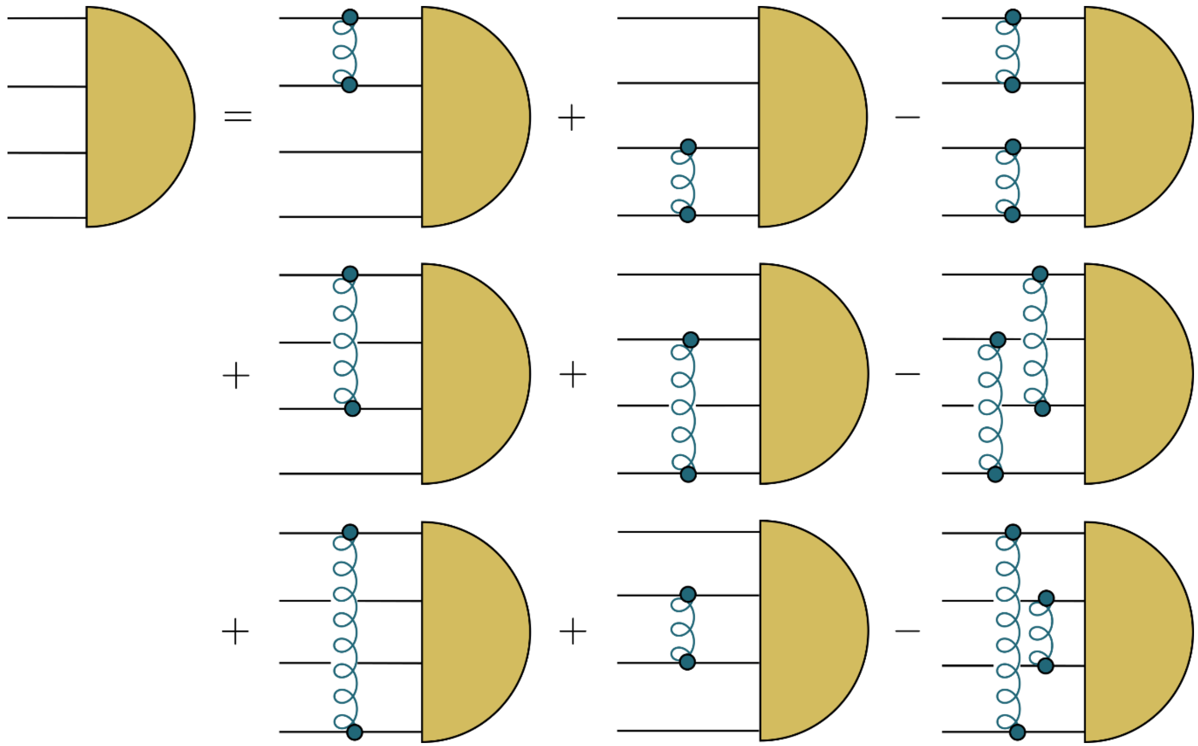


See Kizilersu

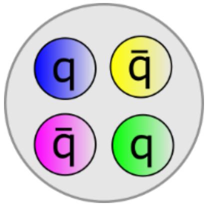
- Strong dynamical enhancement in running of vertex.
- DCSB plays a large role

[Chang, Roberts PRC 85 (2012) 052201]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

Permuted two body kernel



Tetraquarks



Tensor decomposition

512 components

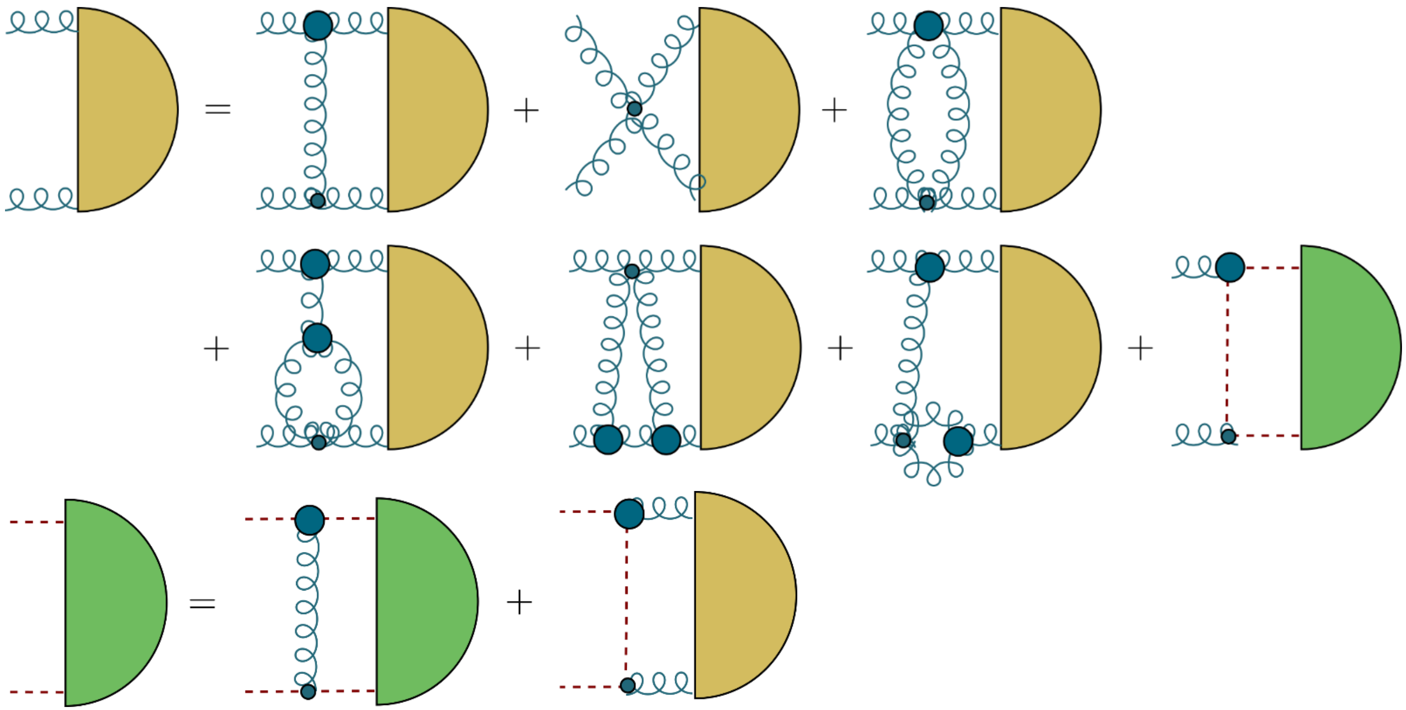
9 kinematic variables

Time-like constituents

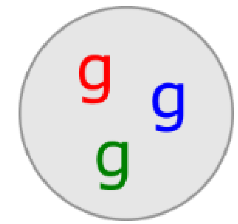
- Analytic continuation
- Resonance structure

[Eichmann, Fischer, Heupel PLB 753 (2016) 282]

Other Applications: Glueballs



Glueballs



Tensor decomposition

(Derive using Helicity formalism)

$J = 0^{+}$: 4 covariants (2 Landau gauge)

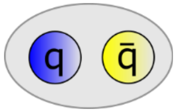
$J = 0^{-}$: 1 covariants

Time-like constituents

- Analytic continuation

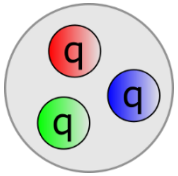
[Strauss, Fischer, Kellermann PRL 109 (2012) 252001]
[Meyers, Swanson PRD 87 (2013) 036009]
[Sanchis-Alepuz, Fischer, Kellermann, von Smekal PRD 92 (2015) 034001]
[Fukamarchi, Kondo, Nishino, Shinohara arXiv:1605.01841]

Mesons $q\bar{q}$



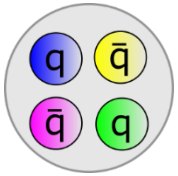
- Only now exploring details of quark-gluon interaction on spectrum
- No longer disconnected from gauge sector. Implicit flavor dependence.

Developing framework

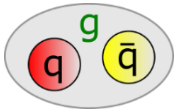


- Unified description of mesons and baryons consistent with symmetries
- Calculation of higher spin and/or excited mesons and baryons

Extensible to other bound-states via nPI

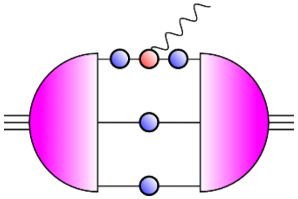


- Baryons
- Tetraquarks
- Glueballs and Hybrid mesons



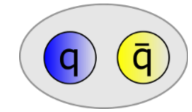
A functional derivative (or two) away …

- Calculation of form-factors, EM transitions and decays



Conclusions

Mesons $q\bar{q}$



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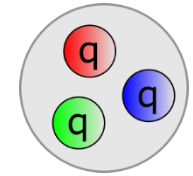

Developing framework

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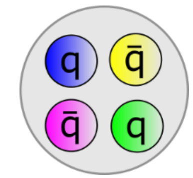
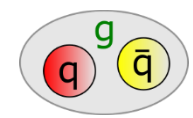


Extensible to other bound-states via nPI

- Baryons
- Tetraquarks
- Glueballs and Hybrid mesons



A functional derivative (or two) away …

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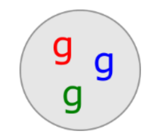
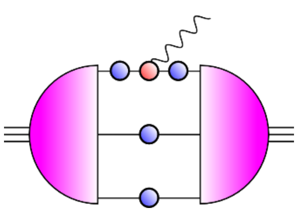
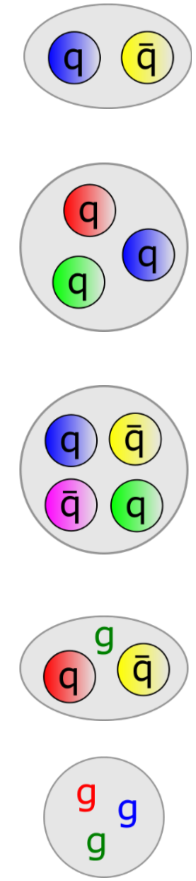




see G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. Fischer for a review



Mesons $q\bar{q}$

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Developing framework

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- Calculation of higher spin and/or excited mesons and baryons

Extensible to other bound-states via nPI

- Baryons
- Tetraquarks
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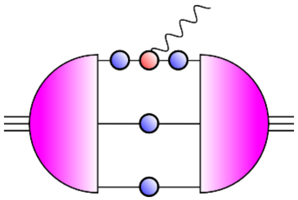
Thank you

A functional derivative (or two) away …

- Calculation of form-factors, EM transitions and decays



see G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. Fischer for a review